CS 184/284A Spring 2021 **Name:** _____

Exam 1 (Online) **SID Number:** _____

March 9th, 2020

Time Limit: 120 minutes

IMPORTANT: PLEASE READ CAREFULLY. NEW INSTRUCTIONS FOR 2021.

- Please carefully read, sign and submit the Honor Code described on the next page.
- You will need your own blank paper, a pen, a web browser, your class Gradescope account, and an internet-connected device capable of taking and uploading photos (a smartphone is ideal).
- You will have 2 hours to complete and submit your exam on Gradescope. You can choose to start and complete the exam anytime during the 24 hours of March 9th 2021, California time (i.e. 12:01am to 11:59pm).
- During the 24 hour period, you must not communicate with anyone about the exam, including posting to or reading from the internet any information about the exam questions. This is a strict requirement.
- This exam is open book, and open internet, subject to the limitations described above.
- This exam is 120 minutes, and has a total of 110 points. There are 14 pages (including this cover page) and 5 parts. Problem difficulty varies throughout the exam, so move on if you find yourself stuck. You are encouraged to show your work and what you know for partial credit.
- For some questions, you will have to handwrite the solution and upload a picture/file to Gradescope. For these, please write out each answer on a separate page and upload a separate file for each question. Uploading time is built into the 2 hours, but if you encounter uploading difficulties you may email your photos to rishi.upadhyay@berkeley.edu with subject 'CS184 Exam'.
- For fairness to students taking the exam at different times, the teaching staff will not answer any questions or post any corrections about the exam during the 24 hour period.

Problem	Points	Score
1	0	
2	20	
3	28	
4	34	
5	28	
Total:	110	

1. Honor Code and Academic Integrity Certification

The Honor Code is the commitment and work of students individually and as a community to uphold honesty and integrity. This commitment is comprehensive. Specifically on exams, it means not giving or receiving any help, or using any resources that are not permitted. The Honor Code requires that students take an active role in seeing to it that others as well as themselves uphold the letter and spirit of the Honor Code.

By uploading my signature I certify that:

- All the work submitted in my name for this exam is my work alone.
- I have not given or received any help during this exam.
- I have not used any un-permitted resources during this exam (see cover page for permitted resources).
- I have timed myself and used no more than the allotted time to write my answers for this exam.
- If I become aware of any Honor Code violations related to this exam, I will inform the course staff immediately.

I understand that a false statement on this declaration is a violation of the Honor Code, and would be subject to the highest level of disciplinary sanction under the University Code of Conduct.

1.i. Certification

UPLOAD PHOTO **On a sheet of your own paper, please sign to provide your certification of this statement, write your name, SID number and the date. Take a photo of your page and upload to Gradescope Exam 1.**

Signature: _____

Name: _____

SID Number: _____

Date: _____

2. (Total : 20 points) True / False

Mark each statement true or false. (1 point each)

(2a) (1 point) **T** Sampling a signal at more than twice its highest frequency will avoid aliasing.

(2b) (1 point) **F** In supersampling the rasterization of N triangles, we supersample each triangle and average down to framebuffer pixels before proceeding to the next triangle.

(2c) (1 point) **F** The Blinn-Phong shading model is a physically-accurate model of reflection.

(2d) (1 point) **F** Rotating by 45 degrees followed by rotating by -30 degrees is not the same as rotating by -30 degrees followed by rotating by 45 degrees.

(2e) (1 point) **T** A perspective projection transform requires a division operation.

(2f) (1 point) **T** The Catmull-Rom spline can be used to interpolate a curve through a sequence of points when the tangent to each point is not known.

(2g) (1 point) **T** The four inputs to cubic Hermite interpolation are the positions and tangent vectors of the end points of the curve.

(2h) (1 point) **F** High-order Bezier curves are popular for interpolating complex paths in animation.

(2i) (1 point) **F** In the Loop subdivision scheme, the number of extraordinary points increases by 4 after every level of subdivision.

