CS 184/284A Spring 2020 **Name:** _____

Exam 1 (Online) **SID Number:** _____

March 10th, 2020

Time Limit: 110 Minutes

IMPORTANT: READ CAREFULLY! THERE ARE MANY NEW CHANGES!

- You will need your own blank paper, a pen, a web browser, your class Gradescope account, and an internet-connected device capable of taking and uploading photos (a smartphone is ideal).
- Please read carefully, sign and submit the Honor Code described on the next page.
- This exam is occurring online. You must not communicate with anyone while you are taking this exam, unless there is an emergency.
- This exam is open book.
- This exam is 110 Minutes, and has a total of 110 points. There are 20 pages (including this cover page) and 7 parts. Note that the last part is optional and carries no points.
- Problem difficulty varies throughout the exam, so move on if you find yourself stuck.
- You will submit your answers for most questions by multiple-choice and typing your text-only answers online in Gradescope Exam 1 part a.
- You will submit your hand-written answers by photo for select questions marked [UPLOAD PHOTO], by uploading to Gradescope Exam 1 part b. You are allowed 15 minutes after the exam is over to upload your photos. If necessary because of uploading problems, you can email your photos to vio@berkeley.edu with subject 'CS184 Exam'.
- Communications: During the exam we will post any live corrections in an online document, and you may submit questions to staff via an online form. Both can be found at [tinyurl.com/cs184mt1clarifications]

Problem	Points	Score
1	0	
2	20	
3	24	
4	24	
5	26	
6	16	
7	0	
Total:	110	

1. Honor Code and Academic Integrity Certification

The Honor Code is the commitment and work of students individually and as a community to uphold honesty and integrity. This commitment is comprehensive. Specifically on exams, it means not giving or receiving any help, or using any resources that are not permitted. The Honor Code requires that students take an active role in seeing to it that others as well as themselves uphold the letter and spirit of the Honor Code.

By uploading my signature I certify that:

- All the work submitted in my name for this exam is my work alone.
- I have not given or received any help during this exam.
- I have not used any un-permitted resources during this exam (see cover page for permitted resources).
- I have timed myself and used no more than 110 minutes to write my answers for this exam.
- If I become aware of any Honor Code violations related to this exam, I will inform the course staff immediately.

I understand that a false statement on this declaration is a violation of the Honor Code, and would be subject to the highest level of disciplinary sanction under the University Code of Conduct.

1.i. Certification

UPLOAD PHOTO **On a sheet of your own paper, please sign to provide your certification of this statement, write your name, SID number and the date. Take a photo of your page and upload to Gradescope Exam 1 part b.**

Signature: _____

Name: _____

SID Number: _____

Date: _____

1.ii. Self-Portrait Photo

UPLOAD PHOTO **Please take a photo of yourself, and upload to Gradescope Exam 1 part b. (*You may decline this task without penalty, and if so, write words to that effect on a piece of paper, and upload a photo of that instead.*)**

2. (Total : 20 points) True / False

Mark each statement true or false. (1 point each)

(2a) (1 point) **F** An arbitrary 2D translation can be represented by a 2×2 matrix.

(2b) (1 point) **T** We get no aliasing from frequencies in the signal that are less than half the sampling frequency.

(2c) (1 point) **F** Diffuse shading is dependent on the view direction.

(2d) (1 point) **T** In 3D, rotation around the origin and uniform scaling are commutative.

(2e) (1 point) **F** In project 1, bilinear filtering involved interpolating across mipmap levels.

(2f) (1 point) **F** A point P is inside a triangle T as long as its barycentric coordinates sum to 1.

(2g) (1 point) **T** After N iterations of loop subdivision, there will be 4^N times as many triangles.

(2h) (1 point) **F** In order to implement loop subdivision for project 2, you had to compute new vertex positions, then flip every existing edge, then split select edges, and then update the vertices' positions.

(2i) (1 point) **T** Cubic Hermite interpolation requires a set of control points and the derivatives at those points.

(2j) (1 point) **F** A cubic Bezier curve with control points in 3D space will always lie within a plane.

(2k) (1 point) **F** The incoming radiance from a ray at a surface is the same as the emitted radiance.

(2l) (1 point) **T** Isotropic light sources emit light uniformly in all directions.

(2m) (1 point) **F** The difference between radiometry and photometry is that radiometry accounts for the human visual system's response.

(2n) (1 point) **T** One advantage of Monte Carlo integration over quadrature-based numerical integration for higher dimensional problems is avoiding the curse of dimensionality.

