Lecture 14:

Material Modeling

Computer Graphics and Imaging
UC Berkeley CS184/284A
Ray Tracer Samples Radiance Along A Ray

(For opaque surfaces)
The light entering the pixel is the sum total of the light reflected off the surface into the ray’s (reverse) direction.
Reflection
(as opposed to transmission, absorption, emission)

Definition: *reflection* is the process by which light incident on a surface interacts with the surface, such that it leaves on the incident (same) side, without change in frequency

Properties

- Color spectrum distribution (later)
- Polarization (not covered in this course)
- Spatioangular distribution (today)
Types of Reflection Functions

Ideal specular
- Perfect mirror reflection

Ideal diffuse
- Equal reflection in all directions

Glossy specular
- Majority of light reflected near mirror direction

Retro-reflective
- Light reflected back towards light source

Diagrams illustrate how light from incoming direction is reflected in various outgoing directions.
The Appearance of Natural Materials

[Courtesy of Prof. Henrik Wann Jensen, UCSD]
What is Material in Computer Graphics?

3D coffee mug model

Rendered

[From TurboSquid, created by artist 3dror]

Rendered
Material == BRDF
BRDF: Bidirectional Reflection Distribution Function

BRDF: $L_o(\omega_i, \omega_o)$
amount of light reflected from incident direction to outgoing direction
What is this material?
Diffuse / Lambertian Material (BRDF)

Uniform colored diffuse BRDF

Textured diffuse BRDF

[Mitsuba renderer, Wenzel Jakob, 2010]
Diffuse / Lambertian Material

Light is equally reflected in each output direction

\[ f_r = c \]

\[ L_o(\omega_o) = \int_{H^2} f_r L_i(\omega_i) \cos \theta_i \, d\omega_i \]

Suppose the incident lighting is uniform:

\[ = f_r L_i \int_{H^2} (\omega_i) \cos \theta_i \, d\omega_i \]

\[ = \pi f_r L_i \]

\[ f_r = \frac{\rho}{\pi} \text{ (albedo — "color")} \]
What is this material?
Glossy material (BRDF)

Copper

Aluminum

[Mitsuba renderer, Wenzel Jakob, 2010]
What is this material?
Ideal reflective / refractive material (BSDF*)

Air ↔ water interface

Air ↔ glass interface (with absorption)

[Mitsuba renderer, Wenzel Jakob, 2010]
Perfect Specular Reflection

[Zátonyi Sándor]
Perfect Specular Reflection

\[ \theta = \theta_o = \theta_i \]

\[ \omega_o + \omega_i = 2 \cos \theta \hat{n} = 2(\omega_i \cdot \hat{n})\hat{n} \]

\[ \omega_o = -\omega_i + 2(\omega_i \cdot \hat{n})\hat{n} \]

Top-down view (looking down on surface)

[Diagram showing specular reflection with vectors and angles]
Perfect Specular Reflection BRDF

\[ L_i(\theta_i, \phi_i) \] \hspace{1cm} \[ L_o(\theta_o, \phi_o) \]

\[ \theta_i \hspace{1cm} \theta_o \]

\[ L_o(\theta_o, \phi_o) = L_i(\theta_i, \phi_i \pm \pi) \]

\[ f_r(\theta_i, \phi_i; \theta_o, \phi_o) = \frac{\delta(\cos \theta_i - \cos \theta_o)}{\cos \theta_i} \delta(\phi_i - \phi_o \pm \pi) \]

• Why cos\(\theta_i\)?

\[ L_o(\theta_o, \phi_o) = \int f_r(\theta_i, \phi_i; \theta_o, \phi_o) L_i(\theta_i, \phi_i) \cos \theta_i \, d\cos \theta_i \, d\phi_i \]

\[ = \int \frac{\delta(\cos \theta_i - \cos \theta_o)}{\cos \theta_i} \delta(\phi_i - \phi_o \pm \pi) L_i(\theta_i, \phi_i) \cos \theta_i \, d\cos \theta_i \, d\phi_i \]

\[ = L_i(\theta_r, \phi_r \pm \pi) \]
Perfect Specular Reflection BRDF
Specular Refraction

In addition to reflecting off surface, light may be transmitted through surface.