(2j) (1 point) **F** In the Loop subdivision scheme, the smooth limit surface that results after a large number of subdivision steps will pass through the vertexes of the original mesh.

(2k) (1 point) **T** A Monte Carlo estimator for a definite integral is unbiased if the expected value of the estimator is equal to the value of the definite integral.

(2l) (1 point) **T** To create a probability density function that is proportional to a given function, we must divide that function by its definite integral over the domain of the function.

(2m) (1 point) **F** In importance sampling, the goal is to avoid sampling a function where its value is large to reduce variance of the sample values.

(2n) (1 point) **T** Photometric quantities are zero for light of far-infrared wavelengths.

(2o) (1 point) **T** The irradiance on your cheek decreases as you tilt it away from the sun.

(2p) (1 point) **F** A Bounding Volume Hierarchy is not an appropriate acceleration structure to use for a scene composed of a detailed teapot in a stadium.

(2q) (1 point) **T** To solve for the intersection of a ray with a sphere, we must solve a quadratic equation.

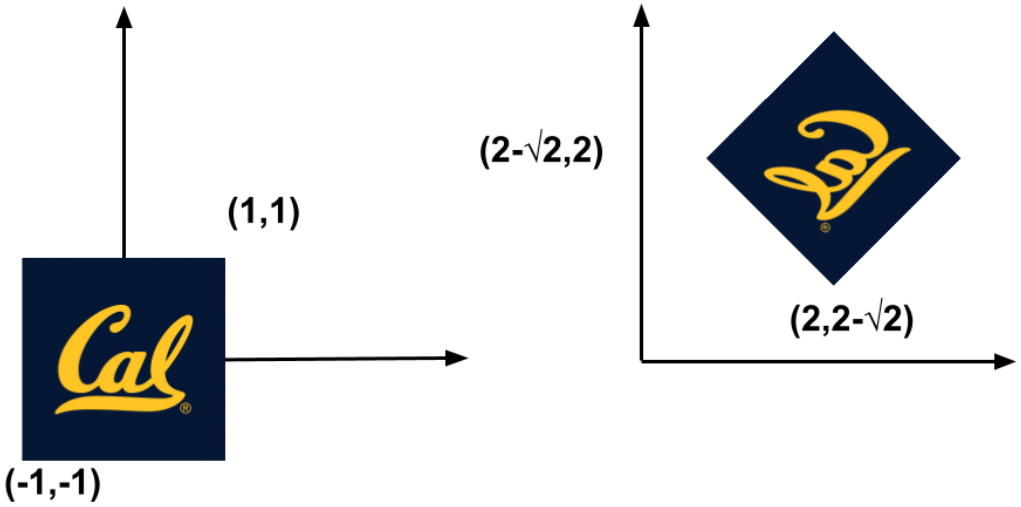
(2r) (1 point) **T** In a K-D tree representing a scene with N triangles, the total number of pointers to triangles in the leaf nodes of the tree can be more than N.

(2s) (1 point) **T** When building a Bounding Volume Hierarchy with 256 objects, if we recursively split at the location of the median element until each leaf node contains exactly one object, we will create a balanced tree with 9 levels.

(2t) (1 point) **T** A Uniform Grid acceleration structure with too many cells will be inefficient because of the computational overhead of traversing many cells without testing for intersection with any scene objects.

3. (Total : 28 points) Graphics Pipeline

(3a) (5 points) Consider these two images:



Which sequences of elementary transformations, applied to the image on the left, would produce the image on the right? (Indicate all that apply.) Recall that 2D rotations are counter-clockwise.

(i) Scale x by -1, Rotate -45°, Translate (2,-2), Rotate 90°

(ii) Rotate 315°, Translate (-2,2), Scale x by -1

(iii) Scale x by -1, Translate (2,2), Rotate 45°

(iv) Rotate 135°, Translate (2,-2), Scale y by -1

(v) Rotate 225°, Translate (2,2) Scale x by -1

Solution: The answers are (i), (ii), and (iv).