(2o) (1 point) **F** When we are trying to integrate over the entire surface area of a sphere, we integrate both θ and ϕ, the polar coordinates used in lecture, from 0 to 2π.

(2p) (1 point) **T** Russian Roulette is a method to probabilistically avoid evaluating a random variable without changing the expected value.

(2q) (1 point) **T** Implicit surface representations make it easy to perform ray-intersection tests.

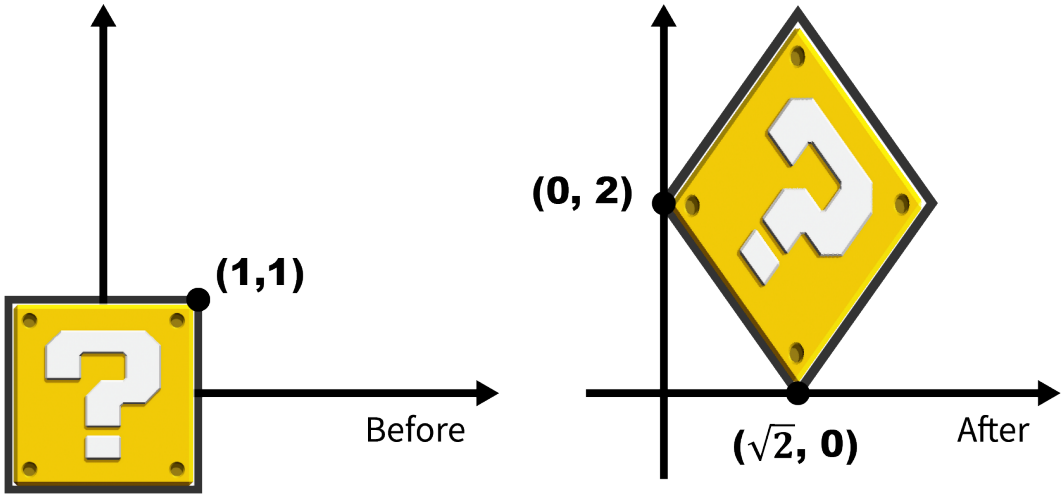
(2r) (1 point) **T** When importance sampling the lighting, we choose directions corresponding to points on the surface of the lights rather than random directions on the hemisphere.

(2s) (1 point) **F** In global illumination, the light directly emitted by a light source is usually considered the 1-bounce radiance.

(2t) (1 point) **T** When testing whether a point is in a 2D triangle, it is sufficient to check that the point is on the same side of all the triangle's edges.

3. (Total : 24 points) Graphics Pipeline

(3a) (4 points) Consider these two images:



Which sequences of elementary transformations, applied to the image on the left, would produce the image on the right? (Indicate all that apply.) Recall that 2D rotations are counter-clockwise.

(i) Rotate 45°, Scale y by $\sqrt{2}$, Translate $(\sqrt{2}, 2)$
(ii) Rotate $-45°$, Scale y by $\sqrt{2}$, Translate $(\sqrt{2}, 2)$
(iii) Rotate 315°, Translate $(\sqrt{2}, \sqrt{2})$, Scale y by $\sqrt{2}$
(iv) Rotate $-45°$, Translate $(\sqrt{2}, 2)$, Scale y by $\sqrt{2}$
(v) Translate $(1, 1)$, Rotate $-45°$, Scale y by $\sqrt{2}$, Translate $(0, 2)$

Solution: The answers are (ii), (iii), and (v).

(3b) (4 points) Determine the values for each entry in the 3×3 matrix representing the transform in the previous problem.

$$\begin{bmatrix} A & B & C \\ D & E & F \\ G & H & I \end{bmatrix}$$

UPLOAD PHOTO **On a sheet of your own paper, write your answer. Take a photo of your page and upload to GRADESCOPE EXAM 1 PART B.**

Solution:

$$\begin{bmatrix} 1 & 0 & \sqrt{2} \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \sqrt{2} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 0 \\ -\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \sqrt{2} \\ -1 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}$$

(3c) (3 points) Mario, an amateur camera-maker, has a camera that is producing spatial aliasing patterns like moire when shooting pictures of clothing with fine patterns. Stumped,

he decides to call you for help. Which of the following might reduce aliasing? Circle all that apply.

(i) Use a lens that is slightly blurrier
(ii) Blur the digital image slightly after each shot
(iii) Use a sensor with smaller pixels
(iv) Use mipmapping

Solution: Circle (i) and (iii)

(3d)3d.i. (2 points) Mario gives up on camera-making and decides to model galaxies instead. Suppose he texture-maps a sphere with a 2D rectangular map of the earth represented in latitude and longitude coordinates. Assume this texture is mipmapped.

