Lecture 2:

Digital Drawing

Computer Graphics and Imaging
UC Berkeley CS184/284

Today: Rasterizing Triangles into Pixels


Life of Pi (2012)

CS184/284A

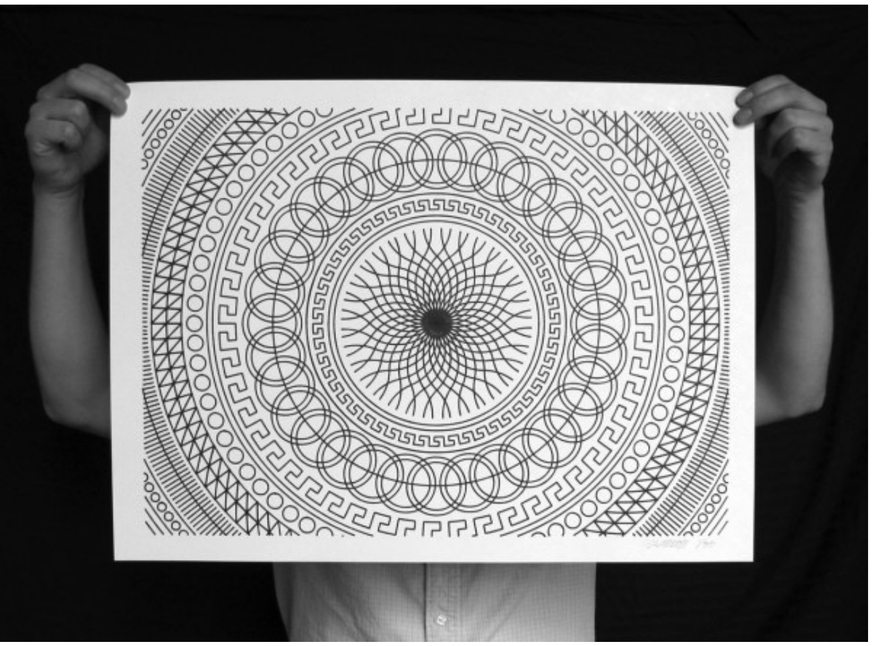
Jonathan Ragan-Kelley & Ren Ng

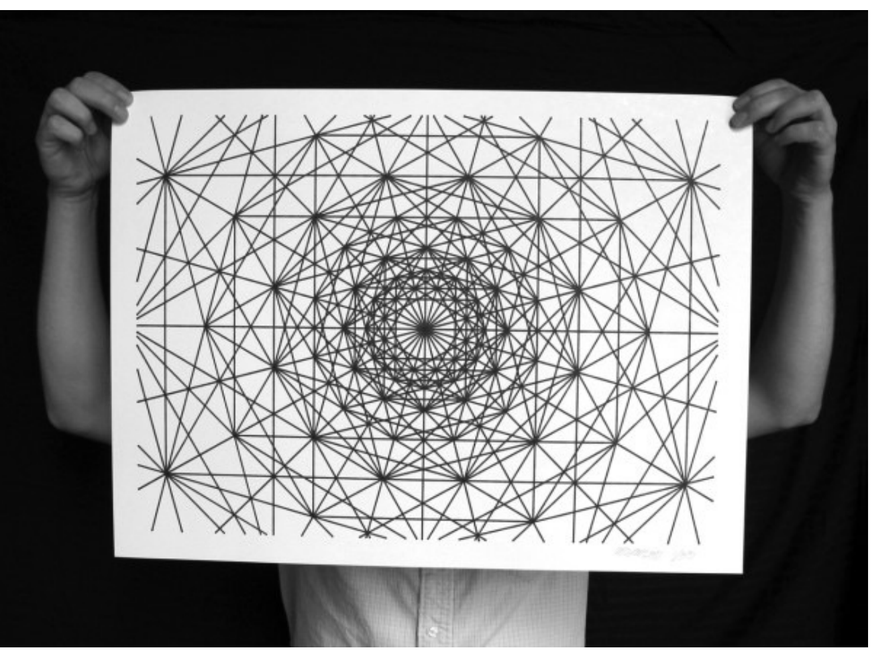
Drawing Machines

Pablo Picasso: drawing machine?

https://www.youtube.com/watch?v=JU9oaD0e7uU

CNC Sharpie Drawing Machine



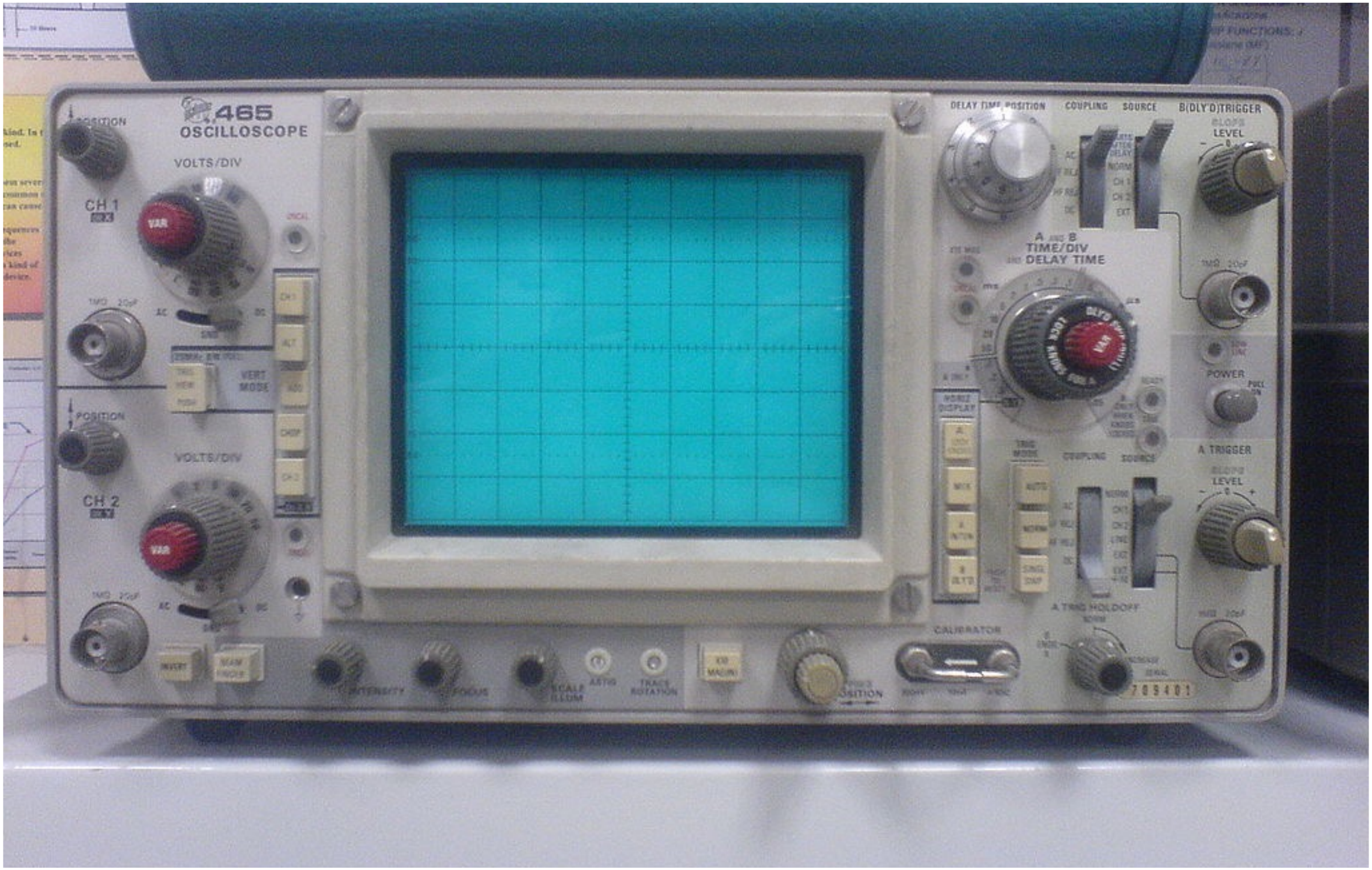


Aaron Panone with Matt W. Moore

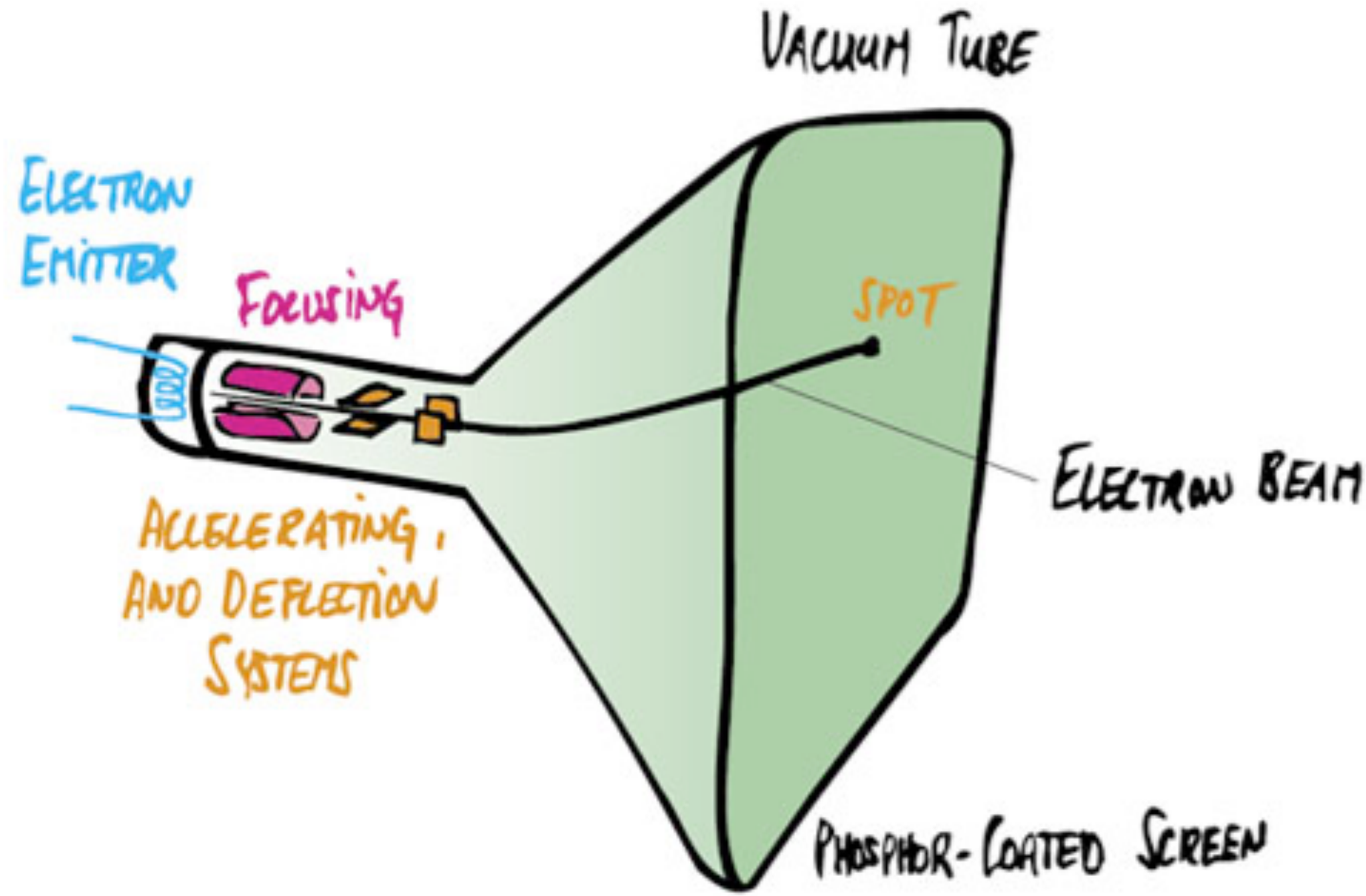
http://44rn.com/projects/numerically-controlled-poster-series-with-matt-w-moore/

Laser Cutters

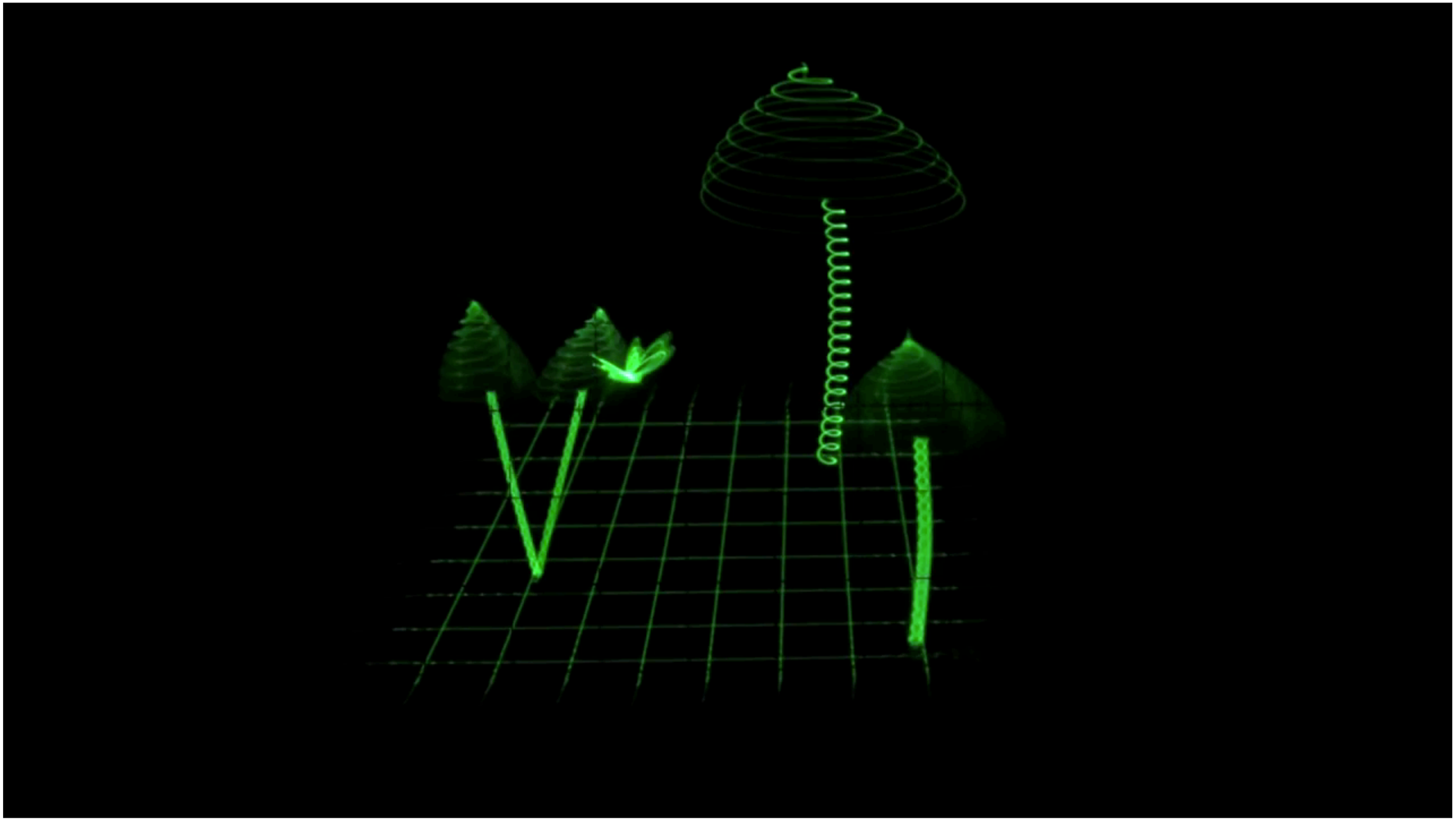
Oscilloscope



Cathode Ray Tube



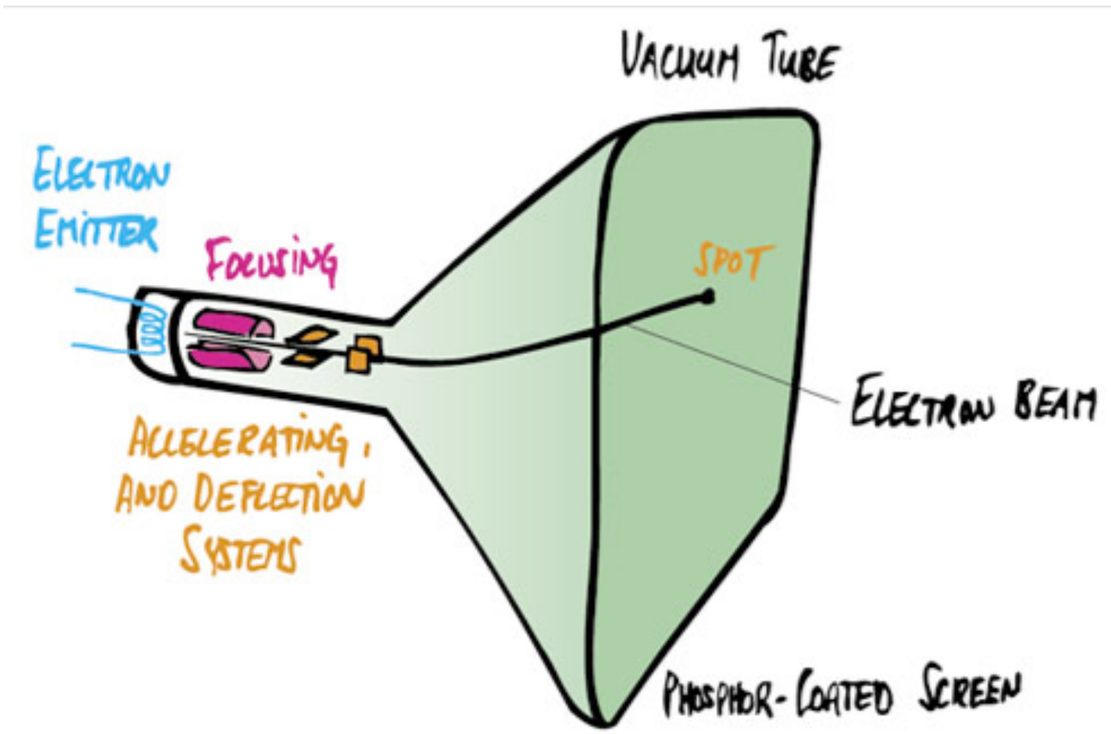
Oscilloscope Art



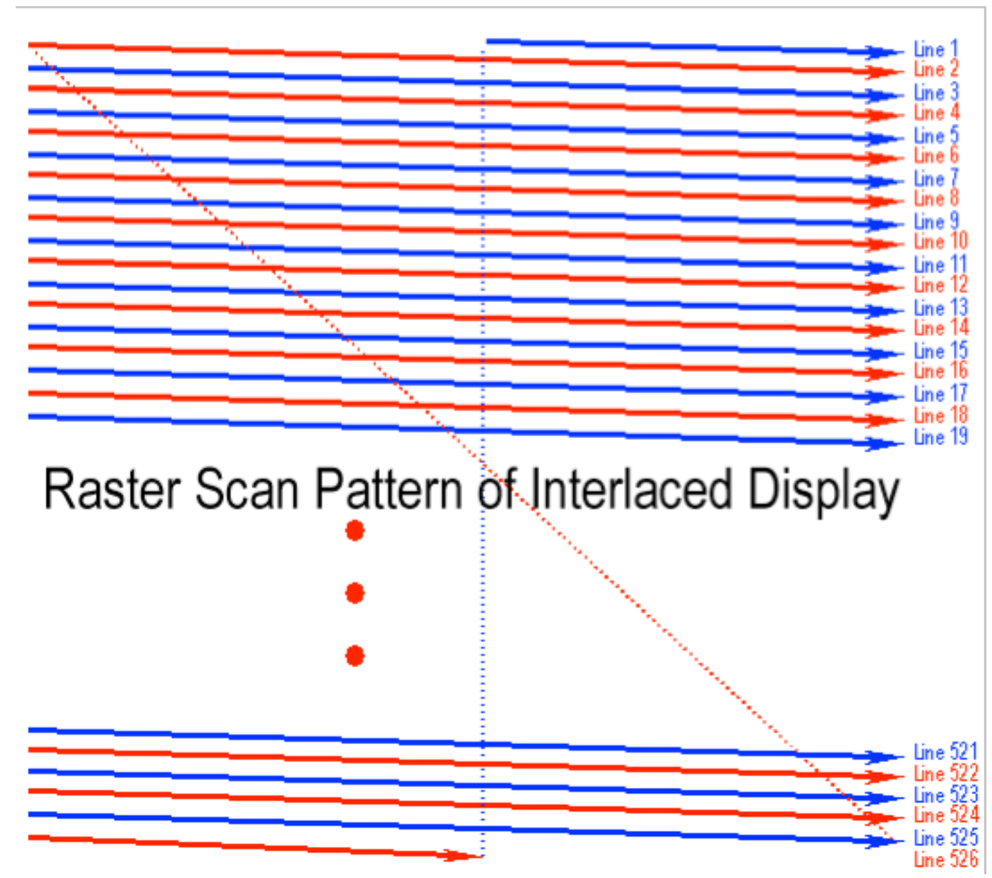
Jerobeam Fenderson

https://www.youtube.com/watch?v=rtR63-ecUNo

Television - Raster Display CRT



Cathode Ray Tube



Raster Scan
(modulate intensity)

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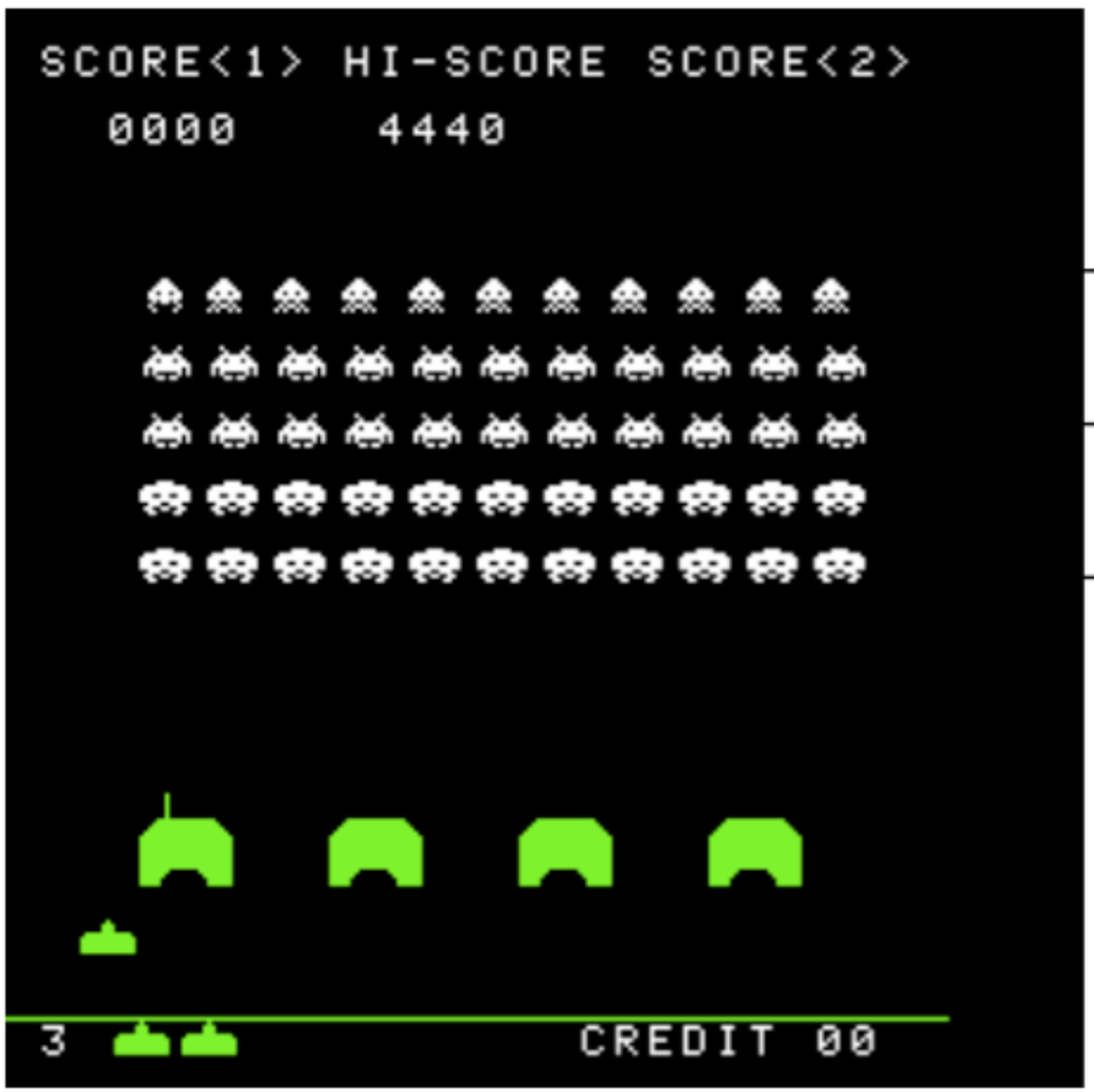
Frame Buffer: Memory for a Raster Display



DAC =
Digital to Analog Convertors

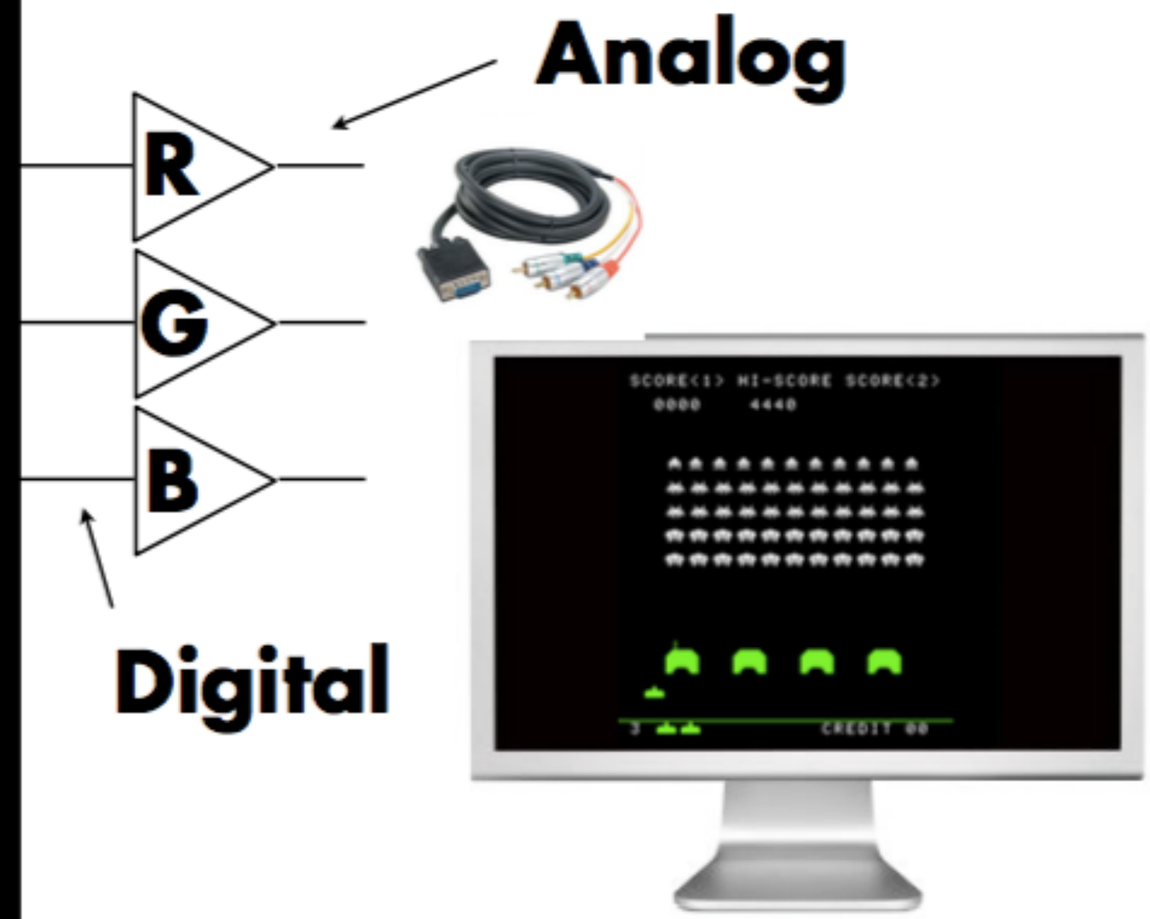


Image = 2D array of colors

A Sampling of Different Raster Displays

Flat Panel Displays

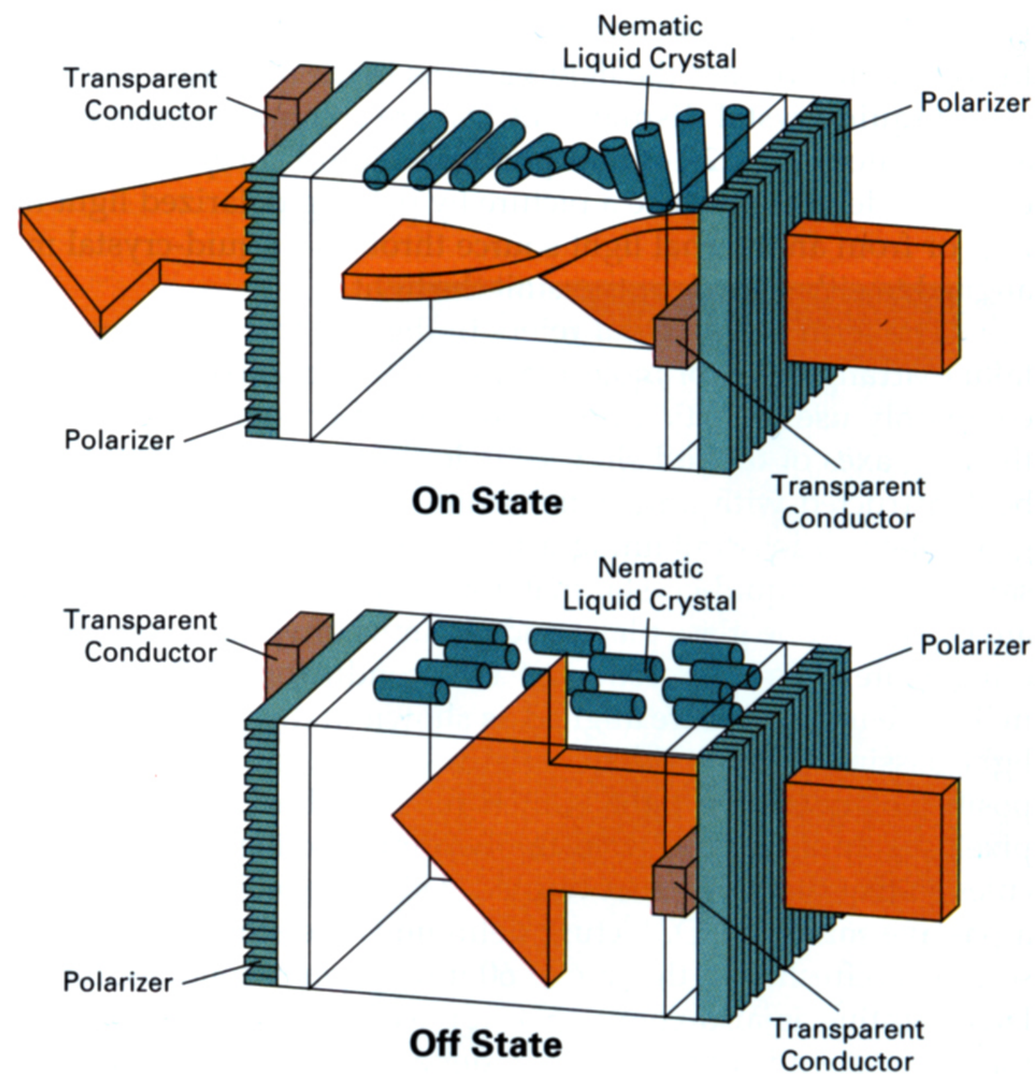
Low-Res LCD Display

LCD (Liquid Crystal Display) Pixel

Principle: block or transmit light by twisting polarization

Illumination from backlight (e.g. fluorescent or LED)

Intermediate intensity levels by partial twist



[H&B fig. 2-16]