(3b) (4 points) Determine the values for each entry in the 3×3 matrix representing the transform in the previous problem. Hint: you may wish to consider the Coordinate System Transform described in lecture.

$$\begin{bmatrix} A & B & C \\ D & E & F \\ G & H & I \end{bmatrix}$$

UPLOAD PHOTO **On a sheet of your own paper, write your answer. Take a photo of your page and upload to GRADESCOPE EXAM 1.**

Solution:

$$\begin{bmatrix} -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 2 \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 2 \\ 0 & 0 & 1 \end{bmatrix}$$

(3c) (4 points) Antialiasing

An amateur camera-maker has prototyped a video camera that is producing time aliasing patterns, such as the wagon-wheel effect, when shooting video with fast moving details. The frame rate is 60 frames per second, and the shutter is open for 0.00025 seconds for each frame (this means that each video frame integrates light over a duration of 0.00025 seconds). Stumped, he decides to call you for help. Which of the following might reduce the time aliasing? Circle all that apply.

(i) Increase the time the shutter is open for each video frame to 0.01 seconds.

(ii) Blur across time by replacing every video frame with the average of the 5 video frames centered at that time.

(iii) Increase the frame rate to create a video with 240 frames per second.

(iv) Increase the frame rate to 240 frames per second and average every 4 frames to produce a 60 frame per second video.

Solution: The following items should be circled: (i) because this would introduce motion blur and reduce the highest temporal frequencies of the video signal; (iii) because the higher temporal sampling rate will raise the Nyquist rate; (iv) because this will simulate motion blur by super-sampling in time. (ii) is analogous to blurring jaggies – it would not eliminate the time aliasing, but would produce a motion-blurred wagon wheel effect.

(3d) Texture Mapping

You are writing an app for browsing photos on your computer. You store the input 12 MP photos (4000 x 3000 pixels) as mipmapped texture maps, and display them by drawing them to the screen in texture-mapped rectangles that have an aspect ratio of width/height = 4/3. In the texture mapping, you use bilinear filtering using only the nearest mipmap level. Assume the mipmap is zero-indexed.

3d.i. (4 points) If the rectangle you draw covers 1800 x 1200 screen pixels, what level of the mipmap will be used?

3d.ii. (4 points) After looking at a few test images, you realize that there is a coding bug. Portrait orientation photos (3000 x 4000 pixels) look squashed vertically. The problem is that the 3000 x 4000 pixel photo is being drawn into the rectangle covering 1800 x 1200 screen pixels. What level of the mipmap is being used in texture-mapping this rectangle? (Note: you should assume that the mipmap level is computed as described in lecture.)

3d.iii. (3 points) Continuing the previous question, in addition to looking squashed, the portrait photos look a little blurry. Explain why this is occurring.

3d.iv. (4 points) You fix the bug, and then you add a gallery view with small images of all the photos. When the user clicks on one of these small image, it animates by slowly increasing in size until the photo fills the screen. In this animation, you notice a visual problem, and determine that it is related to the texture filtering mode that you used (bilinear filtering using the nearest mipmap level). Describe the visual problem, and how you would fix the problem.

Solution:

i.

$$\frac{du}{dx} = \frac{4000}{1800} = 2.22, \frac{dv}{dx} = 0, \frac{du}{dy} = 0, \frac{dv}{dy} = \frac{3000}{1200} = 2.5$$

$$D = \log\left(\max(\sqrt{2.22^2}, \sqrt{2.5^2})\right)$$

$$D = 1.321$$

We will use Level 1.

ii.

$$\frac{du}{dx} = \frac{3000}{1800} = 1.667, \frac{dv}{dx} = 0, \frac{du}{dy} = 0, \frac{dv}{dy} = \frac{4000}{1200} = 3.33$$

$$D = \log\left(\max(\sqrt{1.667^2}, \sqrt{3.33^2})\right)$$

$$D = 1.736$$

We will use Level 2.