Assume he also puts a virtual camera on a satellite that orbits over all points on the earth, looking straight down at only a square mile of the ground. In the resulting pictures from the camera, what part of the earth is visible when sampling from the highest-resolution image in the mipmap?

Solution: At the equator.

3d.ii. (2 points) When the camera flies over the north pole, what image quality problem will it see?

Solution: Overblurring.

3d.iii. (3 points) Mario's brother, Luigi, has a idea to fix this problem. He suggests that cubemap textures, supported by GPUs, might be another way to texture map Mario's spherical model of the earth. As discussed briefly in class, a cubemap contains six images, one for each face of a cube.

Cubemap texture coordinates are parameterized in an interesting way that you need to understand for the next few questions. A cubemap texture coordinate is a *3D* vector (cx, cy, cz) that conceptually points from the center of the cube in ray direction (cx, cy, cz). When a rasterized pixel has cubemap coordinates (cx, cy, cz), it will be colored with the texture value where the corresponding ray hits one of the six faces of the cube.

Will this approach reduce the image quality problems discussed for the latitude-longitude texture? Why?

Solution: Yes, it will solve the problem.
The problem at the north pole was that there were too many points on the 2D texture map (essentially the entire top edge) mapped to the same point on the 3D sphere. This caused overblurring in that area on the sphere. However, with the cubemap approach, there is one defined texture location (the center of the top cubemap face) that maps to the north pole. This means we no longer have overblurring in the textures at the north and, by symmetry, south poles.

3d.iv. (3 points) You can mipmap a cubemap as well – assume Mario does so. In the resulting satellite pictures of his virtual earth, where will he be sampling from the highest-resolution image in the mipmap? State your answer relative to the geometry of the cubemap.

Solution: At the centers of the faces of the cube.

3d.v. (3 points) Assume that to rasterize the earth, Mario represents it in an object coordinate frame such that positions on its surface are on a sphere of unit radius centered at the origin. Consider a vertex at position (x, y, z) on the earth's surface. Write

down a formula for the cubemap texture coordinates for this vertex (hint: review the definition of cubemap 3D texture coordinates in 3d.iii above, and express your answer in terms of x, y and z):

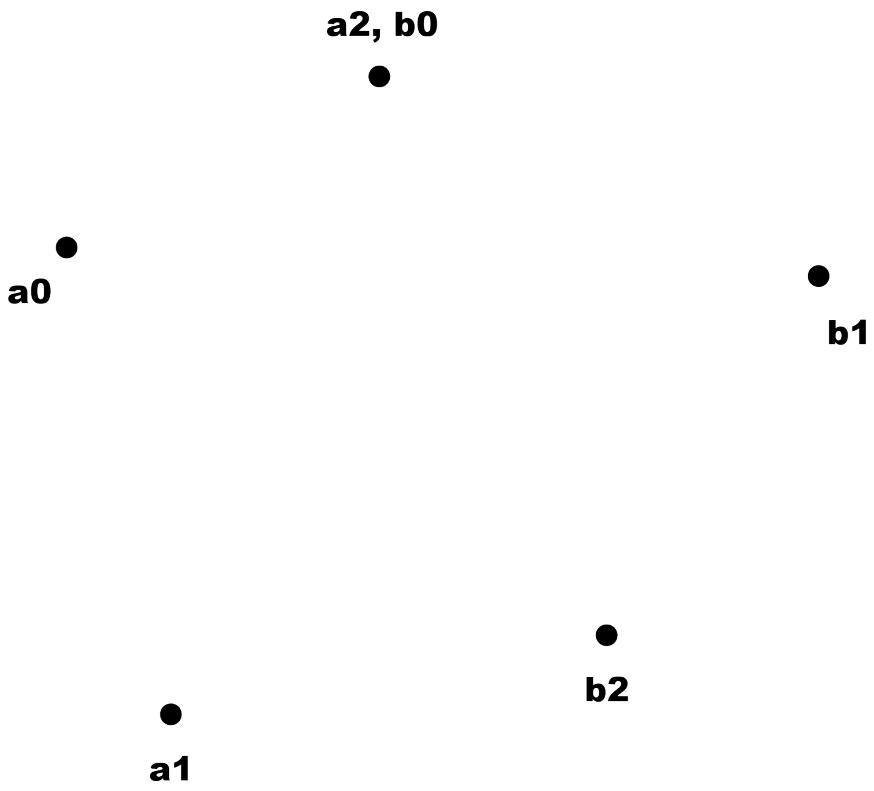
$$(cx, cy, cz) =$$

Solution: $(cx, cy, cz) = (x, y, z)$

4. (Total : 24 points) Geometry

(4a) (4 points) The figure below shows the control points for a Bezier spline composed of two separate quadratic Bezier curves with three control points each. Note that the two curves share a control point $\mathbf{a}_2 = \mathbf{b}_0$. In other words, the endpoint of the first curve is the start point of the second.