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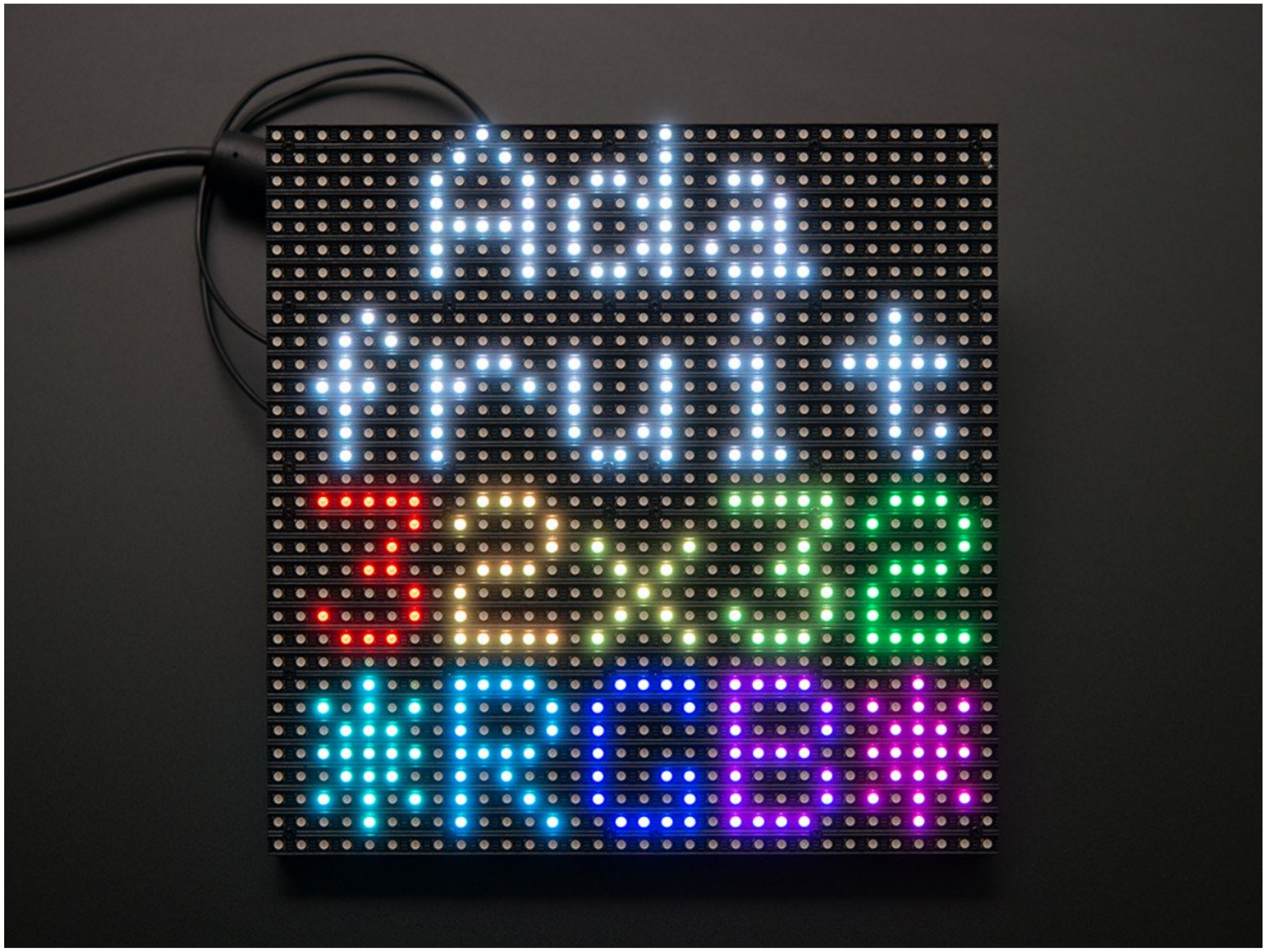
LED Array Display



Light emitting diode array

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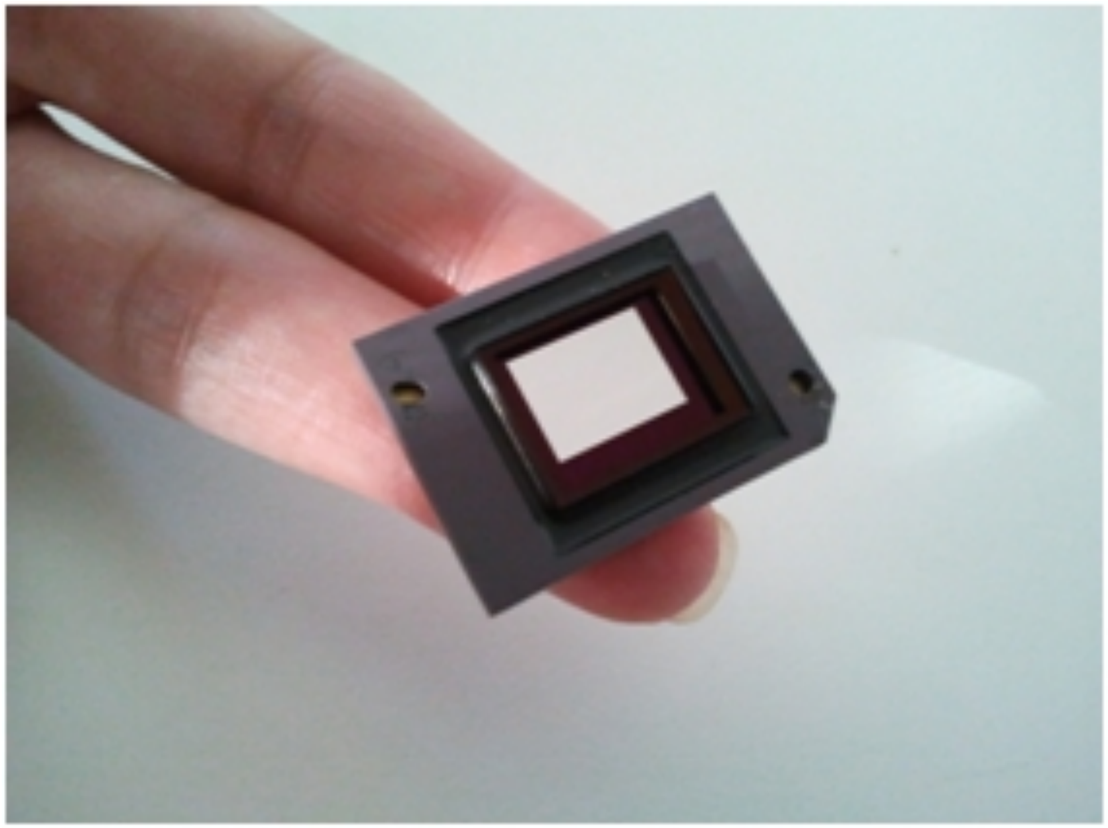
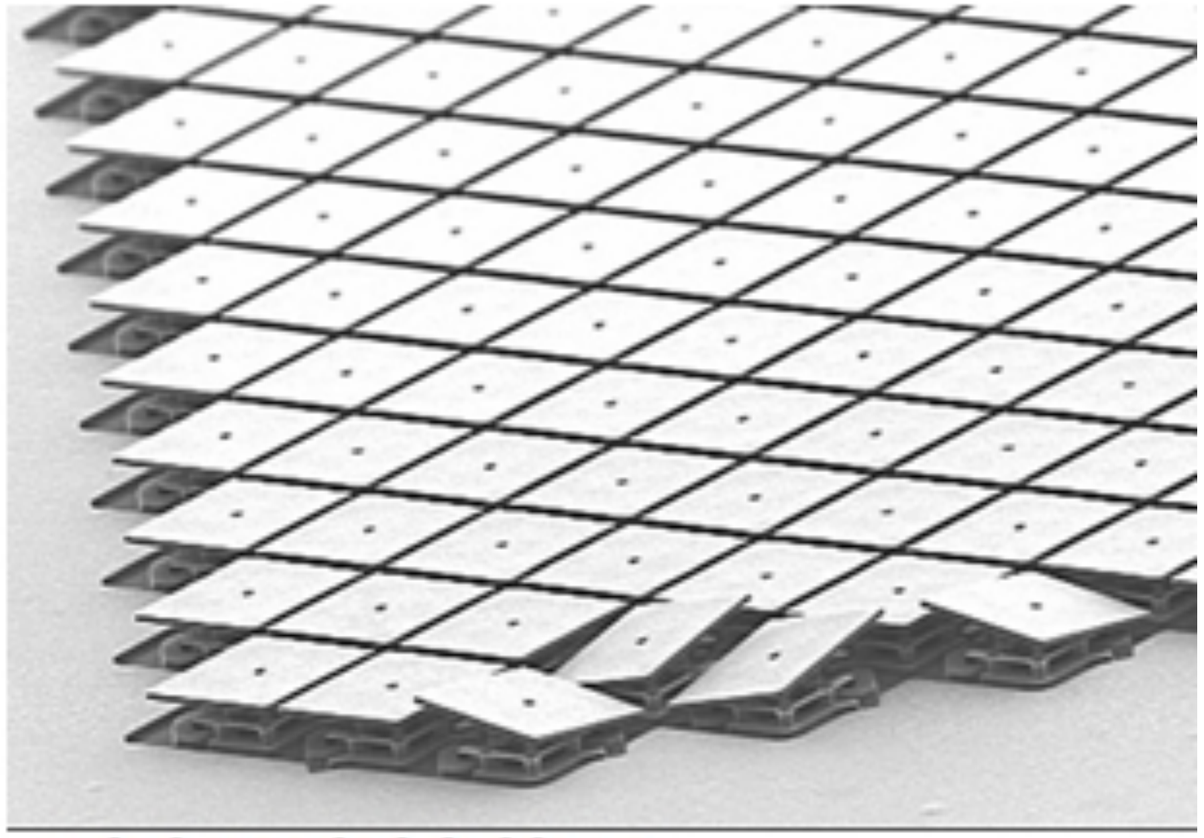
DMD Projection Display



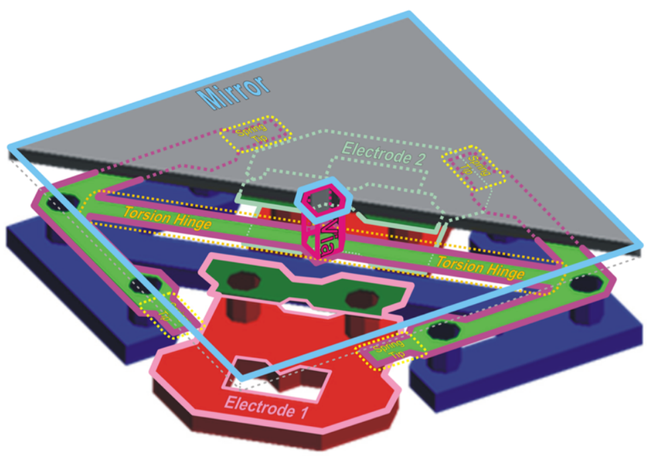
DIGITAL MICRO MIRROR DEVICE **(DMD)**
(**SLM** - Spatial Light Modulator)

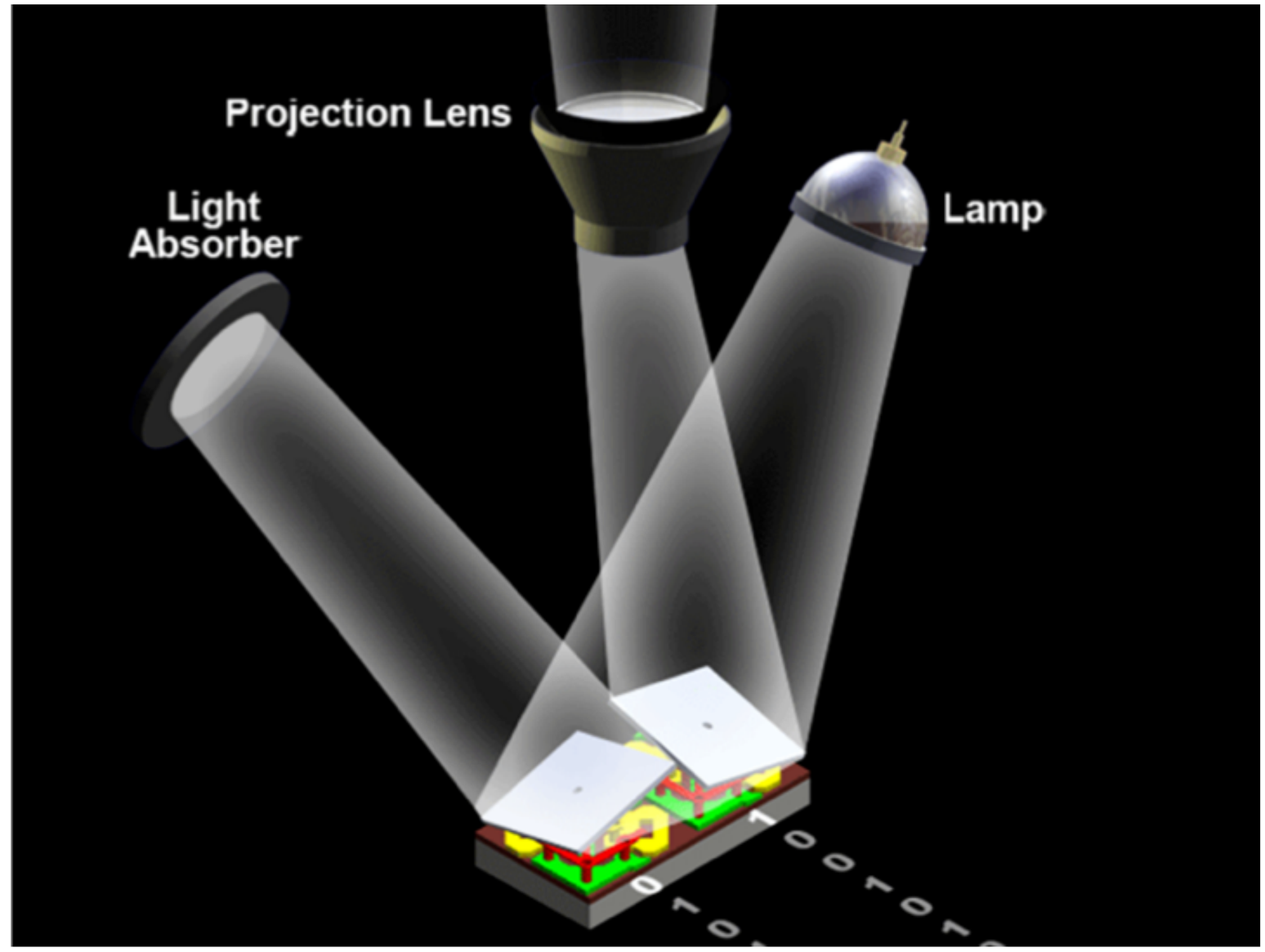


MICRO MIRRORS CLOSE UP

[Y.K. Rabinowitz; EKB Technologies]

DMD Projection Display





[Texas Instruments]

Array of micro-mirror pixels

DMD = Digital Micromirror Device

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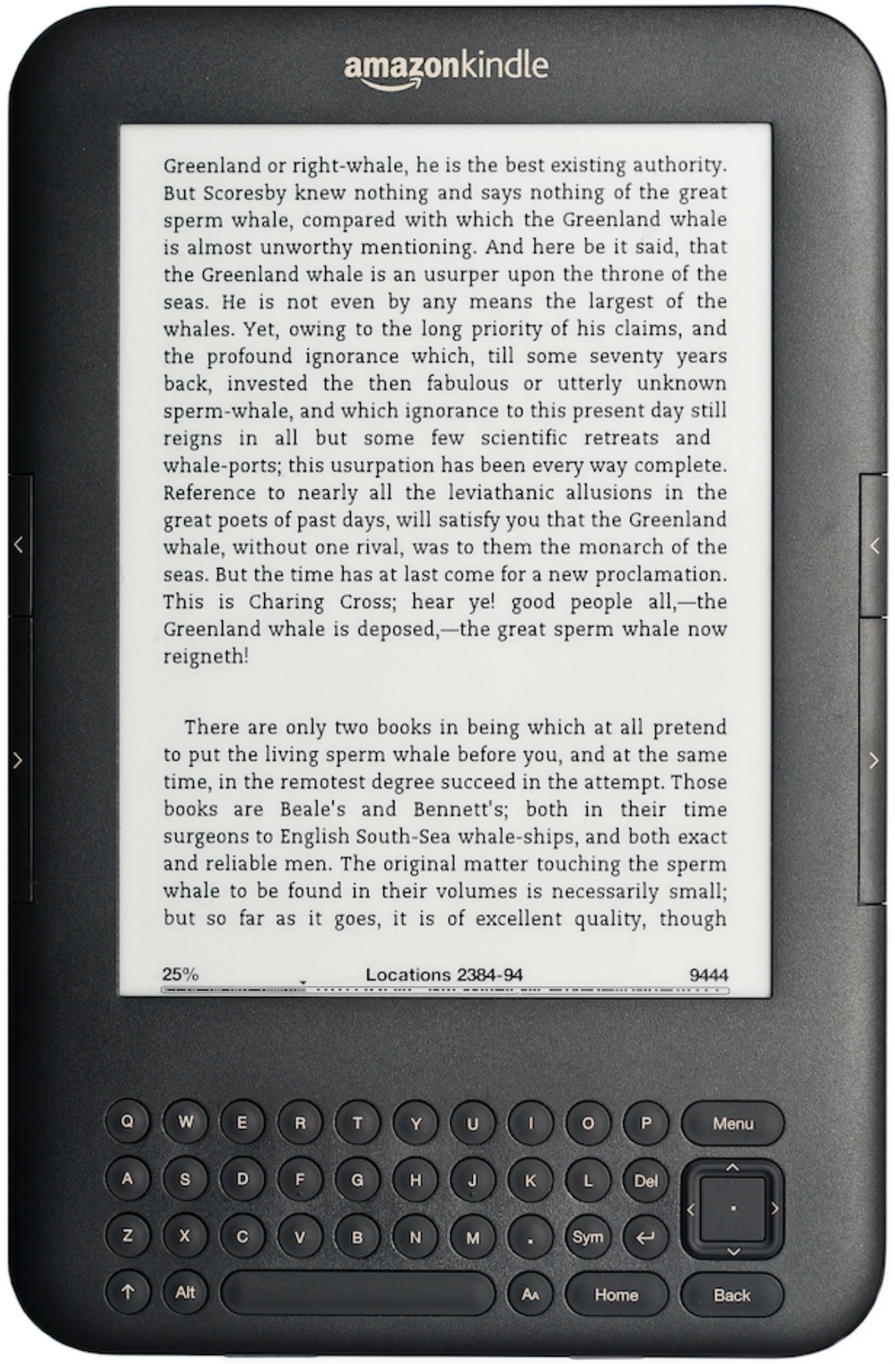
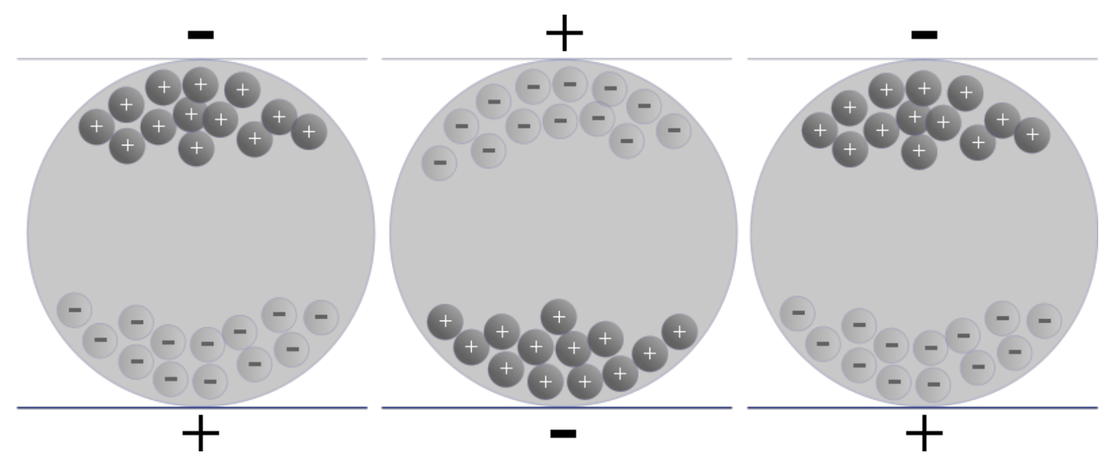
Electrophoretic (Electronic Ink) Display





[Wikimedia Commons —Senarclens]

Drawing to Raster Displays

Polygon Meshes


Life of Pi (2012)

CS184/284A

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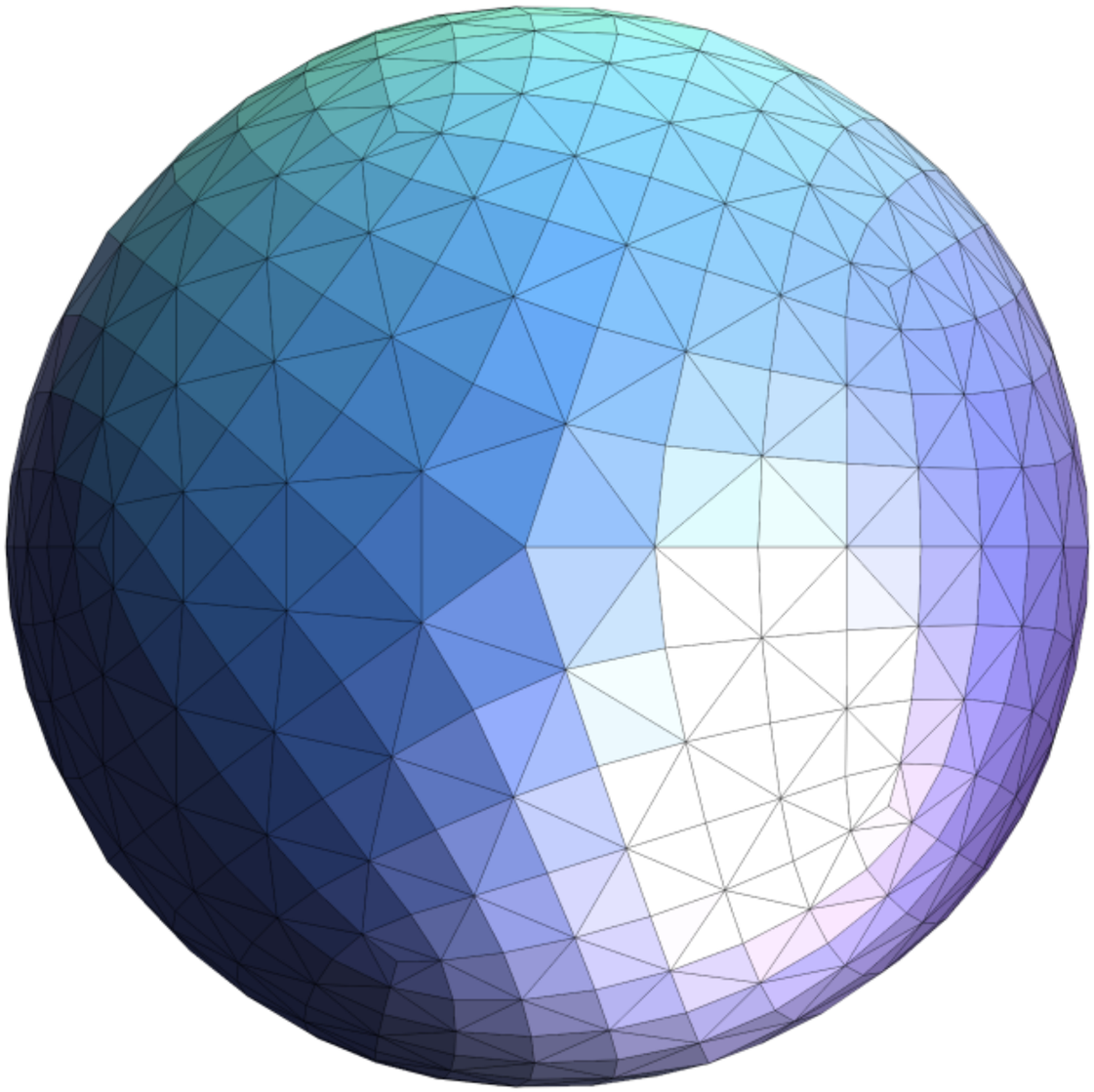
Triangle Meshes



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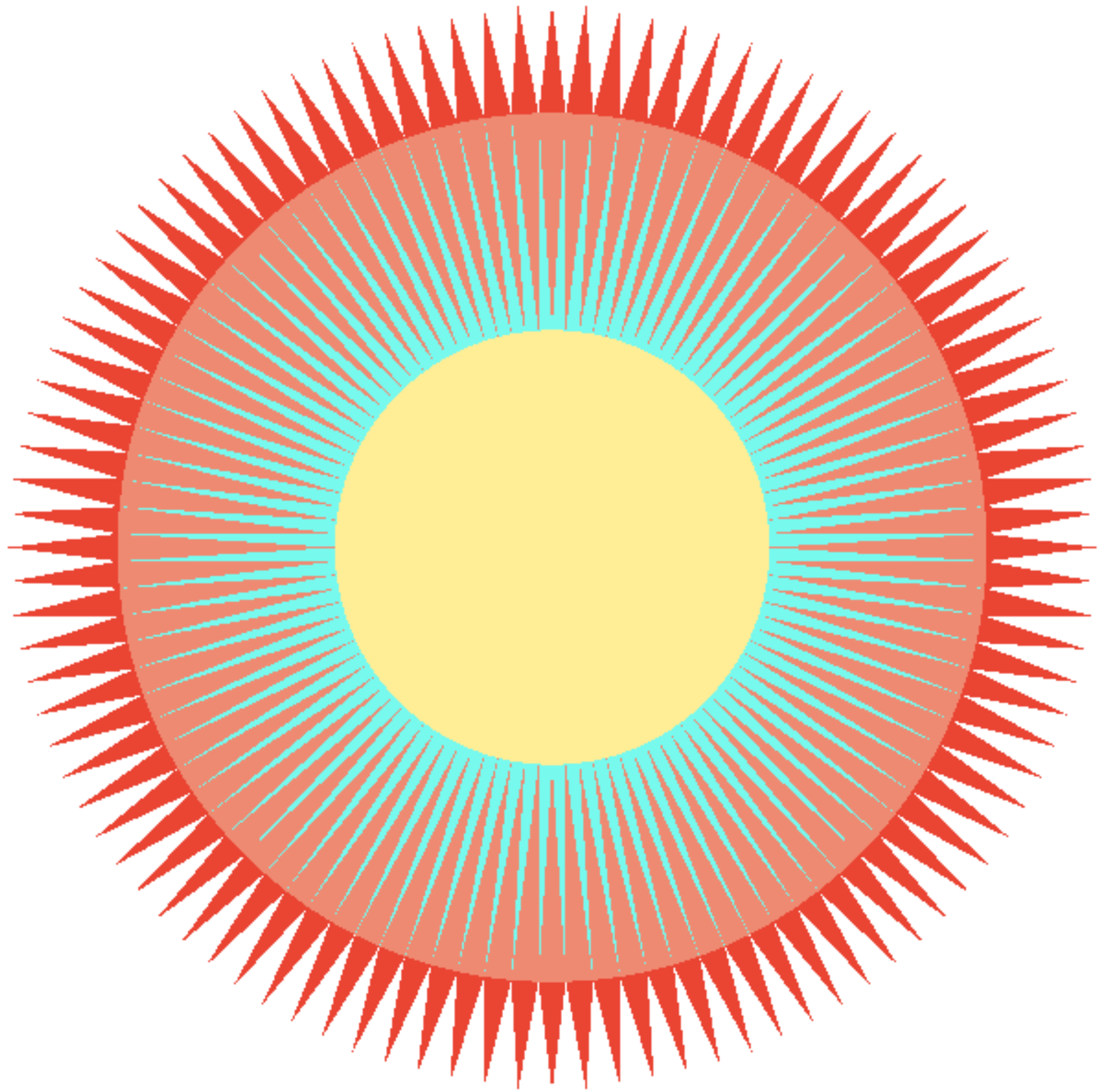
Triangle Meshes



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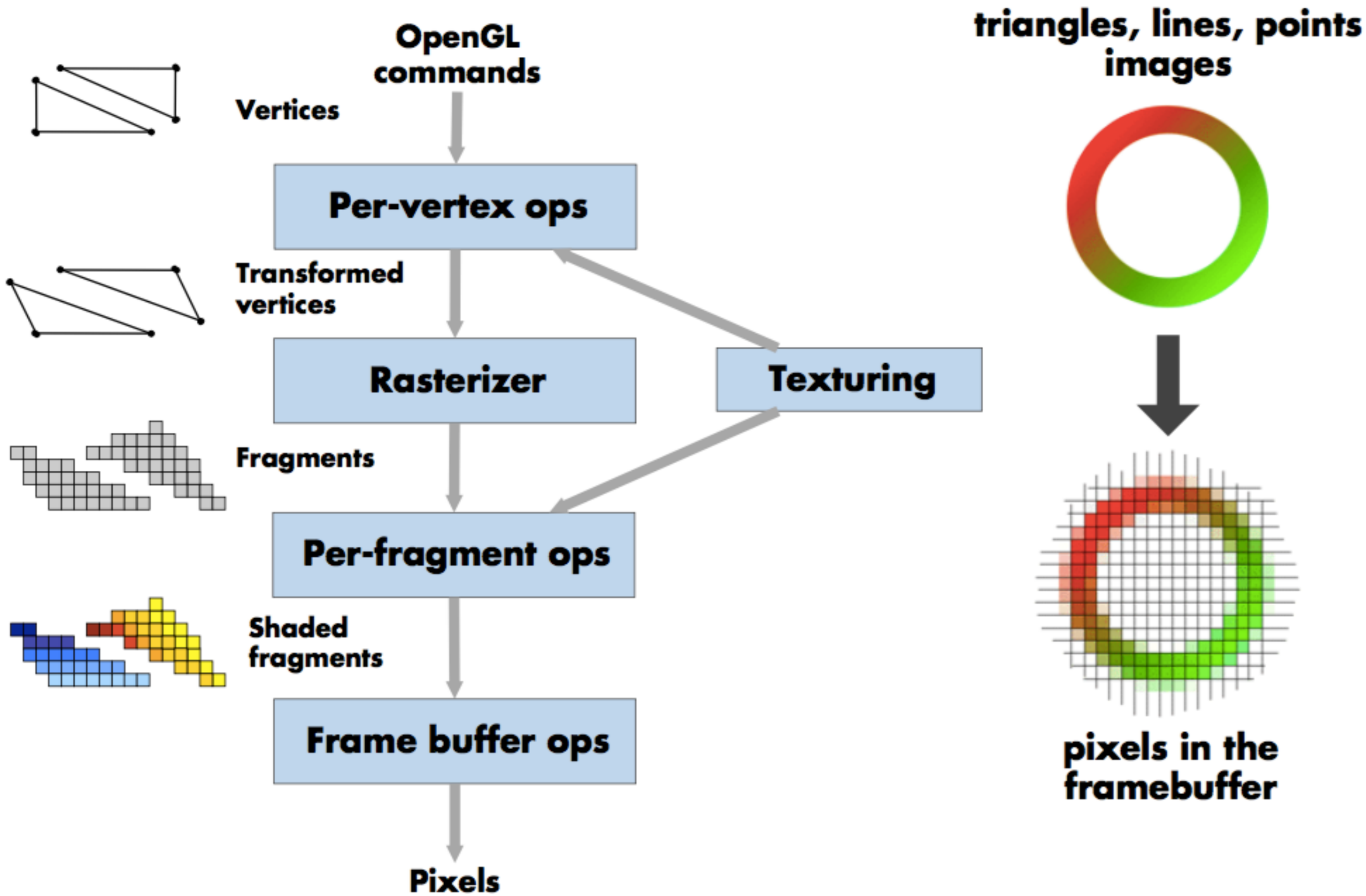
Graphics Pipeline = Abstract Drawing Machine



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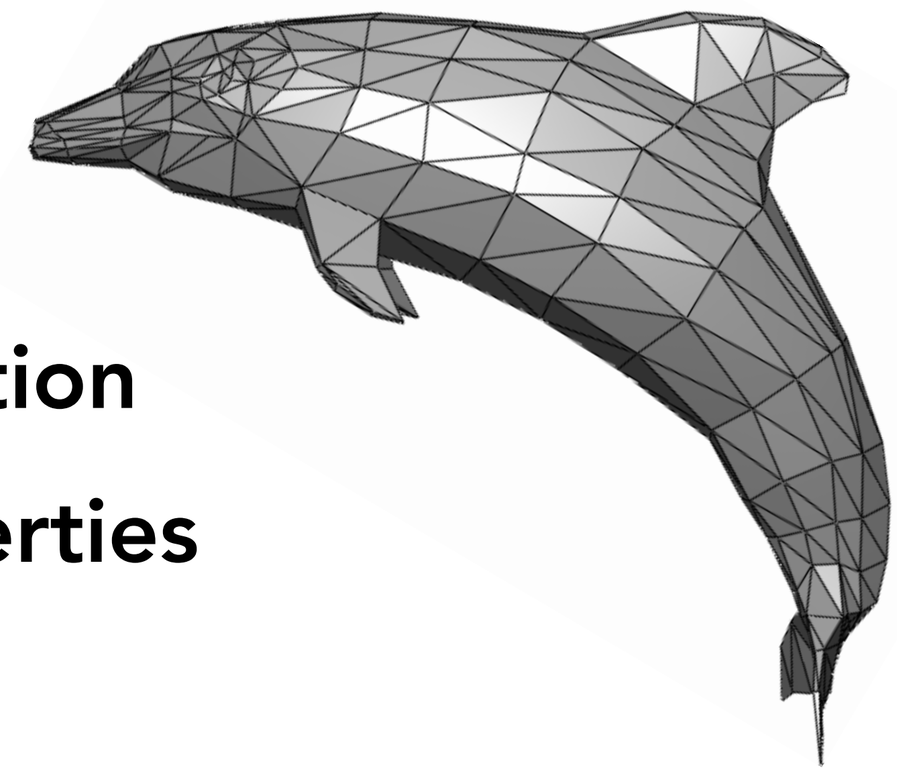
Triangles - Fundamental Area Primitive

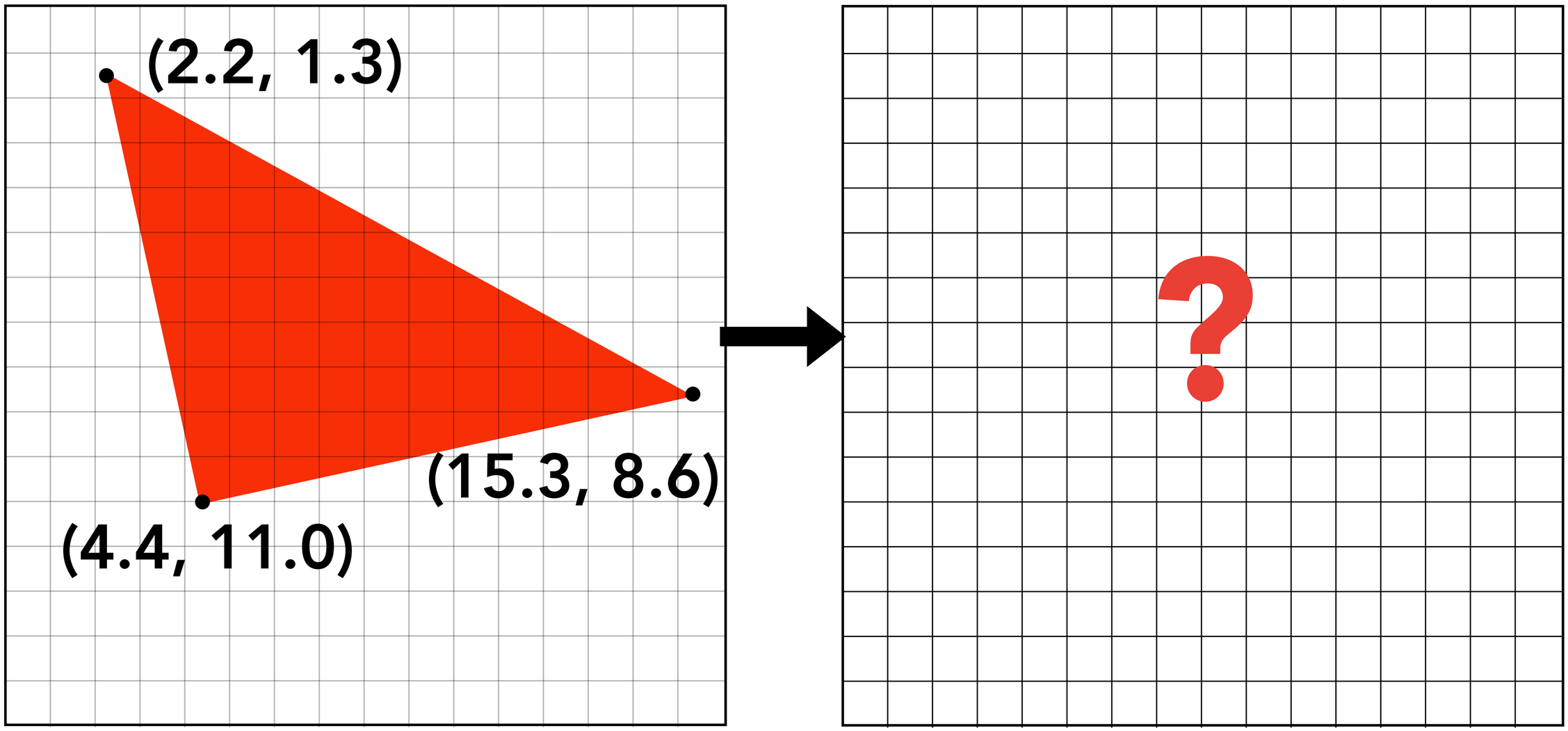
Why triangles?