Light refracts when it enters a new medium.
Snell’s Law

Transmitted angle depends on 
index of refraction (IOR) for incident ray 
index of refraction (IOR) for exiting ray

\[ \eta_i \sin \theta_i = \eta_t \sin \theta_t \]

<table>
<thead>
<tr>
<th>Medium</th>
<th>( \eta^* )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vacuum</td>
<td>1.0</td>
</tr>
<tr>
<td>Air (sea level)</td>
<td>1.00029</td>
</tr>
<tr>
<td>Water (20°C)</td>
<td>1.333</td>
</tr>
<tr>
<td>Glass</td>
<td>1.5-1.6</td>
</tr>
<tr>
<td>Diamond</td>
<td>2.42</td>
</tr>
</tbody>
</table>

* index of refraction is wavelength dependent (these are averages)
Law of Refraction

\[ \eta_i \sin \theta_i = \eta_t \sin \theta_t \]

\[ \cos \theta_t = \sqrt{1 - \sin^2 \theta_t} \]

\[ = \sqrt{1 - \left( \frac{\eta_i}{\eta_t} \right)^2 \sin^2 \theta_i} \]

\[ = \sqrt{1 - \left( \frac{\eta_i}{\eta_t} \right)^2 \left(1 - \cos^2 \theta_i \right)} \]

Total internal reflection:

When light is moving from a more optically dense medium to a less optically dense medium: \( \frac{\eta_i}{\eta_t} > 1 \)

Light incident on boundary from large enough angle will not exit medium.
Snell’s Window/Circle

Total internal reflection

[Livingston and Lynch]
Attendance Time

If you are seated in class, go to this form and sign in:

- https://tinyurl.com/184lecture

Notes:

- Time-stamp will be taken when you submit form. Do it now, won’t count later.
- Don’t tell friends outside class to fill it out now, because we will audit at some point in semester.
- Failing audit will have large negative consequence. You don’t need to, because you have an alternative!
Fresnel Reflection / Term

Reflectance depends on incident angle (and polarization of light)

This example: reflectance increases with grazing angle

[Lafortune et al. 1997]
Fresnel Term (Dielectric, $\eta = 1.5$)
Fresnel Term (Conductor)
Without Fresnel (Fixed Reflectance/Transmission)
Glass with Fresnel Reflection/Transmission
Microfacet Material
Microfacet Reflection

https://twitter.com/Cmdr_Hadfield/status/318986491063828480/photo/1
Microfacet Theory

Rough surface

• Macroscale: flat & rough
• Microscale: bumpy & specular

Individual elements of surface act like mirrors

• Known as Microfacets
• Each microfacet has its own normal
Microfacet BRDF

• Key: the \textit{distribution} of microfacets’ normals
  • Concentrated $\iff$ glossy
  • Spread $\iff$ diffuse
Microfacet BRDF

- What kind of microfacets reflect $\mathbf{w}_i$ to $\mathbf{w}_o$?
  (hint: microfacets are mirrors)

$$f(i, o) = \frac{\mathbf{F}(i, h) \mathbf{G}(i, o, h) \mathbf{D}(h)}{4(n \cdot i)(n \cdot o)}$$

Fresnel term

shadowing-masking term

distribution of normals

$\mathbf{h}$
Microfacet BRDF: Examples

[Autodesk Fusion 360]

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Jonathan Ragan-Kelley & Ren Ng
Isotropic / Anisotropic Materials (BRDFs)

- So far, Point light + Metal = Round / Elliptical highlight
- What can we see inside an elevator?
Isotropic / Anisotropic Materials (BRDFs)

- Key: **directionality** of underlying surface

<table>
<thead>
<tr>
<th>Isotropic</th>
<th>Anisotropic</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image" alt="Surface (normals)" /></td>
<td><img src="image" alt="BRDF (fix wi, vary wo)" /></td>
</tr>
</tbody>
</table>

Isotropic

Anisotropic
Anisotropic BRDFs

Reflection depends on azimuthal angle $\phi$

$$f_r(\theta_i, \phi_i; \theta_r, \phi_r) \neq f_r(\theta_i, \theta_r, \phi_r - \phi_i)$$

Results from oriented microstructure of surface, e.g., brushed metal
Anisotropic BRDF: Brushed Metal

- How is the pan brushed?
Anisotropic BRDF: Nylon

[Westin et al. 1992]
Anisotropic BRDF: Velvet

[Westin et al. 1992]
Anisotropic BRDF: Velvet

[https://www.youtube.com/watch?v=2hjoW8TYTd4]
Properties of BRDFs

- Non-negativity

\[ f_r(\omega_i \rightarrow \omega_r) \geq 0 \]