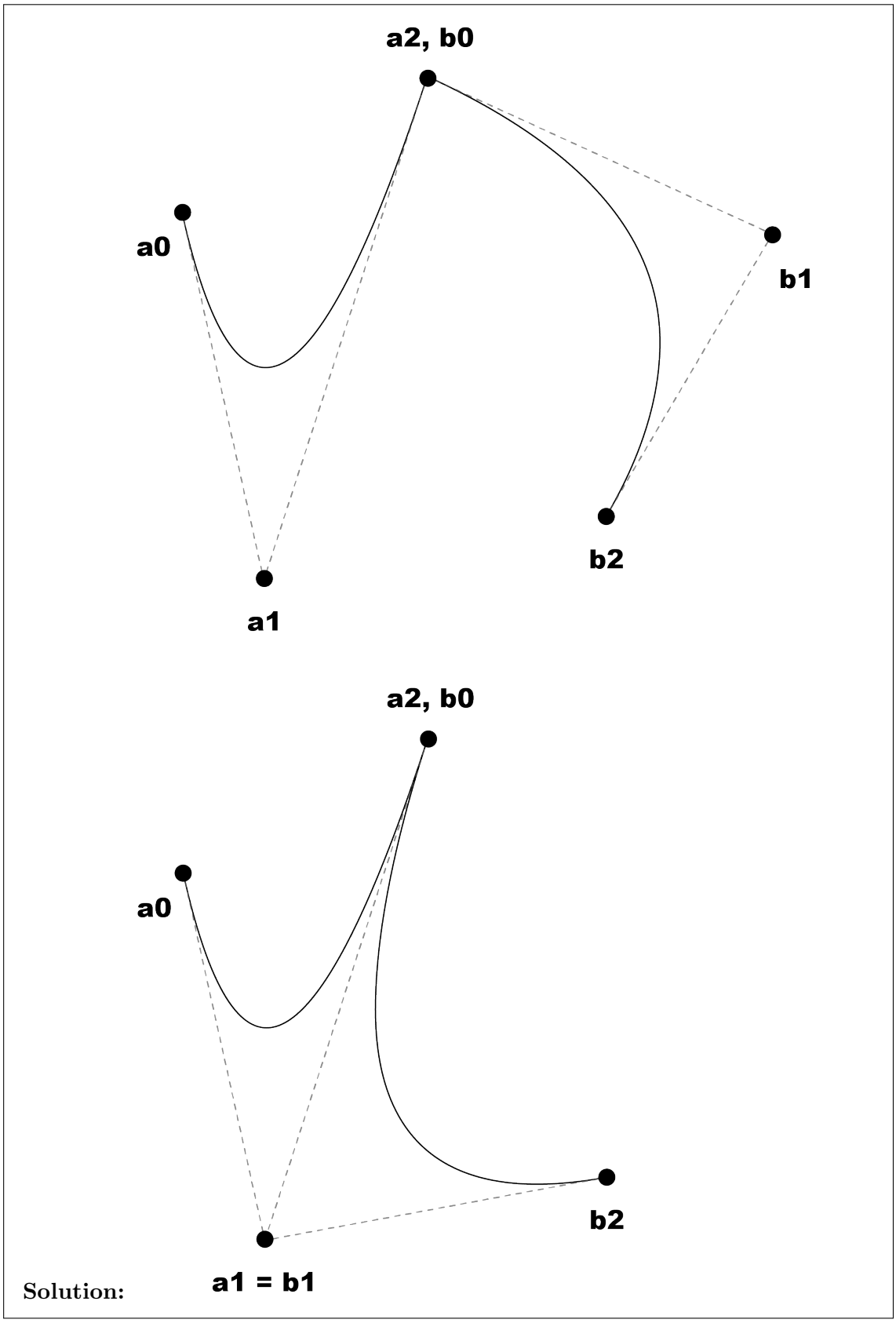
- Use the de Casteljau algorithm to identify the parametric midpoint of each curve. Draw all intermediate points in the algorithm, but you do not have to label them.
- Sketch the rest of both curves.
- Note that the spline is continuous at the joint between the two curves, but its derivative is not.