- Most basic polygon
  - Break up other polygons
  - Optimize one implementation
- Triangles have unique properties
  - Guaranteed to be planar
  - Well-defined interior
  - Well-defined method for interpolating values at vertices over triangle (barycentric interpolation)



Drawing a Triangle To The Framebuffer ("Rasterization")

What Pixel Values Approximate a Triangle?



Input: position of triangle vertices projected on screen

Output: set of pixel values approximating triangle

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Today, Let's Start With A Simple Approach: Sampling

Sampling a Function

Evaluating a function at a point is sampling.

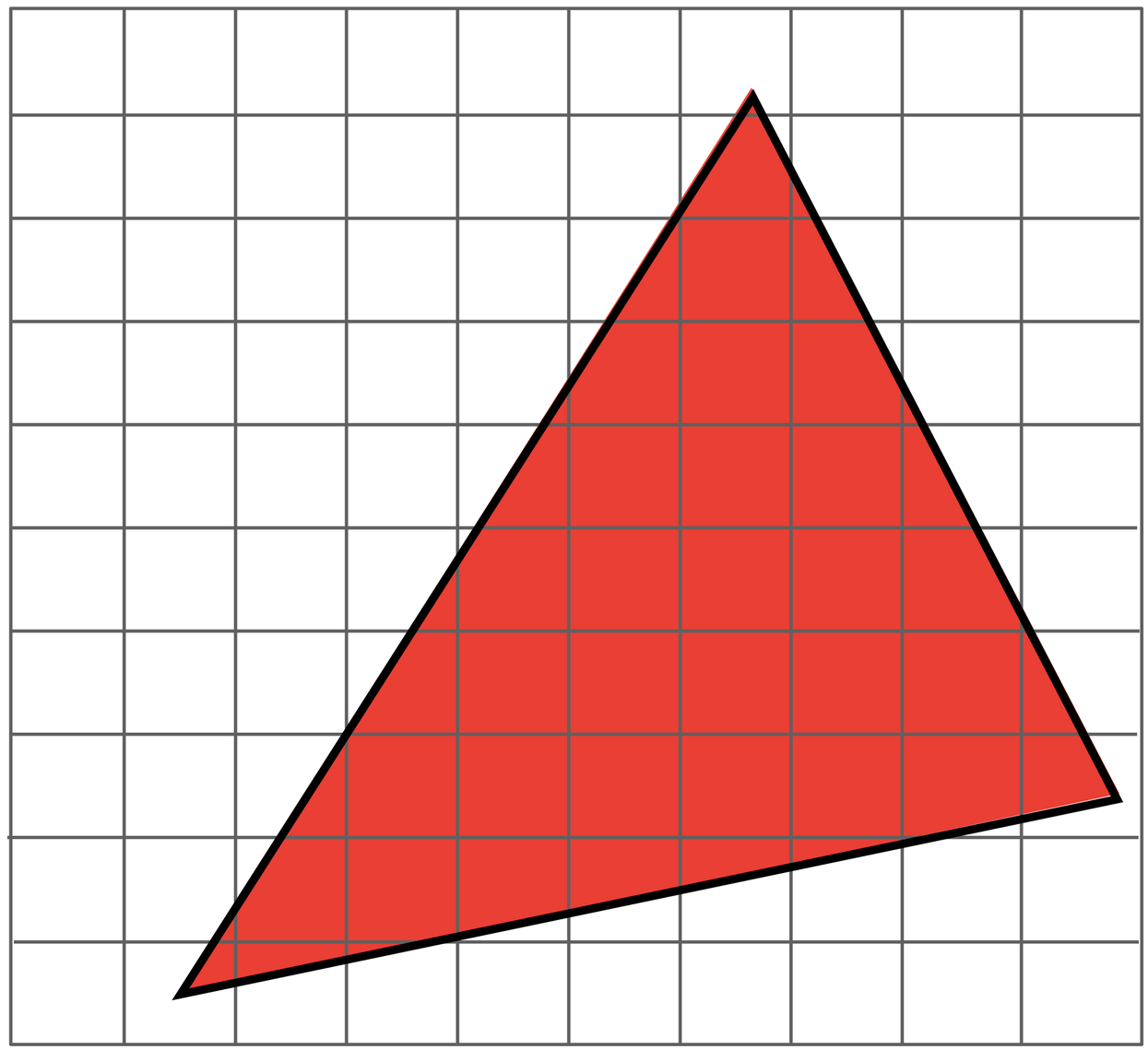
We can discretize a function by periodic sampling.

```
for( int x = 0; x < xmax; x++ )
    output[x] = f(x);
```

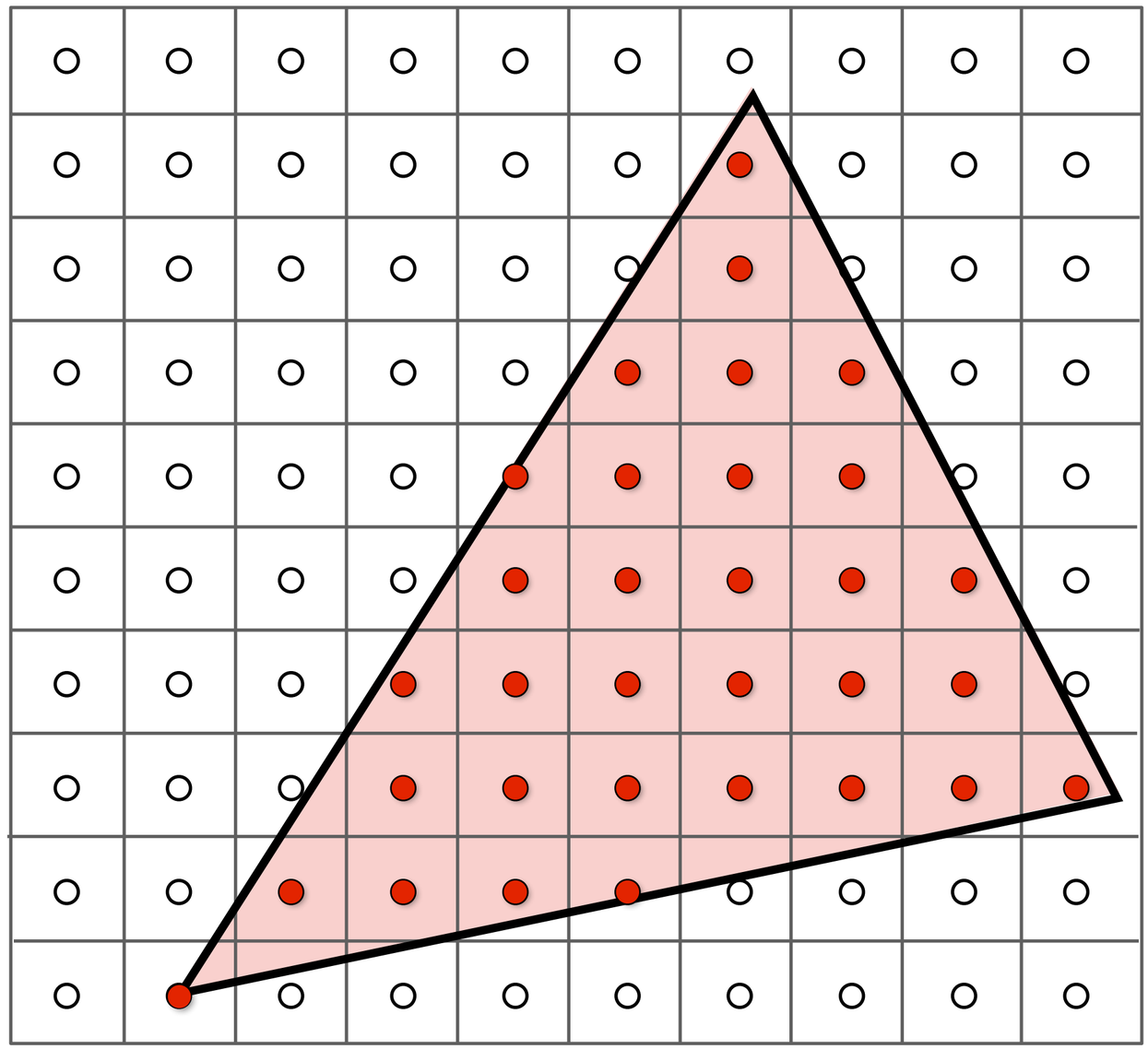
Sampling is a core idea in graphics. We'll sample time (1D), area (2D), angle (2D), volume (3D) …

We'll sample N-dimensional functions, even infinite dimensional functions.

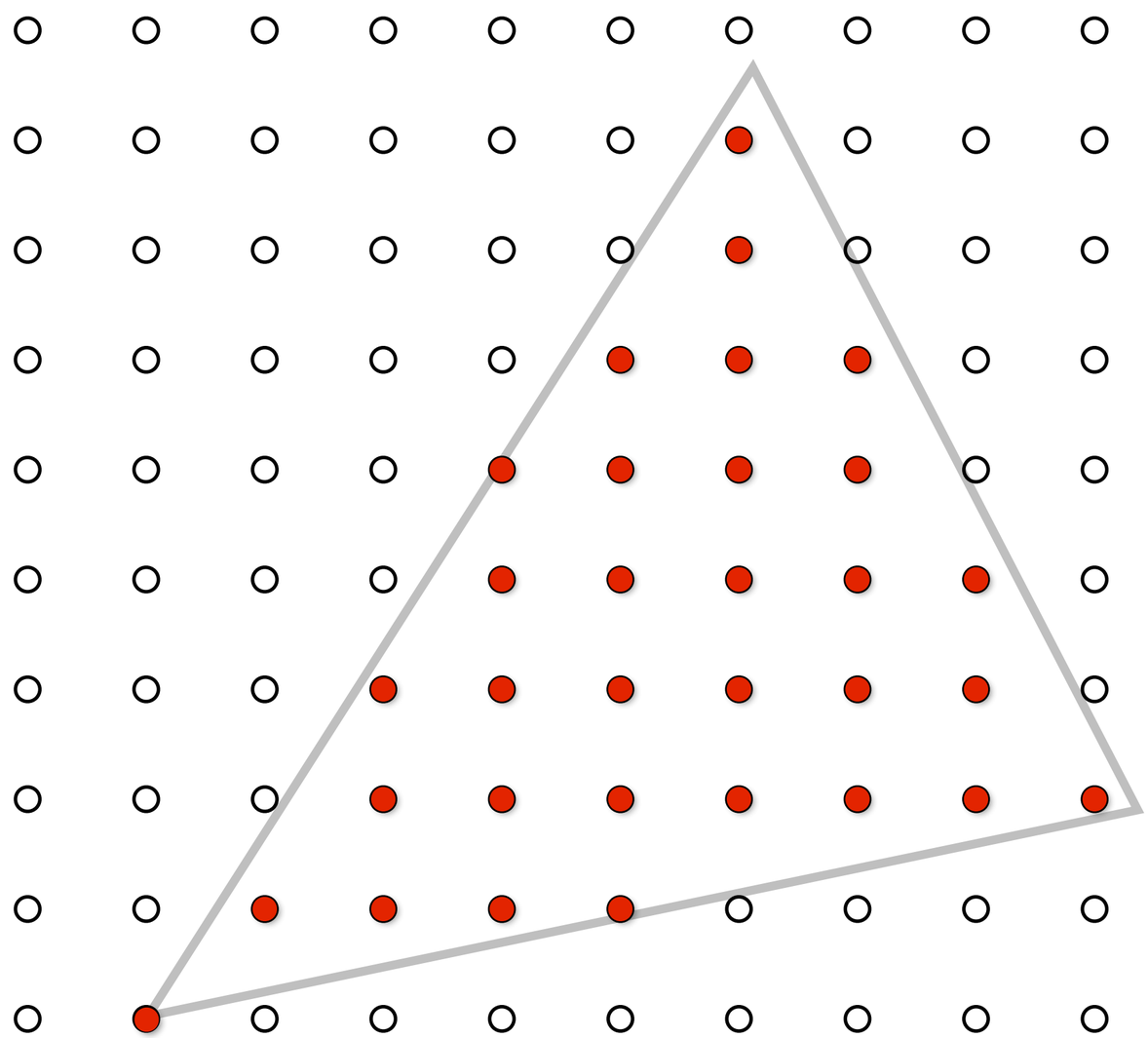
Let's Try Rasterization As 2D Sampling



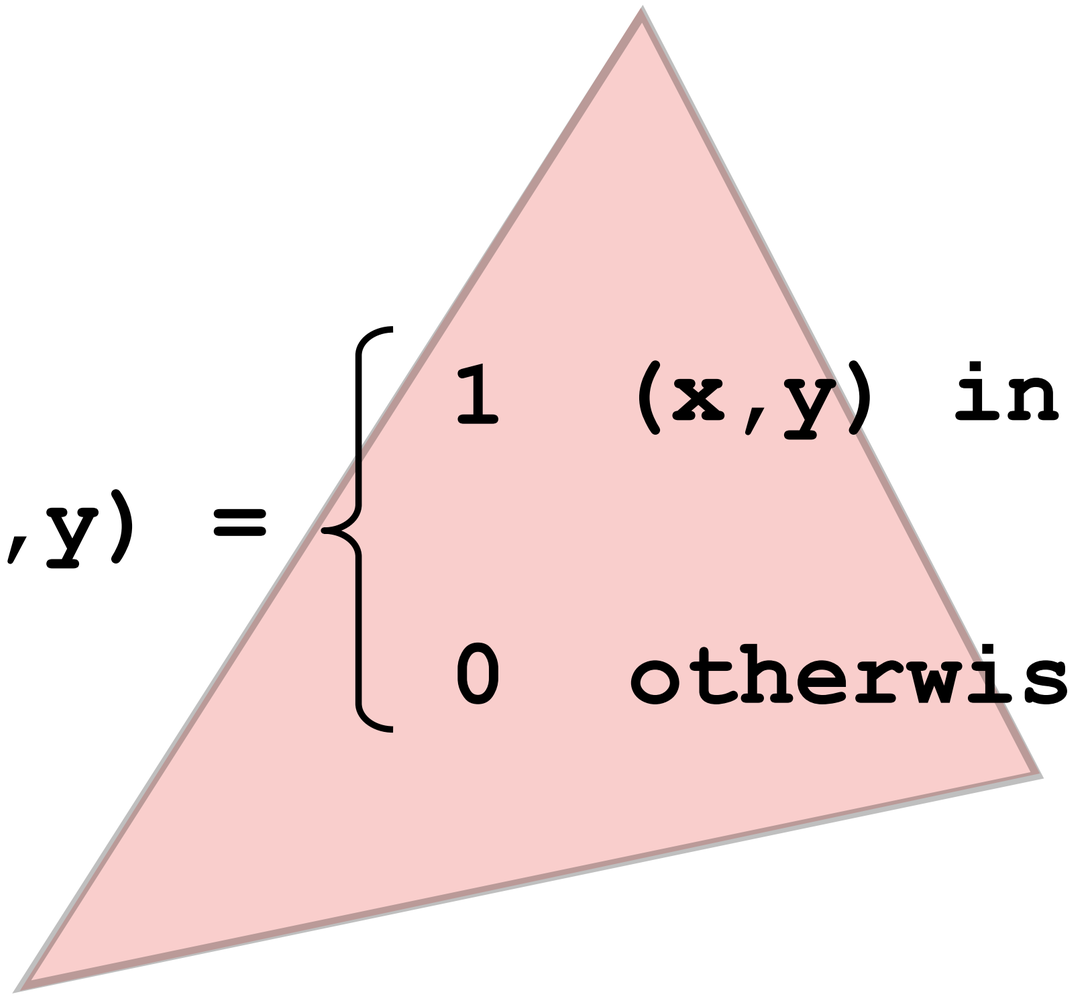
Sample If Each Pixel Center Is Inside Triangle



Sample If Each Pixel Center Is Inside Triangle



Define Binary Function: `inside(tri,x,y)`



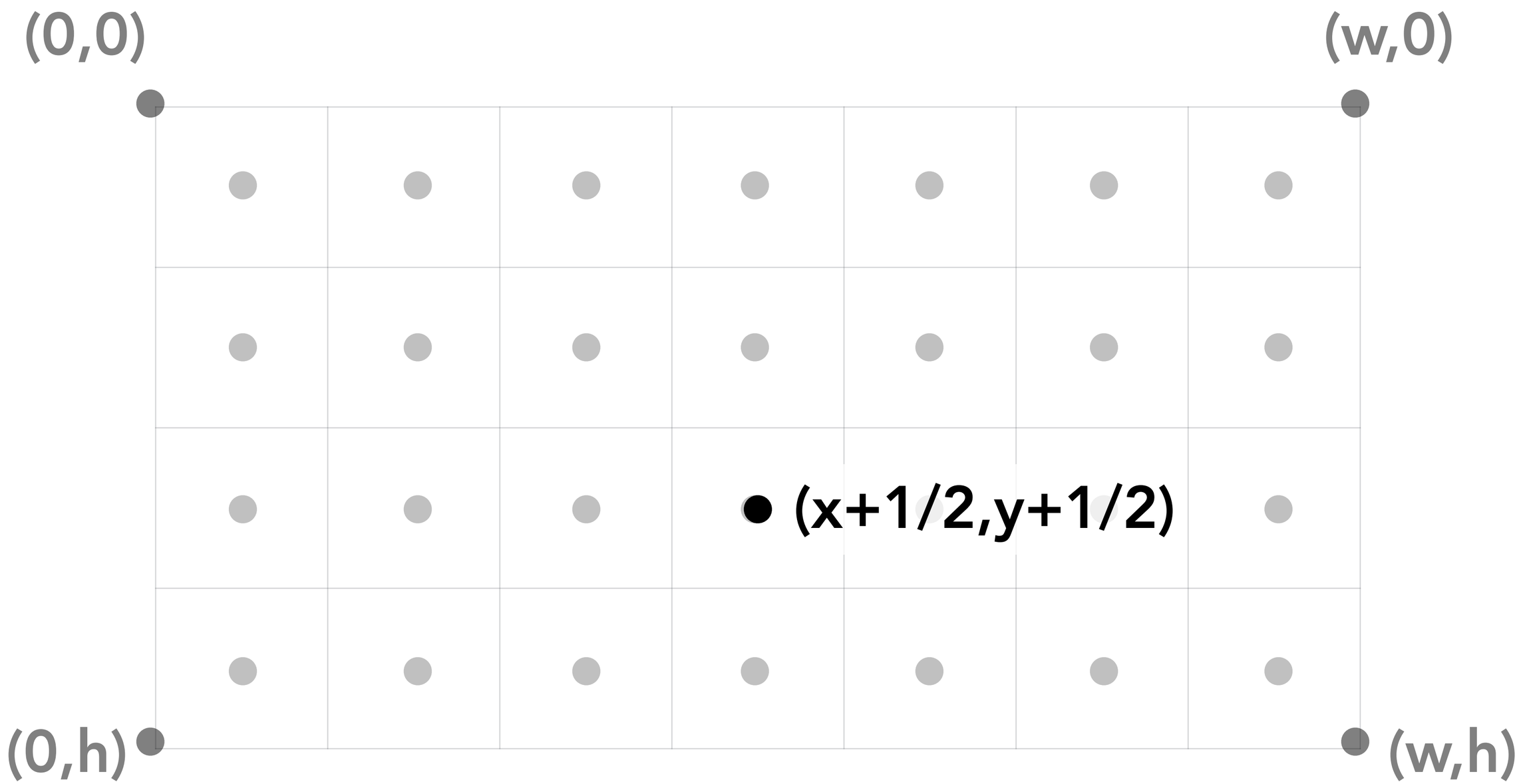
$$\texttt{inside(t,x,y)} = \begin{cases} 1 & \texttt{(x,y) in triangle t} \\ 0 & \texttt{otherwise} \end{cases}$$

Rasterization = Sampling A 2D Indicator Function

```
for( int x = 0; x < xmax; x++ )
  for( int y = 0; y < ymax; y++ )
    Image[x][y] = f(x + 0.5, y + 0.5);
```

Rasterize triangle `tri` by sampling the function
`f(x,y) = inside(tri,x,y)`

Implementation Detail: Sample Locations



Sample location for pixel (x,y)

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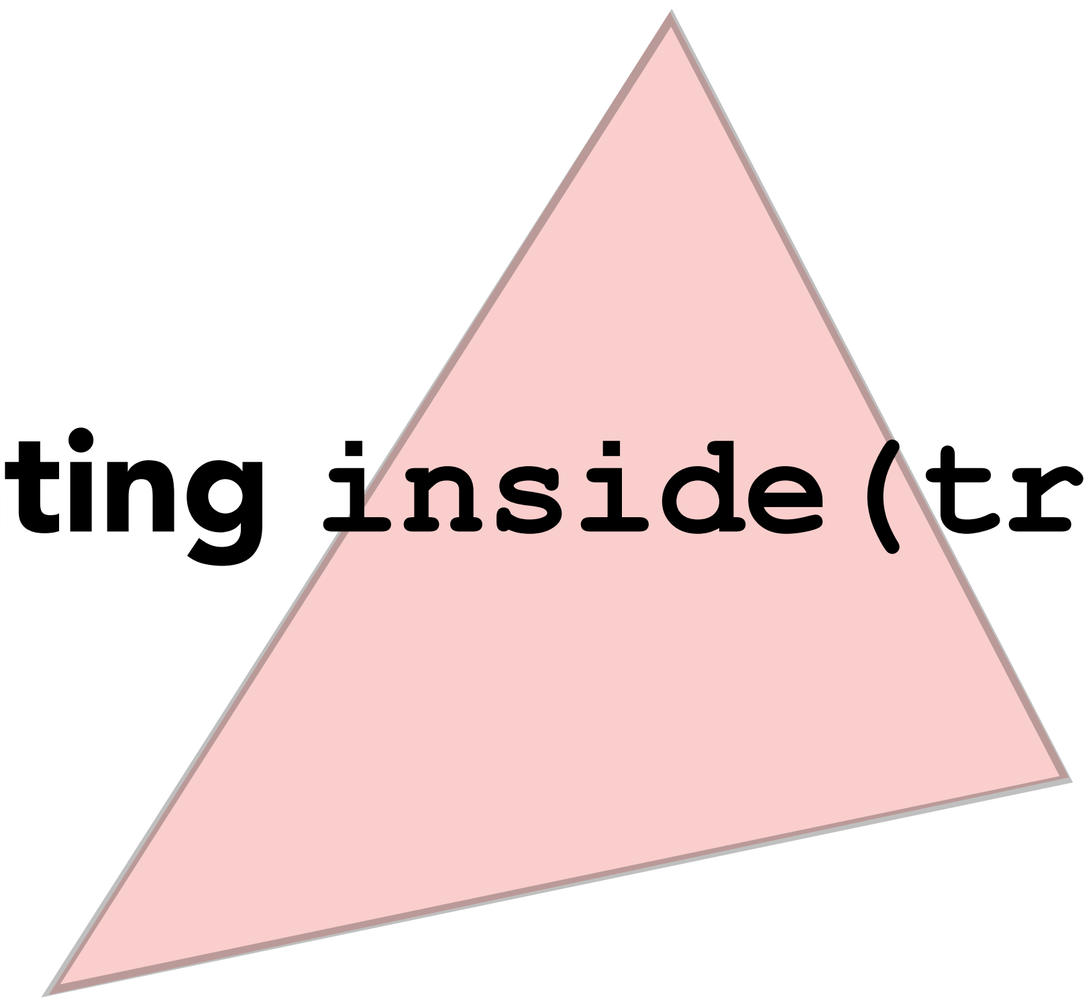
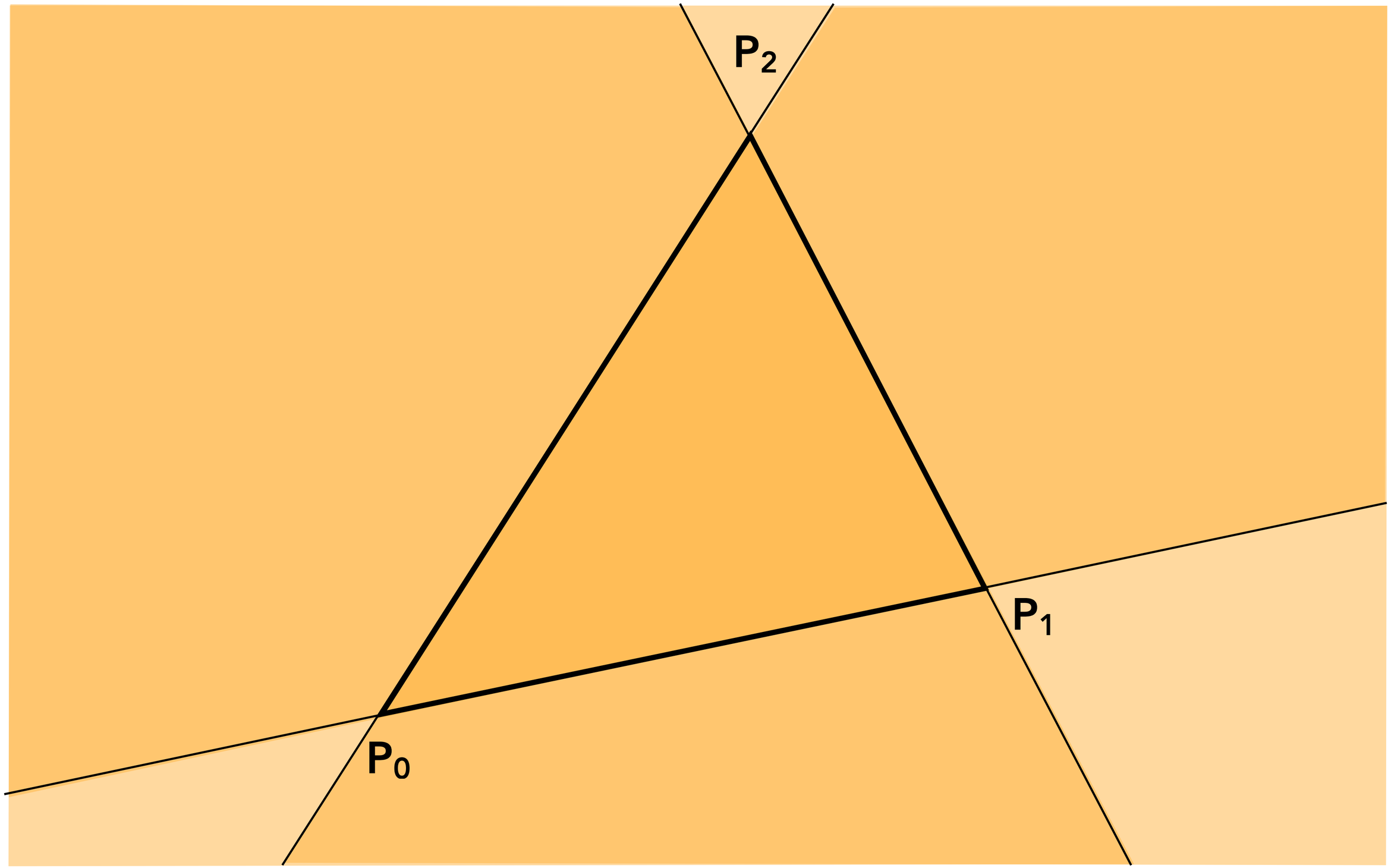
Evaluating `inside(tri,x,y)`



Triangle = Intersection of Three Half Planes



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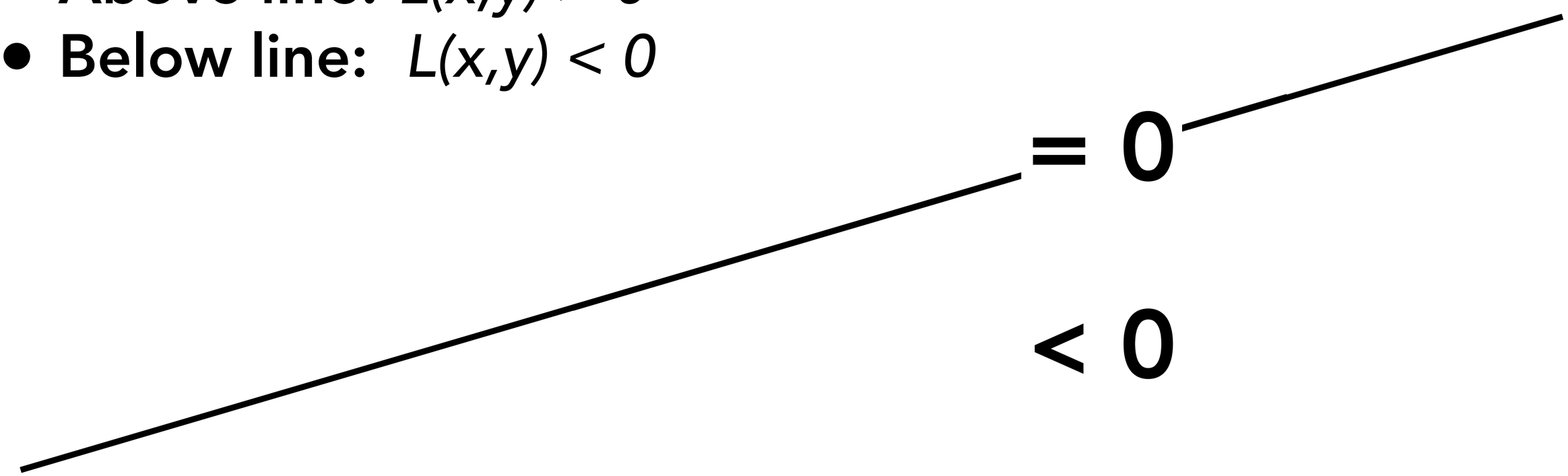
Each Line Defines Two Half-Planes

