CS 184/284A Spring 2025
Exam 1

March 10th, 2025
Time Limit: 110 minutes

Name: _____

SID Number: _____

Room: _____

Instructions:

- Please wait to look past this page until official start time.
- You must carefully read and sign the Honor Code below.
- Initial the top of every page, in case pages separate while scanning.
- This exam is closed book, except for one 8.5×11 page of notes (double sided), handwritten.
- This exam is: 110 minutes long, 100 points, 14 pages (including this cover page) and 5 parts.
- Problem difficulty varies, so move on if feeling stuck on a problem.
- Answer each question in the space provided. Partial credit may be given on certain problems.
- ChatGPT helped a little with this exam, but all text was finalized by human and full responsibility is taken by your instructor.
- No corrections will be given during the exam. State any assumptions you require on page 14 (the Assumptions box).

Question	Points
1	20
2	25
3	20
4	20
5	15
Total:	100

Honor Code and Academic Integrity Certification

The Honor Code is the commitment and work of students individually and as a community to uphold honesty and integrity. This commitment is comprehensive. Specifically on exams, it means not giving or receiving any help, or using any resources that are not permitted. The Honor Code requires that students take an active role in seeing to it that others as well as themselves uphold the letter and spirit of the Honor Code.

By signing I certify that:

- All the work submitted in my name for this exam is my work alone.
- I have not given or received any help during this exam.
- I have not used any resources during this exam aside from my own page of notes.
- I will inform course staff immediately about any Honor Code violations I become aware of.

I understand that a false statement on this declaration is a violation of the Honor Code, and would be subject to the highest level of disciplinary sanction under the University Code of Conduct.

Signature: _____

Date: _____

1. (Total : 20 points) True / False

   Mark each statement True or False. (1 point each)

   (1a) T ◯ F ◯ Antialiasing can be accomplished by filtering out frequencies above the Nyquist frequency before sampling.

   (1b) T ◯ F ◯ In supersampled rasterization of N triangles, we must supersample and downsample each triangle into the framebuffer, one-by-one.

   (1c) T ◯ F ◯ Rotating and then translating is the same as translating and then rotating.

   (1d) T ◯ F ◯ Scaling and then rotating is the same as rotating and then scaling.

   (1e) T ◯ F ◯ A mipmap requires $\frac{4}{3}$ the amount of memory compared to storing just the highest resolution texture image.

   (1f) T ◯ F ◯ When starting to rasterize a new frame, we reset values in the Z-buffer to zero distance.

   (1g) T ◯ F ◯ In Catmull-Rom spline interpolation given N points, the resulting curve passes through all the given control points, except the first and last.

   (1h) T ◯ F ◯ If we use the de Casteljau algorithm to evaluate one point on a cubic Bezier curve, we will be given four points and calculate six interpolated points.

   (1i) T ◯ F ◯ If we perform an Edge Collapse mesh operation in the middle of a manifold mesh, we will delete two faces.

   (1j) T ◯ F ◯ A cubic Bézier curve is always contained within the convex hull of its control points.

   (1k) T ◯ F ◯ Mesh simplification techniques always preserve the exact shape of the original model.

   (1l) T ◯ F ◯ Computing the intersection between a ray and a sphere involves solving a quadratic equation.

   (1m) T ◯ F ◯ In a Bounding Volume Hierarchy, the bounding box stored at a given node is equal to the larger of the bounding boxes for its two children.

   (1n) T ◯ F ◯ Irradiance is measured in watts per square meter.

   (1o) T ◯ F ◯ A hemisphere subtends 4π steradians.

   (1p) T ◯ F ◯ We can obtain an unbiased estimate of the area under a curve from a to b by averaging random samples of the function over $[a, b]$ and multiplying by $(b - a)$.

   (1q) T ◯ F ◯ With effective importance sampling, the variance of our randomized integral estimate goes down.