UPLOAD PHOTO On a sheet of your own paper, copy the diagram above and draw your answer. Take a photo and upload to GRADESCOPE EXAM 1 PART B

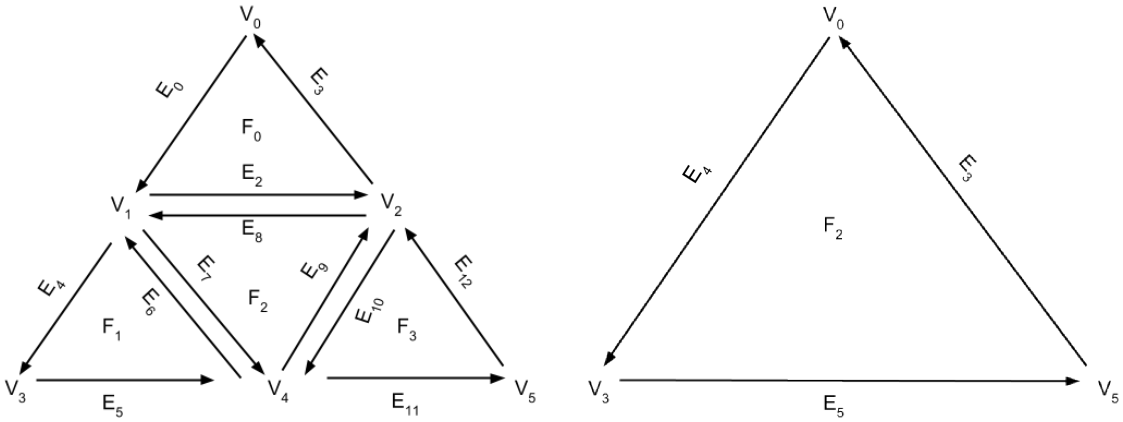
(4b) (2 points) Referring to your answer for the previous question, where would you move control point $\mathbf{b}_1$ to produce a spline with the *sharpest* possible point where the two Bezier curves join? Draw the resulting control points and two curves, after applying this change. (Assume that the changed $\mathbf{b}_1$ must lie within the polygon defined by the other control points.)

UPLOAD PHOTO **On a sheet of your own paper, draw your answer. Take a photo and upload to Gradescope Exam 1 part b.**



Solution:

(4c) (4 points) Consider the following initial (left) and final (right) half-edge meshes.



For this problem, please use `halfedge()`, `vertex()`, `face()`, `twin()`, `next()` as the calls to both get and set those items. For example, given a halfedge pointer $\texttt{E}_2$, you could set its twin to the halfedge belonging to some vertex $\texttt{V}_1$ by writing:
$\texttt{E}_2 \rightarrow \texttt{twin}() = \texttt{V}_1 \rightarrow \texttt{halfedge}()$

Link, an aspiring 3D artist at Hyrule Inc., is trying to modify the mesh on the left to look like the one on the right. He is initially only given a pointer to the halfedge $\texttt{E}_4$. How can Link correctly assign $\texttt{E}_4$'s new face, using the fewest number of calls? Assume he cannot access any elements that are not shown. Write this in one line.

Solution: $\texttt{E}_4 \rightarrow \texttt{face}() = \texttt{E}_4 \rightarrow \texttt{next}() \rightarrow \texttt{next}() \rightarrow \texttt{twin}() \rightarrow \texttt{face}()$

(4d) (4 points) Again considering the above meshes, what is the fewest number of elements that must have their `next()` reassigned in order to *guarantee* that Link gets the correct mesh on the right? Assume there are no changes outside of the parts of the mesh shown.

Solution: 2, $\texttt{E}_3$ and $\texttt{E}_5$

(4e) Geometric Acceleration Structures

Zelda, a software developer at Hyrule Inc., is asked to write an algorithm for collision detection in 2D between a box and an object that is the union of a list of triangles. Collision detection is a problem where we test whether two objects intersect.