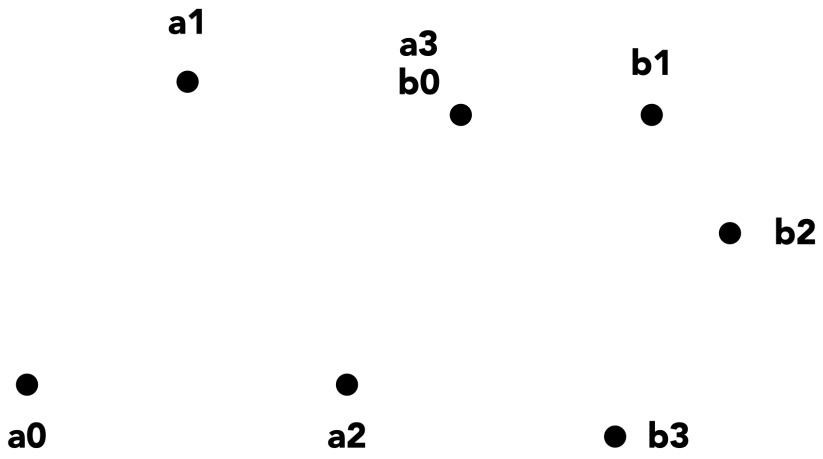
iii. Because of the screen dimensions, we are using a higher level, lower resolution mipmap. This leads to a blurrier image.

iv. The problem is that using the nearest mipmap level leads to jumps in the blurriness as the image grows. One way to fix this would be to use tri-linear interpolation instead for a smooth transition.

4. (Total : 34 points) Geometry

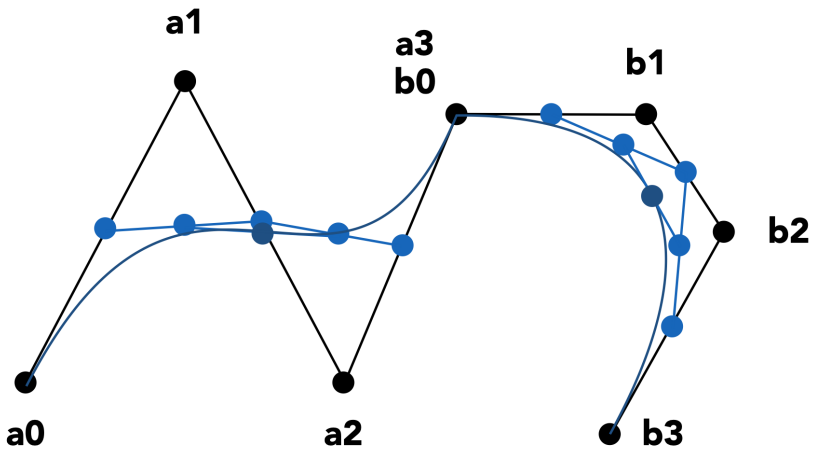
(4a) (6 points) The figure below shows the control points for a Bezier spline composed of two cubic Bezier curve with four control points each. Note that the two curves share a control point $\mathbf{a}_3 = \mathbf{b}_0$. In other words, the endpoint of the first curve is the start point of the second.

- Use the de Casteljau algorithm to identify the parametric midpoint of each curve. Draw all intermediate points in the algorithm, but you do not have to label them.
- Sketch the rest of both curves.
- Note that the spline is continuous at the joint between the two curves, but its derivatives are not.