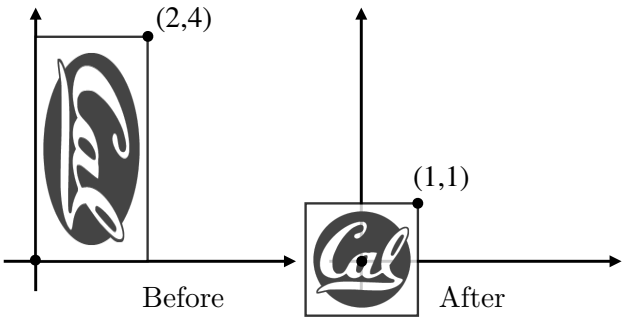
   (1r) T ◯ F ◯ A constant-valued BRDF represents a material with a matte appearance.

   (1s) T ◯ F ◯ Russian Roulette is a termination strategy used in path tracing to prevent rays from bouncing indefinitely while maintaining an unbiased result.

   (1t) T ◯ F ◯ Uniformly sampling directions over a hemisphere is the most efficient strategy for global illumination calculations.

2. (Total : 25 points) Graphics Pipeline

(2a) [10 points] Transforms



Following the conventions in lecture, which of the following 2D transforms will produce the result depicted in the image above. Graph paper is included at the end of the exam (optional, and you may detach the graph paper page), though these questions can be answered without. Mark each option as True or False (2 points each).

2a.i. T ◯ F ◯ $\begin{bmatrix} 0 & -\frac{1}{2} & 1 \\ 1 & 0 & -1 \\ 0 & 0 & 1 \end{bmatrix}$

2a.ii. T ◯ F ◯ Scale($\frac{1}{2}$,1) · Rotate(90°) · Translate(−1,−2)

2a.iii. T ◯ F ◯ · Scale(2,1) · Translate(2,−1) · Rotate(90°)

2a.iv. T ◯ F ◯ $\begin{bmatrix} 0 & 1 & 1 \\ -2 & 0 & 2 \\ 0 & 0 & 1 \end{bmatrix}^{-1}$

2a.v. T ◯ F ◯ $\begin{bmatrix} \frac{1}{2} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos(90°) & -\sin(90°) & 0 \\ \sin(90°) & \cos(90°) & 0 \\ 0 & 0 & 1 \end{bmatrix}$

(2b) Volume Rasterization: Regular and Randomized.

In class, we studied rasterization of 2D shapes into framebuffer comprised of pixels in a 2D grid. In this problem, you will consider the analagous problem of rasterizing a 3D shape into a volume buffer comprised of *voxels* in a 3D grid. The pixels are like little squares, and voxels are like little cubes. You can use the following structure:

```
struct VolumeBuffer {
    // pre-allocated memory for width * height * depth elements
    int width, height, depth;
    vector<float>   voxels;
};
```

All we know is that the 3D shape is white inside a 3D space over a black background, and we have a procedure that computes whether a point `(x,y,z)` is inside the shape:

```
// You do NOT need to implement this function.
// Rather, you should call this function from the code you write below.
bool InsideVolume(float x, float y, float z) {
    /* Implementation hidden */
}.
```

Assume that the coordinate system is analogous to lecture and the first homework; that is, the origin is in the front, top left corner of the volume buffer grid, and each voxel is one unit. And for simplicity, we will assume floating-point grayscale voxel grid values, where 0 is black (empty space) and 1 is white (inside the volume).

Lastly, for some of the functions you write below, you will need a random number generator that returns a value between 0 and 1 with uniform probability:

```
// You should call this function from the code you write below.
float rand01() { /* Implementation hidden */  }.
```

2b.i. [7 points] Volume Rasterization by Point Sampling

In your first implementation, write a function that rasterizes the 3D shape defined by the `InsideVolume` function into the given `VolumeBuffer`. You do not need to clear `vb.voxels`, and you should overwrite anything in `vb.voxels`.

```
void RasterizeVolumePointSampling(VolumeBuffer & vb) {
    // Your code goes here









}  // end of function
```