Zelda decides to start with a simple, brute-force implementation, and asks you to help her write it. She gives you the following code for triangles and boxes:

```
// It is not safe to go alone. Here, take these.

struct Vector2D {
    float x, y;
};

struct Triangle {
    Vector2D v1, v2, v3;
};

// An axis-aligned box
struct Box {
    Vector2D lowerLeft, upperRight;
};

struct Object {
    // An object is defined as the union of area inside these triangles
    vector<Triangle> triangles;
};
```

Zelda also writes the following helper function that tests whether the given box and triangle intersect. You do NOT need to implement this function.

```
boolean boxTriangleTest(const Box &b, const Triangle &tri) {

    /* Implementation Hidden */

}
```

4e.i. (2 points) First, write a function that tests whether a box intersects the object composed of triangles. You should use the supplied helper function.

```
boolean boxObjectTest(const Box &b, const Object &obj) {

    /* Your code answer goes here */

}
```

Solution:

```
boolean boxObjectTest(const Box &b, const Object &obj) {
    /* Your code here */
    for(Triangle tri: obj->triangles) {
        if (boxTriangleTest(b, tri)) {
            return true;
        }
```

```
        }
        return false;
    }
```

4e.ii. (8 points) Next, Zelda decides to improve her implementation using a Bounding Volume Hierarchy. For this part, assume that the object is provided to you as a BVH data structure, as follows:

```
struct BVHNode {
    Box boundingBox;
    boolean isLeafNode;

    // Only contain valid child pointers if not a leaf node
    BVHNode* left;
    BVHNode* right;

    // Only contains triangles if a leaf node
    vector<Triangle> triangles;
};
```

Zelda also writes the following helper function that tests whether two given boxes intersect. You do NOT need to implement this function.

```
boolean boxBoxTest(const Box &b1, const Box &b2) {
    /* Implementation Hidden */
}
```

Now, write code to compute whether a box intersects with the object, given as a BVH:

```
boolean boxObjectTestBVH(const Box &b,  BVHNode *rootNode) {



    /* Your code answer goes here */



}
```

Solution:

```
boolean boxObjectTestBVH(const Box &b,  BVHNode *rootNode) {
    Box bvhBox = rootNode->boundingBox;
    if (!boxBoxTest(bvhBox, b)) {
        return false;
    }
    if (rootNode->isLeafNode) {
        struct Object o = {rootNode->triangles};
        return boxObjectTest(b, o);
    } else {
        return boxObjectTestBVH(b, rootNode->left)
                || boxObjectTestBVH(b, rootNode->right);
    }
}
```

5. (Total : 26 points) Monte Carlo Integration and Radiometry

(5a) (4 points) Radiant Intensity

Consider a small isotropic light source with a total luminous flux of 1000 lumens. You create a reflector so that somehow all the light exits evenly through only one face of a cube surrounding the light. What is the luminous intensity of the light before and after you do this? Use the simplest units possible, and explain your work.

UPLOAD PHOTO On a sheet of your own paper, write your answer. Take a photo of your page and upload to GRADESCOPE EXAM 1 PART B.

Solution:
Before: $\frac{1000}{4\pi} = \frac{250}{\pi}$ candelas.
After: $\frac{1500}{\pi}$ candelas for the reflector case. The light is zero candelas when viewed from directions corresponding to the other five faces, since all light exits from one face. The steradians corresponding to one face is $\frac{4\pi}{6}$, and $\frac{1000}{\frac{4\pi}{6}} = \frac{1500}{\pi}$ candelas

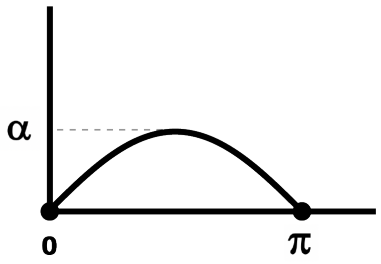
(5b) (4 points) Photometric Units

Which of these units is NOT equivalent to Lux (circle all that apply)?

(i) Watt / meter^2
(ii) Lumen / meter^2
(iii) Candela / meter^2
(iv) Nit $\cdot$ steradians

Solution: Watt / meter^2 and Candela / meter^2

(5c) (6 points) Random Sampling



Lucas is writing a program to do Monte Carlo integral estimation of a function $f(x)$ for x between 0 and π, and wants to use importance sampling. His psychic powers give him a hunch that $f(x)$ rises and falls over the interval, so he decides to try importance sampling with a probability density function proportional to $\sin(x)$ as shown in the function graphed above.

Given v, a uniform random value between 0 and 1, derive an expression for a random value x (as a function of v) that satisfies Lucas' desired probability density function.

UPLOAD PHOTO On a sheet of your own paper, write your answer. Take a photo of your page and upload to GRADESCOPE EXAM 1 PART B.