Implicit line equation

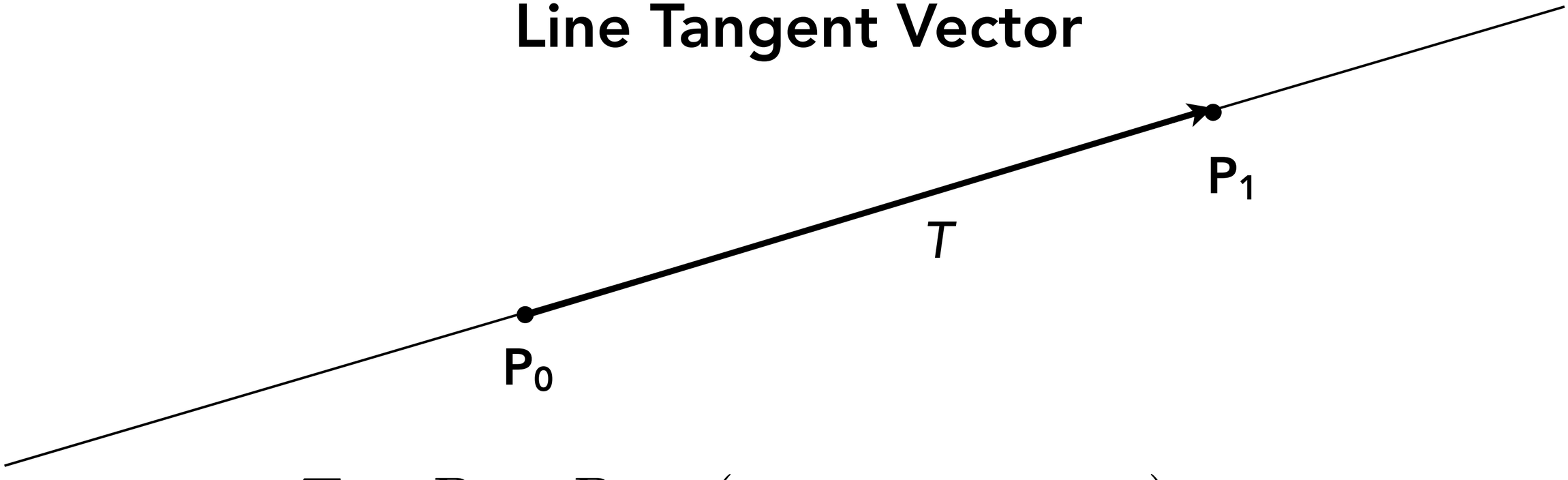
- $L(x,y) = Ax + By + C$

- On line: $L(x,y) = 0$
- Above line: $L(x,y) > 0$
- Below line: $L(x,y) < 0$



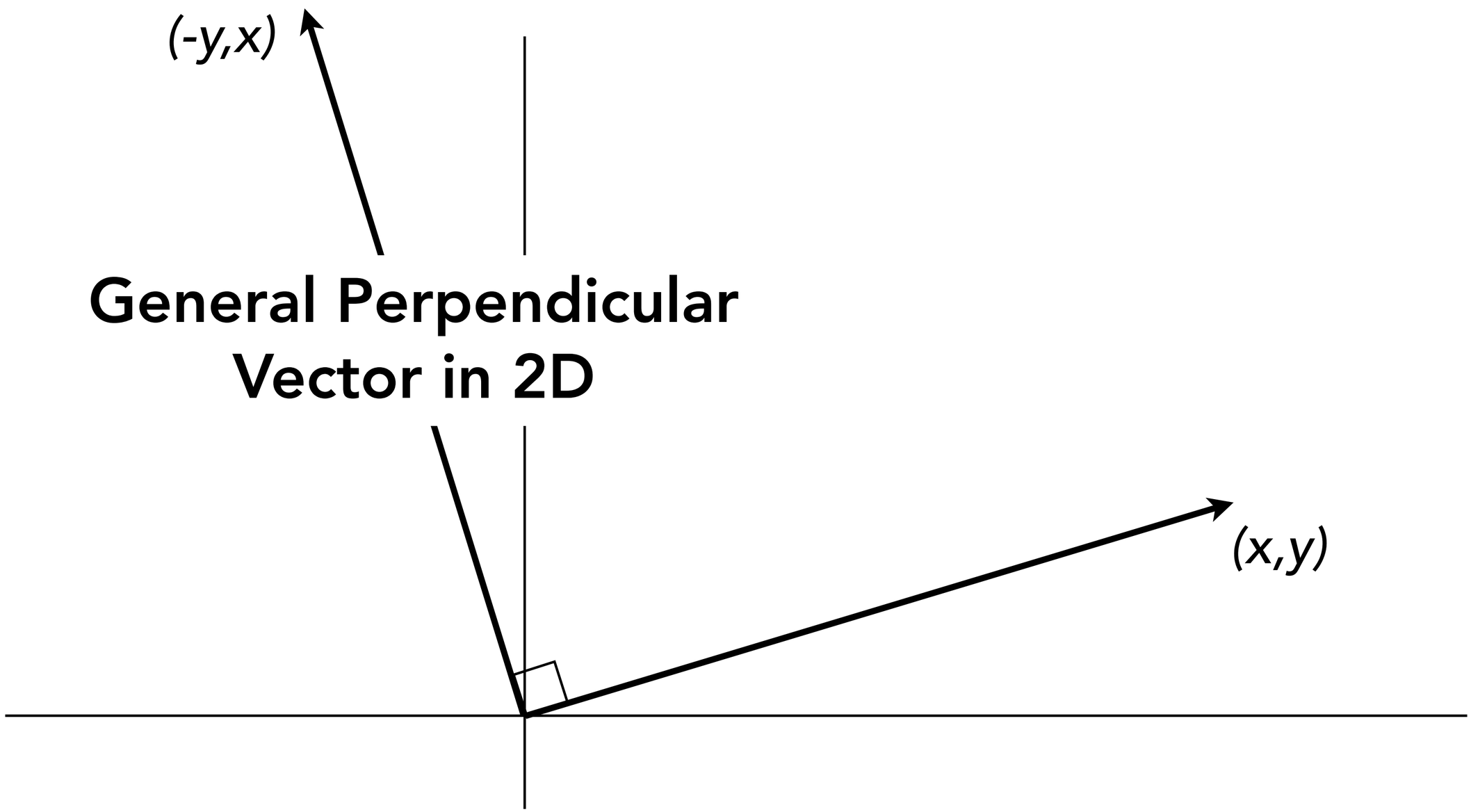
Line Equation Derivation

Line Tangent Vector



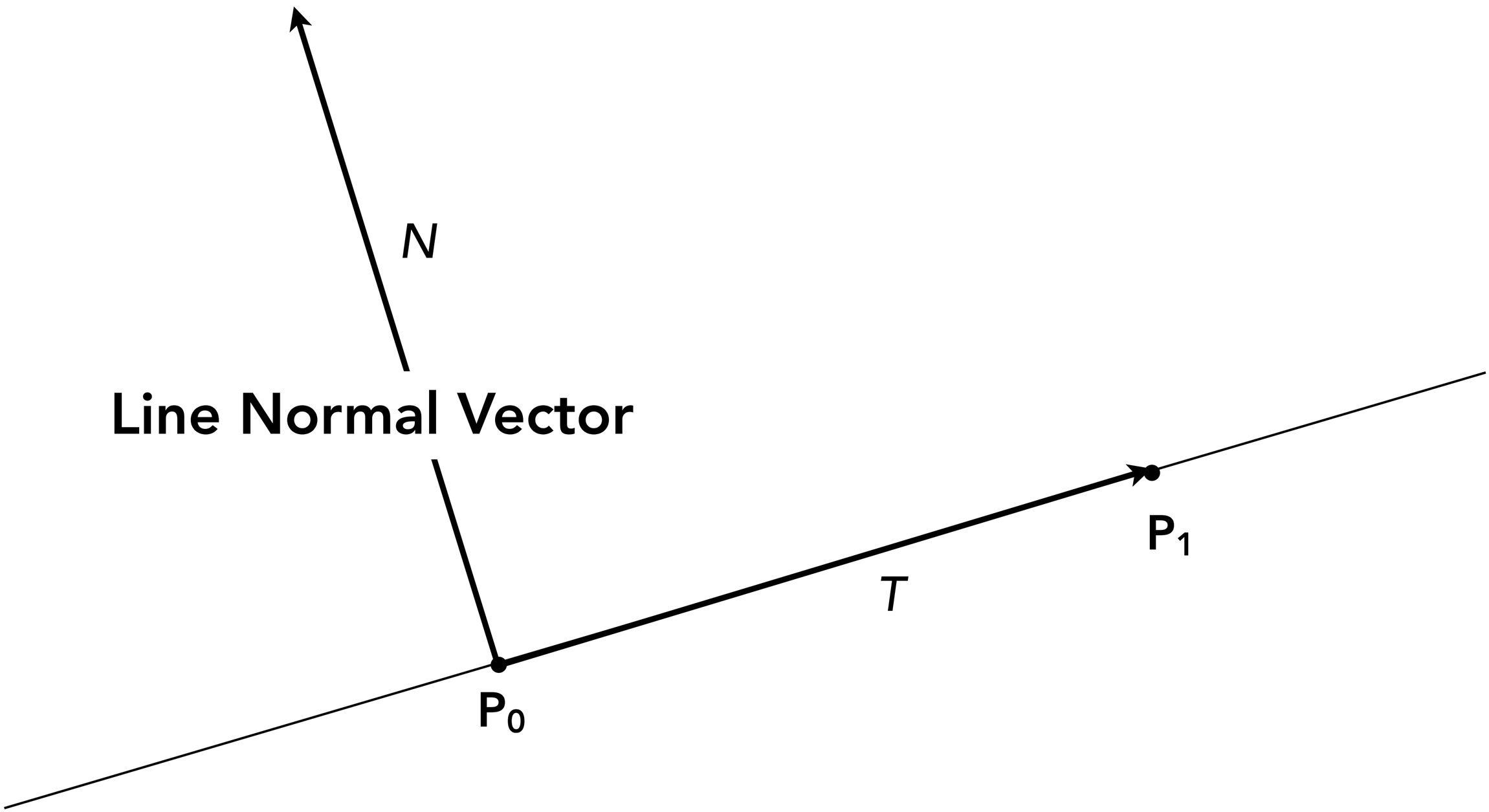
$$T = P_1 - P_0 = (x_1 - x_0, y_1 - y_0)$$

Line Equation Derivation



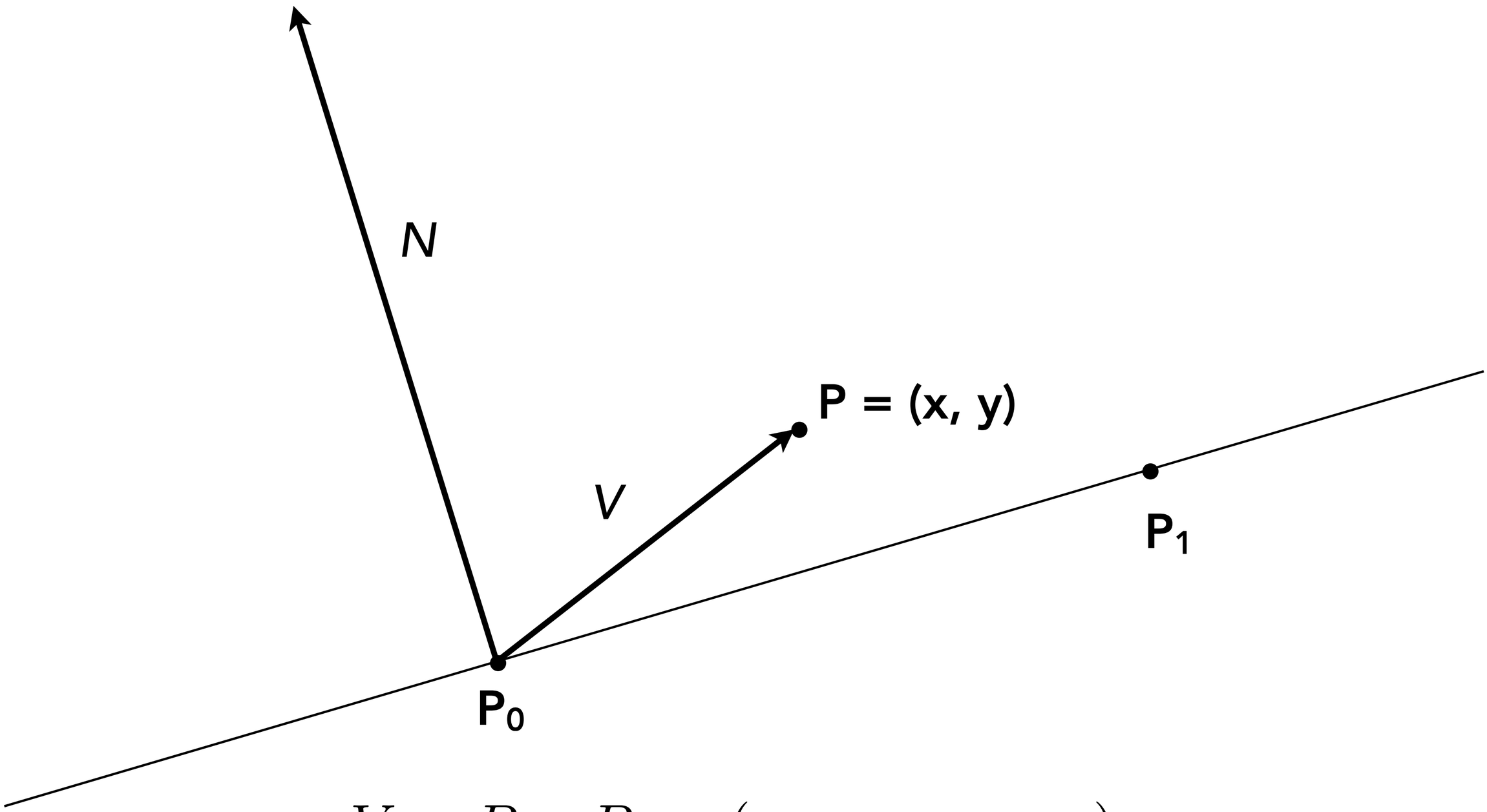
$$\mathrm{Perp}(x, y) = (-y, x)$$

Line Equation Derivation



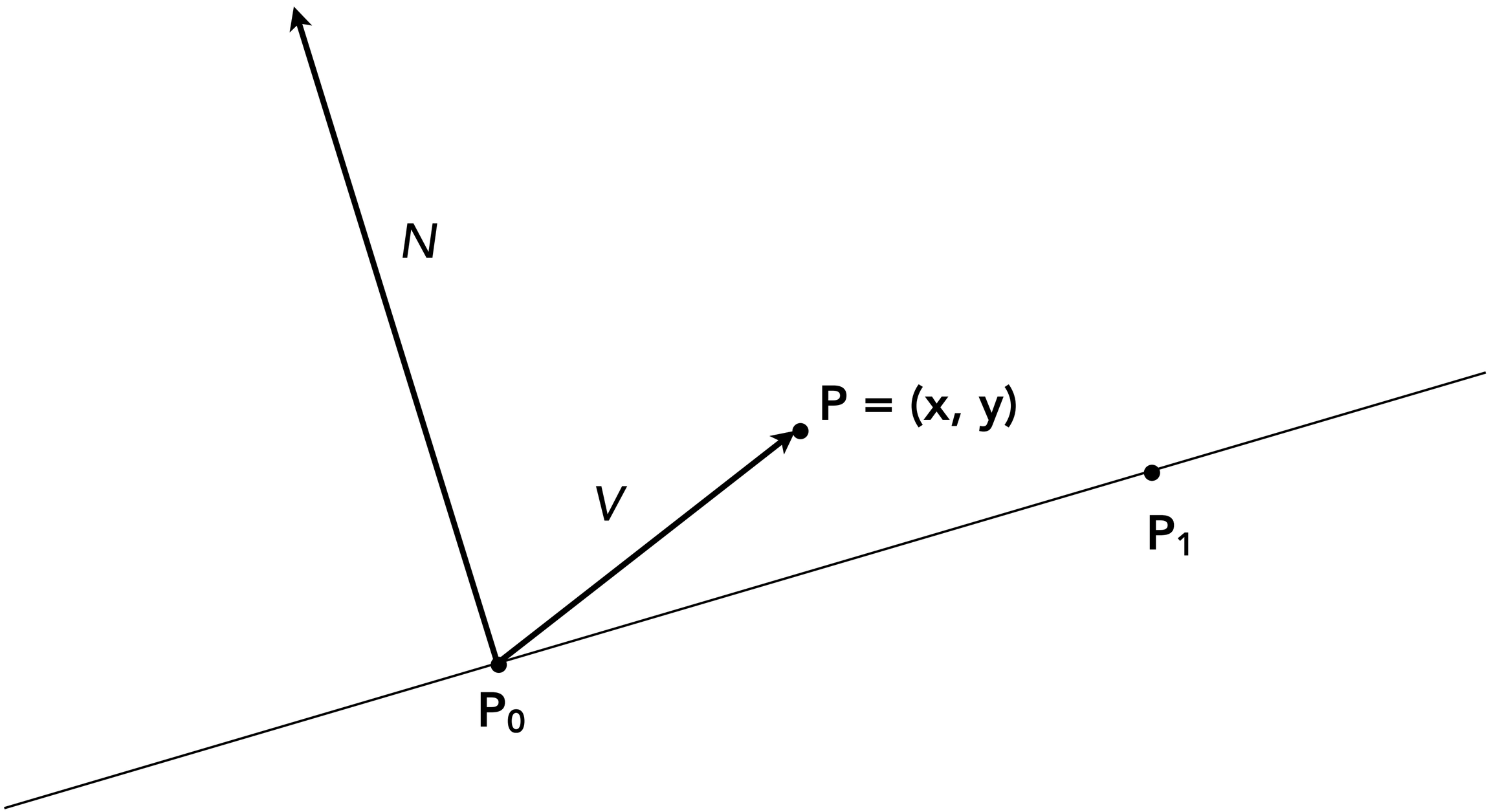
$$N = \mathrm{Perp}(T) = (-(y_1 - y_0), x_1 - x_0)$$

Line Equation Derivation



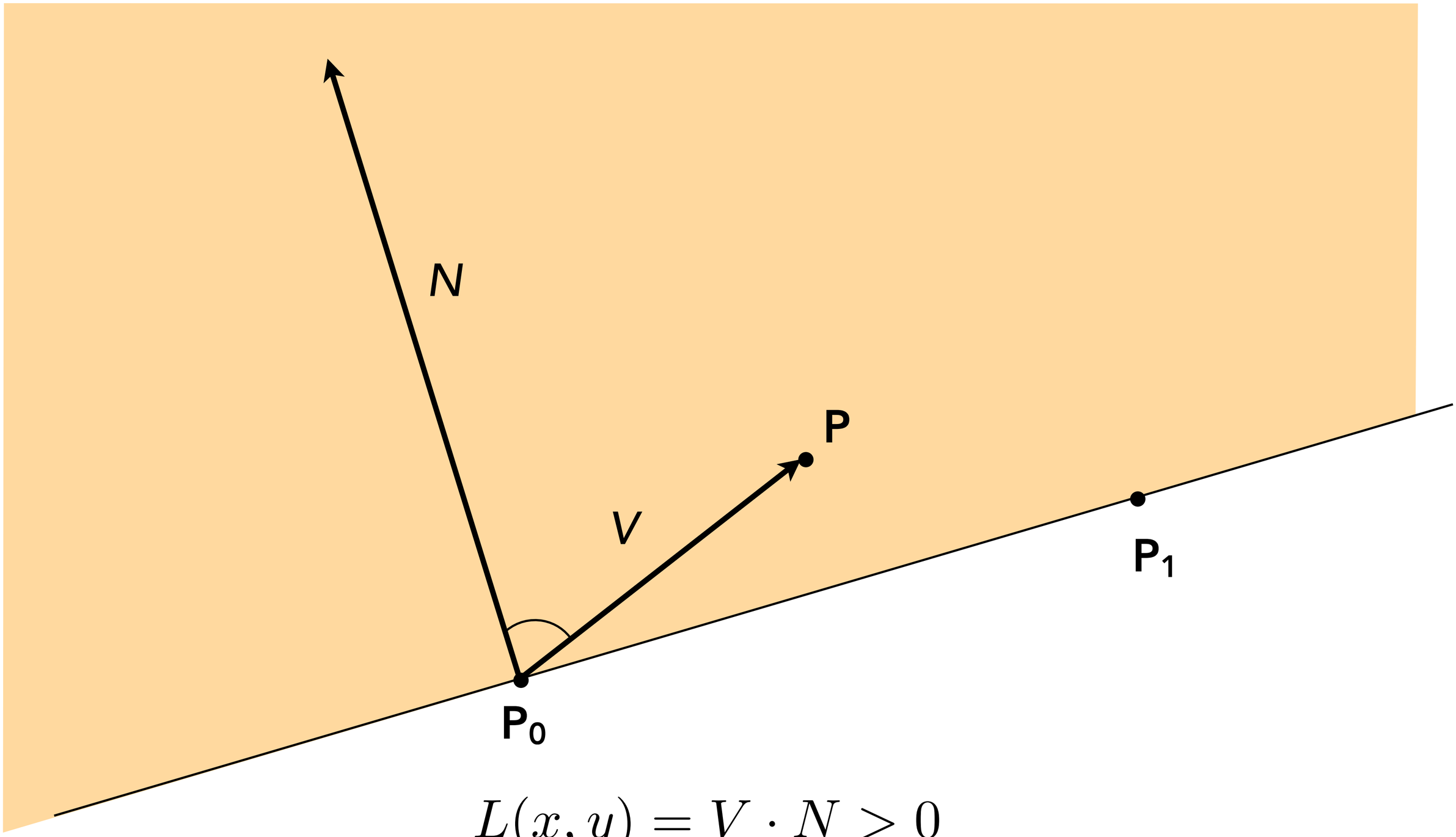
$$V = P - P_0 = (x - x_0, y - y_0)$$

Line Equation



$$L(x, y) = V \cdot N = -(x - x_0)(y_1 - y_0) + (y - y_0)(x_1 - x_0)$$

Line Equation Tests



$$L(x, y) = V \cdot N > 0$$

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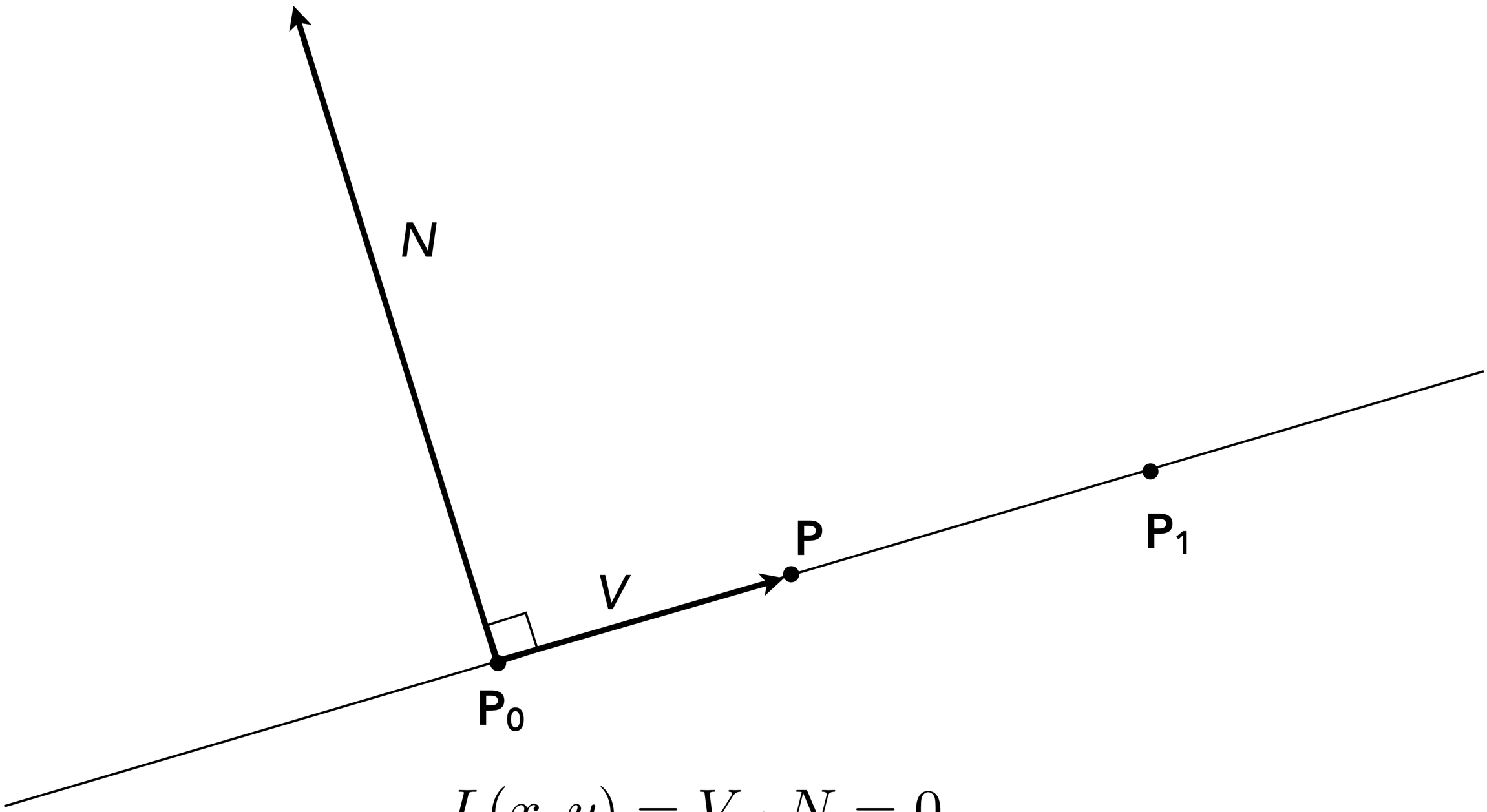
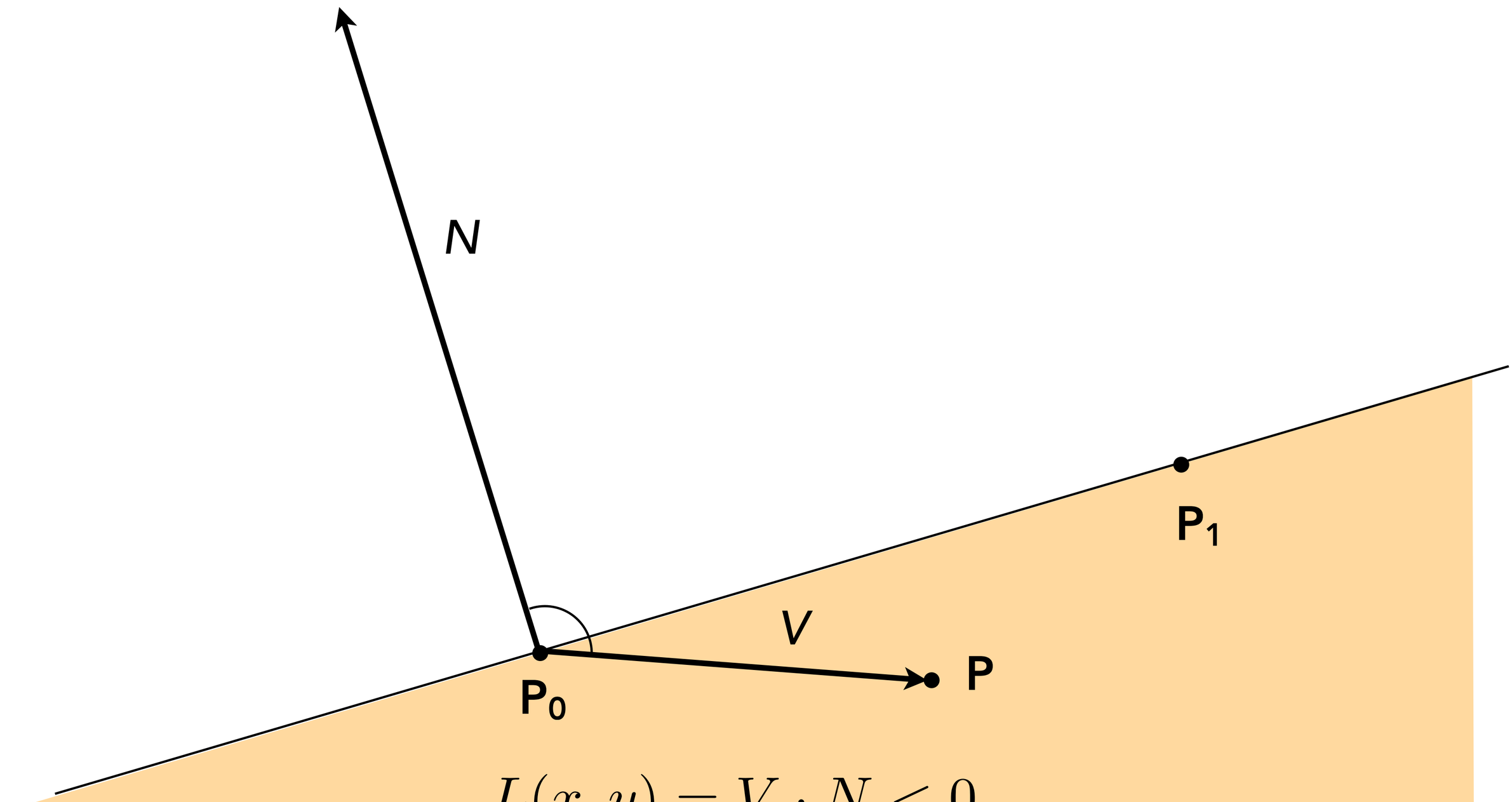
Line Equation Tests



$$L(x, y) = V \cdot N = 0$$

Line Equation Tests



$$L(x, y) = V \cdot N < 0$$

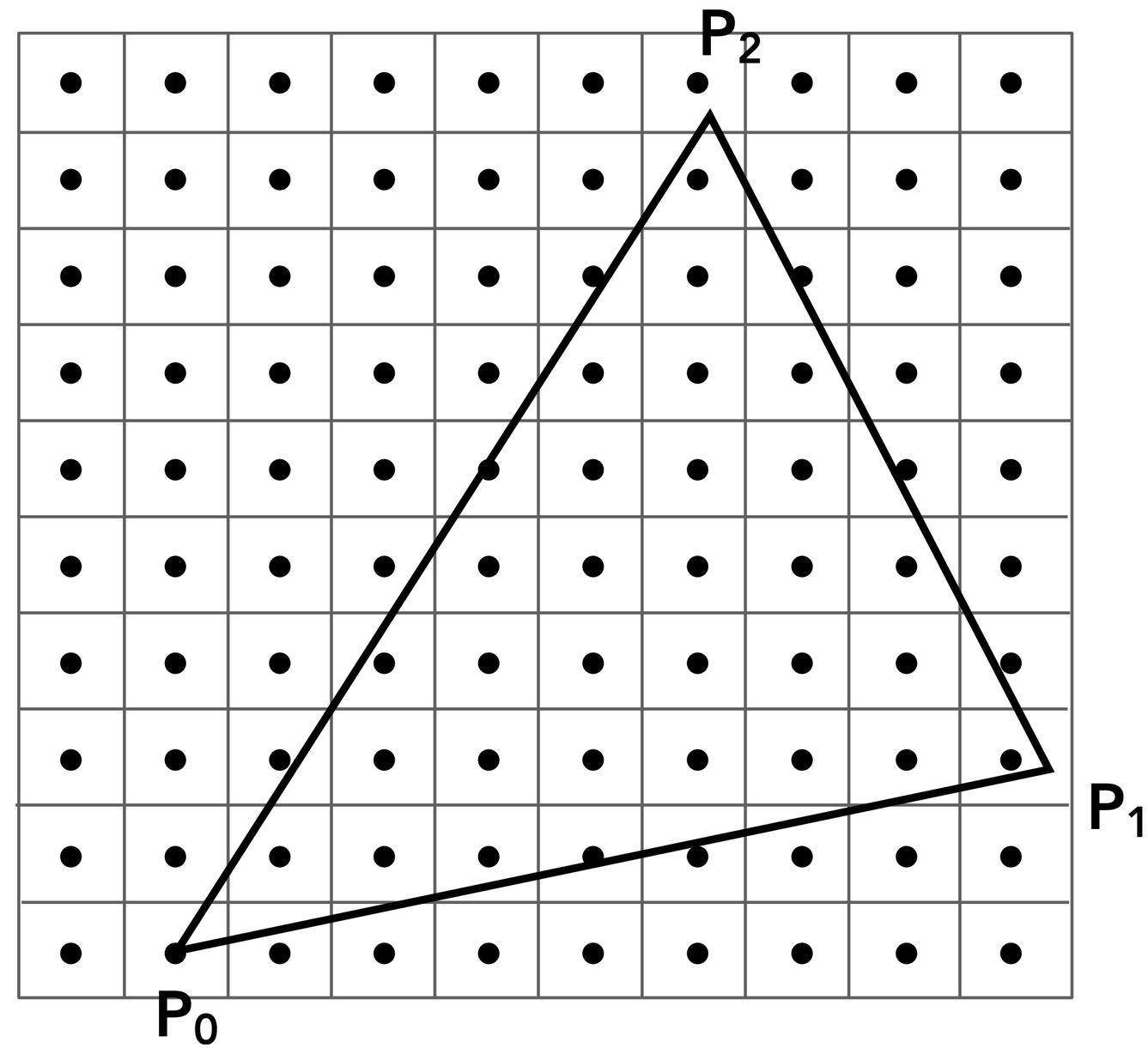
Point-in-Triangle Test: Three Line Tests

$P_i = (X_i, Y_i)$

$dX_i = X_{i+1} - X_i$
$dY_i = Y_{i+1} - Y_i$

$L_i(x, y) = -(x - X_i)\, dY_i + (y - Y_i)\, dX_i$
$\quad = A_i\, x + B_i\, y + C_i$

$L_i(x, y) = 0$: point on edge
< 0 : outside edge
> 0 : inside edge



Compute line equations from pairs of vertices

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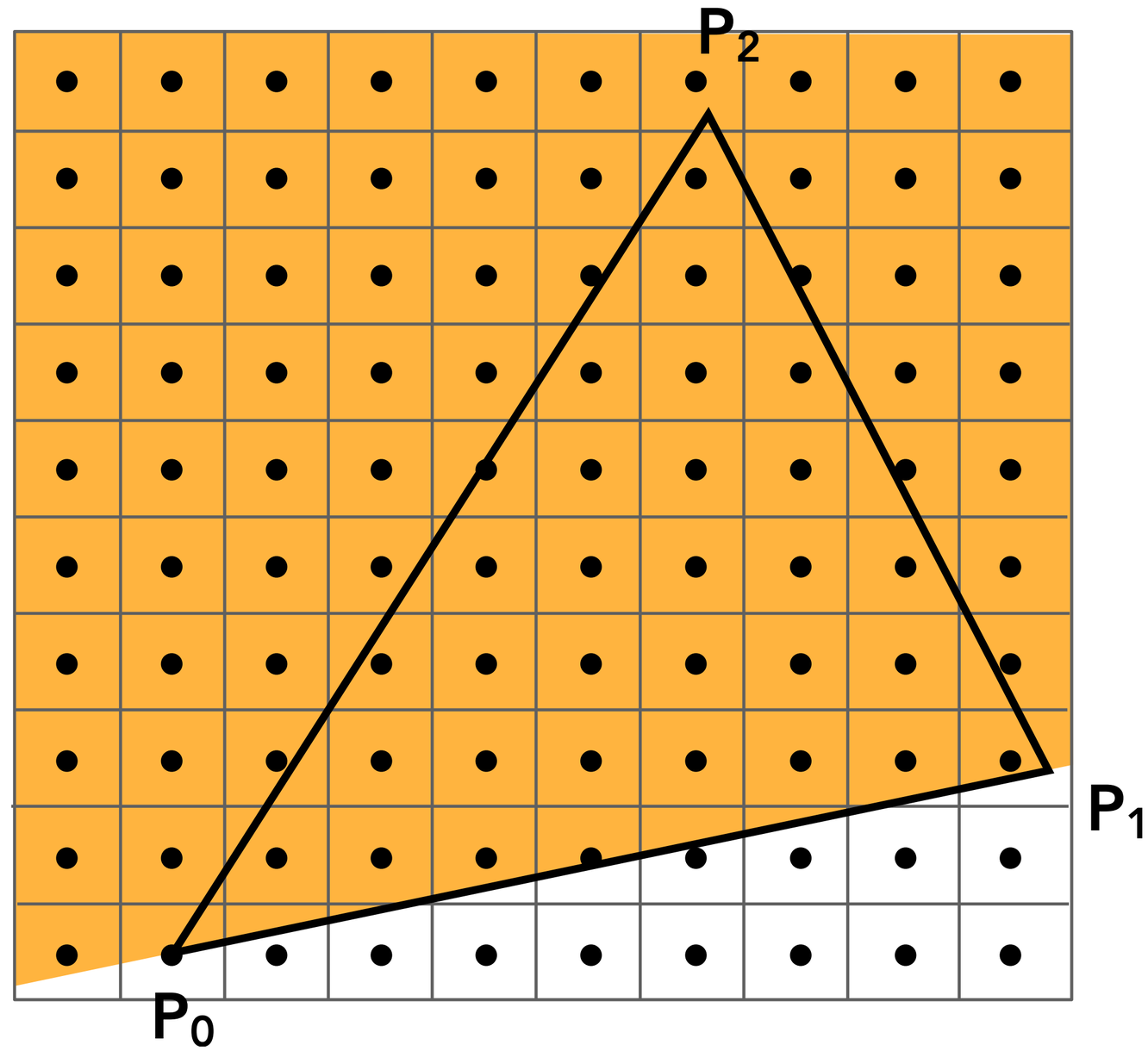
Point-in-Triangle Test: Three Line Tests

$P_i = (X_i, Y_i)$

$dX_i = X_{i+1} - X_i$
$dY_i = Y_{i+1} - Y_i$

$L_i(x, y) = -(x - X_i)\, dY_i + (y - Y_i)\, dX_i$
$= A_i\, x + B_i\, y + C_i$

$L_i(x, y) = 0$: point on edge
< 0 : outside edge
> 0 : inside edge



$L_0(x, y) > 0$

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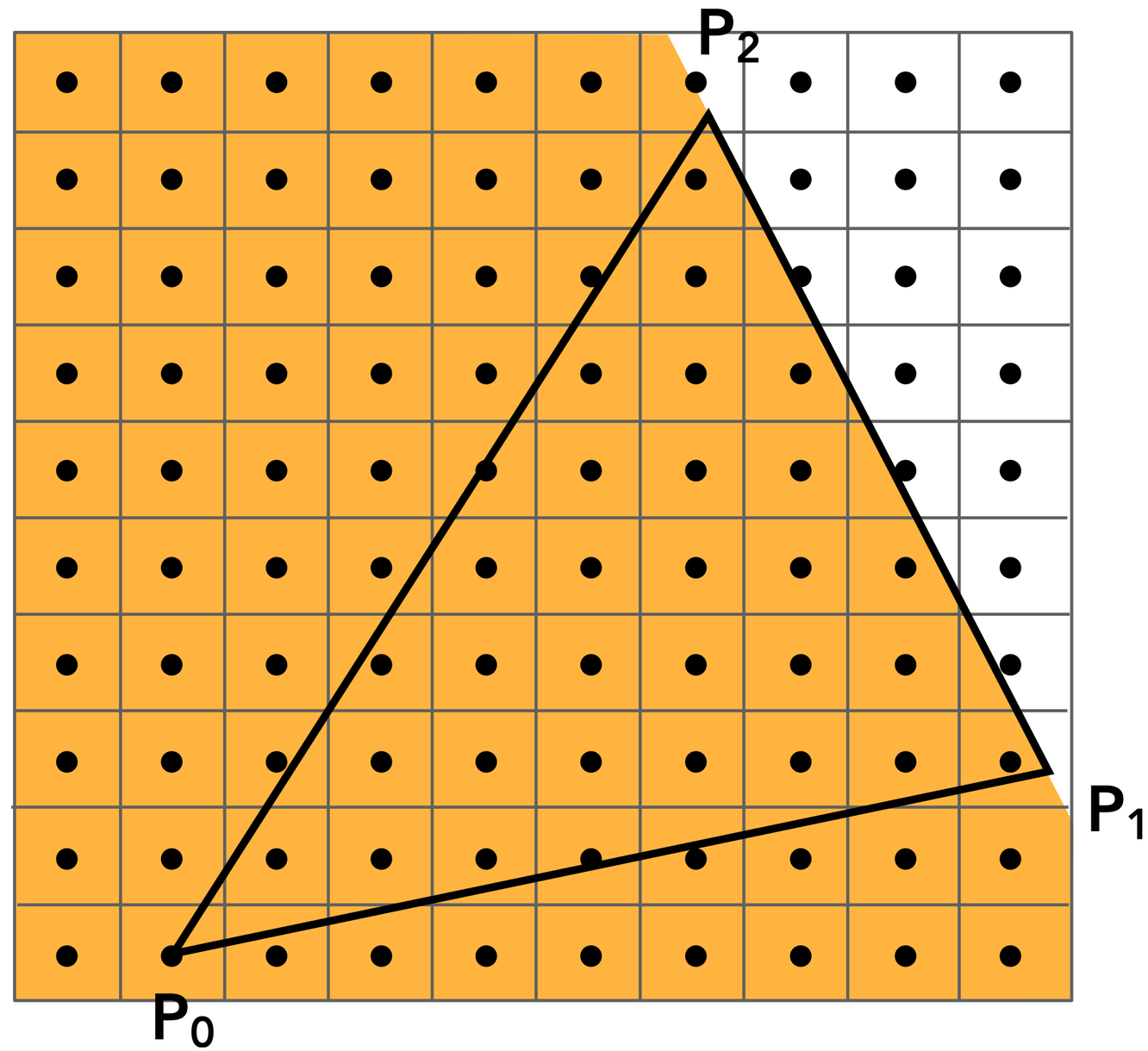
Point-in-Triangle Test: Three Line Tests

$P_i = (X_i, Y_i)$

$dX_i = X_{i+1} - X_i$
$dY_i = Y_{i+1} - Y_i$

$$L_i(x, y) = -(x - X_i)\, dY_i + (y - Y_i)\, dX_i$$
$$= A_i\, x + B_i\, y + C_i$$

$L_i(x, y) = 0$: point on edge
< 0 : outside edge
> 0 : inside edge



$L_1(x, y) > 0$

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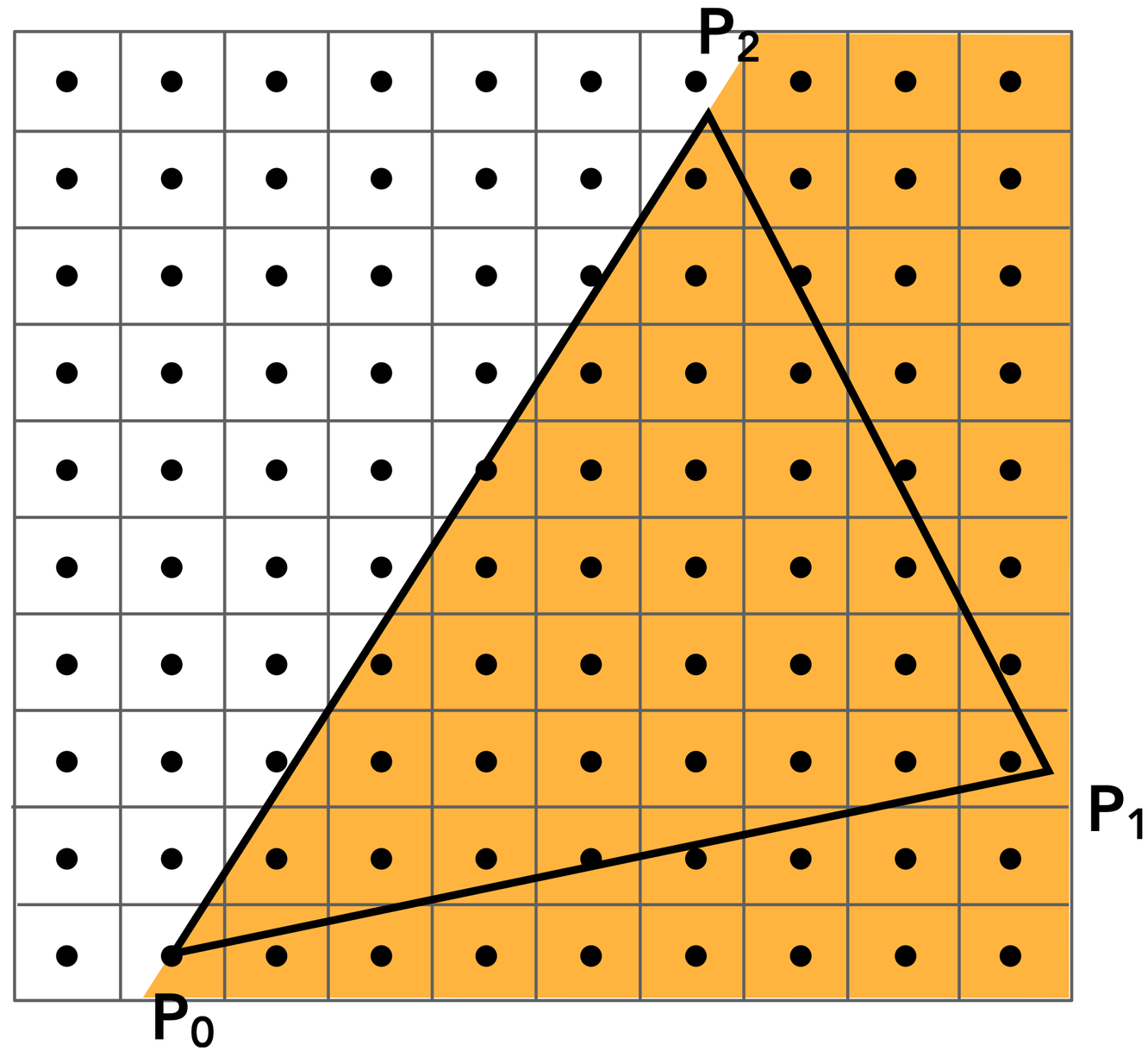
Point-in-Triangle Test: Three Line Tests

$P_i = (X_i, Y_i)$

$dX_i = X_{i+1} - X_i$
$dY_i = Y_{i+1} - Y_i$

$L_i(x, y) = -(x - X_i)\, dY_i + (y - Y_i)\, dX_i$
$= A_i\, x + B_i\, y + C_i$

$L_i(x, y) = 0$: point on edge
< 0 : outside edge
> 0 : inside edge



$L_2(x, y) > 0$

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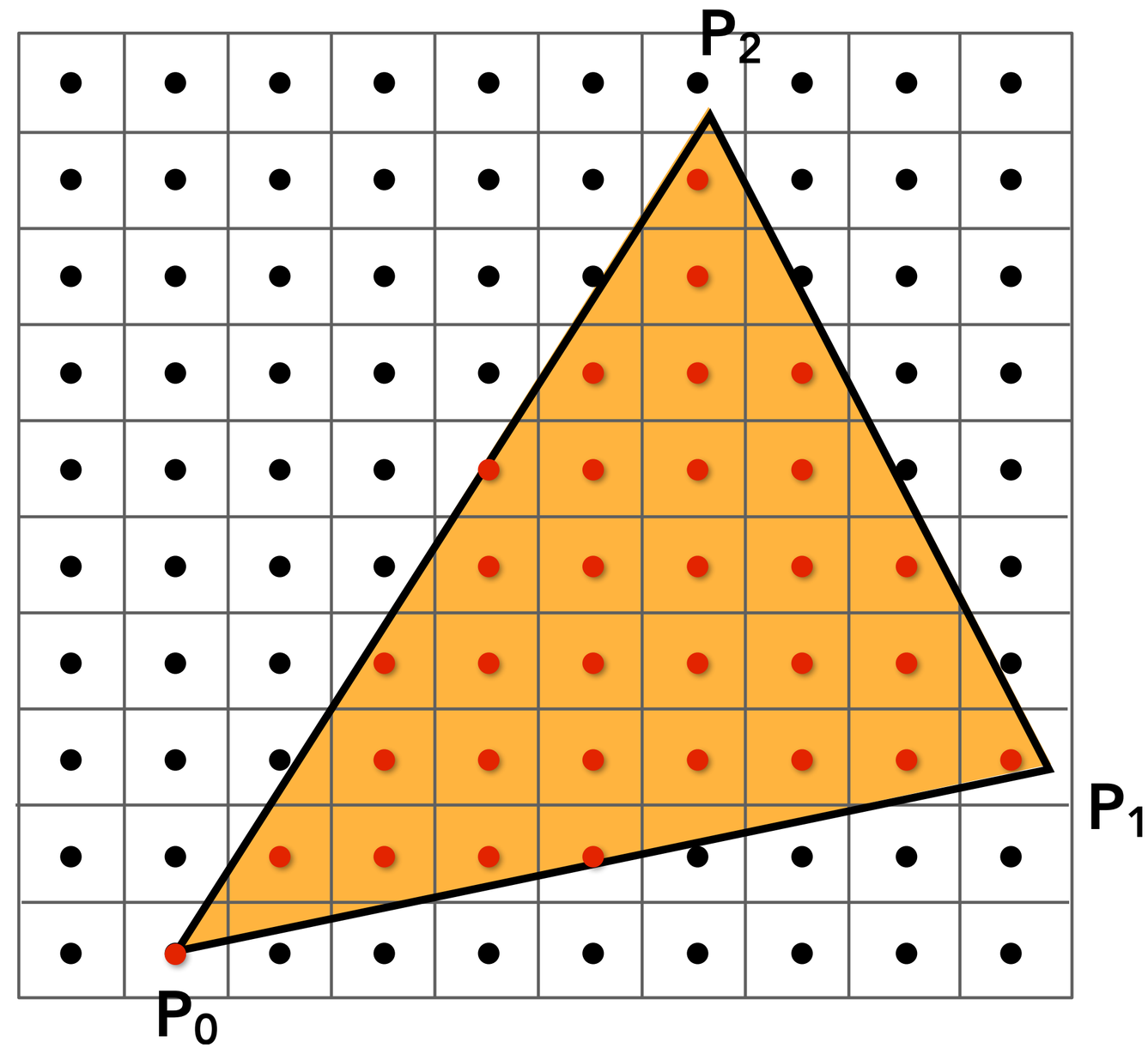
Point-in-Triangle Test: Three Line Tests

Sample point $s = (sx, sy)$ is inside the triangle if it is inside all three lines.

$$
\begin{aligned}
&inside(sx, sy) = \\
&\quad L_0(sx, sy) > 0 \;\&\& \\
&\quad L_1(sx, sy) > 0 \;\&\& \\
&\quad L_2(sx, sy) > 0;
\end{aligned}
$$

Note: actual implementation of $inside(sx,sy)$ involves ≤ checks based on edge rules



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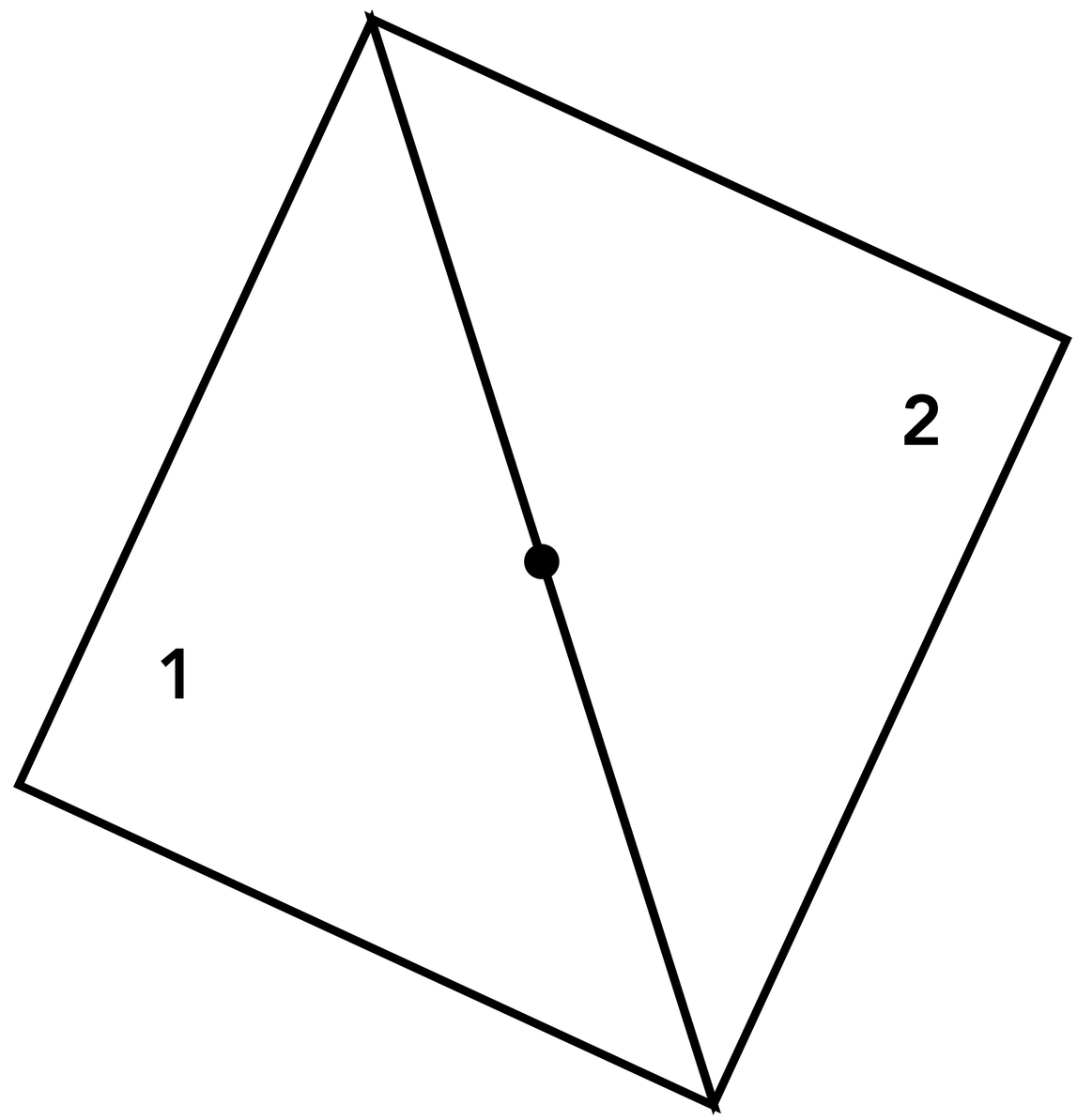
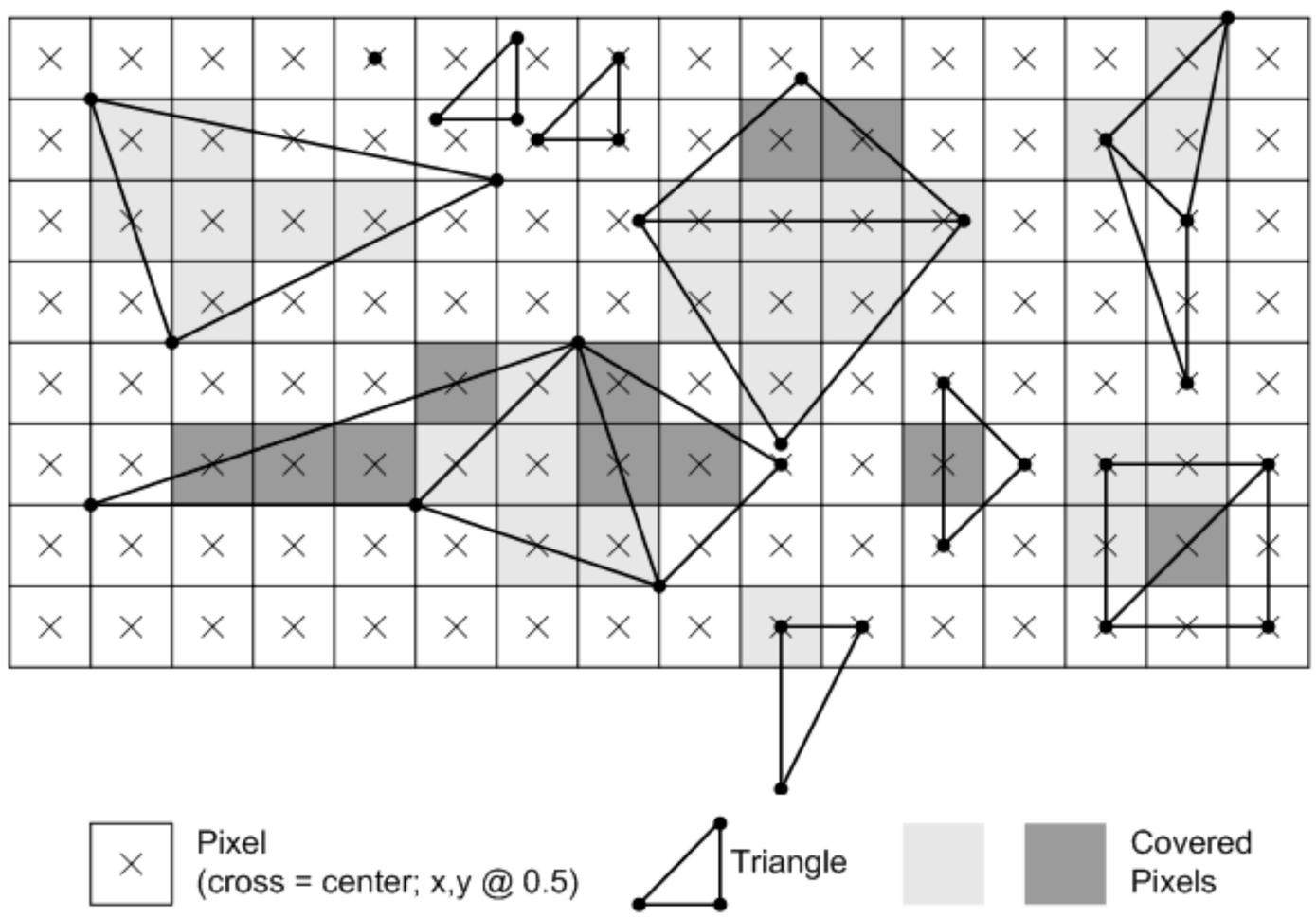
Edge Cases (Literally)

Is this sample point covered by triangle 1, triangle 2, or both?



OpenGL/Direct3D Edge Rules

When sample point falls on an edge, the sample is classified as within triangle if the edge is a "top edge" or "left edge"

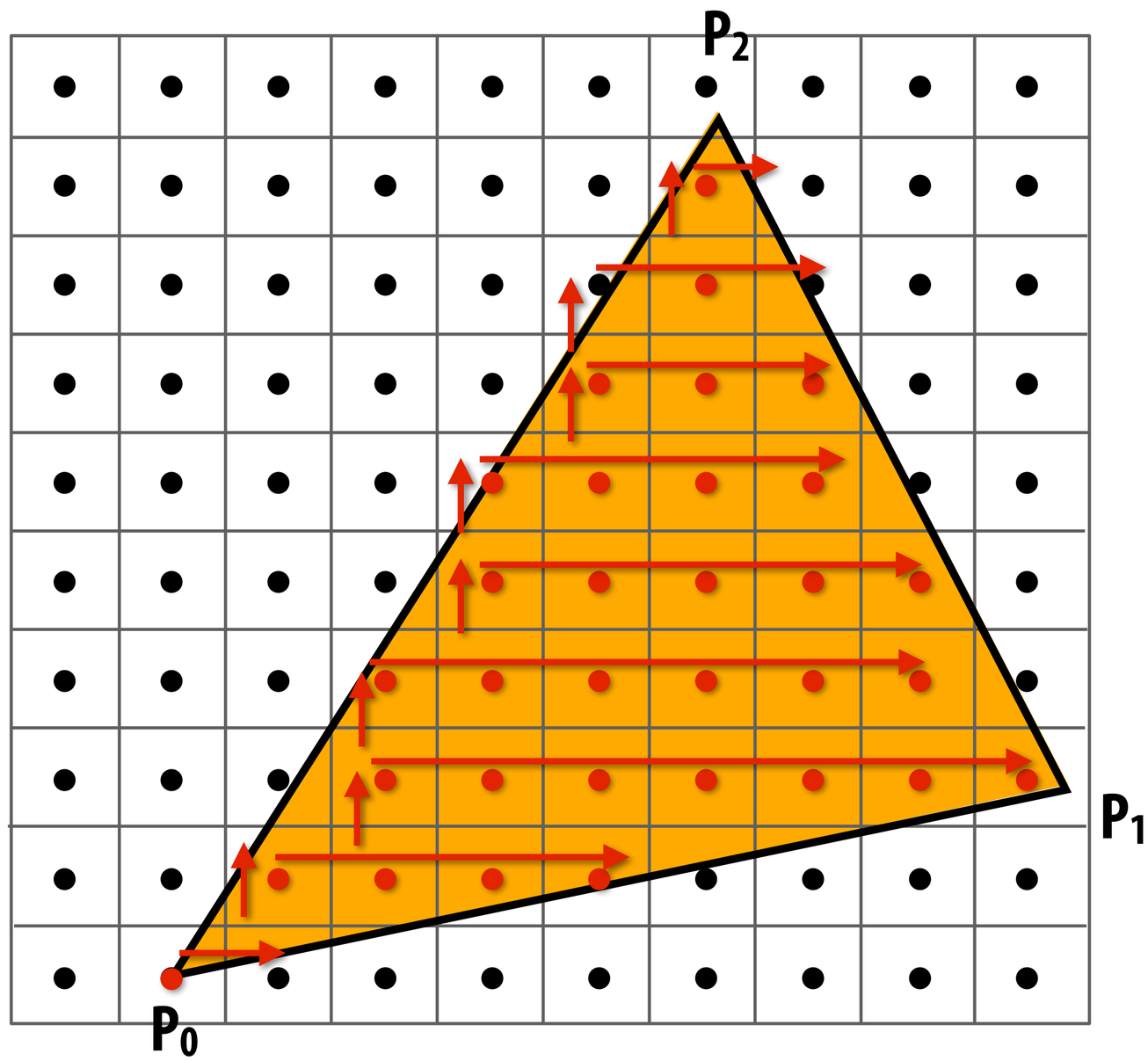


Source: Direct3D Programming Guide, Microsoft

Top edge: horizontal edge that is above all other edges

Left edge: an edge that is not exactly horizontal and is on the left side of the triangle. (triangle can have one or two left edges)

Incremental Triangle Traversal (Faster?)



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Modern Approach: Tiled Triangle Traversal

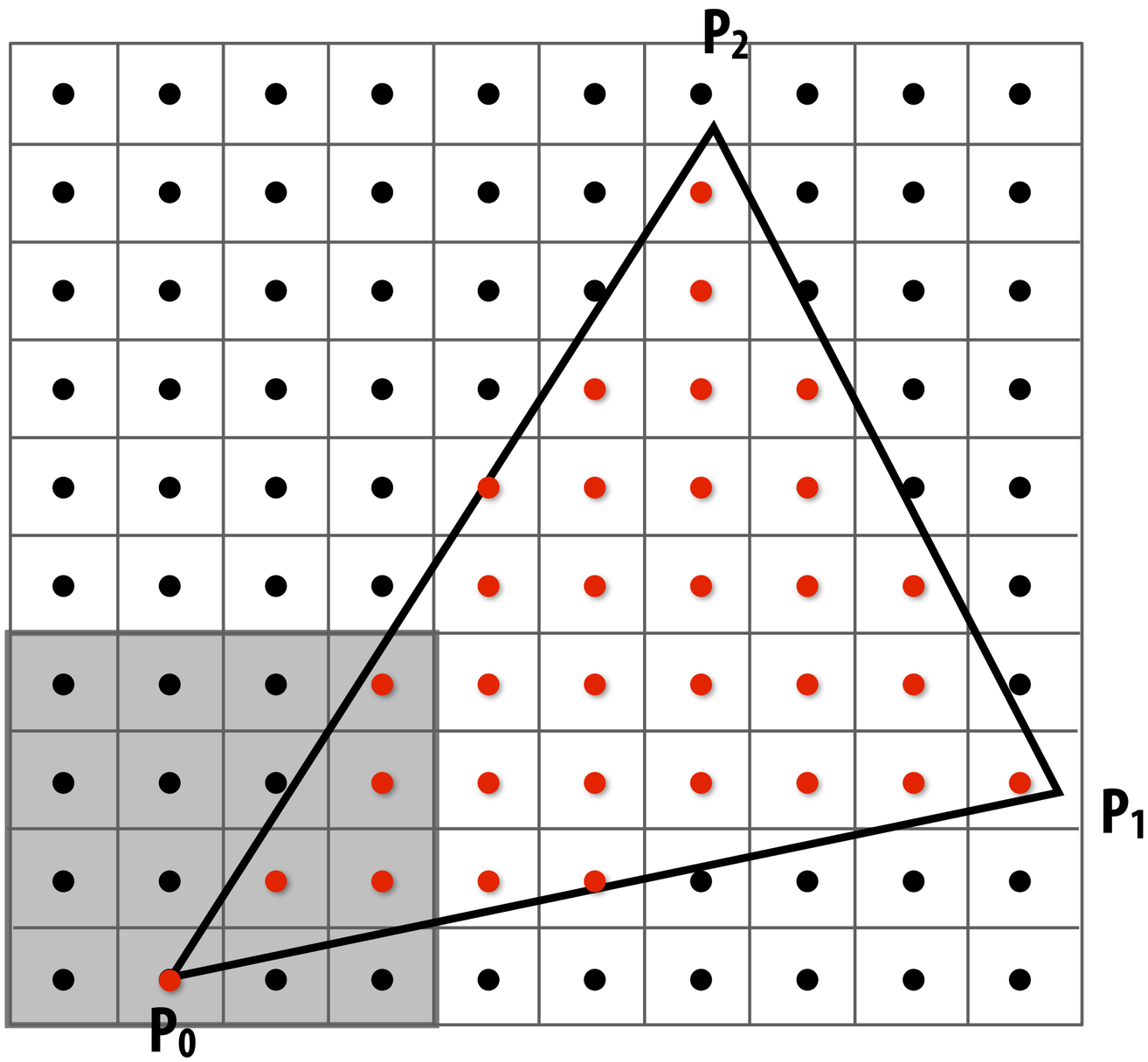
Traverse triangle in blocks

Test all samples in block in parallel

Advantages:

- Simplicity of wide parallel execution overcomes cost of extra point-in-triangle tests (most triangles cover many samples, especially when super-sampling)

- Can skip sample testing work: entire block not in triangle ("early out"), entire block entirely within triangle ("early in")

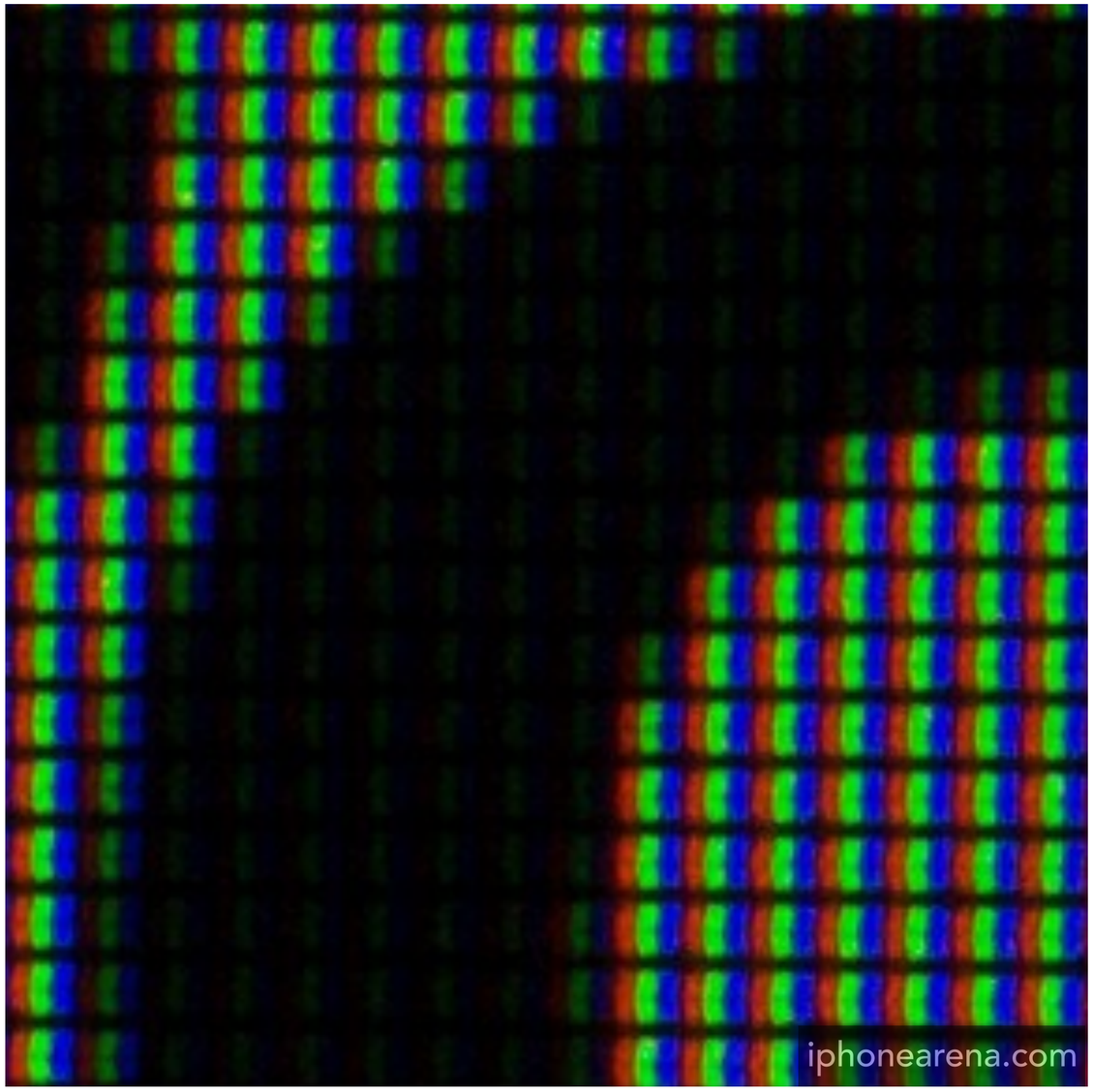


All modern GPUs have special-purpose hardware for efficient point-in-triangle tests

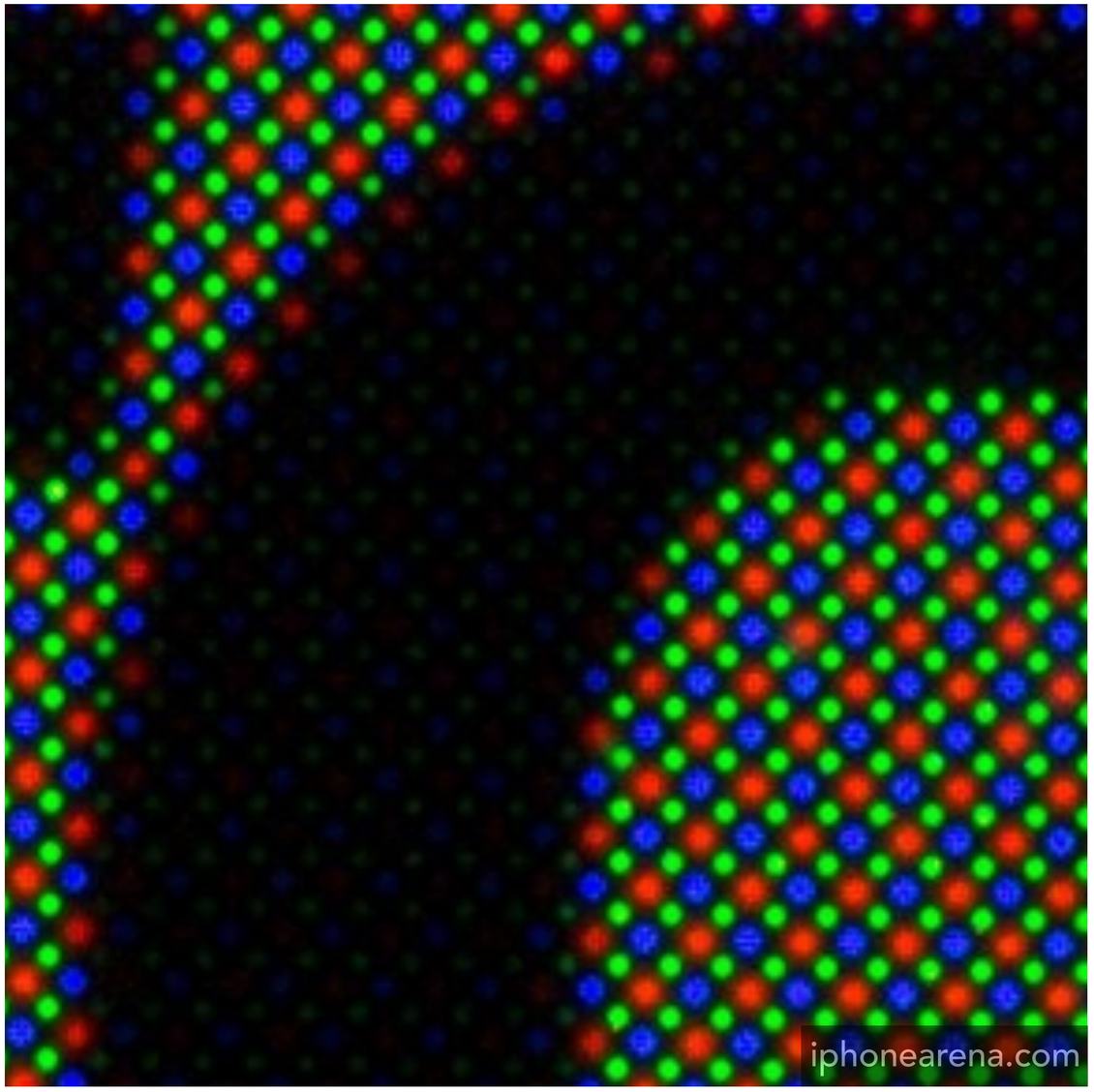
3 minute break

Signal Reconstruction on Real Displays

Real LCD/OLED Screen Pixels (Closeup)



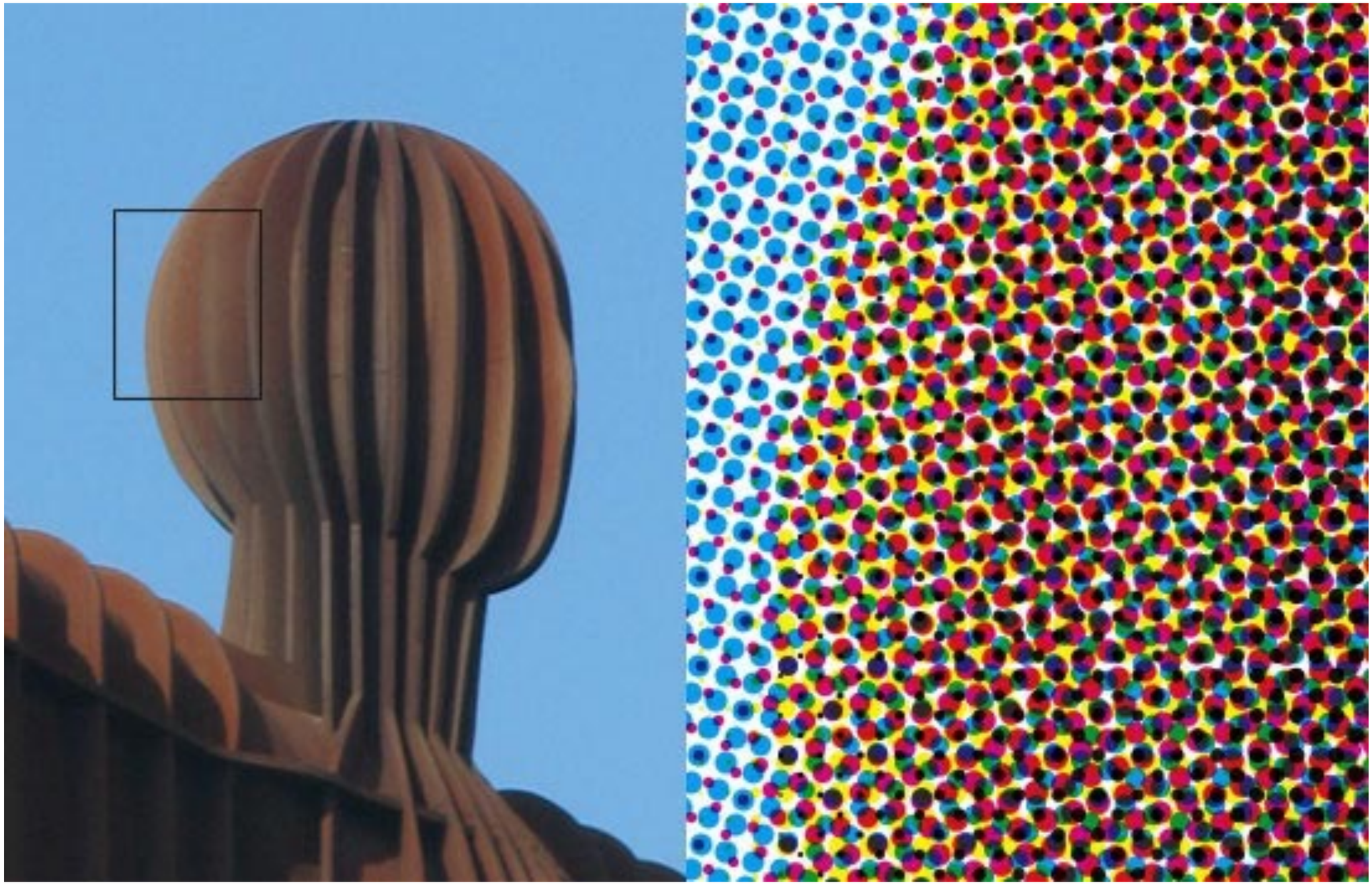
iPhone 6S



Galaxy S5

Notice R,G,B pixel geometry! But in this class, we will assume a colored square full-color pixel.

Aside: What About Other Display Methods?

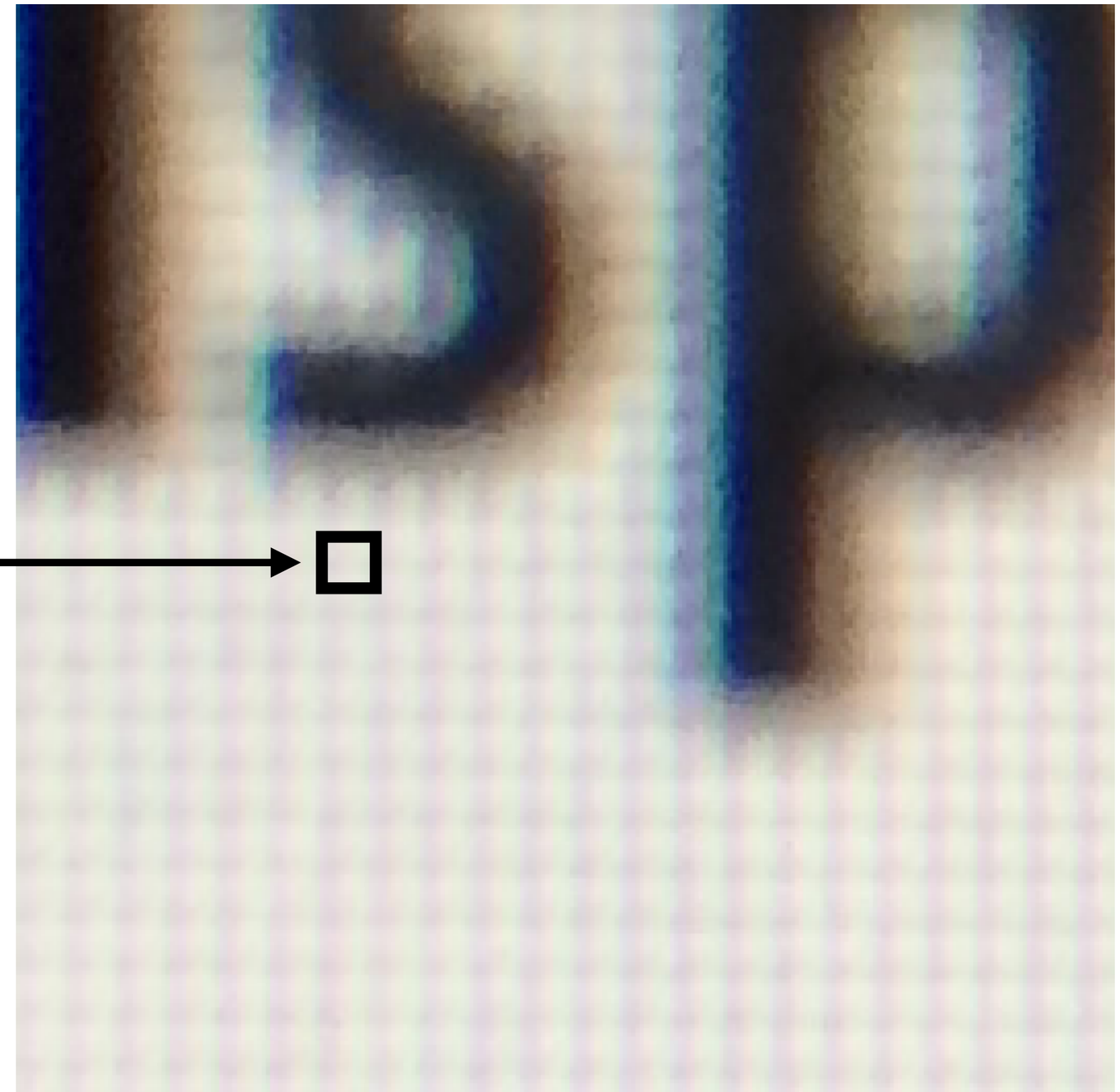


Color print: observe half-tone pattern

Assume Display Pixels Emit Square of Light

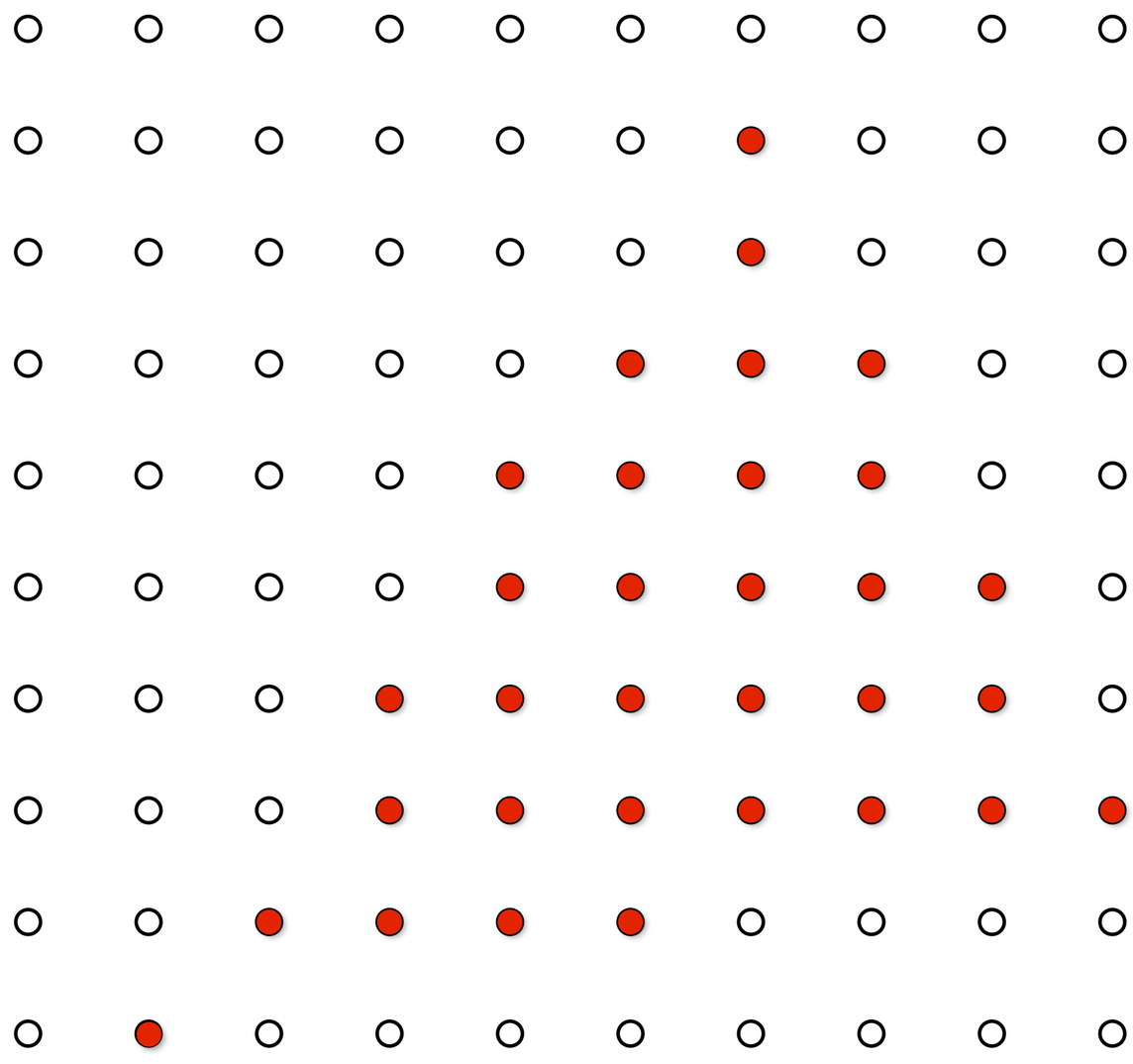
Each image sample sent to the display is converted into a little square of light of the appropriate color: (a pixel = picture element)

LCD pixel on laptop

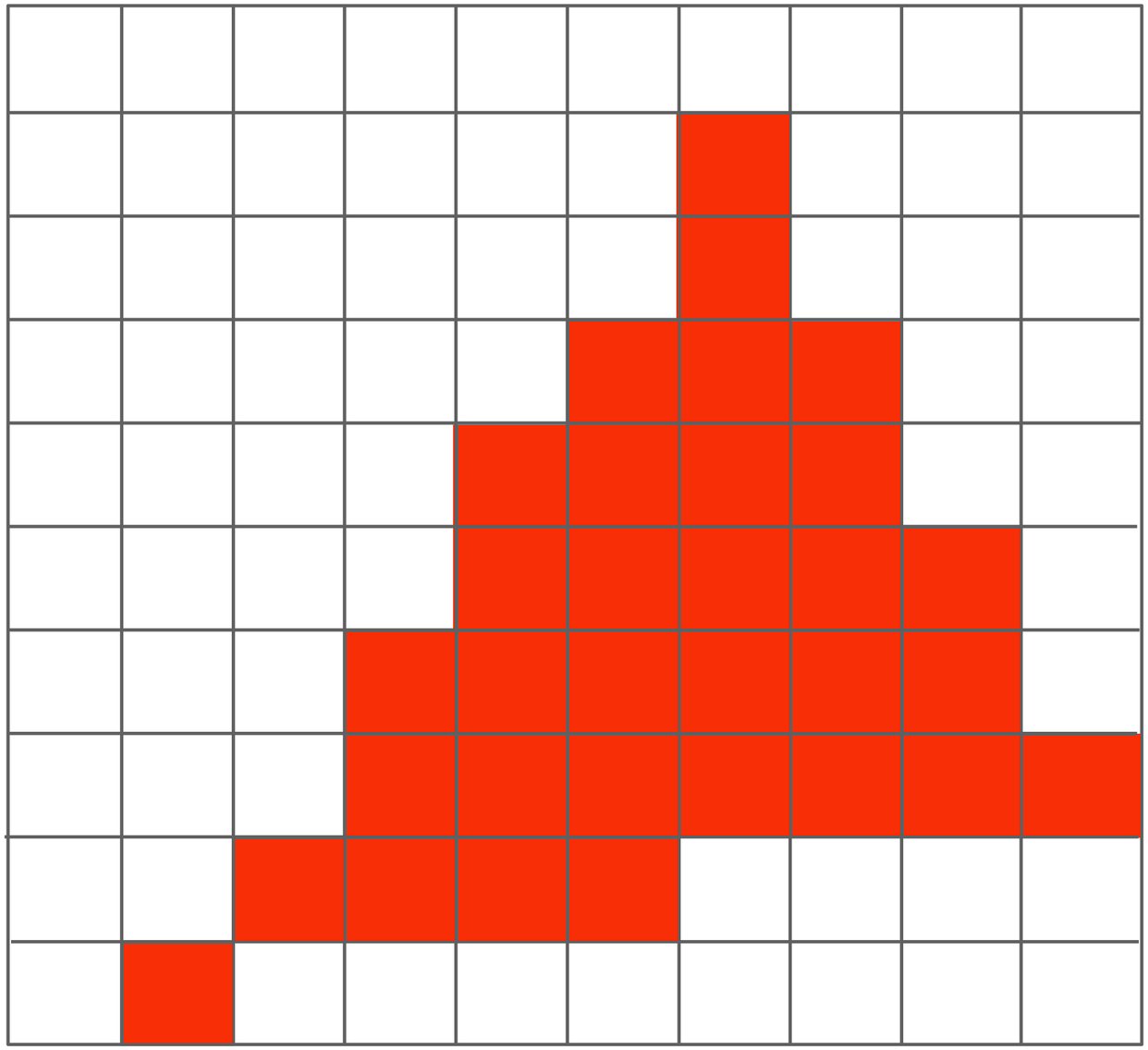


* LCD pixels do not actually emit light in a square of uniform color, but this approximation suffices for our current discussion

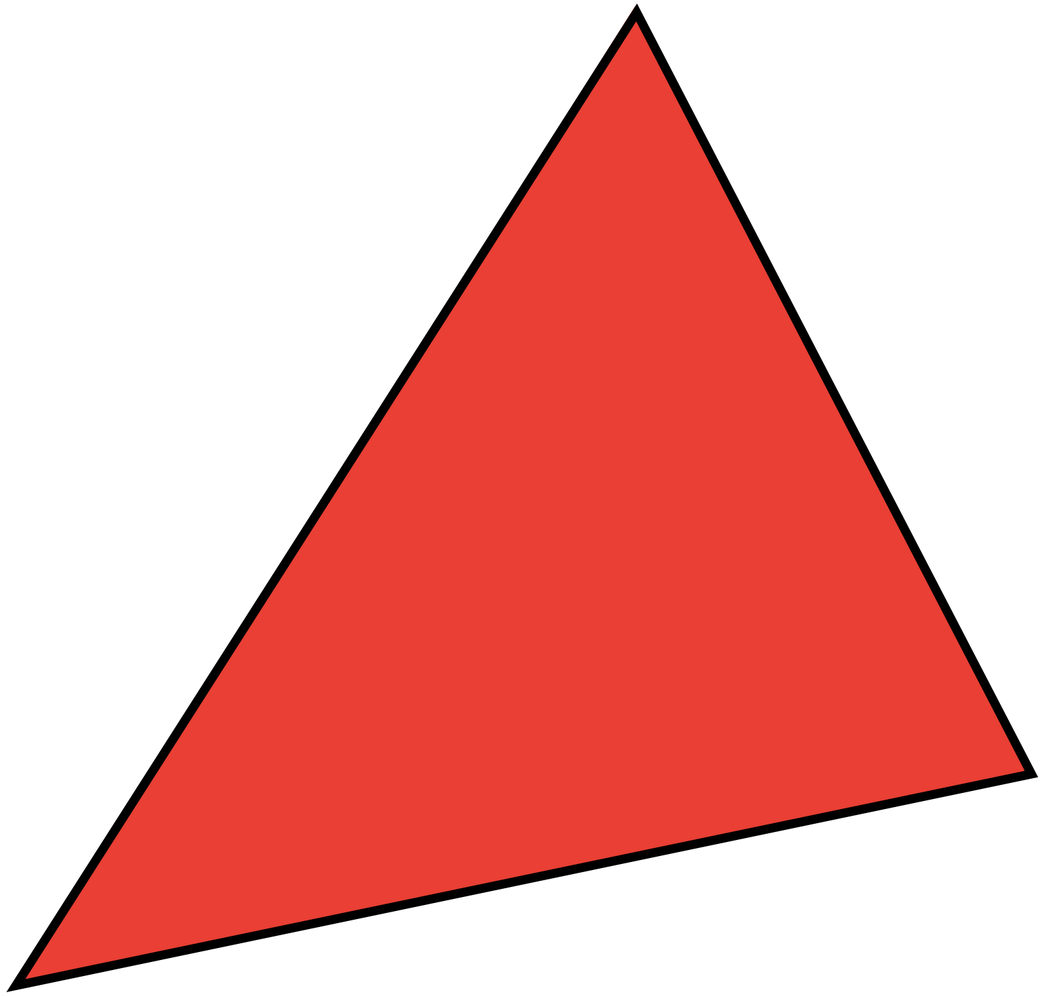
So, If We Send The Display This Sampled Signal



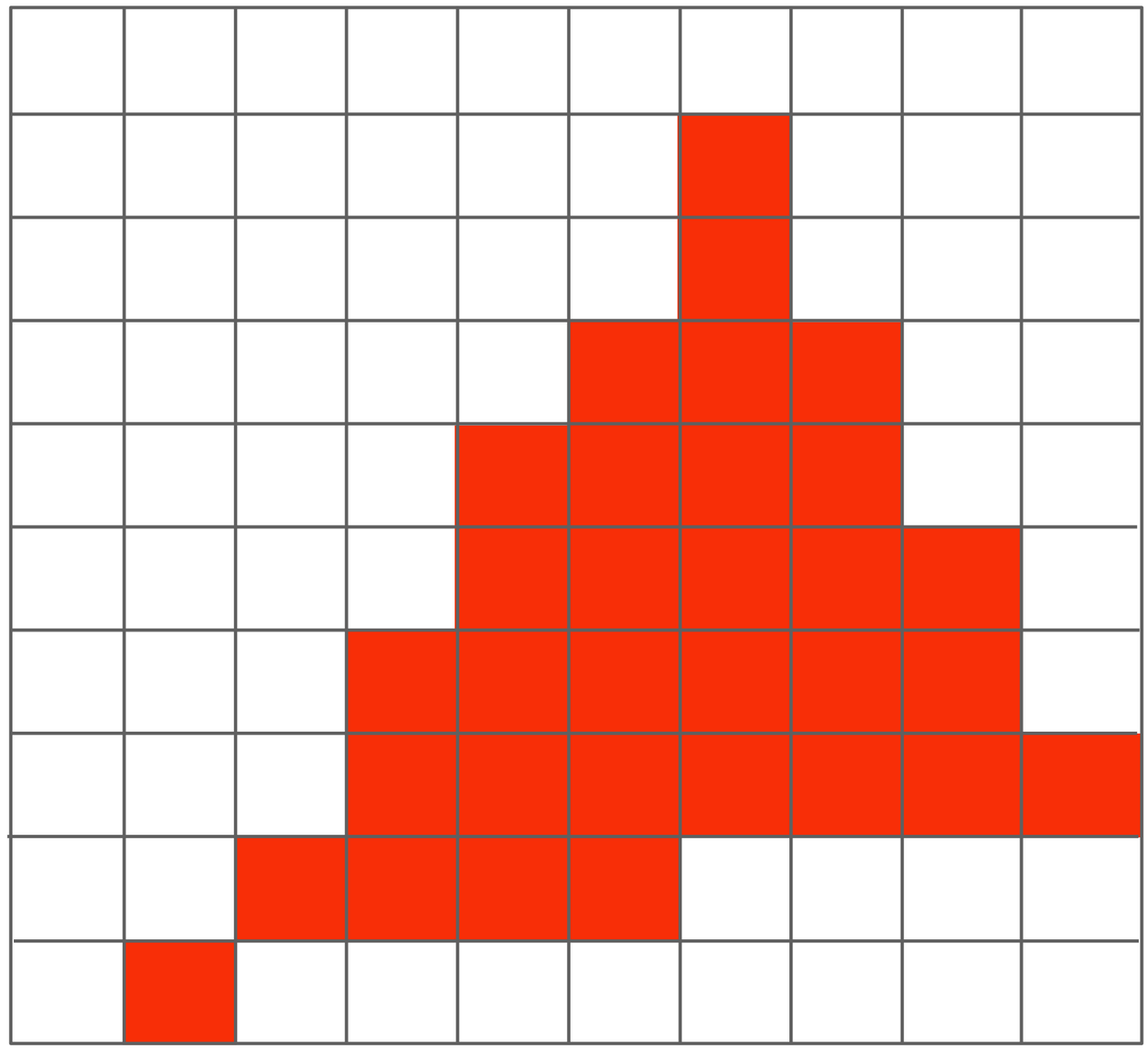
The Display Physically Emits This Signal



Compare: The Continuous Triangle Function



What's Wrong With This Picture?



Jaggies!

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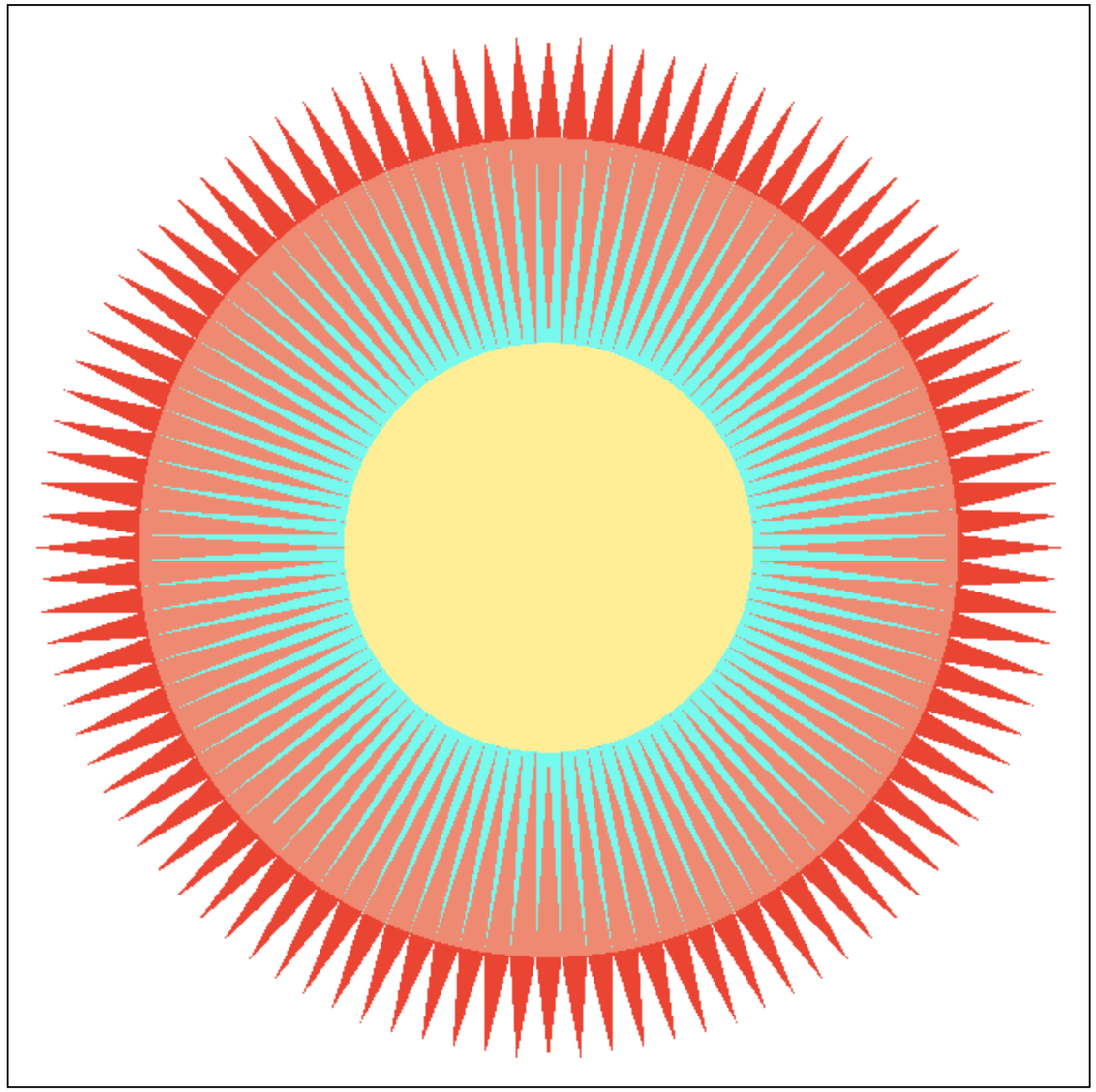
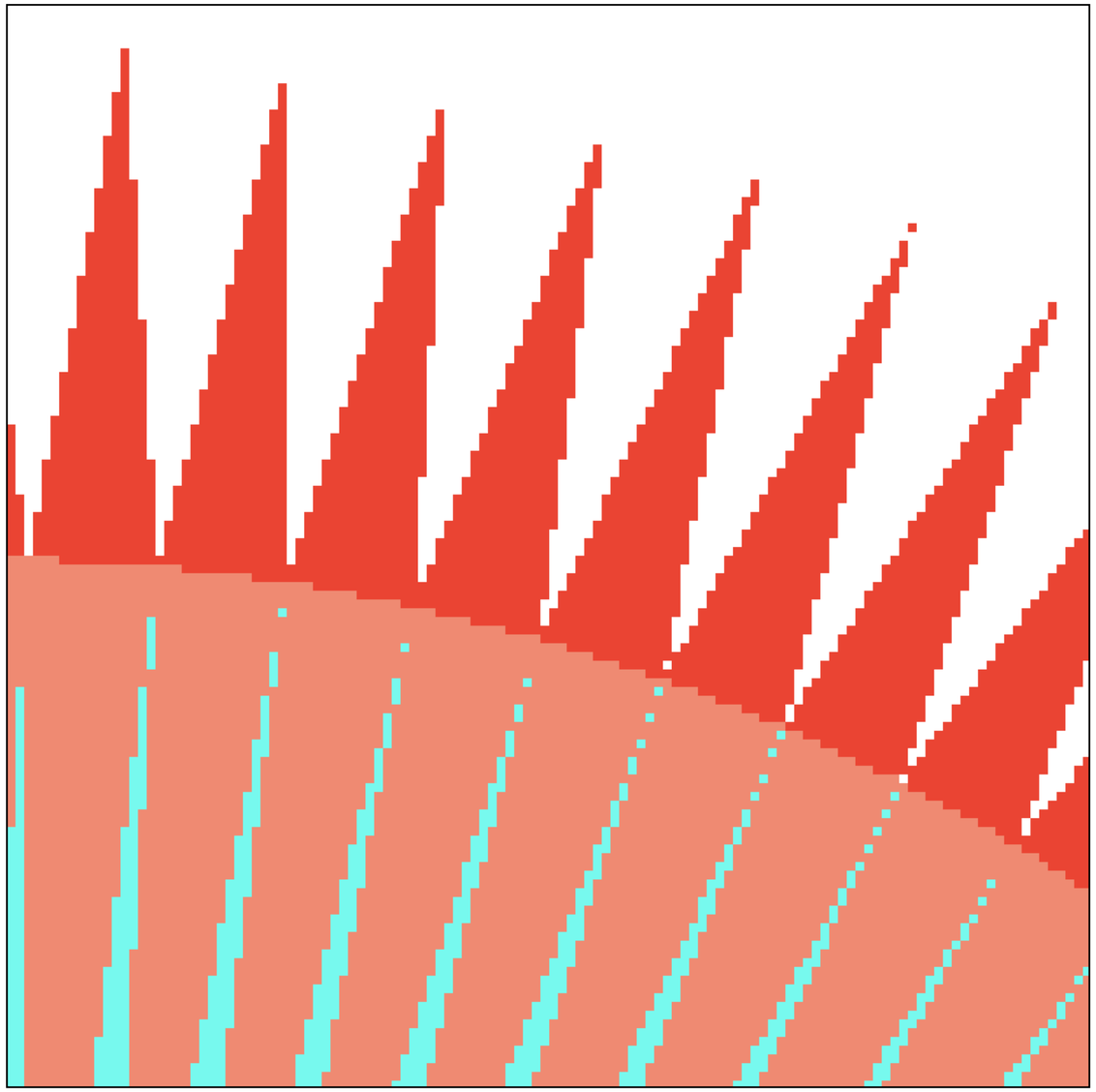
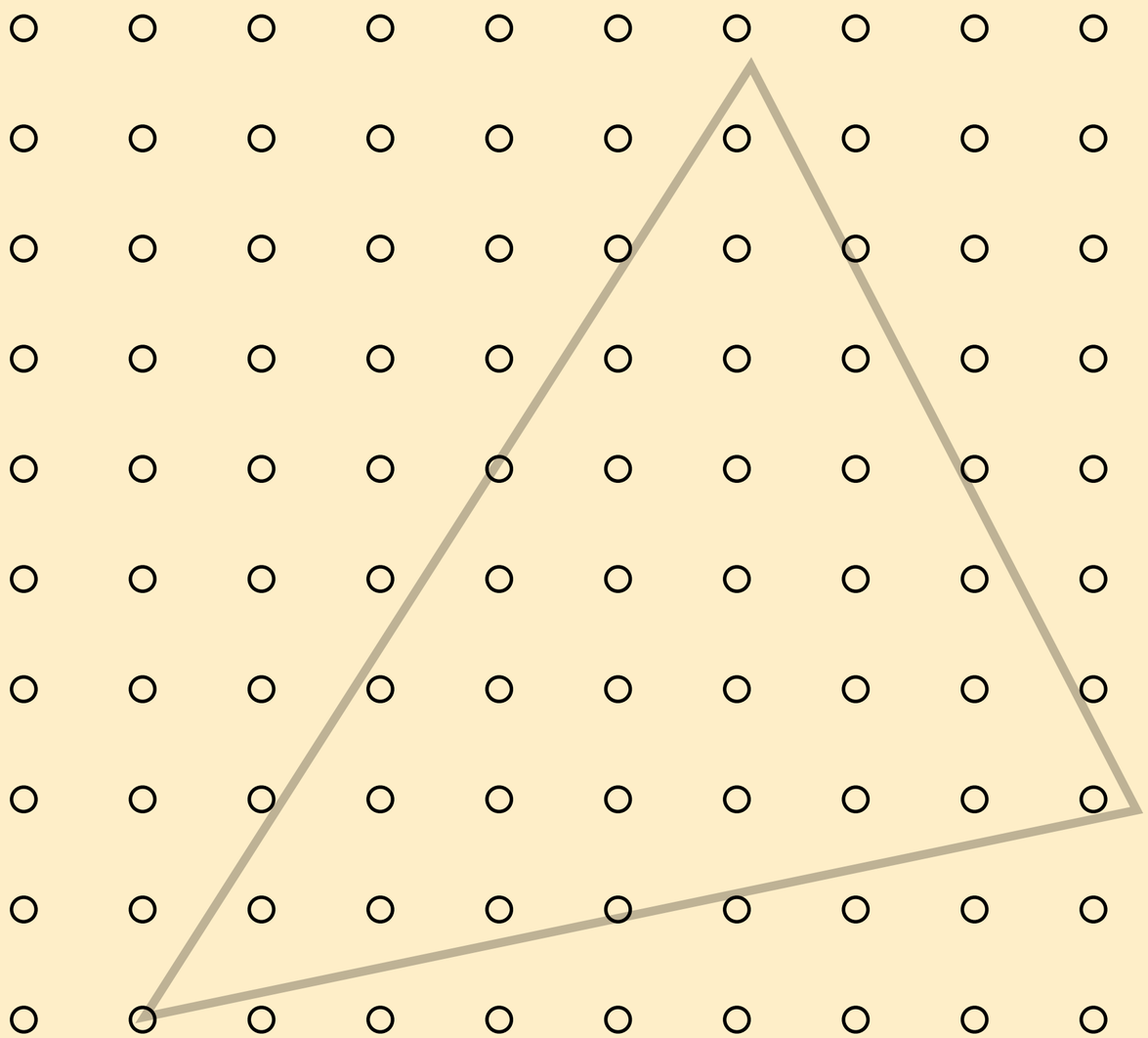
Jaggies (Staircase Pattern)





Is this the best we can do?

Discussion: What Value Should a Pixel Have?



Potential topics for your pair discussion:

- Ideas for "higher quality" pixel formula?
- What are all the relevant factors?
- What's right/wrong about point sampling?
- Why do jaggies look "wrong"?

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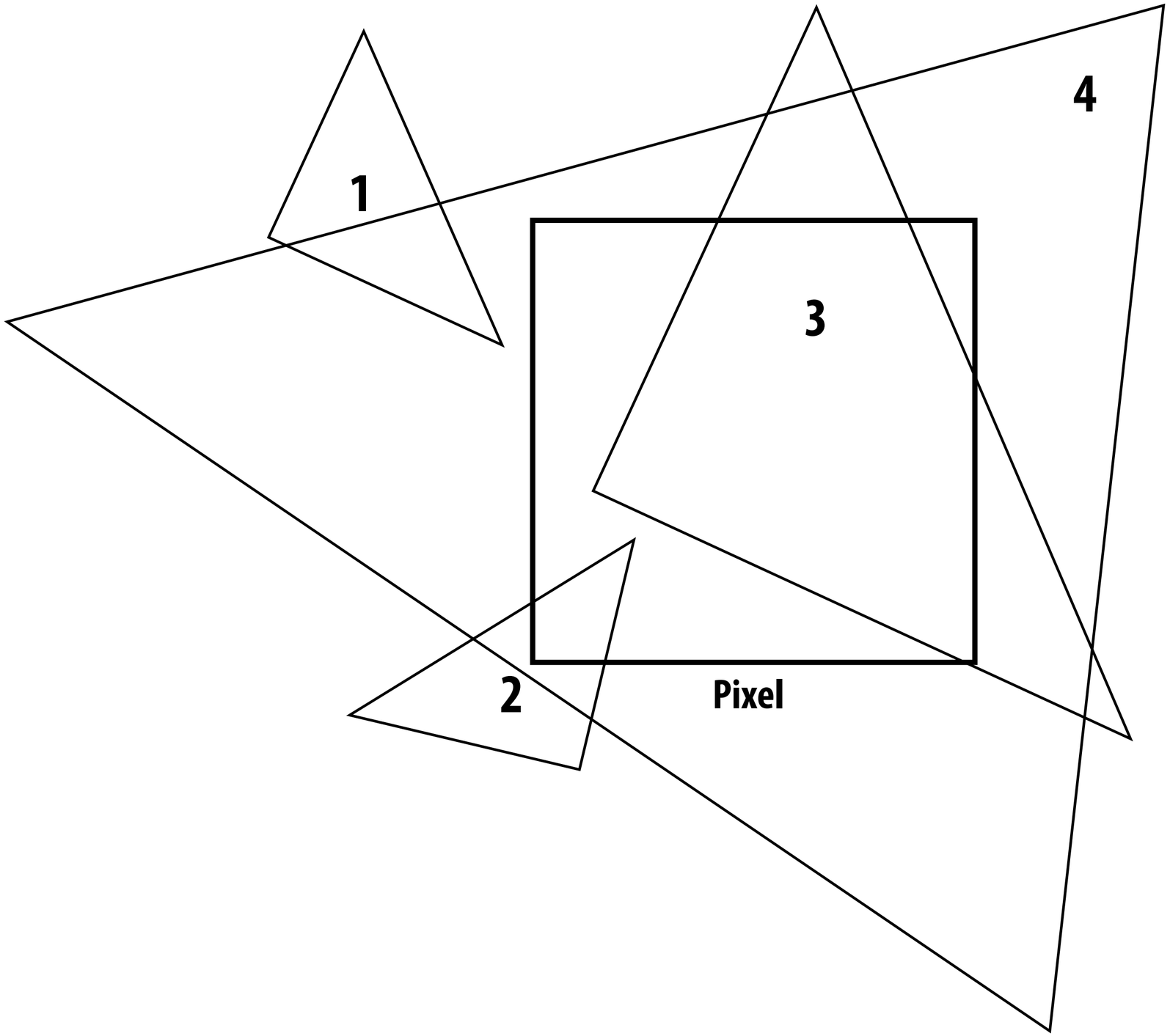
What does it mean for a pixel to be covered by a triangle?

Question: which triangles "cover" this pixel?



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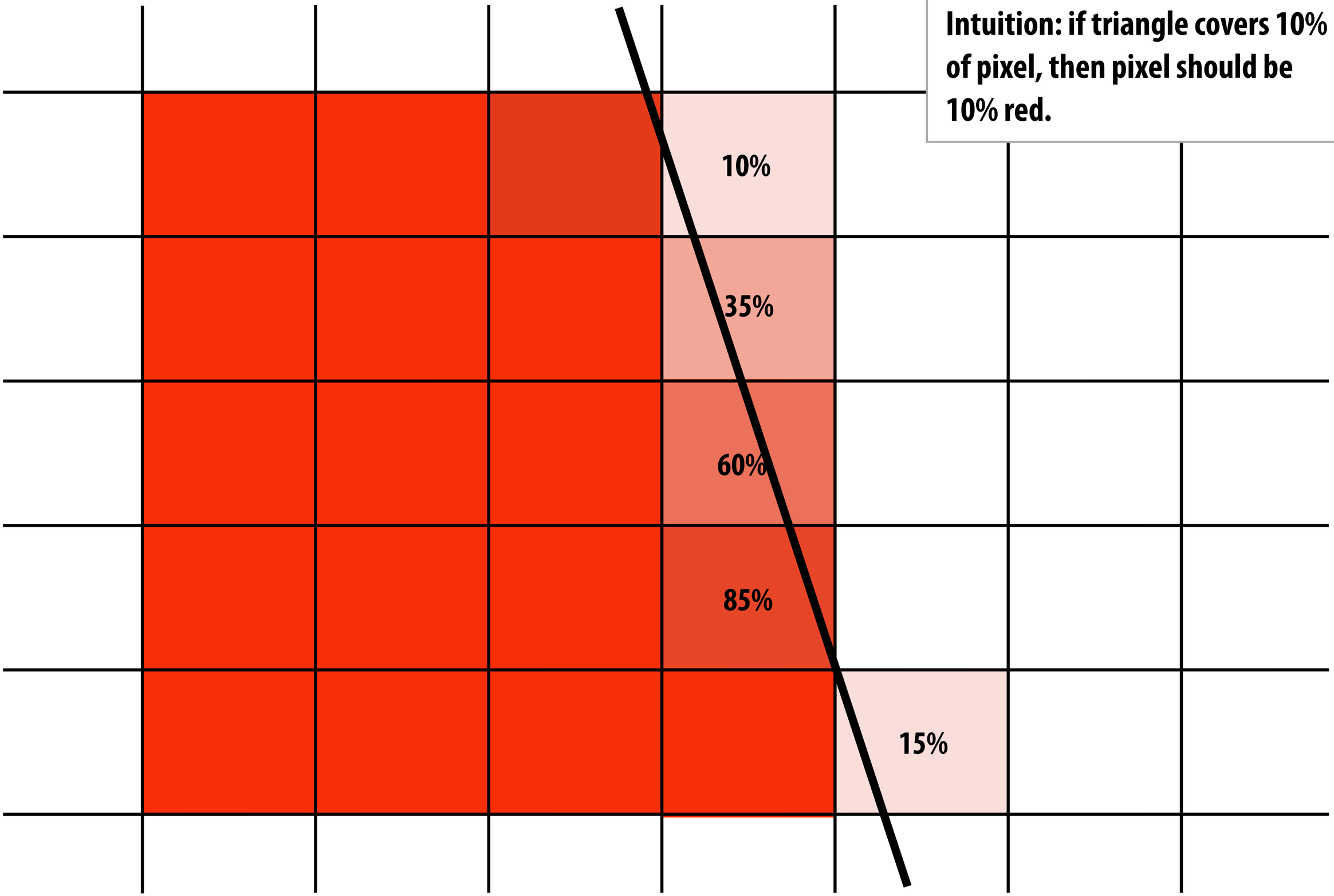
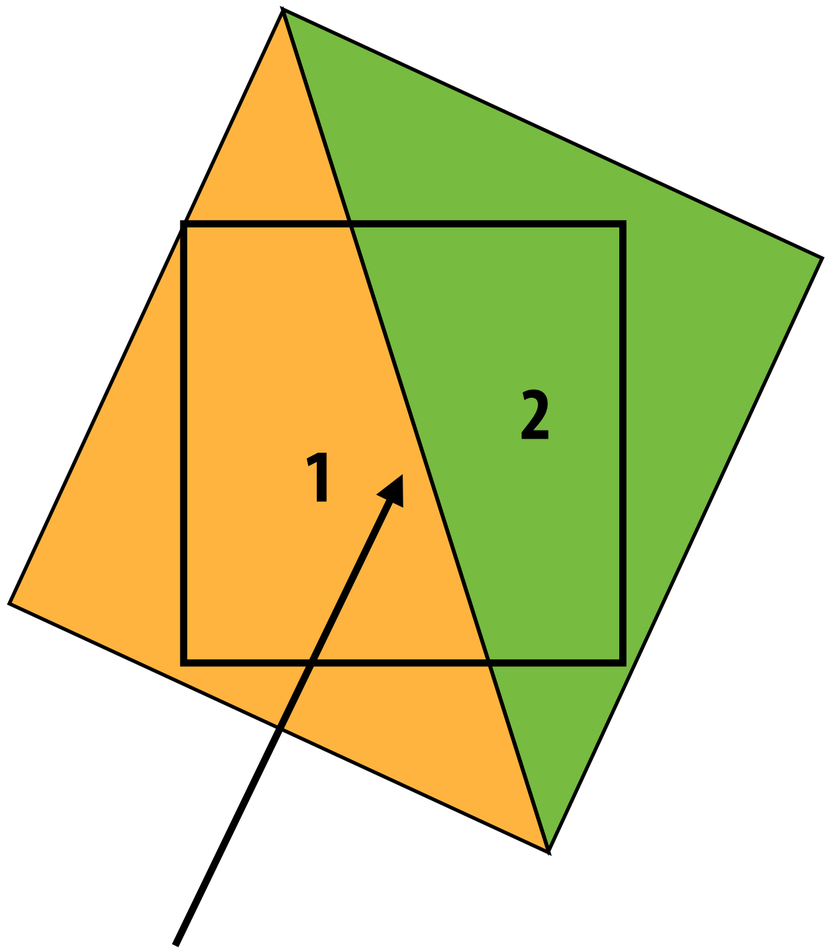
One option: compute fraction of pixel area covered by triangle, then color pixel according to this fraction.

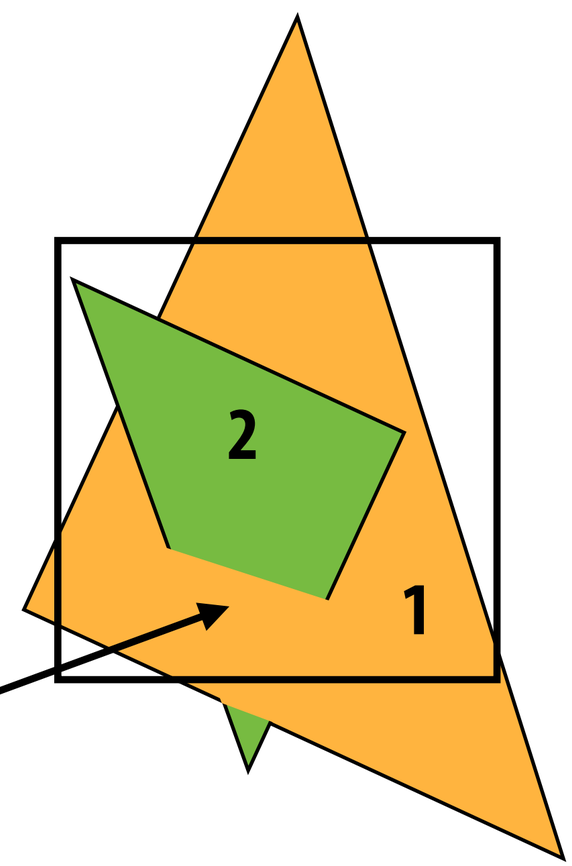


Intuition: if triangle covers 10% of pixel, then pixel should be 10% red.

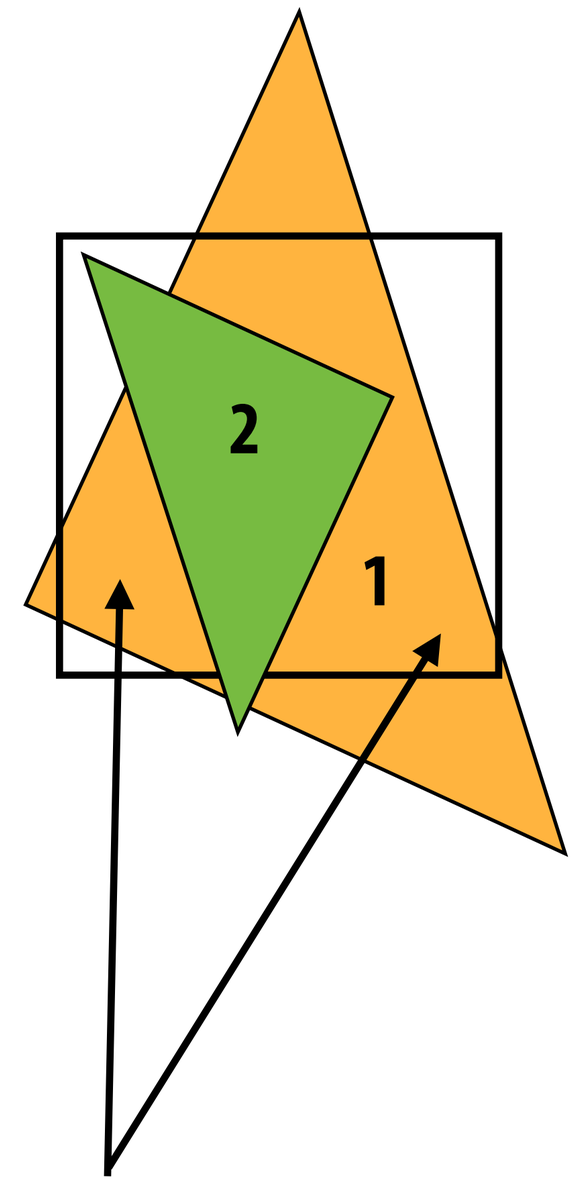
Analytical coverage schemes get tricky when considering occlusion of one triangle by another



Pixel covered by triangle 1, other half covered by triangle 2



Interpenetration of triangles: even trickier



Two regions of triangle 1 contribute to pixel. One of these regions is not even convex.

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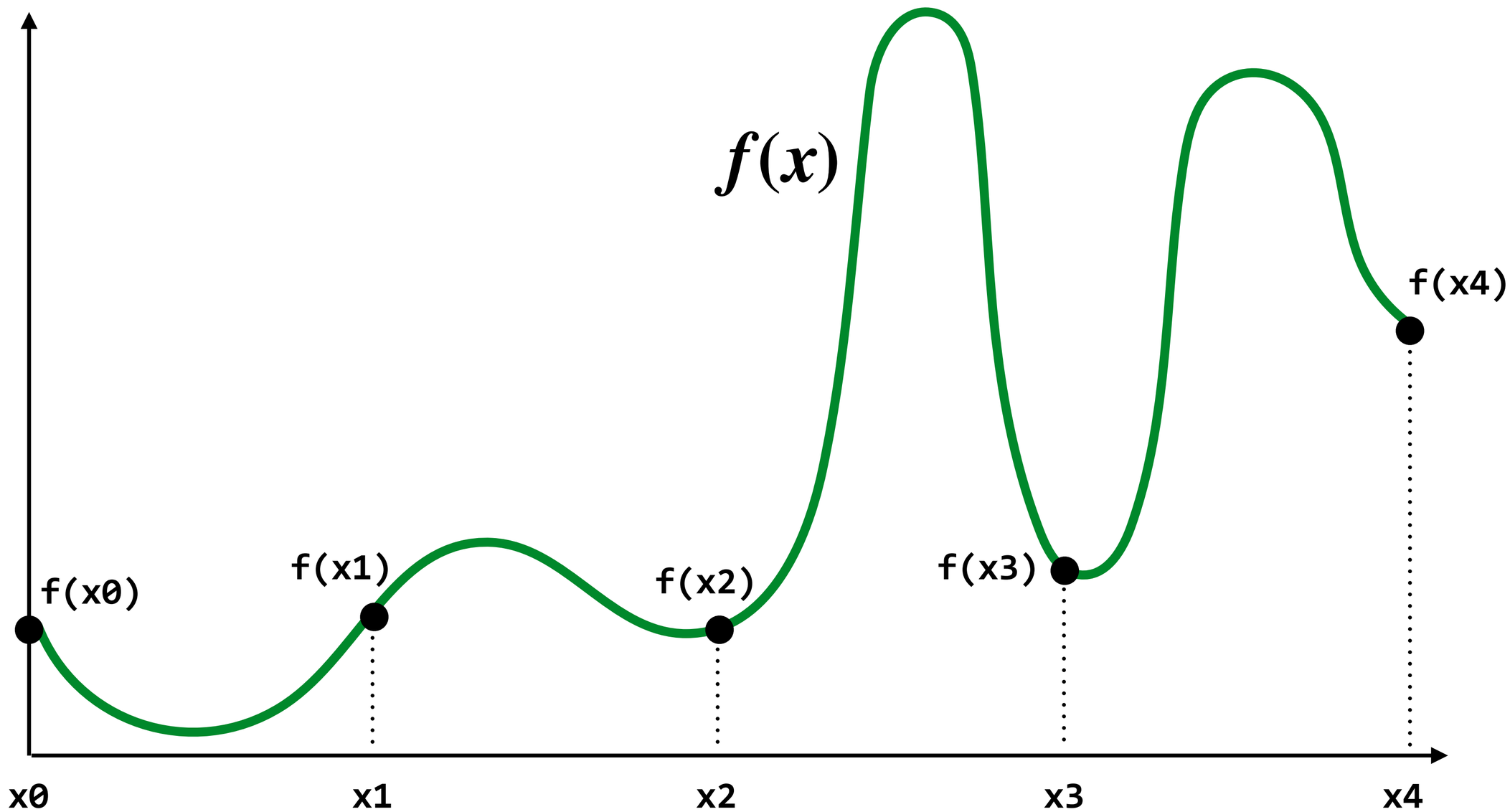
Sampling: taking measurements a signal

Below: five measurements ("samples") of $f(x)$



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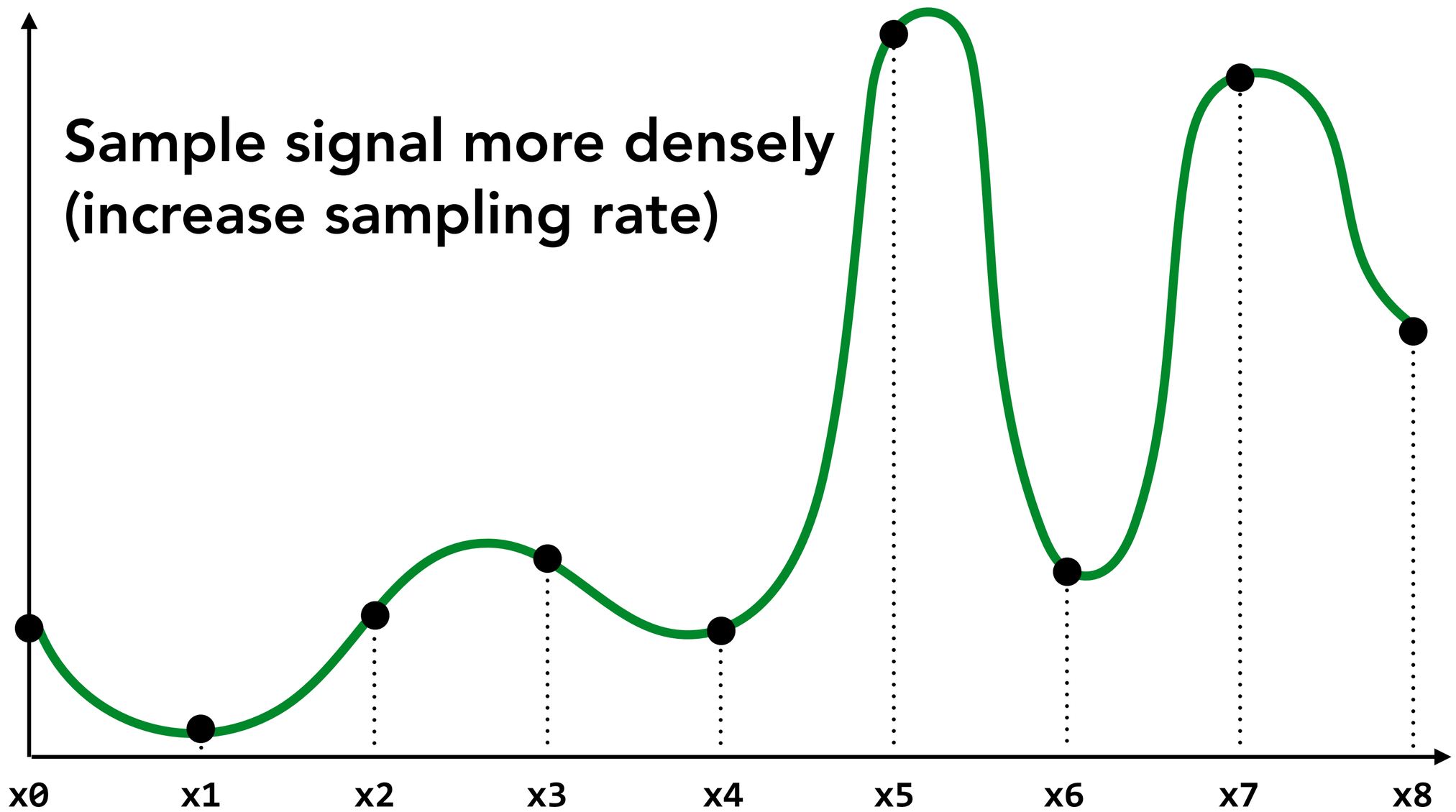
How can we represent a sampled signal more accurately?



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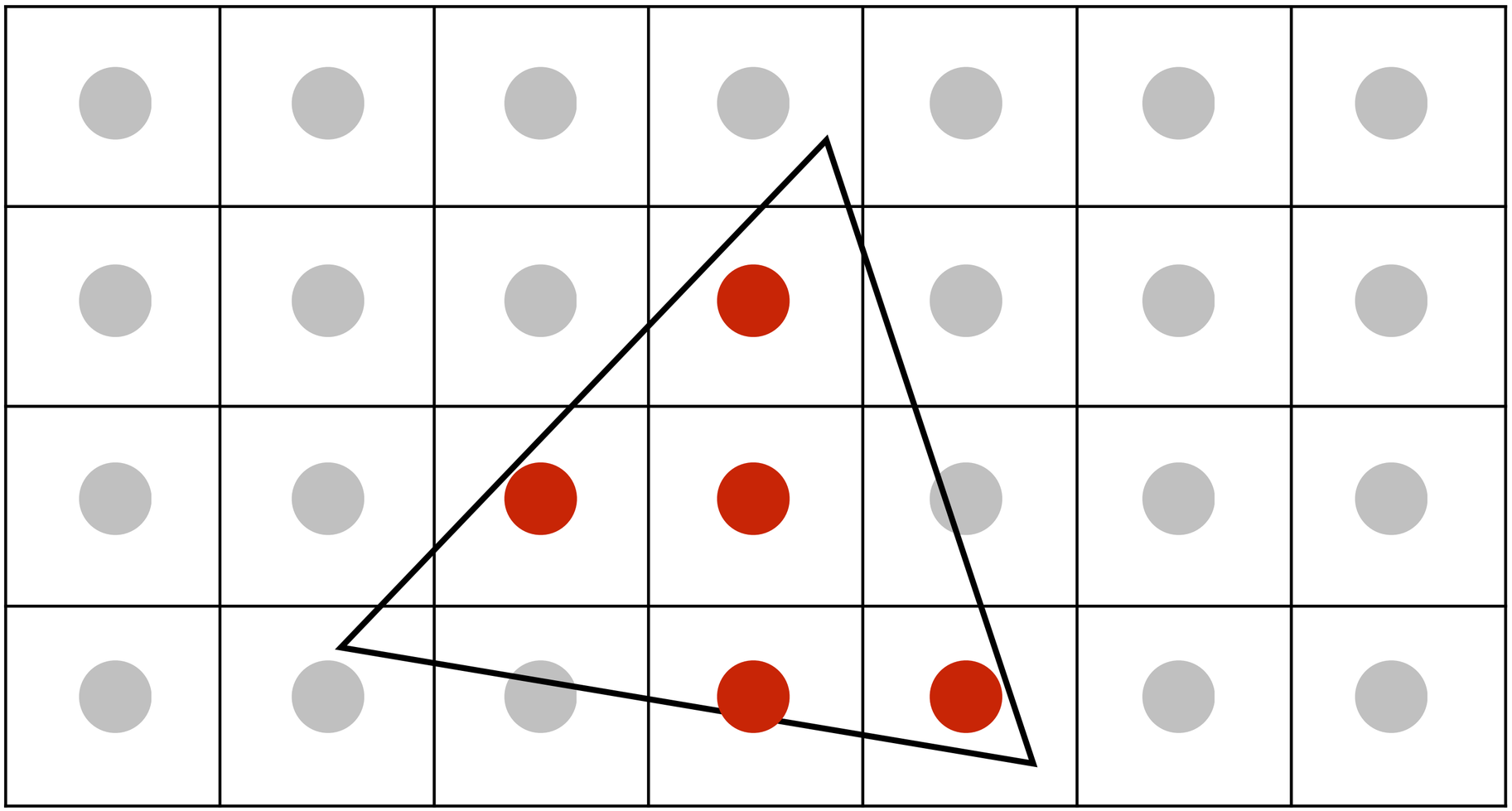
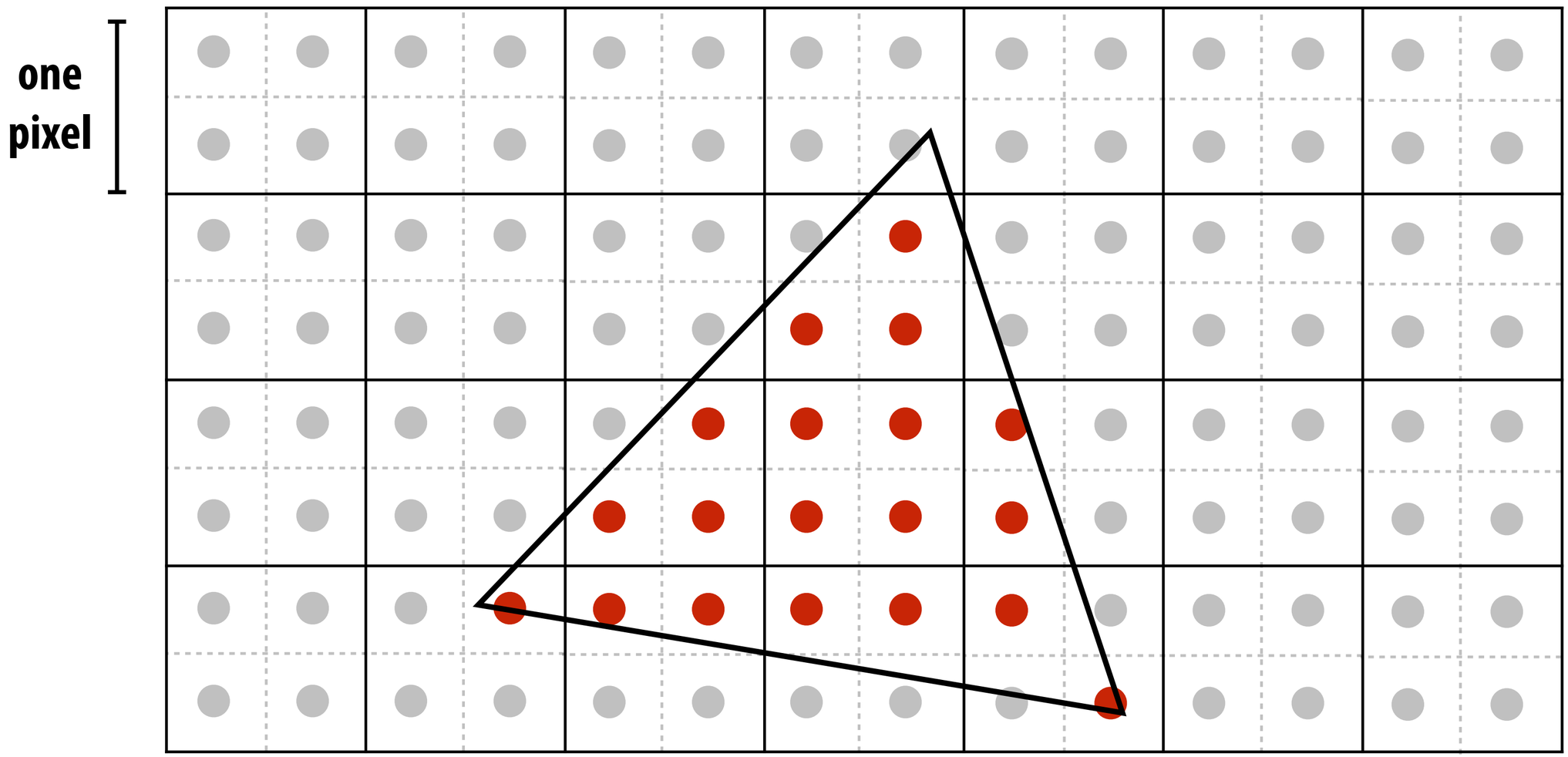
Point sampling: one sample per pixel



Supersampling: step 1

Take NxN samples in each pixel

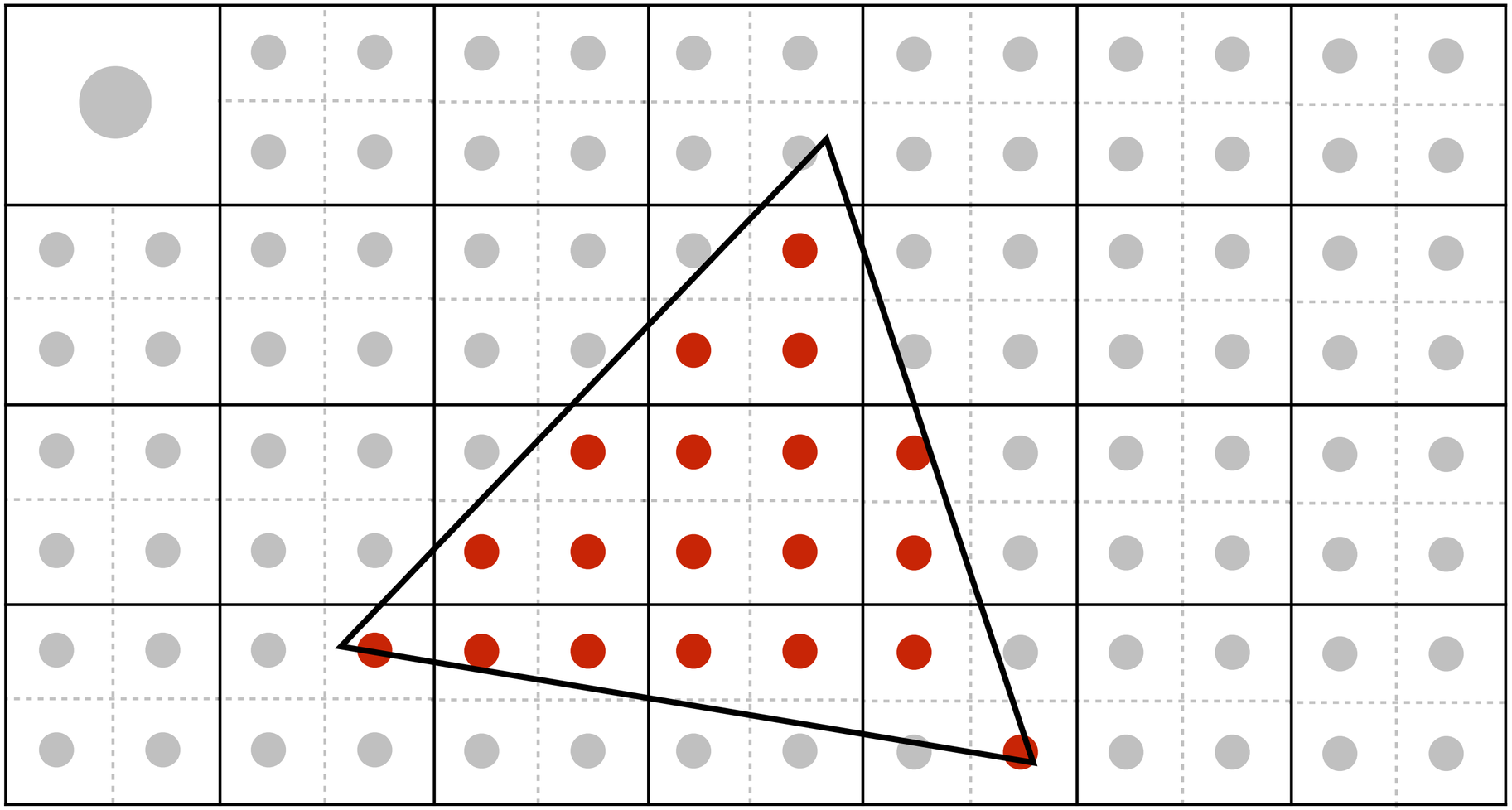
(but… how do we use these samples to drive a display, since there are four times more samples than display pixels!)



2x2 supersampling

Supersampling: step 2

Average the NxN samples “inside” each pixel



Averaging down

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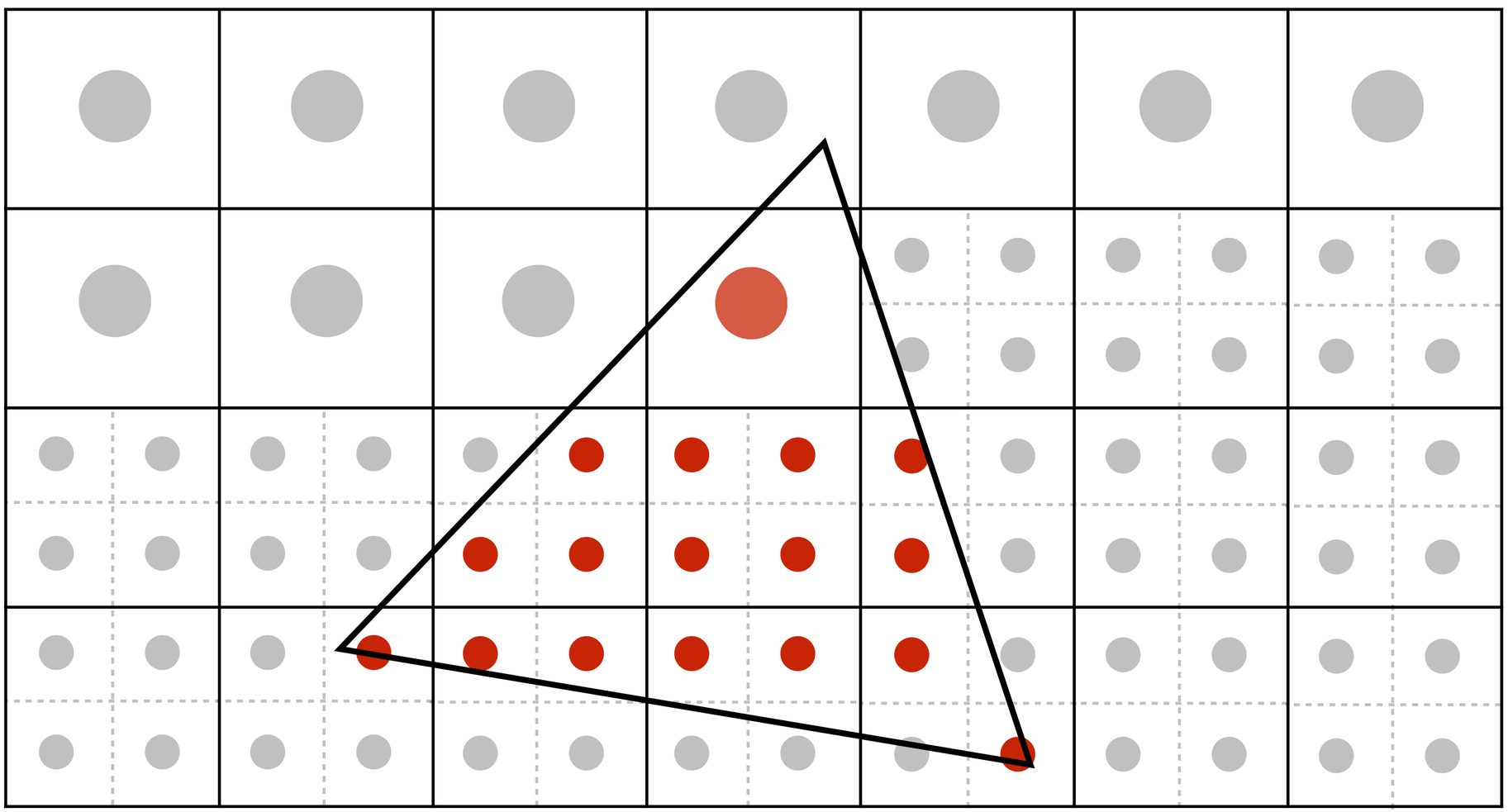
Supersampling: step 2

Average the NxN samples “inside” each pixel



Averaging down

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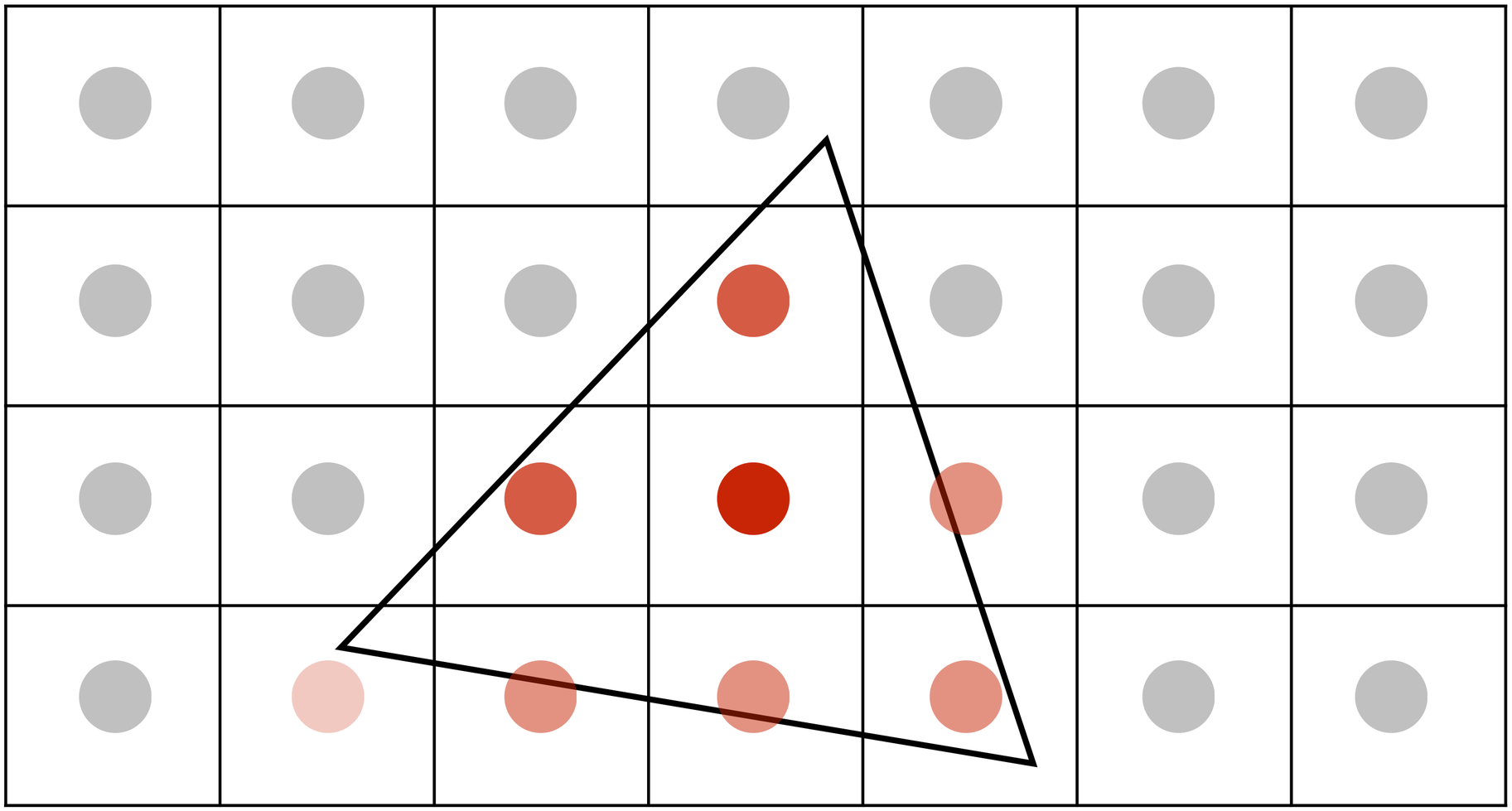
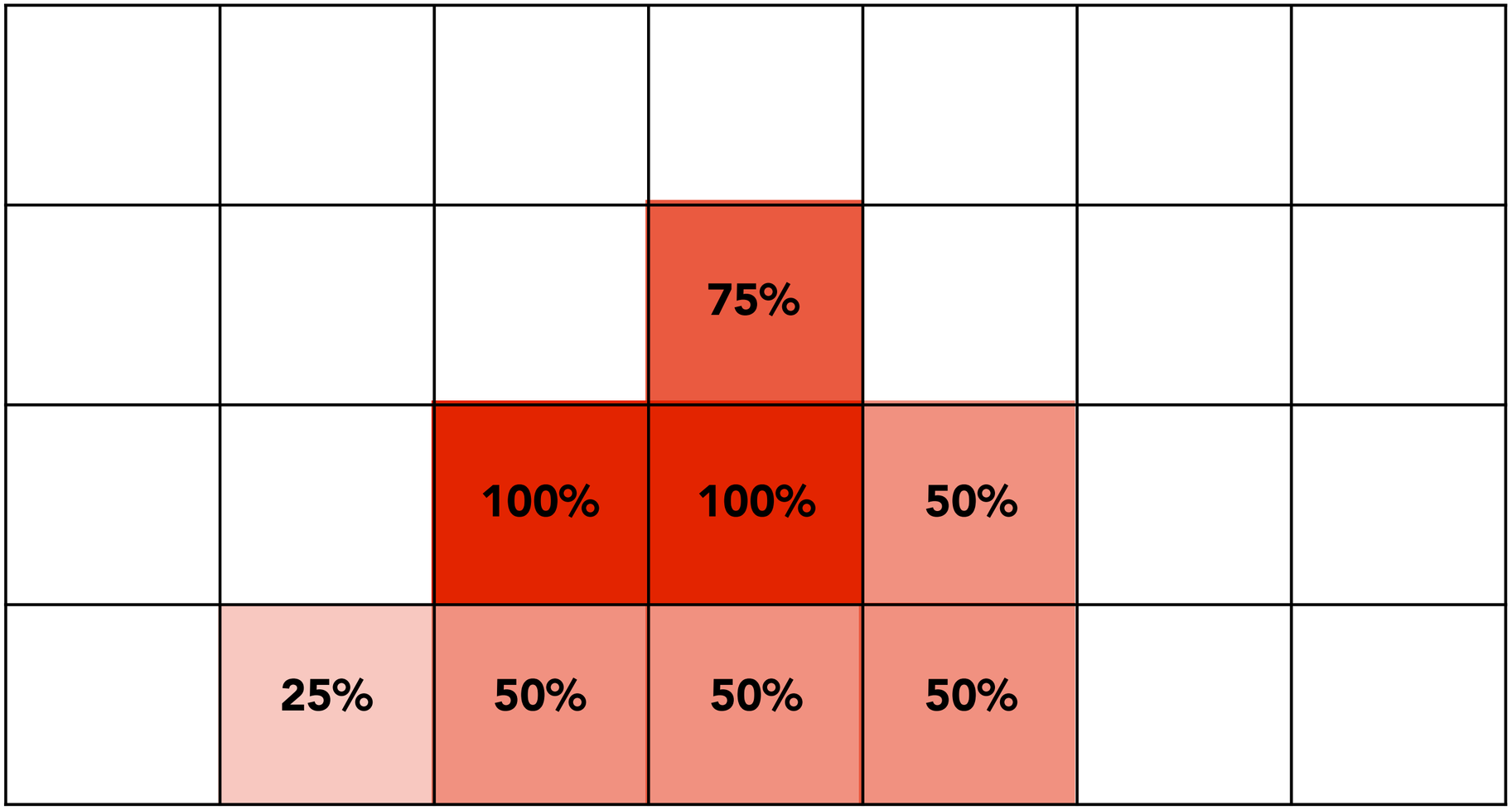
Supersampling: step 2

Average the NxN samples “inside” each pixel

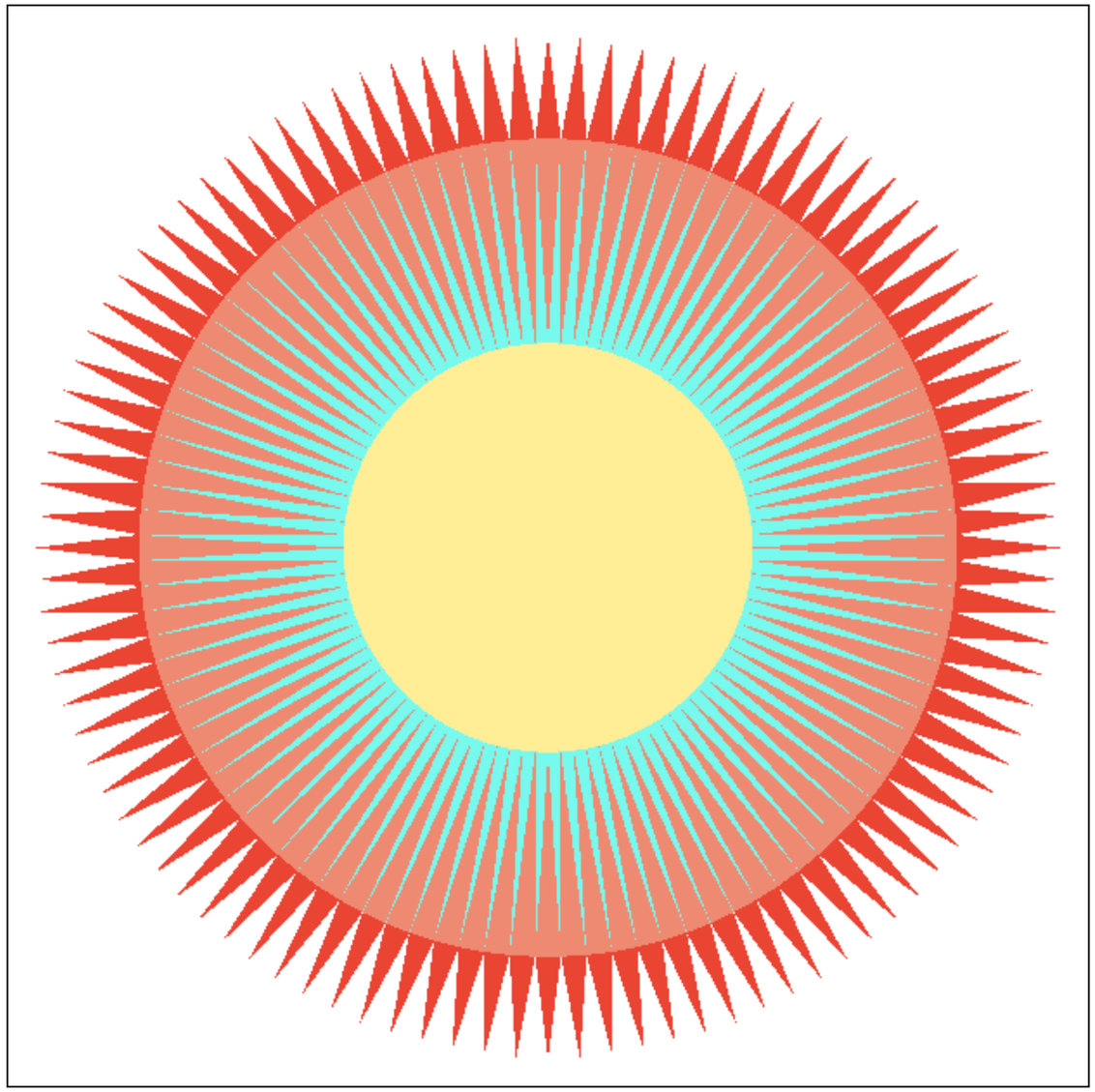


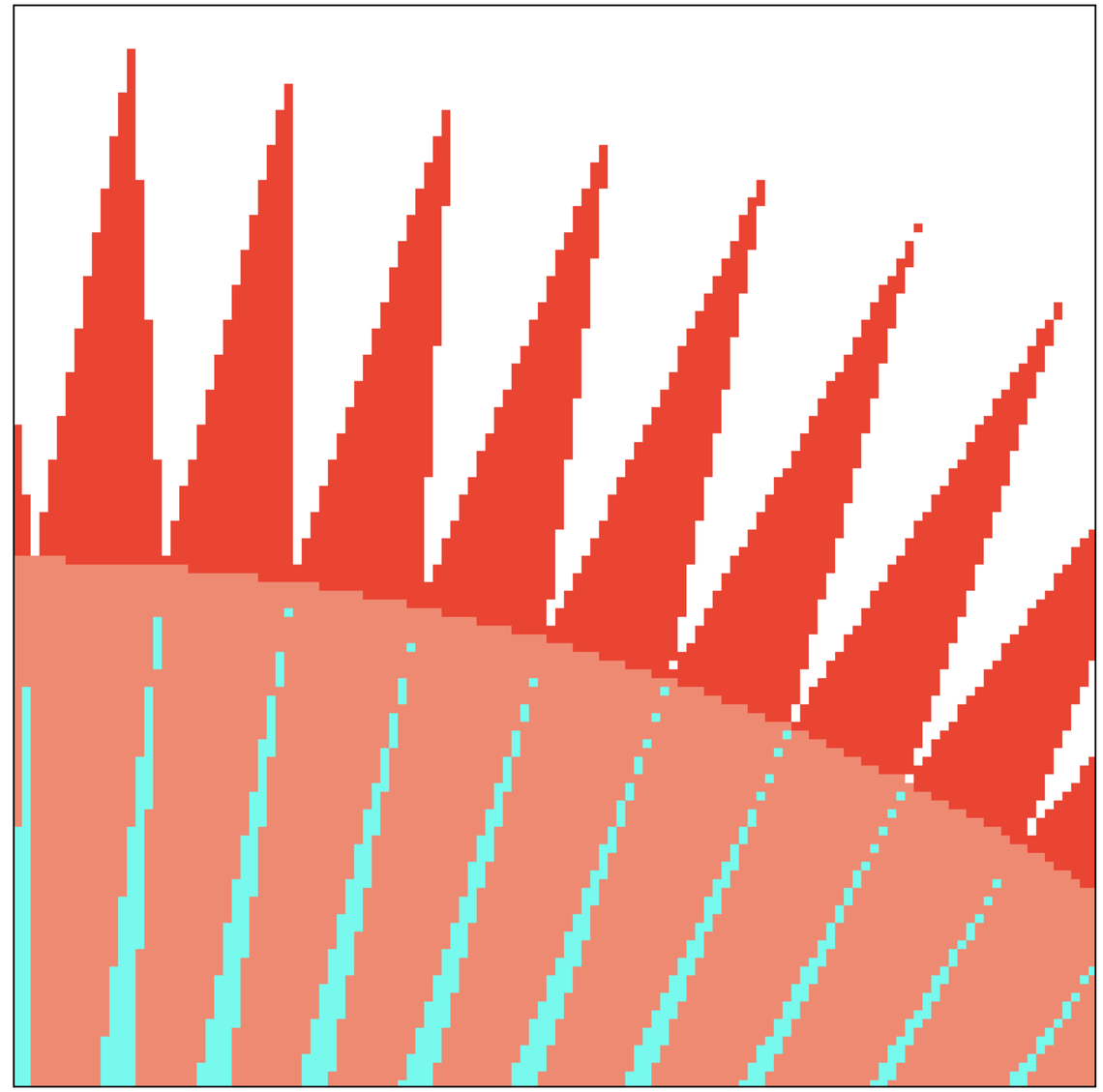
Supersampling: result

This is the corresponding signal emitted by the display



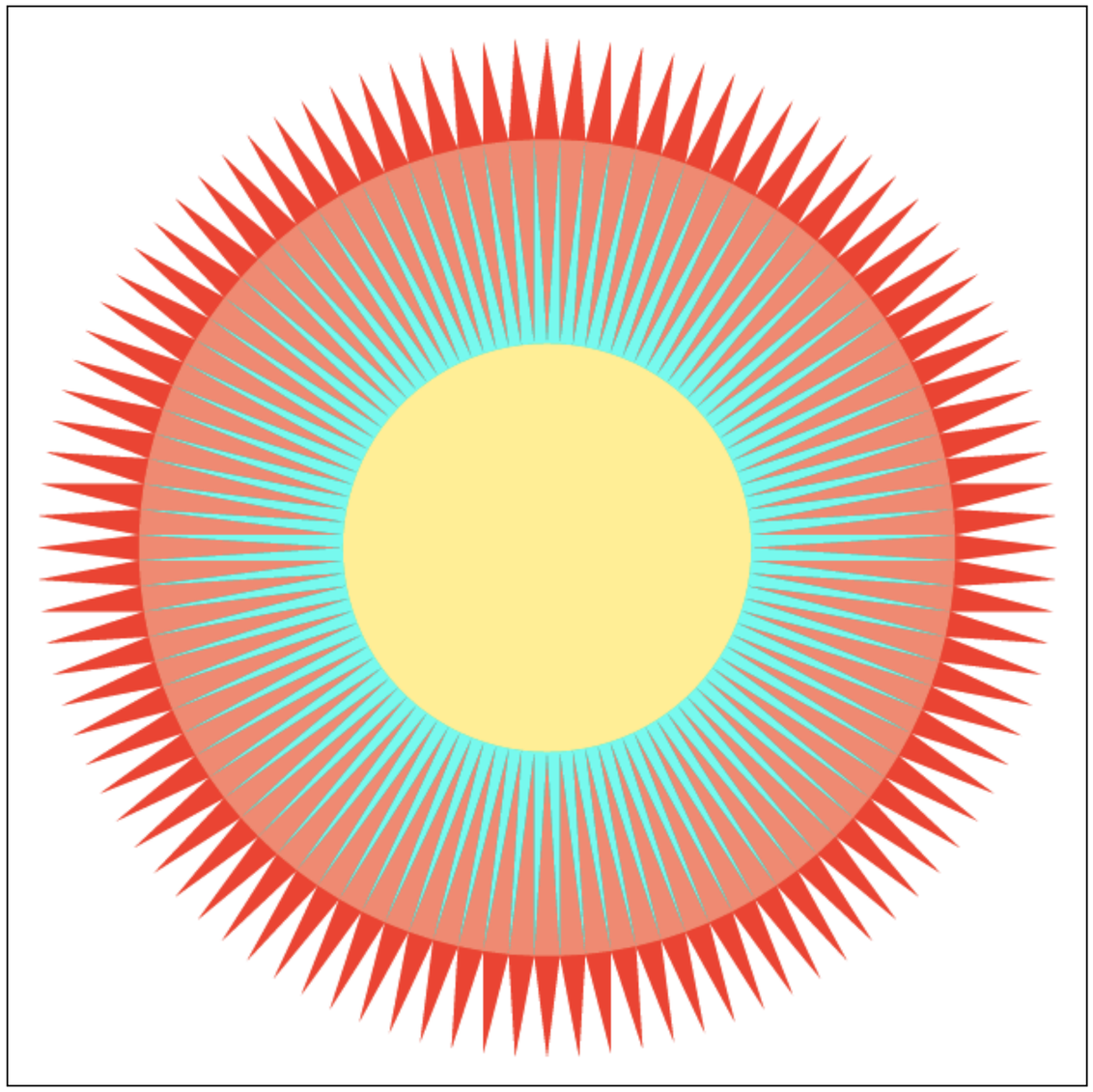
Point sampling

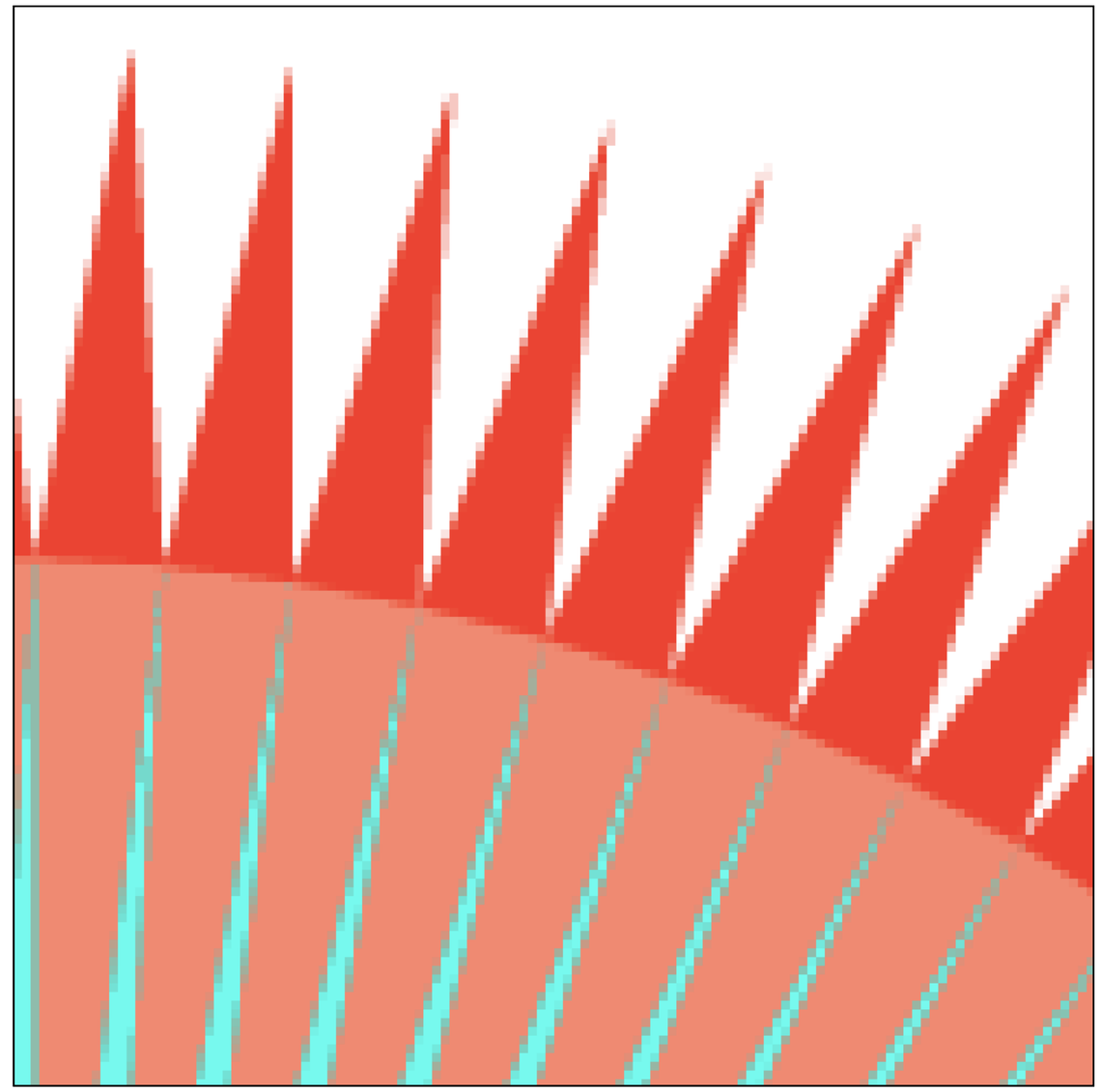




One sample per pixel

4x4 supersampling + downsampling





Pixel value is average of 4x4 samples per pixel

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Next time: *Antialiasing*

Understanding what just happened in a more principled way

Things to Remember

Drawing machines

- Many possibilities
- Why framebuffers and raster displays?
- Why triangles?

We posed rasterization as a 2D sampling process

- Test a binary function `inside(triangle,x,y)`
- Evaluate triangle coverage by 3 point-in-edge tests
- Finite sampling rate causes "jaggies" artifact (next time we will analyze in more detail)

Acknowledgments

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