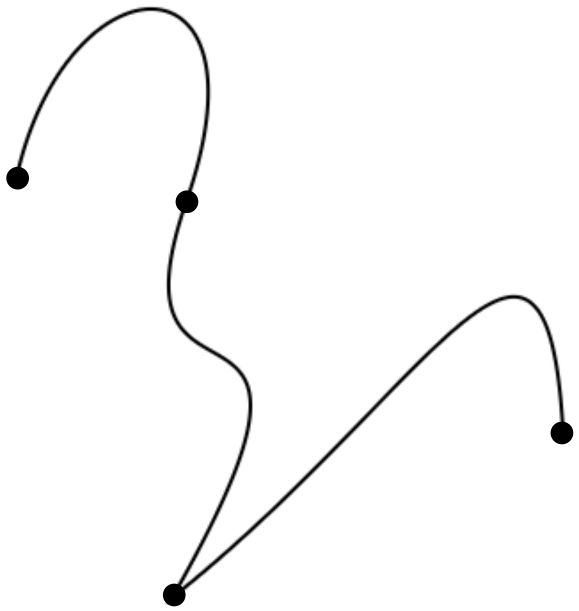
2b.ii. [8 points] Volume Rasterization with Uniform Monte Carlo Supersampling
We can improve the quality of the previous answer with randomized supersampling. For simplicity, let's use `N` supersamples in total per voxel (note: not `N x N x N`). Write a function that performs rasterization this way, using `N` random (x, y, z) locations drawn from a uniform probability density over each voxel. You should use the `rand01` helper function defined above. (Comment: this estimate of each voxel's value is equal to a basic Monte Carlo estimator with N samples, for the integral of the `InsideShape(x,y,z)` function over the volume of that voxel.)

```
void RasterizeVolumeSupersampledUniformMCSampling(VolumeBuffer & vb, int N) {
    // Your code goes here









}
```

3. (Total : 20 points) Geometry

(3a) [3 points] Cubic Bezier Splines

The following diagram shows a spline composed of three cubic Bezier curves. The spline is C^1 continuous at the second point from the left in the diagram. Draw the missing control points for the three cubic Bezier curves at their approximate locations.



(3b) Catmull-Clark Subdivision Surfaces: The Soccer Ball

A classic soccer ball is modeled as a truncated icosahedron. As shown above, this polyhedron consists of 12 pentagons and 20 hexagons. In this problem, we will study this mesh, and use it as a starting control cage for Catmull-Clark subdivision.

3b.i. [1 point] What is the valence for any given vertex in the soccer ball? Recall that the valence of a vertex is the number of faces that touch it.

$N_V =$ [ ]

3b.ii. [3 points] How many edges are there in total in the mesh? Hint: for each of the two polygon face types, how many edges around that type of polygon; and note that each edge is counted in two neighboring polygons.

$E =$ [ ]

3b.iii. [3 points] How many vertexes are there? Hint: for each of the two polygon face types, how many vertexes surround that type of polygon; and note that each vertex is counted in N_V neighboring polygons (from part 3b.i).

$V =$ [ ]

3b.iv. [1 point] Recall Euler's formula for a closed, convex polyhedron:

$$V - E + F = 2.$$

The total number of faces in the soccer ball is $F = 12 + 20 = 32$. Verify that Euler's formula works for your answers for E and V above:

Check:

3b.v. [5 points] Now let's consider applying Catmull-Clark subdivision to this mesh. Recall that in Catmull-Clark, each subdivision's topological change is to add a new vertex in the middle of each face, and connect it to a new vertex in the middle of each surrounding edge.
Draw one level of subdivision, clearly showing the new vertexes and edges on this illustration showing the wireframe of the mesh. You can subdivide just the polygons that are fully visible.

Credit: gureke, sketchfab

3b.vi. [1 point] After the first level of subdivision, how many faces in the mesh are non-quad faces (i.e. have a different number of surrounding edges than 4)?

3b.vii. [3 points] There are a total of 32 faces in the original mesh. How many faces, F_N are there after N rounds of subdivision?

$F_1 =$

$F_2 =$

$F_N =$

4. (Total : 20 points) Radiometry and Photometry

(4a) [4 points] Photometric Units

Claim: the following units are all the same as the Candela. Mark true or false on each one.

4a.i. T ◯ F ◯ Nit $\cdot\, m^2$

4a.ii. T ◯ F ◯ W/steradian

4a.iii. T ◯ F ◯ Lux $\cdot\, m^2$/steradian

4a.iv. T ◯ F ◯ Lumen $\cdot$ steradian

(4b) [5 points] Solid Angle

The magical planet of Angsol is completely transparent. You take the elevator down to the center of the planet and look up at the surface. I paint a large circle on the surface that is 10 meters in radius. From your perspective, this circle appears like the size of the sun on earth (the sun subtends a solid angle of approximately 6 x 10^{-5} steradians). What is the planetary radius R of Angsol? Express your answer exactly (you do not necessarily need to compute a decimal answer) with the correct units, and show your work for partial credit.

$R =$ [ ]

(4c) Photometry for Comfortable Reading

4c.i. [4 points] To comfortably read a book, the illuminance on the pages should be at least 450 lux. In an ideal setup, a reading lamp is mounted directly above the reading surface at a height of 1 meter. Assuming that the lamp is an ideal isotropic emitter, determine the minimum luminous flux Φ that the lamp must produce. Express your answer exactly (you do not necessarily need to compute a decimal answer) with the correct units, and show your work for partial credit.

$\Phi =$ [ ]

4c.ii. [4 points] Unfortunately, your available light bulb is rated at 200π lumens. To still achieve 450 lux on the reading surface, by what height H must the bulb be mounted (directly above the surface)? Express your answer exactly (you do not necessarily need to compute a decimal answer) with the correct units, and show your work for partial credit.

$H =$ [ ]

4c.iii. [3 points] Your friends decide that the bulb (rated at 200π lumens) is too dim when mounted at a reduced height. They raise the light bulb back to the original height of 1 meter and install a lampshade with a mirrored underside so that it reflects and concentrates all of the bulb's light onto a circular reading area of radius $\frac{1}{4}$ m. What is the average illuminance on the reading area? Express your answer exactly (you do not necessarily need to compute a decimal answer) with the correct units, and show your work for partial credit.

$E =$ [ ]

5. (Total : 15 points) Monte Carlo Integration

(5a) Monte Carlo Integration with a Non-Uniform Distribution

Consider the following Monte Carlo integration problem, where we wish to integrate a function $F(x)$ over the interval $[-\pi/2, \pi/2]$, which is known to have a humped shape near the origin. The goal is to compute:

$$I = \int_{-\pi/2}^{\pi/2} F(x)\, dx$$

We are given a proposal distribution $G(x) = \cos(x)$ and will perform importance sampling proportional to this function to estimate the integral of $F(x)$.

5a.i. [9 points] Sampling Proportional to $G(x)$: First, we need to sample from $G(x)$. Follow the inversion method described in class: normalize $G(x)$ to get the PDF $p_G(x)$, compute its cumulative distribution function (CDF), $P_G(x)$, and invert the CDF to get a function that, given a uniform random variable $u \in [0, 1]$, $P_G^{-1}(u)$ is a random sample in the domain $[-\pi/2, \pi/2]$ that is chosen randomly with a probability that is proportion to $G(x)$. Derive expressions for $p_G(x)$, $P_G(x)$ and $P_G^{-1}(u)$.

$p_G(x) =$

$P_G(x) =$

$P_G^{-1}(x) =$

5a.ii. [6 points] Monte Carlo Estimator for $I = \int_{-\pi/2}^{\pi/2} F(x)dx$:
Assume that we have a sequence of uniformly random samples X_i drawn from $[-\pi/2, \pi/2]$. As discussed in class, a basic Monte Carlo estimate for I is

$$\hat{I}_{\text{basic}} = \frac{\pi}{N} \sum_{i=1}^{N} F(X_i).$$

Now, use your results from the previous part to instead derive an expression for an importance-sampled Monte Carlo estimator. Use the sequence of randomly samples X_i. Hint: transform these uniformly random samples into random samples drawn from the PDF $p_G(x)$.

$\hat{I}_{\text{importance}} =$

Assumptions Box

Congratulations for making it to the end of the exam! If there's anything else you want us to know, or you feel like there was an ambiguity in the exam, please put it in the box below. For ambiguities, you must qualify your answer and provide an answer for both interpretations. You will only receive credit if it is a genuine ambiguity and both of your answers are correct. We will only look at ambiguities if you request a regrade.

For example, "If question X is asking about A, then my answer is Y, but if the question is asking about B, then my answer is Z".