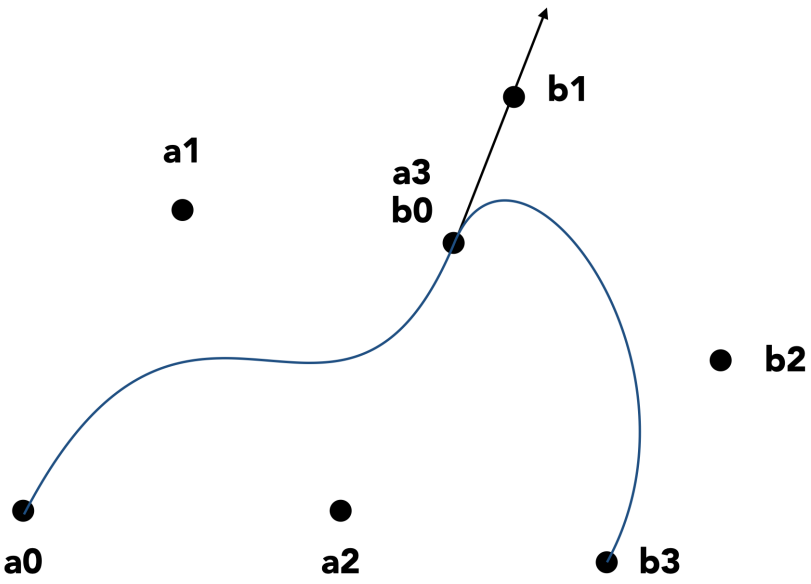
UPLOAD PHOTO **On a sheet of your own paper, copy the diagram above and draw your answer. Take a photo and upload to GRADESCOPE EXAM 1**

Solution:



(4b) (4 points) Referring to your answer for the previous question, draw the set of all points where you could move control point $\mathbf{b}_1$ to produce a spline that it is smooth (continuous first derivative) where the two cubic Bezier curves join. Move $\mathbf{b}_1$ to one of these locations, and draw the resulting two curves.

UPLOAD PHOTO On a sheet of your own paper, copy the diagram above and draw your answer. Take a photo and upload to GRADESCOPE EXAM 1



Solution: The half-line indicated, starting from $\mathbf{b}_0$ and extending away from $\mathbf{a}_2$ is the set of points where $\mathbf{b}_1$ must be moved to produce a spline that it is smooth where the two cubic bezier curves join.

(4c) Triangle Mesh Processing

I have a triangle mesh with N total triangles, and a given vertex v in the mesh has a total of E edges surrounding it. I would like to find the longest of these edges.

4c.i. (4 points) If my mesh is represented as a half-edge data structure, what is the asymptotic complexity of finding this longest edge? Briefly justify your answer.

Answer: _____

Solution: O(E) because we just need to visit neighbor vertices using h.next().vertex() and update h as h.twin().next() until we arrive back at h.

4c.ii. (4 points) If my mesh is represented as a simple list of triangles, what is the asymptotic complexity of finding this longest edge? Briefly justify your answer.

Answer: _____

Solution: O(N) because we need to check every triangle and see if it contains the

input vertex as one of its vertices. If it does, we check the length of the triangle's edges that connect to that vertex, updating the longest one found.

4c.iii. (4 points) If my mesh is represented as a triangle-neighbor data structure (each triangle contains pointers to its 3 vertices and pointers to the three neighboring triangles, and each vertex contains a pointer to its position and a pointer to a triangle that it belongs to), what is the asymptotic complexity of finding this longest edge? Briefly justify your answer.

Answer: _____

Solution: O(E) because we just need to visit neighbor vertices by starting at the triangle that the input vertex points to, checking the length of the two edges that connect to the input vertex, then repeatedly visiting the neighboring triangle that has not been visited and also still contains the input vertex as one of its vertices. We stop when we return to the first triangle, having visited E triangles.

(4d) Geometric Acceleration Structures

As part of a 2D particle simulation, you need to write code to count the number of particles within a given radius from a target point.

4d.i. (2 points) First, you try the simple, exhaustive approach. Write a function that counts the number of particles within a given radius by iterating over all particles.

```
struct Point2D {
    float x, y;
};

int countParticlesWithinRadius(const vector<Point2D> &particles,
                               const Point2D &center,
                               float radius) {

    /* Your code answer goes here */

}
```

Solution:

```
int countParticlesWithinRadius(const vector<Point2D> &particles,
                               const Point2D &center,
                               float radius) {

    float radius2 = radius * radius;
    int count = 0;

    for (auto pt : particles)
    {
        float dx = center.x - pt.x;
        float dy = center.y - pt.y;
        if (dx * dx + dy * dy < radius2) count++;
    }
```

```
}
```

4d.ii. (6 points) Next, you decide to try an implementation using a Bounding Volume Hierarchy. For this part, assume that the particle cloud is provided to you within a BVH data structure, as follows:

```
// An axis-aligned box
struct Box {
    Point2D lowerLeft, upperRight;
};

struct BVHNode {
    Box boundingBox;
    boolean isLeafNode;

    // Only contain valid child pointers if not a leaf node
    BVHNode* left;
    BVHNode* right;

    // Only contains particles if a leaf node
    vector<Particle2D> particles;
};
```

You are also given the following helper function that tests whether the box intersects with the disk of given center and radius. You do NOT need to implement this function.

```
boolean doBoxAndDiskIntersect(const Box &box,
                              const Point2D &diskCenter,
                              float diskRadius) {
    /* Implementation Hidden */
}
```

Now, write code to count the number of particles within a given radius, where the particles are provided in the BVH. You may call this function recursively and write any helper functions you want.

```
int countParticlesWithinRadius(BVHNode *rootNode,
                               const Point2D &center,
                               float radius) {



    /* Your code answer goes here */



}
```

Solution:

```
int countParticlesWithinRadius(BVHNode *rootNode,
```

```
                                    const Point2D &center,
                                    float radius) {

    if (!doBoxAndDiskIntersect(rootNode->boundingBox, center, radius)) return 0;

    if (rootNode->isLeafNode)
    {
        float radius2 = radius * radius;
        int count = 0;

        for (auto pt : rootNode->particles)
        {
            float dx = center.x - pt.x;
            float dy = center.y - pt.y;
            if (dx * dx + dy * dy < radius2) count++;
        }

        return count;
    }
    else
    {
        return
            countParticlesWithinRadius(rootNode->left, center, radius) +
            countParticlesWithinRadius(rootNode->right, center, radius);
    }

}
```

4d.iii. (2 points) Under what situations will the exhaustive implementation be slower than the BVH-based implementation? Describe a simple example.

Solution:
If there is a lot of particles spanning a large area compared the the area spanned by the radius.
(Or many other ways)

4d.iv. (2 points) Under what situations will the exhaustive implementation be faster than the BVH-based implementation? Describe a simple example.

Solution:
If the particles are dense and the search radius is large, this may take more work just doing bounding box intersections. Or if the BVH is very inefficiently constructed. (Or many more)

5. (Total : 28 points) Monte Carlo Integration and Radiometry

(5a) (5 points) Solid Angle

The magical planet of Solan has a radius of 10 kilometers and is completely transparent. You take the elevator down to the center of the planet and look up at the surface. I draw a circle on the surface that from your perspective appears like the size of the sun on earth (the sun subtends a solid angle of approximately 6 x 10^{-5} steradians). What is the radius of the circle I've drawn on the surface of Solan?

Solution:

$$6 * 10^{-5} = \frac{\pi * r^2}{(10^4)^2}$$

$$r = 43.7 \text{ meters}$$

(5b) Photometry for Reading Books

UPLOAD PHOTO On a sheet of your own paper, write your answer for this question. Take a photo of your page and upload to GRADESCOPE EXAM 1.

5b.i. (4 points) To comfortably read a book, the light level should be at least 500 lux on the pages. Your friends have a dark room where all the light comes directly from a light bulb hanging 2 meters directly above their favorite reading table. Assuming that the light bulb is isotropic and that you can ignore light reflecting off the walls, how bright a light bulb your friends need to install? Use the simplest units possible, and explain your work.

Solution: If the light bulb is X lumens, the illuminance on the table is $\frac{X}{4\pi * 2^2}$. If this is 500 lux, solving for X gives $25,133$ lumens. This is very bright!

5b.ii. (3 points) Unfortunately, all your friends have is an 800 lumen light bulb, the typical light bulb we saw in lecture. This is too dim. You advise your friends that they could lower the light bulb so that it is closer to the table. What height does the light bulb need to be? (Let's say the 800 lumen bulb is Y times the required luminous flux defined in the previous sub-question, where $Y < 1$. You may use Y in your answer if you wish.)

Solution: We still want 500 lux on the pages, so we can set up an equation similar to the last part:

$$500 = \frac{800}{4\pi r^2}$$

$$r = 0.3568 \text{ m}$$

5b.iii. (3 points) Your friends redecorate their room, and replace the reading table with a nice reclining armchair. While reading a book, it is exactly the same height as the table surface, but now it is tilted at 45 degrees because they lean backwards while they are reading. They mention that it seems dark again when they are reading. What is the reason, and what is the new illuminance (as a percentage of the illuminance before they redecorated)?

Solution: Lambert's Law applies to the tilted pages. It is sin(45) darker, about 30% less power.

(5c) (5 points) Random Sampling

You are writing a program to do Monte Carlo integral estimation of a function $f(t)$ for t between 0 and 2, and decide to use importance sampling. You have a hunch that $f(t)$ rises smoothly over the interval, so you decide to try importance sampling with a probability density function proportional to $f(x) = x$.

Given v, a uniform random value between 0 and 1, derive an expression for a random value t (as a function of v) that satisfies the desired probability density function.

UPLOAD PHOTO On a sheet of your own paper, write your answer. Take a photo of your page and upload to GRADESCOPE EXAM 1.

Solution:

$$v \sim U(0,1)$$

$$\begin{aligned} f(x) &= \alpha x \\ \int_0^2 f(x)dx &= \int_0^2 \alpha x \\ &= \alpha \frac{x^2}{2}|_0^2 \\ &= 2\alpha \\ \implies \alpha &= \frac{1}{2} \\ \implies f(x) &= \frac{1}{2}x \\ F(x) &= \int_0^x \frac{1}{2}x \\ &= \frac{x^2}{4}|_0^x \\ &= \frac{x^2}{4} = v \\ x &= 2\sqrt{v} \end{aligned}$$

(5d) (8 points) Monte Carlo Estimators

For each row in the table below, you are given a definite integral to evaluate with Monte Carlo integration, the probability density function (PDF) to draw random samples from and the number of samples to take. Your job is to write down an equation for the corresponding unbiased Monte Carlo estimator in the last column. The first two rows are completed as examples for you. In this question, H^2 denotes the hemisphere and S^2 denotes the sphere. Simplify your answer as much as possible, and show your work for partial credit.

	Definite Integral	**Number of Samples**	**Random Sampling**	**Estimator**
	$\int_a^b f(x)dx$	1	Random X drawn uniformly at random from $[a, b]$	$(b-a)f(X)$
	$\int_a^b f(x)dx$	N	Random X_i drawn uniformly at random from $[a, b]$	$\frac{b-a}{N}\sum_{i=1}^{N} f(X_i)$
(a)	$\int_a^b f(x)p(x)dx$	1	Random X drawn from pdf $p(x)$ over $[a, b]$	
(b)	$\int_0^\pi f(x)g(x)dx$	N	Random X_i drawn from $[0, \pi]$ with probability proportional to $\sin(x)$	
(c)	$\int_{S^2} f(\omega)d\omega$	1	Random ω drawn uniformly over S^2	

UPLOAD PHOTO **On a sheet of your own paper, write your answer for each row. Take a photo of your page and upload to Gradescope Exam 1.**

Solution:

(a) $f(X)$

(b) $\frac{1}{N}\sum_{i=1}^{N}\frac{2f(X_i)g(X_i)}{\sin(X_i)}$

(c) $4\pi f(\omega)$