Solution:

$$
\begin{aligned}
v &\sim U(0, \pi) \\
x &\sim f(x)
\end{aligned}
$$

$$
\begin{aligned}
f(x) &= \alpha \sin(x) \\
\int_0^\pi f(x)dx &= \int_0^\pi \alpha \sin(x)dx \\
&= -\alpha \cos(x)|_0^\pi \\
&= -\alpha(\cos(\pi) - \cos(0)) \\
&= -\alpha(-1 - 1) \\
&= 2\alpha = 1 \\
\implies \alpha &= \frac{1}{2} \\
\implies f(x) &= \frac{1}{2} \sin(x)
\end{aligned}
$$

$$
\begin{aligned}
F(x) &= \int_0^x \frac{1}{2} \sin(x)dx \\
&= -\frac{1}{2} \cos(x)|_0^x \\
&= -\frac{1}{2}(\cos(x) - \cos(0)) \\
&= -\frac{1}{2}(\cos(x) - 1) \\
&= \frac{1}{2}(1 - \cos(x)) = v \\
\implies x &= \arccos(1 - 2v)
\end{aligned}
$$

(5d) (4 points) Solid Angle

As a vanguard for the attack against Earth, the alien overlord Giygas sends a satellite to our solar system. The satellite uses a flat, disk-shaped solar sail that is 10 meters in diameter.

When Ness, an inhabitant of earth, saw the satellite yesterday, the satellite's sail was facing directly towards earth, and was 400 million kilometers away. Today, Ness observes that the satellite is 1600 million kilometers away. Ness' science-y friend Jeff estimates that today, the sail subtends a solid angle of only $\frac{1}{32}$ compared to yesterday. What angle away from the earth is the satellite's sail pointing today? Briefly explain your answer.

UPLOAD PHOTO On a sheet of your own paper, write your answer. Take a photo of your page and upload to GRADESCOPE EXAM 1 PART B.

Solution: The solid angle formula is $\frac{A}{r^2}$. The radius of the satellite is 4 times as large as before, meaning the denominator will be 16 times as large. Therefore, in order for the solid angle today to be 1/32 of the angle yesterday, the area must be half as large. By Lambert's law, the visible area is proportional to the cosine of the angle so $\cos(\theta) = 0.5$, so $\theta = 60°$

(5e) (8 points) Monte Carlo Estimators

For each row, you are given a definite integral to evaluate with Monte Carlo integration, the probability density function (PDF) to draw random samples from and the number of samples to take. Your job is to write down an equation for the corresponding unbiased estimator. The first two rows are completed as examples for you. In this question, H^2 denotes the hemisphere and S^2 denotes the sphere.

	Definite Integral	**Number of Samples**	**PDF**	**Estimator**
	$\int_a^b f(x)dx$	1	X_i drawn uniformly at random from $[a, b]$	$(b-a)f(X_i)$
	$\int_a^b f(x)dx$	N	X_i drawn uniformly at random from $[a, b]$	$\frac{b-a}{N}\sum_{i=1}^{N} f(X_i)$
(i)	$\int_{H^2} f(\omega)d\omega$	N	ω_i drawn from pdf $p(\omega_i)$ over H^2	
(ii)	$\int_{S^2} f(\omega)d\omega$	N	ω_i drawn uniformly at random from S^2	
(iii)	$\int_{S^2} f(\omega)d\omega$	1	ω_i drawn from pdf $p(\omega_i)$ over S^2	
(iv)	$\int_{-\pi}^{\pi}\int_{-\pi}^{\pi} f(x,y)dxdy$	N	X_i,Y_i drawn uniformly at random from $[-\pi, \pi]$	

UPLOAD PHOTO On a sheet of your own paper, write your answer for each row. Take a photo of your page and upload to GRADESCOPE EXAM 1 PART B.

Solution:

(a) $\frac{1}{N}\sum_{i=1}^{N}\frac{f(\omega_i)}{p(\omega_i)}$

(b) $\frac{4\pi}{N}\sum_{i=1}^{N} f(\omega_i)$

(c) $\frac{f(\omega_i)}{p(\omega_i)}$

(d) $\frac{4\pi^2}{N}\sum_{i=1}^{N} f(x_i, y_i)$

6. (Total : 16 points) Rendering

(6a) (10 points) GLaDOS writes a ray tracing program in which rays bounce randomly inside a diffuse sphere of unit radius with a hole of surface area A. Let X be the number of bounces before the average ray exits. Please fill in the following blanks. Show your work. For parts ii and iii, you can refer to your answer in part i as the probability p.