- Linearity

\[ L_r(p, \omega_r) = \int_{H^2} f_r(p, \omega_i \rightarrow \omega_r) L_i(p, \omega_i) \cos \theta_i \, d\omega_i \]

[Sillion et al. 1990]
Properties of BRDFs

- Reciprocity

\[ f_r(\omega_r \rightarrow \omega_i) = f_r(\omega_i \rightarrow \omega_r) \]

- Energy conservation

\[ \forall \omega_r \int_{H^2} f_r(\omega_i \rightarrow \omega_r) \cos \theta_i \, d\omega_i \leq 1 \]
Properties of BRDFs

- Isotropic vs. anisotropic
- If isotropic, \( f_r(\theta_i, \phi_i; \theta_r, \phi_r) = f_r(\theta_i, \theta_r, \phi_r - \phi_i) \)
- Then, from reciprocity,
  \[
  f_r(\theta_i, \theta_r, \phi_r - \phi_i) = f_r(\theta_r, \theta_i, \phi_i - \phi_r) = f_r(\theta_i, \theta_r, |\phi_r - \phi_i|)
  \]
Measuring BRDFs
Measuring BRDFs: Motivation

Avoid need to develop / derive models

- Automatically includes all of the scattering effects present

Can accurately render with real-world materials

- Useful for product design, special effects, ...

Theory vs. practice:

[Bagher et al. 2012]
Image-Based BRDF Measurement

[Marschner et al. 1999]
Measuring BRDFs: gonioreflectometer

Spherical gantry at UCSD
Measuring BRDFs

General approach:

```plaintext
foreach outgoing direction wo
  move light to illuminate surface with a thin beam from wo
for each incoming direction wi
  move sensor to be at direction wi from surface
  measure incident radiance
```

Improving efficiency:

- Isotropic surfaces reduce dimensionality from 4D to 3D
- Reciprocity reduces # of measurements by half
- Clever optical systems...
Challenges in Measuring BRDFs

- Accurate measurements at grazing angles
  - Important due to Fresnel effects
- Measuring with dense enough sampling to capture high frequency specularities
- Retro-reflection
- Spatially-varying reflectance, ...
Representing Measured BRDFs

Desirable qualities

• Compact representation
• Accurate representation of measured data
• Efficient evaluation for arbitrary pairs of directions
• Good distributions available for importance sampling
Tabular Representation

Store regularly-spaced samples in 
\((\theta_i, \theta_o, |\phi_i - \phi_o|)\)

- Better: reparameterize angles to better match specularities

Generally need to resample measured values to table

Very high storage requirements

MERL BRDF Database
[Matusik et al. 2004]
90*90*180 measurements
Advanced Appearance Models
Translucent Materials: Jade
Translucent Materials: Skin
Translucent Materials: Leaves
Subsurface Scattering

Visual characteristics of many surfaces caused by light entering at different points than it exits

- Violates a fundamental assumption of the BRDF

[Jensen et al 2001]

[Donner et al 2008]
Scattering Functions

- Generalization of BRDF; describes exitant radiance at one point due to incident differential irradiance at another point:
  \[ S(x_i, \omega_i, x_o, \omega_o) \]

- Generalization of reflection equation integrates over all points on the surface and all directions (!)
  \[
  L(x_o, \omega_o) = \int_A \int_{H^2} S(x_i, \omega_i, x_o, \omega_o) L_i(x_i, \omega_i) \cos \theta_i \, d\omega_i \, dA
  \]
Cloth

- A collection of twisted fibers!
- Two levels of twist
- Woven or knitted
Cloth: Render as Surface

- Given the weaving pattern, calculate the overall behavior
- Render using a BRDF

[Sadeghi et al. 2013]
Render as Surface — Limitation

[Westin et al. 1992]

✓  x
Cloth: Render as Participating Media

- Properties of individual fibers & their distribution -> scattering parameters
- Render as a participating medium
Cloth: Render as Actual Fibers

- Render every fiber explicitly!

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[Kai Schroder]

Lingqi Yan
Cloth: Demo

[Shuang et al. 2012]
Hair Appearance
Fiber Model

[Marschner et al. 2003]
Things to Remember

Materials (BRDFs)

• Diffuse, Glossy, ideal specular
• Fresnel reflection / Fresnel term
• Microfacet BRDFs
• Anisotropic BRDFs
• Properties of BRDFs
• Measured BRDFs
Acknowledgments

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