6a.i. The probability that $X = 0$ is:

6a.i. _____

6a.ii. The probability that $X = 2$ is:

6a.ii. _____

6a.iii. The expected value of X is:

6a.iii. _____

UPLOAD PHOTO **On a sheet of your own paper, write your answer. Take a photo of your page and upload to GRADESCOPE EXAM 1 PART B.**

Solution:
i. $p = \frac{A}{4\pi}$ ii. $p(1-p)^2$ iii. $\frac{1-p}{p}$
Let $\bar{p} = 1 - p$
$E[X] = (\bar{p} + 2\bar{p}^2 + 3\bar{p}^3 + \dots)p$
$\bar{p}E[X] + (\bar{p}^2 + \bar{p}^3 + \dots)p = (2\bar{p}^2 + 3\bar{p}^3 + \dots)p$
$\bar{p}E[X] + \bar{p}^2 = E[x] - \bar{p}p$
$E[X] = \frac{1-p}{p}$

(6b) (6 points) In path tracing, we discussed two types of importance sampling: sampling over the surface of light sources, or sampling the BRDF in the directions where it is large.

GLaDOS is considering a point on the surface of a material at which there is a very shiny BRDF and light source that subtends a large solid angle. To obtain a more accurate image, would it be better for GLaDOS to importance sample the lights or the BRDF, and why?

Solution: Better to sample the BRDF. In importance sampling we want the pdf to sample proportional to the integrand function. In this case, we have an integral over directions where the integrand is the product of the intensity of the lights and the magnitude of the BRDF. We are told the lights have large support in this domain, and the BRDF being shiny means it has narrow support. The integrand, being the product of these two functions, has a support that is closer in size to the BRDF's smaller support.

7. **Optional Challenge Questions (No Points)**

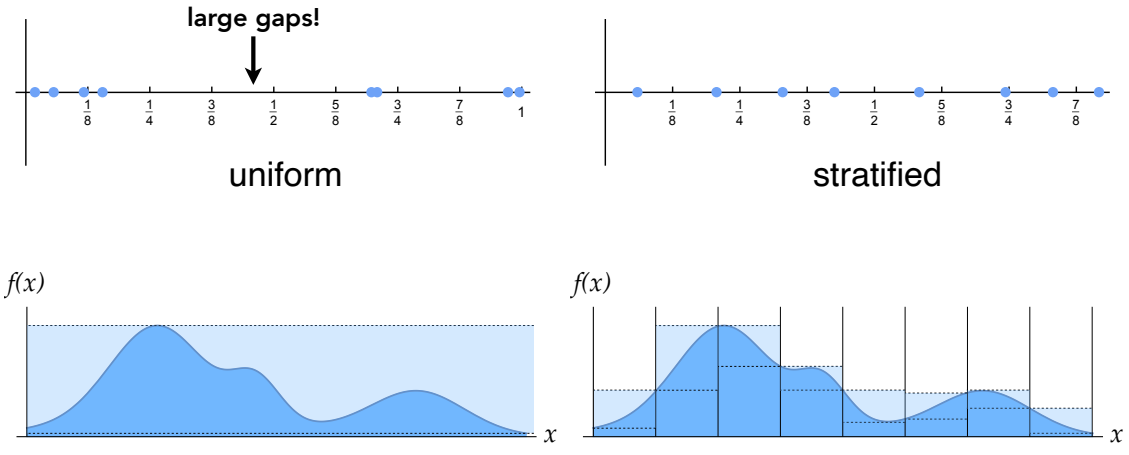
Important: These problems are entirely optional. They **will NOT contribute any points to your exam score.** Do not attempt these problems until you are completely satisfied with the rest of your exam. Any responses to these questions will be read by the instructor.

(7a) General 2D collision detection. Extend your answer for the collision detection problem. Write a function that will test for intersection between two objects, each provided as a BVH.

UPLOAD PHOTO On a sheet of your own paper, write your answer. Take a photo of your page and upload to GRADESCOPE EXAM 1 PART B.

Solution:

(7b) Stratified sampling is a technique that can reduce variance in Monte Carlo integration. It is based on splitting the integration domain into subsets, and sampling randomly within each subset. The diagram below compares drawing N samples on a line interval, using either uniform random sampling, or stratified random sampling where we split the interval into N buckets. Uniform random sampling results in large gaps between samples and clumping, which stratified sampling avoids. It is a fact that the variance of Monte Carlo integration stratified sampling is never higher than uniform random sampling (it is often lower). Prove this fact for the case of drawing N samples to estimate $\int_0^1 f(x)dx$. Hint: look at the bottom right diagram, and consider how the variance of $f(x)$ within each bucket ("stratum") compares to the variance of $f(x)$ across [0,1].



UPLOAD PHOTO On a sheet of your own paper, write your answer. Take a photo of your page and upload to GRADESCOPE EXAM 1 PART B.

Solution: