

Course Roadmap

Rasterization Pipeline

Core Concepts

- Sampling
- Antialiasing
- Transforms

Geometric Modeling

Core Concepts

- Splines, Bezier Curves
- Topological Mesh Representations
- Subdivision, Geometry Processing

Lighting & Materials

Core Concepts

- Measuring Light
- Unbiased Integral Estimation
- Light Transport & Materials

Cameras & Imaging

• Rasterization

• Transforms & Projection

• Texture Mapping

• Visibility, Shading, Overall Pipeline

• Intro to Geometry

• Curves and Surfaces

• Geometry Processing

• Ray-Tracing & Acceleration

• Radiometry & Photometry

Today

• Monte Carlo Integration

• Global Illumination & Path Tracing

• Material Modeling



Lecture 11:

Measuring Light: Radiometry and Photometry

**Computer Graphics and Imaging
UC Berkeley CS184/284A**

Radiometry

Measurement system and units for illumination

Measure the spatial properties of light

- New terms: Radiant flux, intensity, irradiance, radiance

Perform lighting calculations in a physically correct manner

Assumption: geometric optics model of light

- Photons travel in straight lines, represented by rays

Light

Visible electromagnetic spectrum

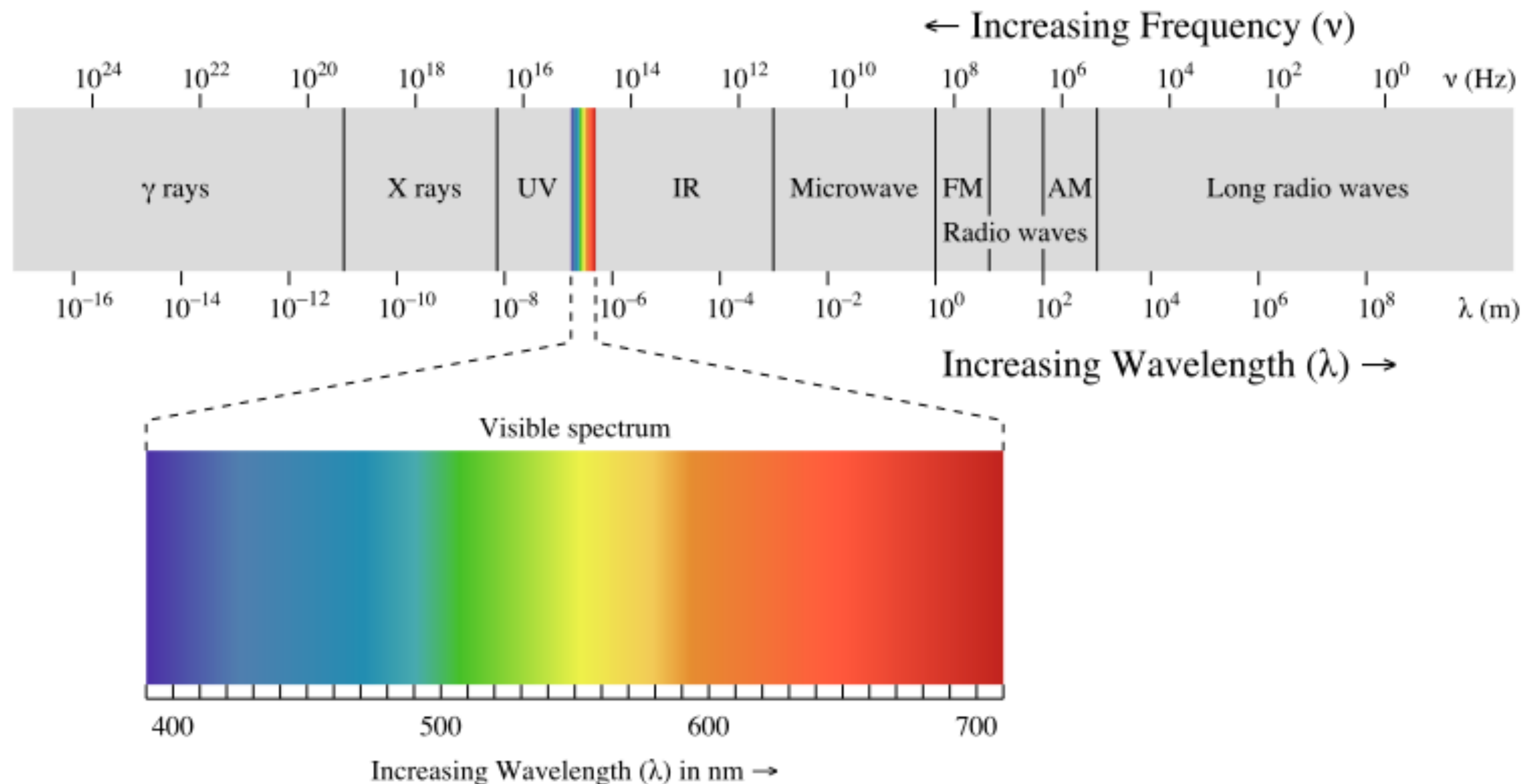


Image credit: Licensed under CC BY-SA 3.0 via Commons
https://commons.wikimedia.org/wiki/File:EM_spectrum.svg#/media/File:EM_spectrum.svg

Lights: How Do They Work?



**Cree 11 W LED light bulb
(60W incandescent
replacement)**

Physical process converts energy into photons

- Each photon carries a small amount of energy

Over some amount of time, light consumes some amount of energy, Joules

- Some is turned into heat, some into photons

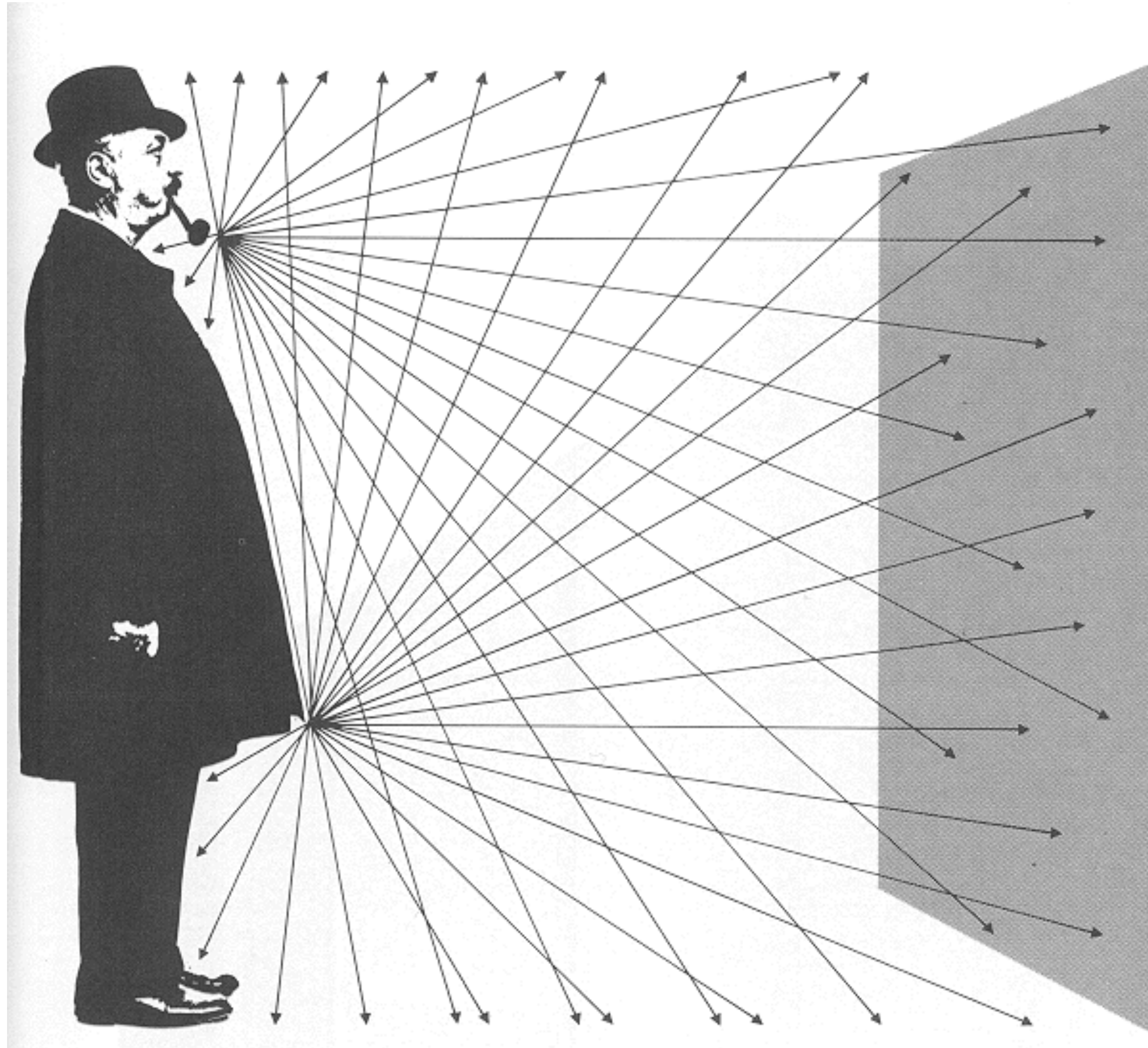
Energy of photons hitting an object ~ exposure

- Film, sensors, sunburn, solar panels, ...

Graphics: generally assume "steady state" flow

- Rate of energy consumption is constant, so flux (power) and energy are often interchangeable

Flux – How Fast Do Photons Flow Through a Sensor?



From London and Upton

Radiant Energy and Flux (Power)

Radiant Energy and Flux (Power)

Definition: Radiant (luminous*) energy is the energy of electromagnetic radiation. It is measured in units of joules, and denoted by the symbol:

$$Q \text{ [J = Joule]}$$

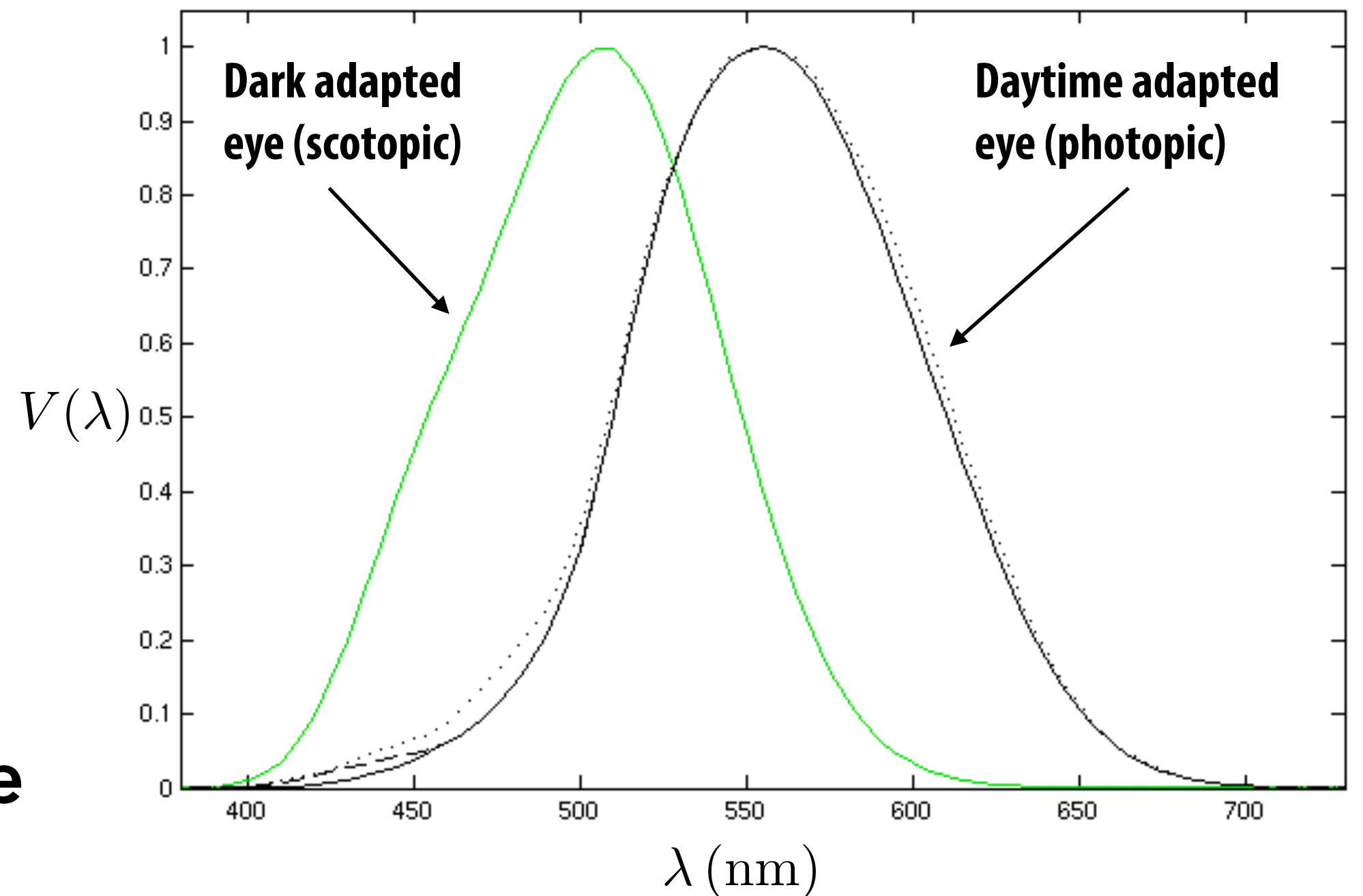
Definition: Radiant (luminous*) flux is the energy emitted, reflected, transmitted or received, per unit time.

$$\Phi \equiv \frac{dQ}{dt} \text{ [W = Watt] [lm = lumen]}^*$$

* Definition slides will provide photometric terms in parentheses and give photometric units

Photometry

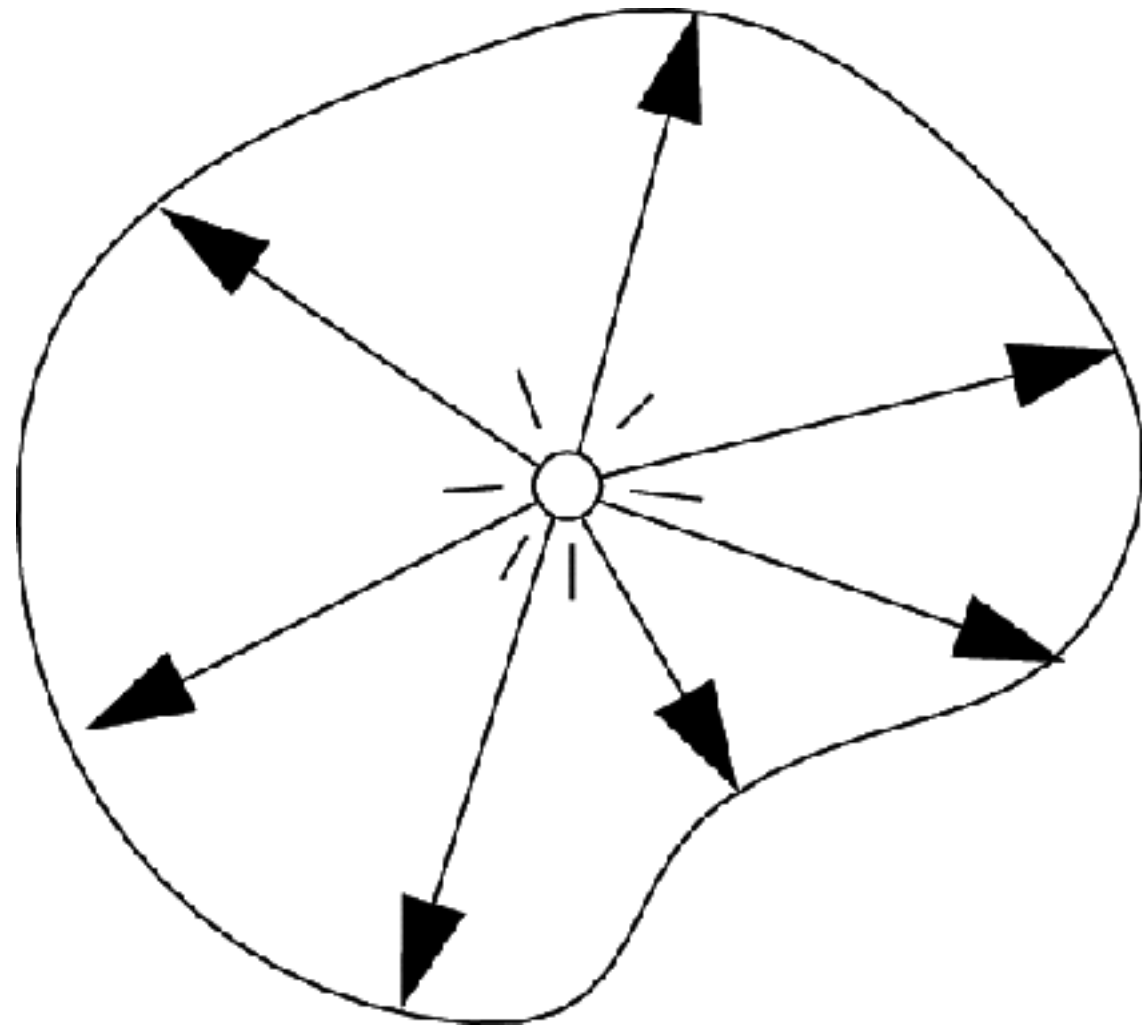
- All radiometric quantities have equivalents in photometry
- Photometry: accounts for response of human visual system
- E.g. Luminous flux Φ_v is the photometric quantity that corresponds to radiant flux Φ_e : integrate radiant flux over all wavelengths, weighted by eye's luminous efficiency curve $V(\lambda)$



<https://upload.wikimedia.org/wikipedia/commons/a/a0/Luminosity.png>

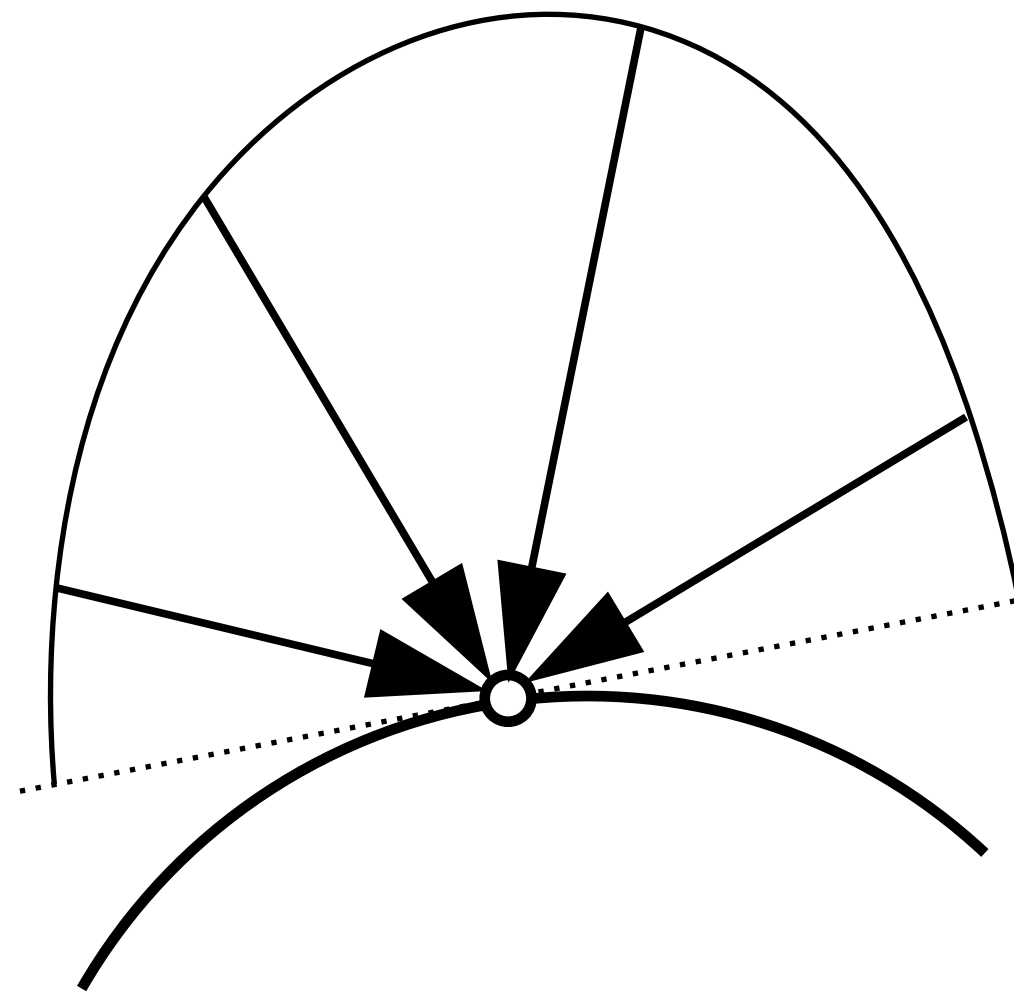
$$\Phi_v = \int_0^{\infty} \Phi_e(\lambda) V(\lambda) d\lambda$$

Example Light Measurements of Interest



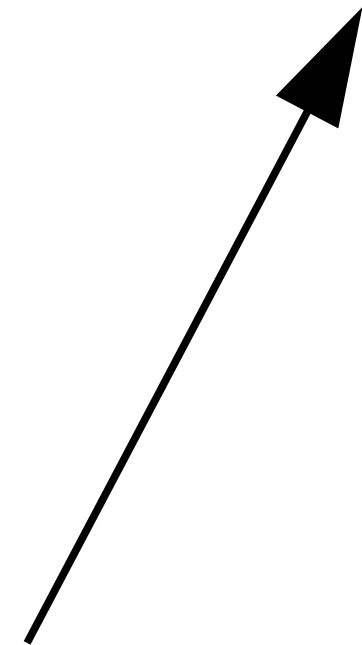
Light Emitted
From A Source

"Radiant Intensity"



Light Falling
On A Surface

"Irradiance"



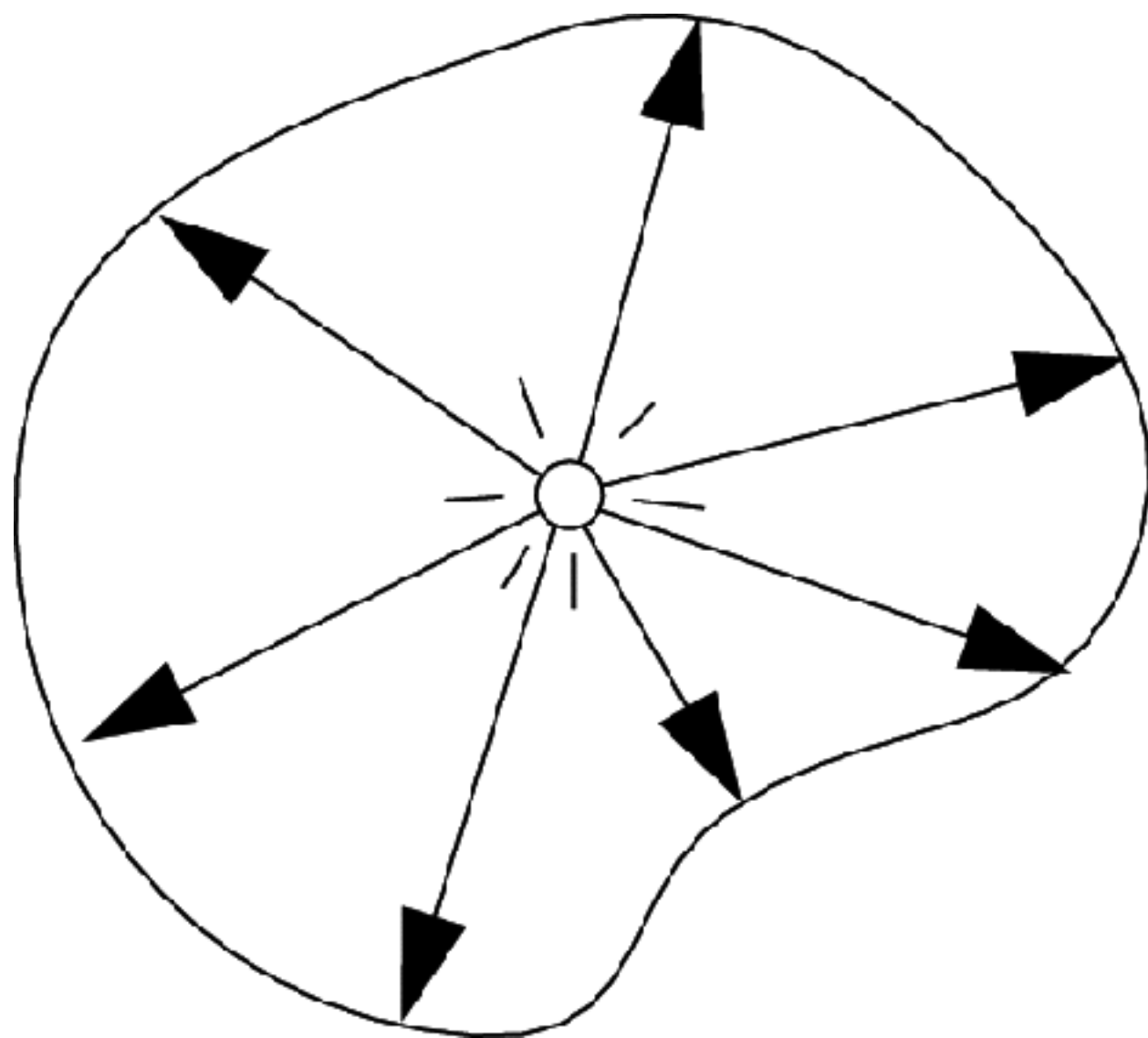
Light Traveling
Along A Ray

"Radiance"

Radiant Intensity

Radiant Intensity

Definition: The radiant (luminous) intensity is the power per unit solid angle emitted by a point light source.



$$I(\omega) \equiv \frac{d\Phi}{d\omega}$$

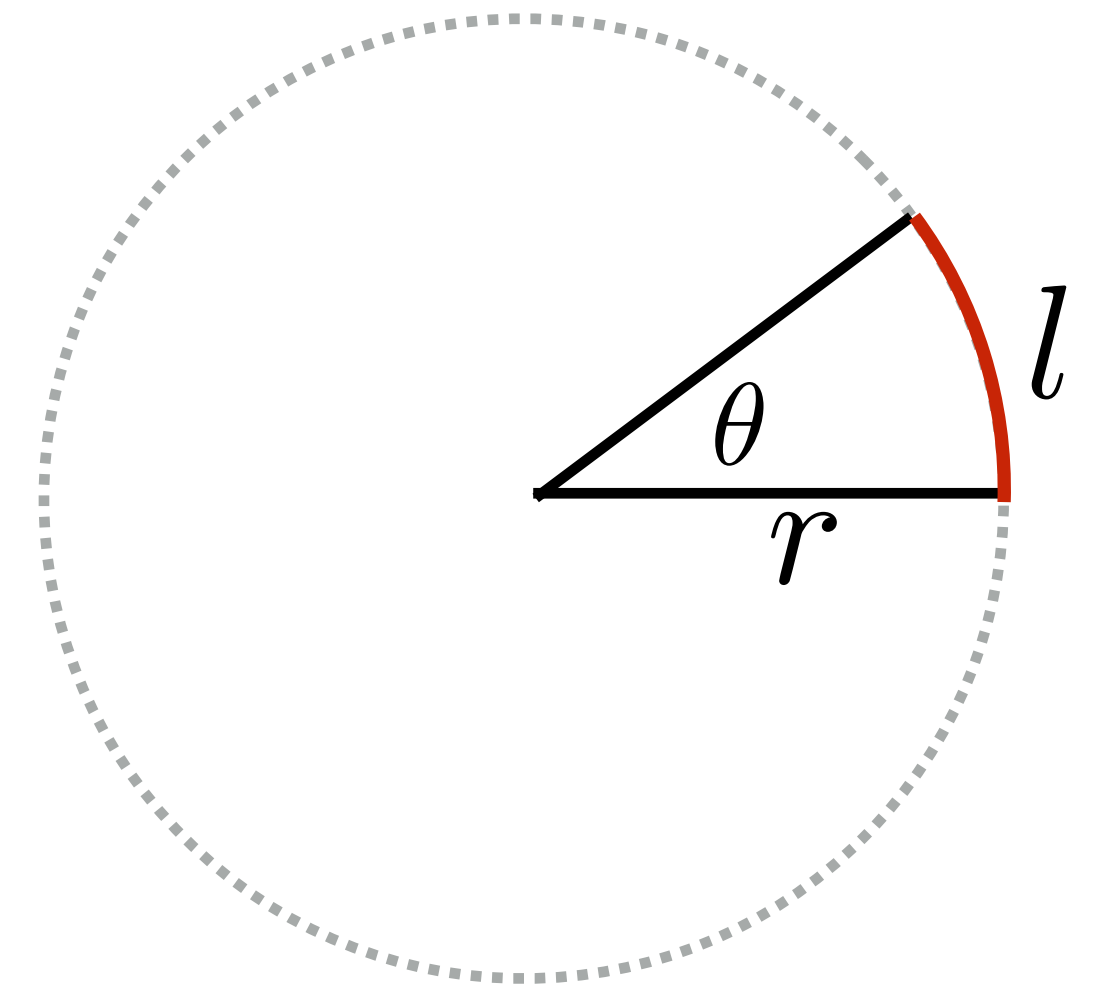
$$\left[\frac{\text{W}}{\text{sr}} \right] \left[\frac{\text{lm}}{\text{sr}} \right] = \text{cd} = \text{candela}$$

The candela is one of the seven SI base units.

Angles and Solid Angles

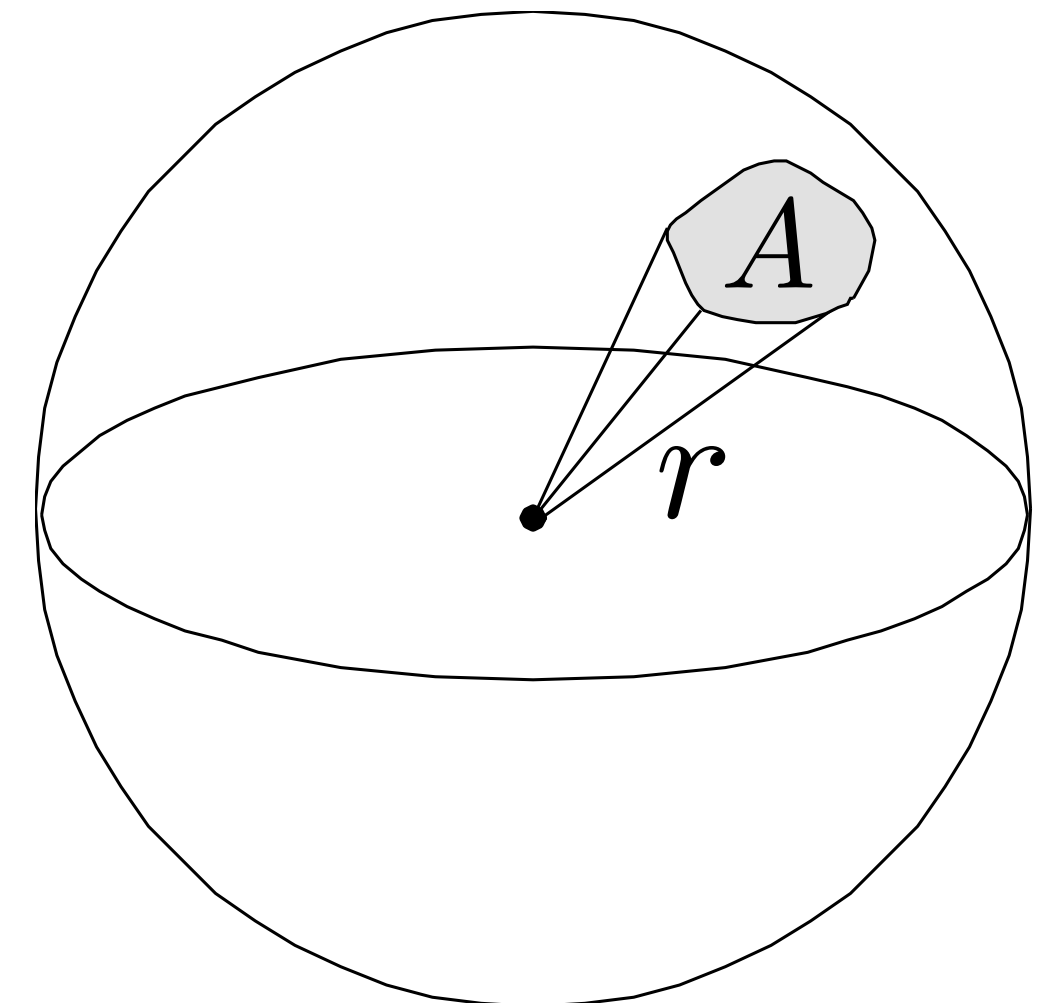
Angle: ratio of subtended arc length on circle to radius

- $\theta = \frac{l}{r}$
- Circle has 2π **radians**



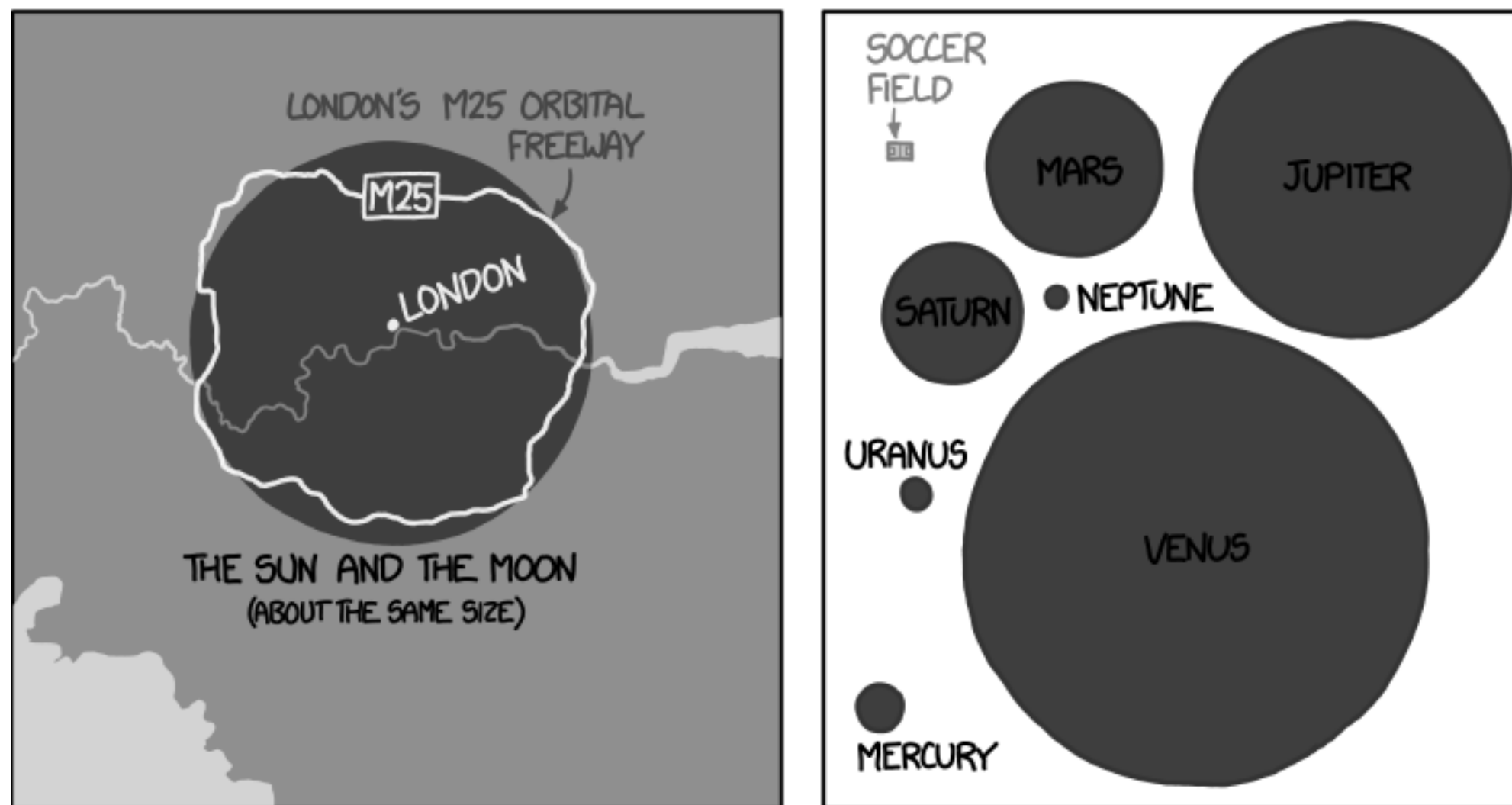
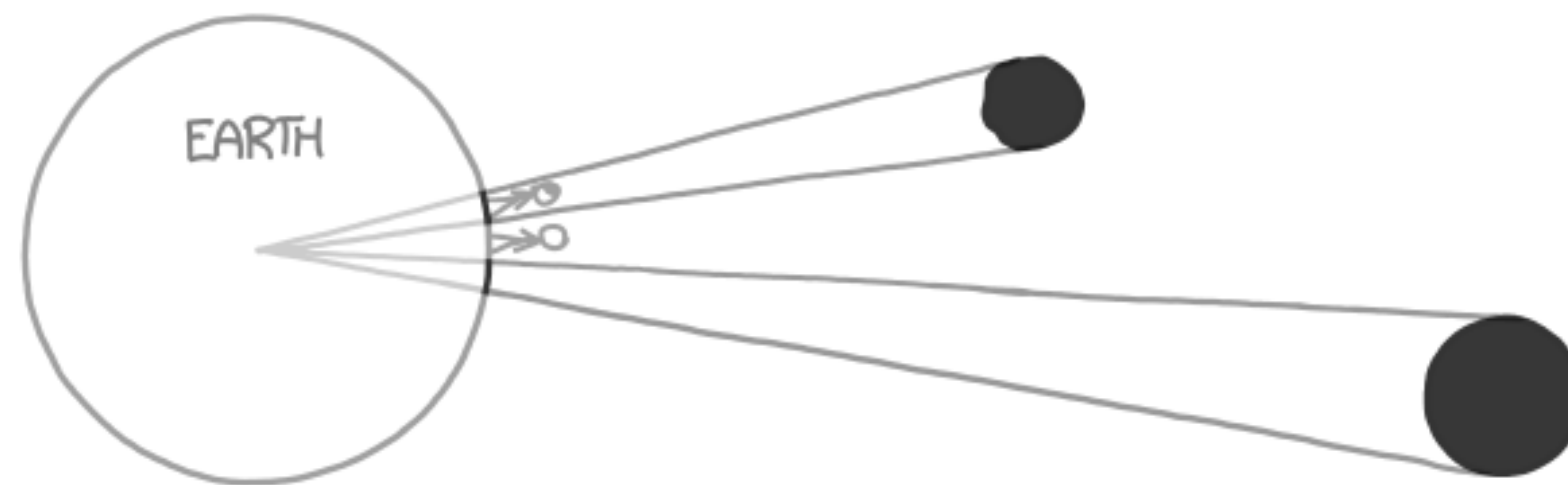
Solid angle: ratio of subtended area on sphere to radius squared

- $\Omega = \frac{A}{r^2}$
- Sphere has 4π **steradians**



Solid Angles in Practice

THE SIZE OF THE PART OF EARTH'S SURFACE
DIRECTLY UNDER VARIOUS SPACE OBJECTS

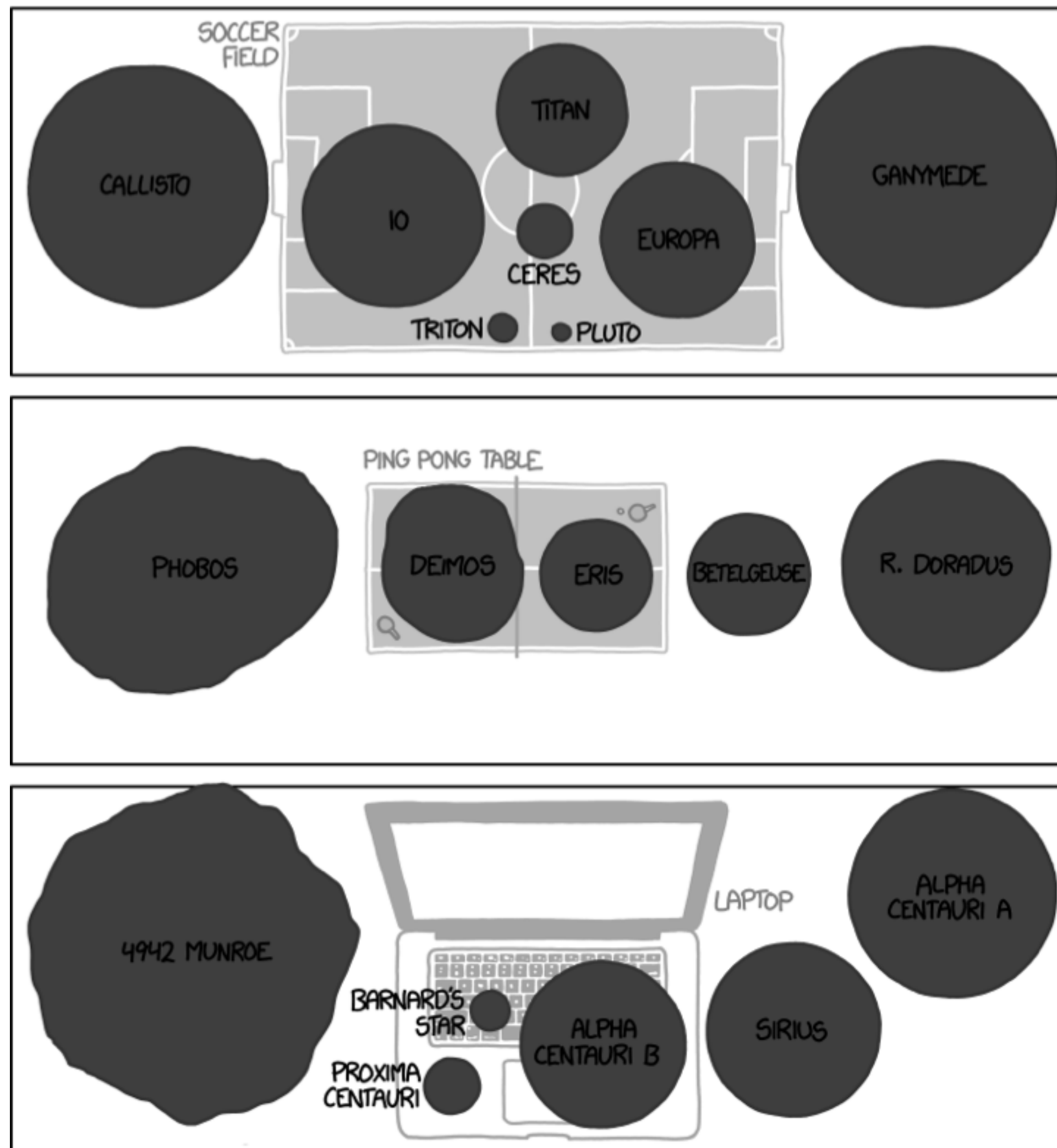


- Sun and moon both subtend $\sim 60\mu$ sr as seen from earth
- Surface area of earth: $\sim 510\text{M km}^2$
- Projected area:

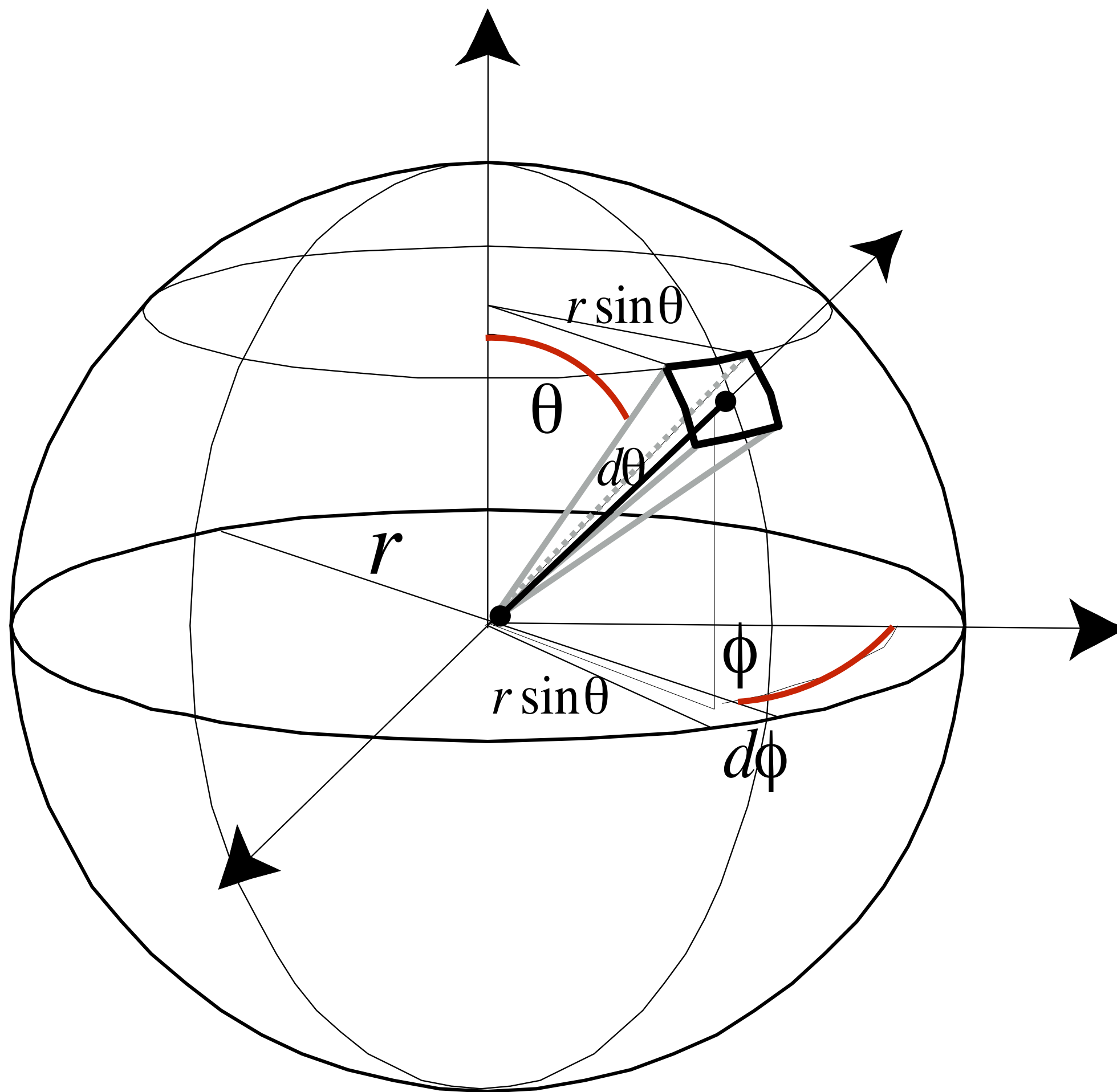
$$510\text{Mkm}^2 \frac{60\mu\text{sr}}{4\pi\text{sr}} = 510 \frac{15}{\pi} \approx 2400\text{km}^2$$

<http://xkcd.com/1276/>

Solid Angles in Practice



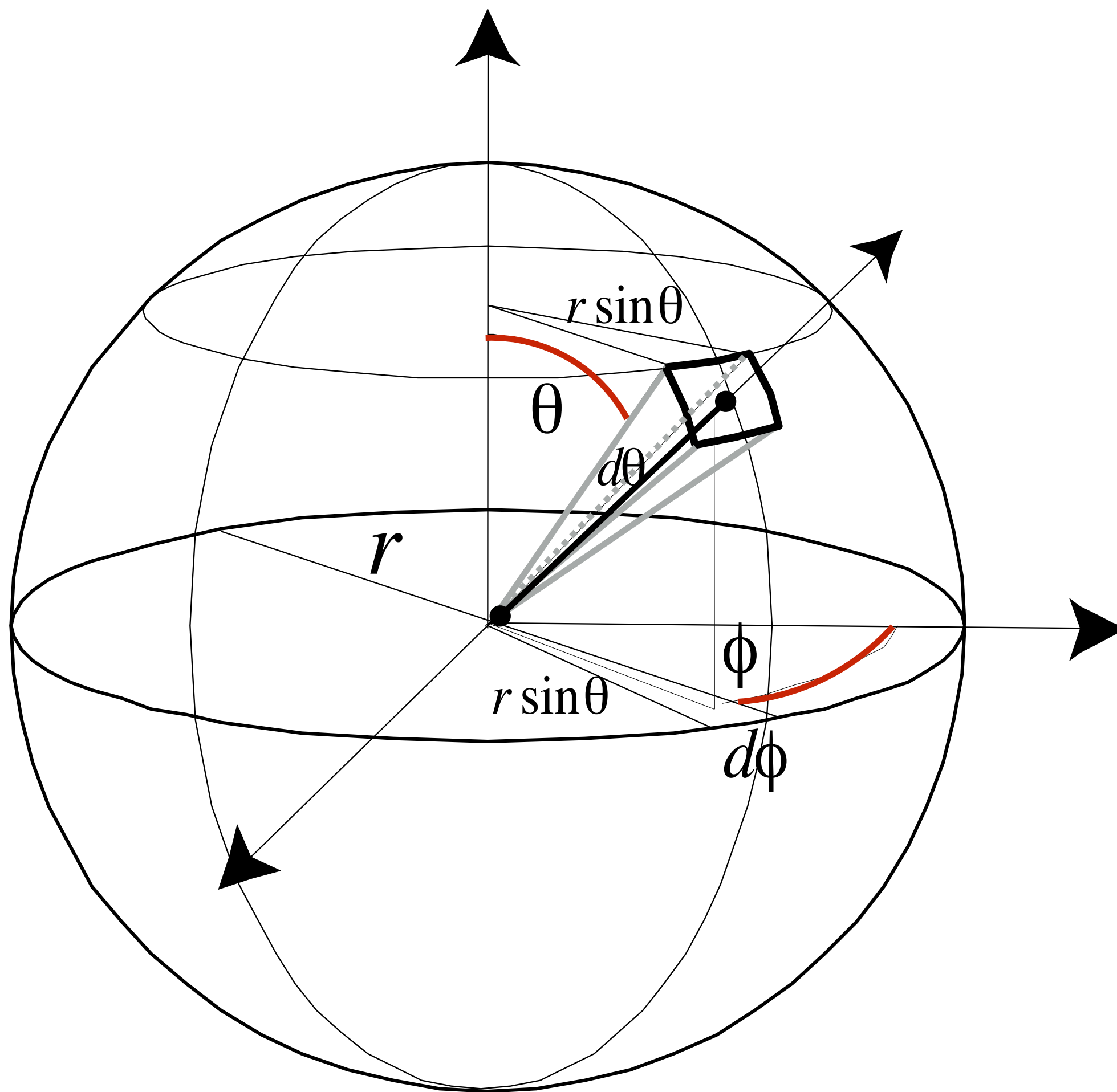
Differential Solid Angles



$$\begin{aligned} dA &= (r d\theta)(r \sin \theta d\phi) \\ &= r^2 \sin \theta d\theta d\phi \end{aligned}$$

$$d\omega = \frac{dA}{r^2} = \sin \theta d\theta d\phi$$

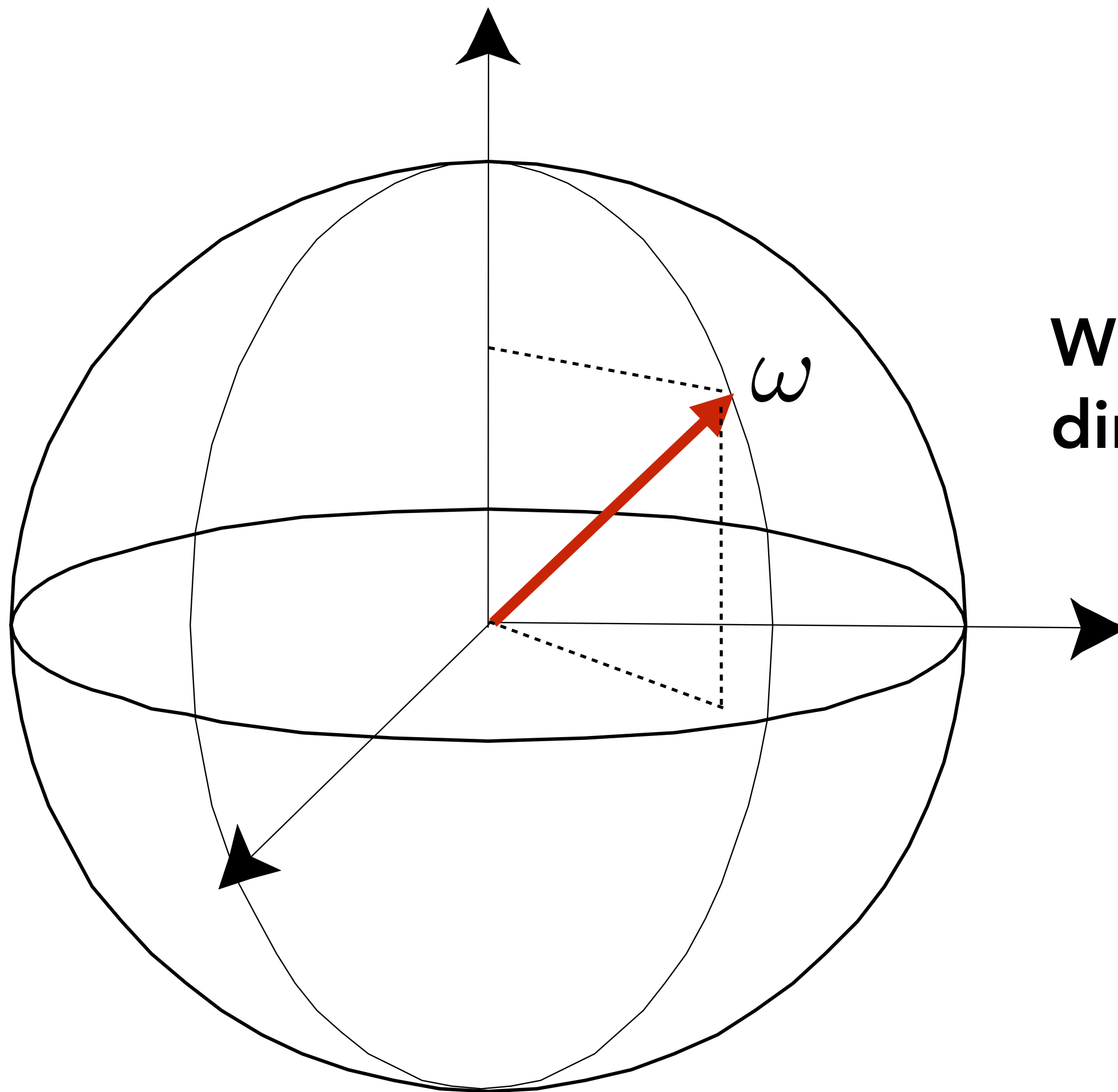
Differential Solid Angles



Sphere: S^2

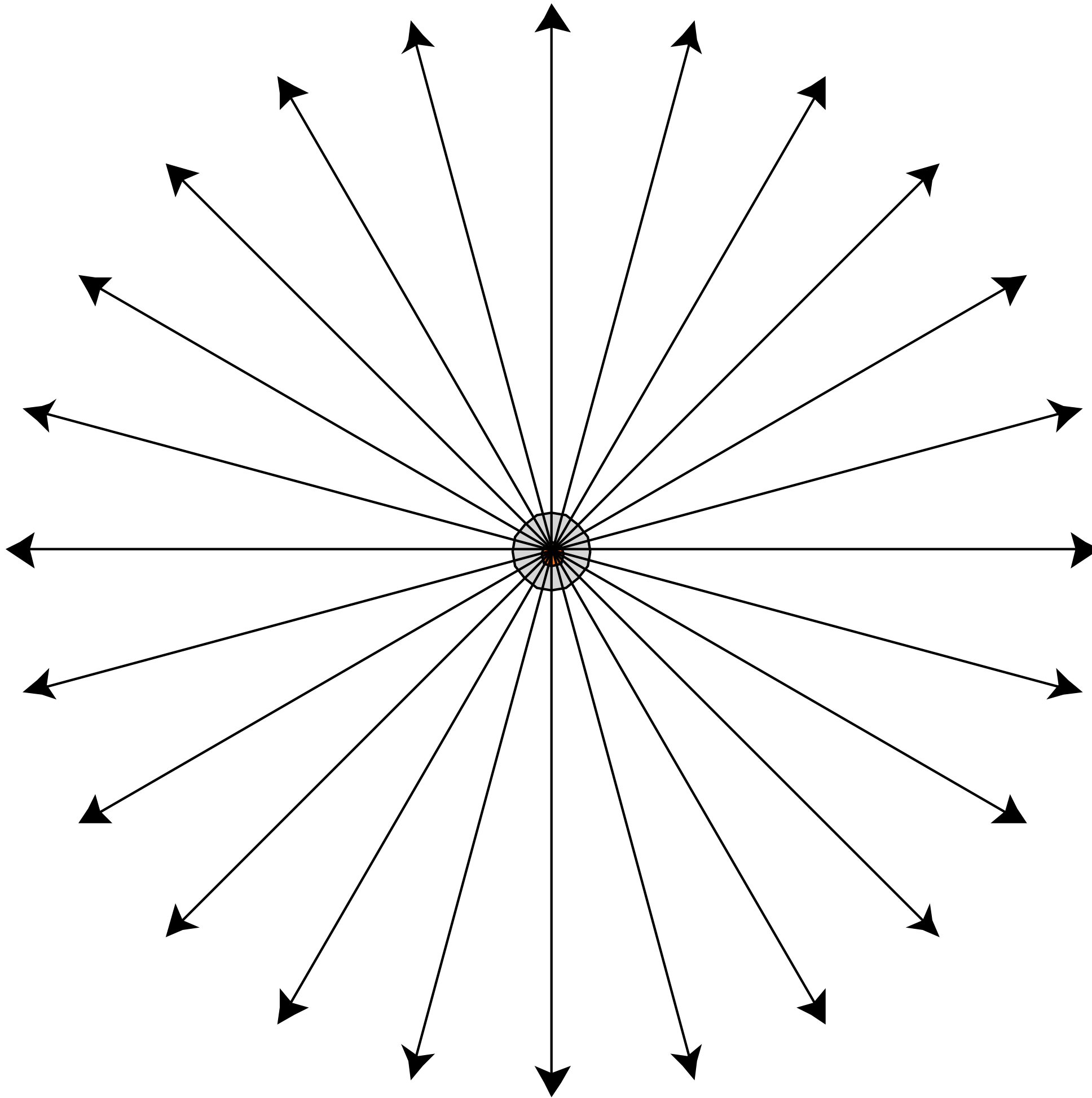
$$\begin{aligned}\Omega &= \int_{S^2} d\omega \\ &= \int_0^{2\pi} \int_0^\pi \sin \theta d\theta d\phi \\ &= 4\pi\end{aligned}$$

ω as a direction vector



Will use ω to denote a
direction vector (unit length)

Isotropic Point Source



$$\Phi = \int_{S^2} I \, d\omega$$
$$= 4\pi I$$

$$I = \frac{\Phi}{4\pi}$$

Modern LED Light

Output: 815 lumens

(11W LED replacement
for 60W incandescent)

Luminous intensity?

Assume isotropic:

Intensity = $815 \text{ lumens} / 4\pi \text{ sr}$
= 65 candelas

If focused into 20° diameter
cone. Intensity = ??



Spectral Power Distribution

Describes distribution of energy by wavelength

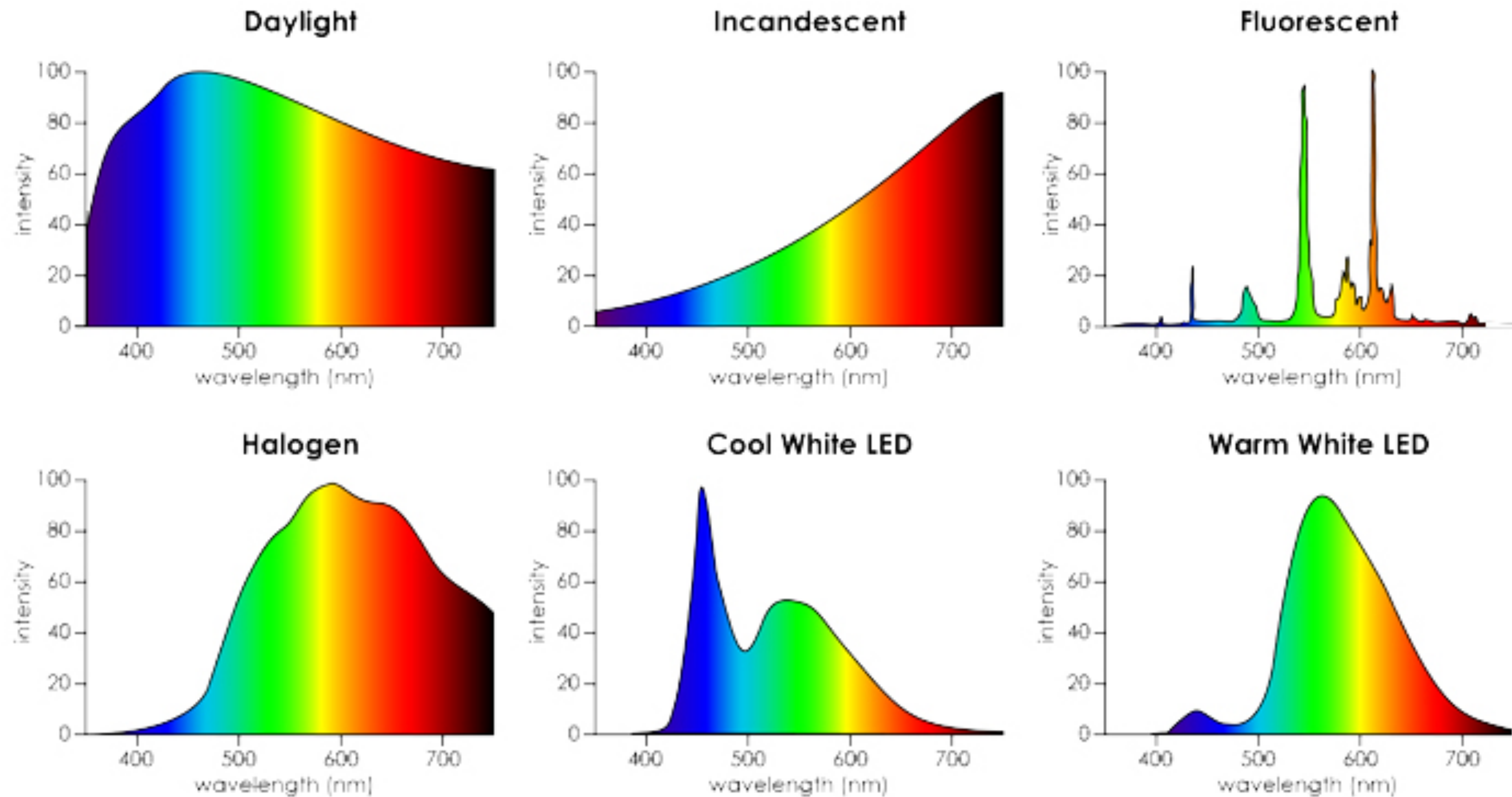
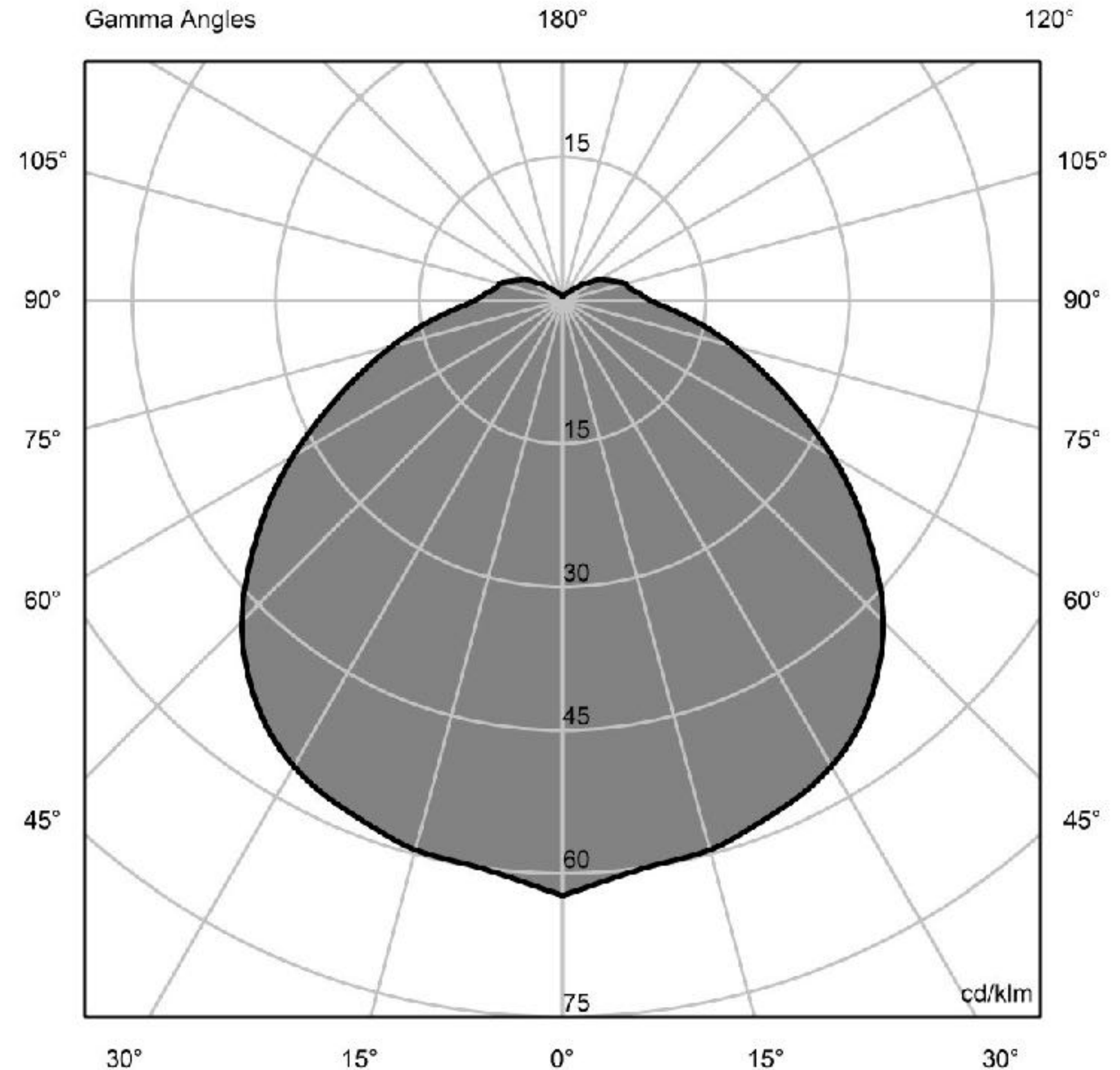


Figure credit:  **admesy**
ADVANCED MEASUREMENT SYSTEMS

Light Fixture Measurements - Goniometric Diagram



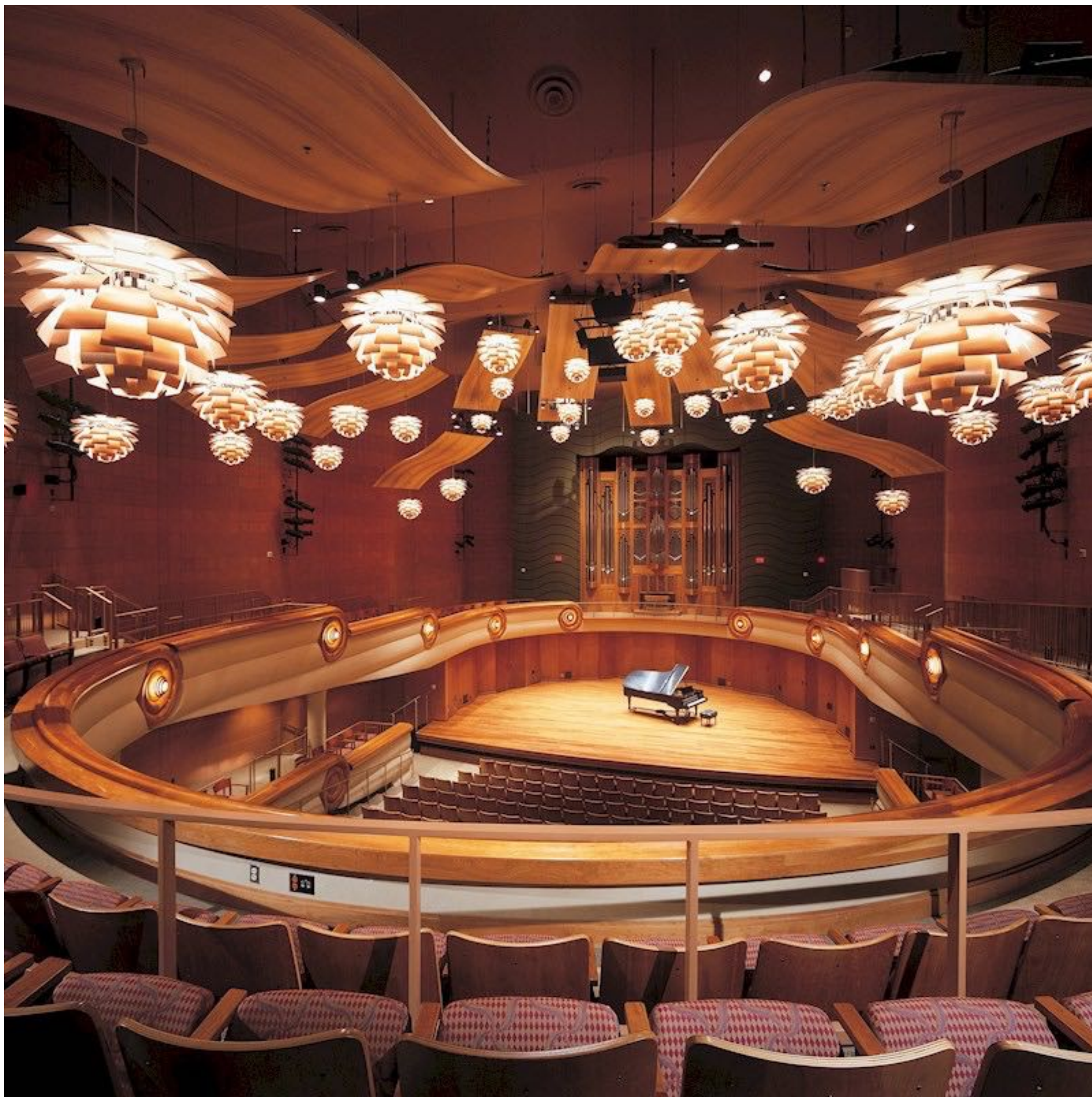
Poul Henningsen's Artichoke Lamp

CS184/284A

Polar AKA Goniometric Diagram

<http://www.louispoulsen.com/>

Ren Ng



<http://www.louispoulsen.com/>

PH Artichoke Lamps in Rivercenter for the Performing Arts, Georgia

Rendering with Goniometric Diagrams



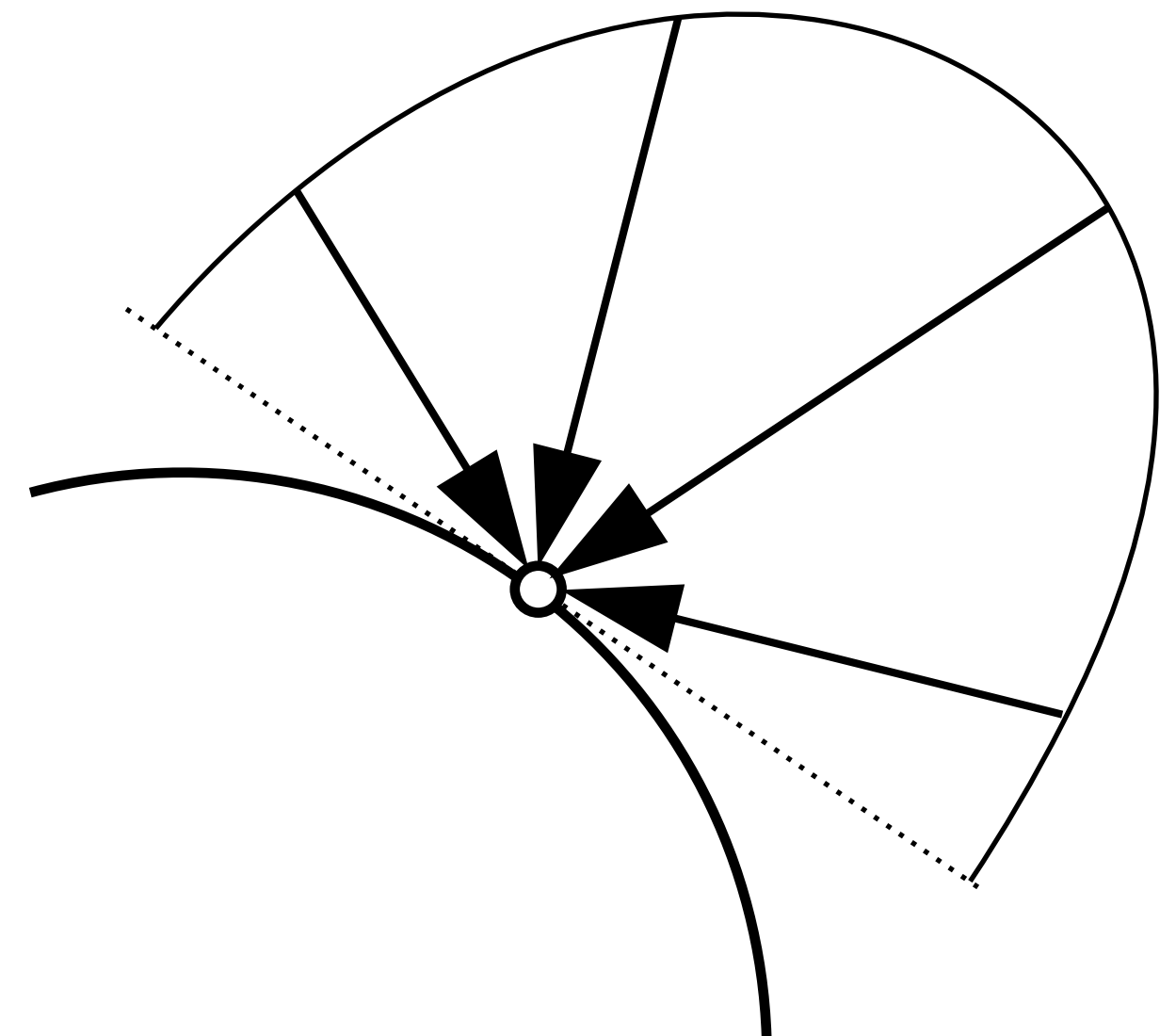
Irradiance

Irradiance

Definition: The irradiance (illuminance) is the power per unit area incident on a surface point.

$$E(\mathbf{x}) \equiv \frac{d\Phi(\mathbf{x})}{dA}$$

$$\left[\frac{\text{W}}{\text{m}^2} \right] \quad \left[\frac{\text{lm}}{\text{m}^2} = \text{lux} \right]$$



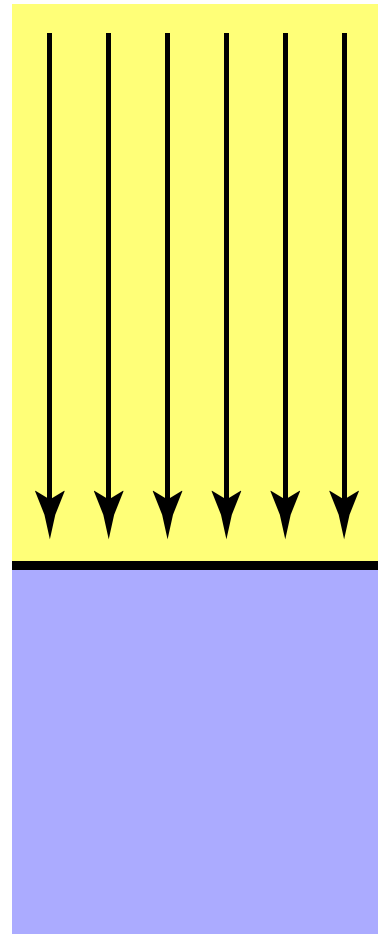
Typical Values of Illuminance [lm/m²]

Brightest sunlight	120,000 lux
Overcast day (midday)	15,000
Interior near window (daylight)	1,000
Residential artificial lighting	300
Sunrise / sunset	40
Illuminated city street	10
Moonlight (full)	0.02
Starlight	0.0003



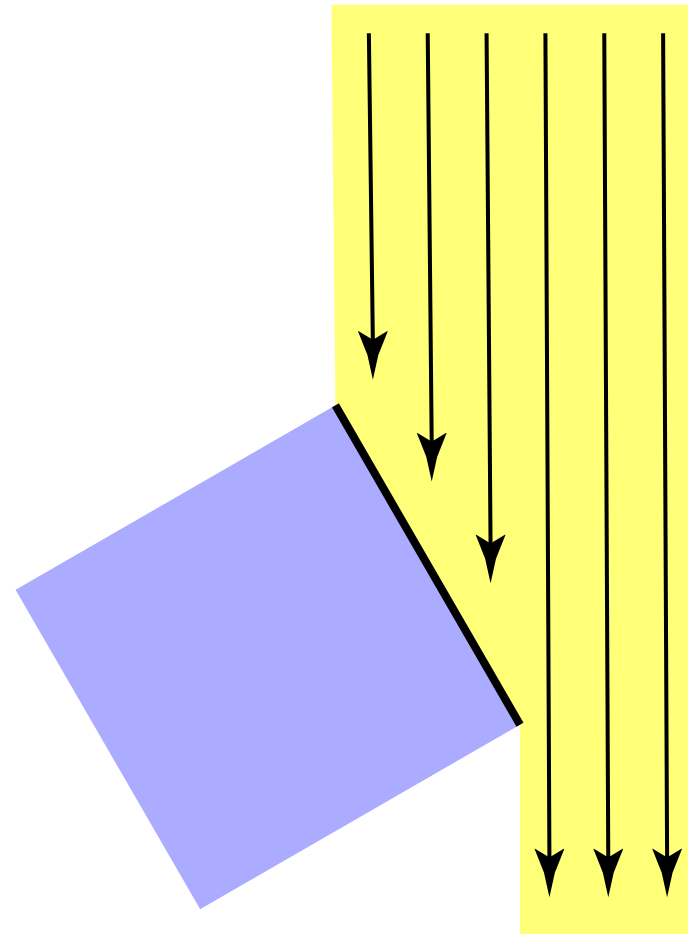
Light meter

Lambert's Cosine Law



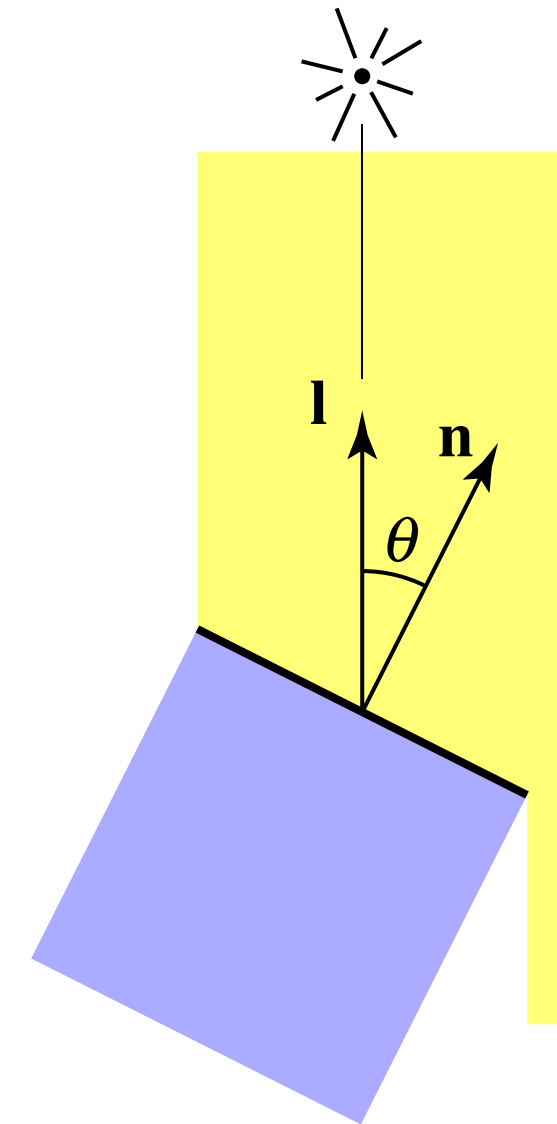
Top face of cube receives a certain amount of power

$$E = \frac{\Phi}{A}$$



Top face of 60° rotated cube receives half power

$$E = \frac{1}{2} \frac{\Phi}{A}$$

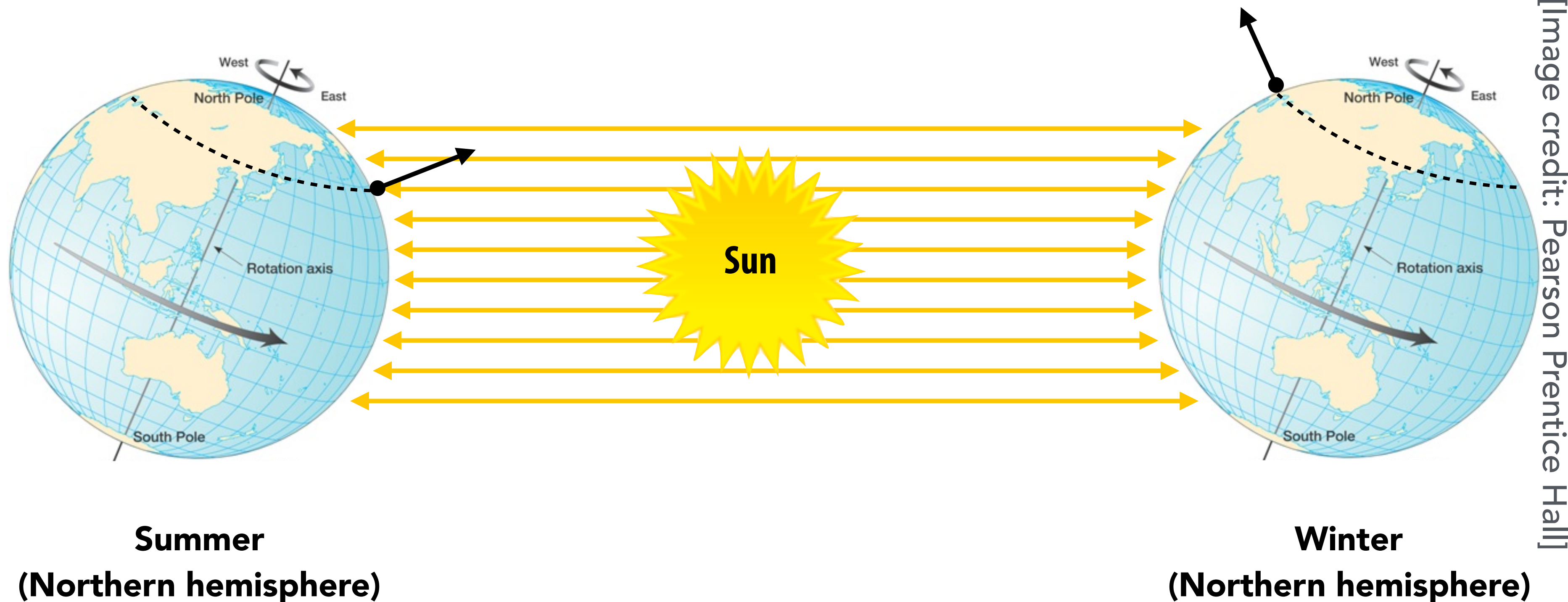


In general, power per unit area is proportional to $\cos \theta = l \cdot n$

$$E = \frac{\Phi}{A} \cos \theta$$

Irradiance at surface is proportional to cosine of angle between light direction and surface normal.

Why Do We Have Seasons?

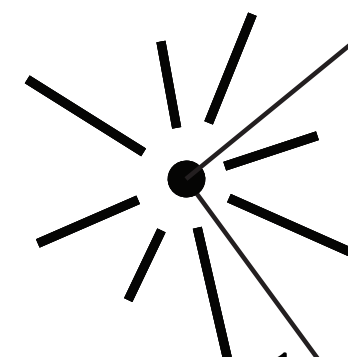


Earth's axis of rotation: $\sim 23.5^\circ$ off axis

Irradiance Falloff

Assume light is emitting
flux Φ in a uniform
angular distribution

Compare irradiance at
surface of two spheres:



1

intensity
here: E

$$E = \frac{\Phi}{4\pi}$$

r

intensity
here: E/r^2

$$E' = \frac{\Phi}{4\pi r^2} = \frac{E}{r^2}$$

Attendance

If you are seated in class, go to this form and sign in:

- <https://tinyurl.com/184lecture>

Notes:

- Time-stamp will be taken when you submit form.
Do it now, won't count later.
- Don't tell friends outside class to fill it out now, because we will audit at some point in semester.
- Failing audit will have large negative consequence.
You don't need to, because you have an alternative!

Radiance

Radiance

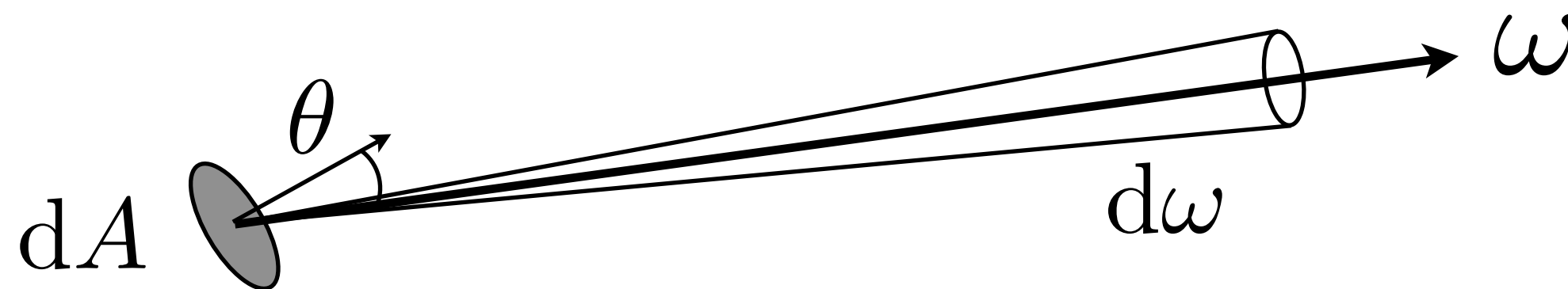


Light Traveling Along A Ray

1. Radiance is the fundamental field quantity that describes the distribution of light in an environment
 - Radiance is the quantity associated with a ray
 - Rendering is all about computing radiance
2. Radiance is invariant along a ray in a vacuum

Surface Radiance

Definition: The radiance (luminance) is the power emitted, reflected, transmitted or received by a surface, per unit solid angle, per unit projected area.



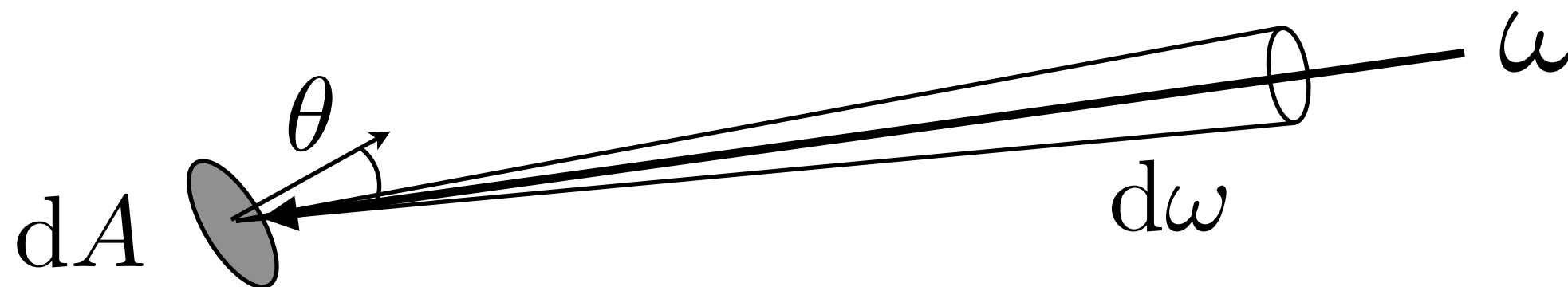
$$L(p, \omega) \equiv \frac{d^2 \Phi(p, \omega)}{d\omega \, dA \cos \theta}$$

$\cos \theta$ accounts for projected surface area

$$\left[\frac{\text{W}}{\text{sr m}^2} \right] \left[\frac{\text{cd}}{\text{m}^2} = \frac{\text{lm}}{\text{sr m}^2} = \text{nit} \right]$$

Incident Surface Radiance

Equivalent: Incident surface radiance (luminance) is the irradiance per unit solid angle arriving at the surface.

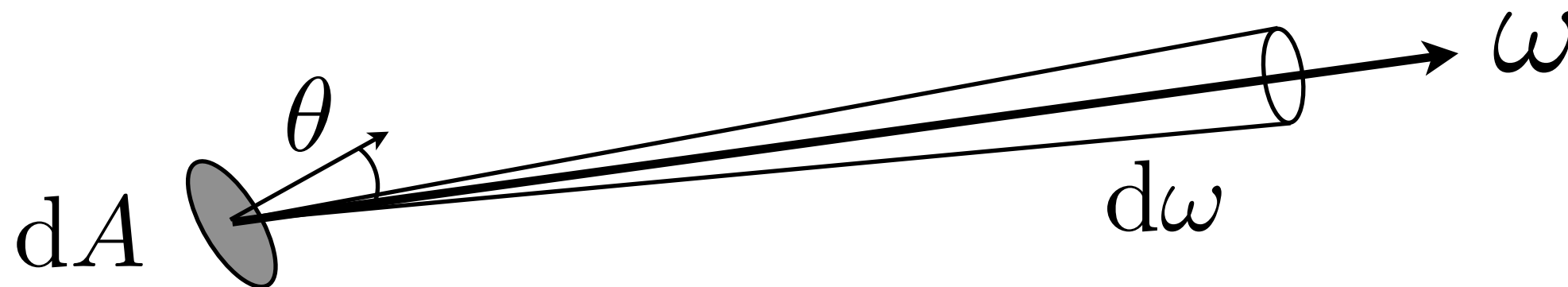


$$L(p, \omega) = \frac{dE(p)}{d\omega \cos \theta}$$

i.e. it is the light arriving at the surface along a given ray (point on surface and incident direction).

Exiting Surface Radiance

Equivalent: Exiting surface radiance (luminance) is the intensity per unit projected area leaving the surface.

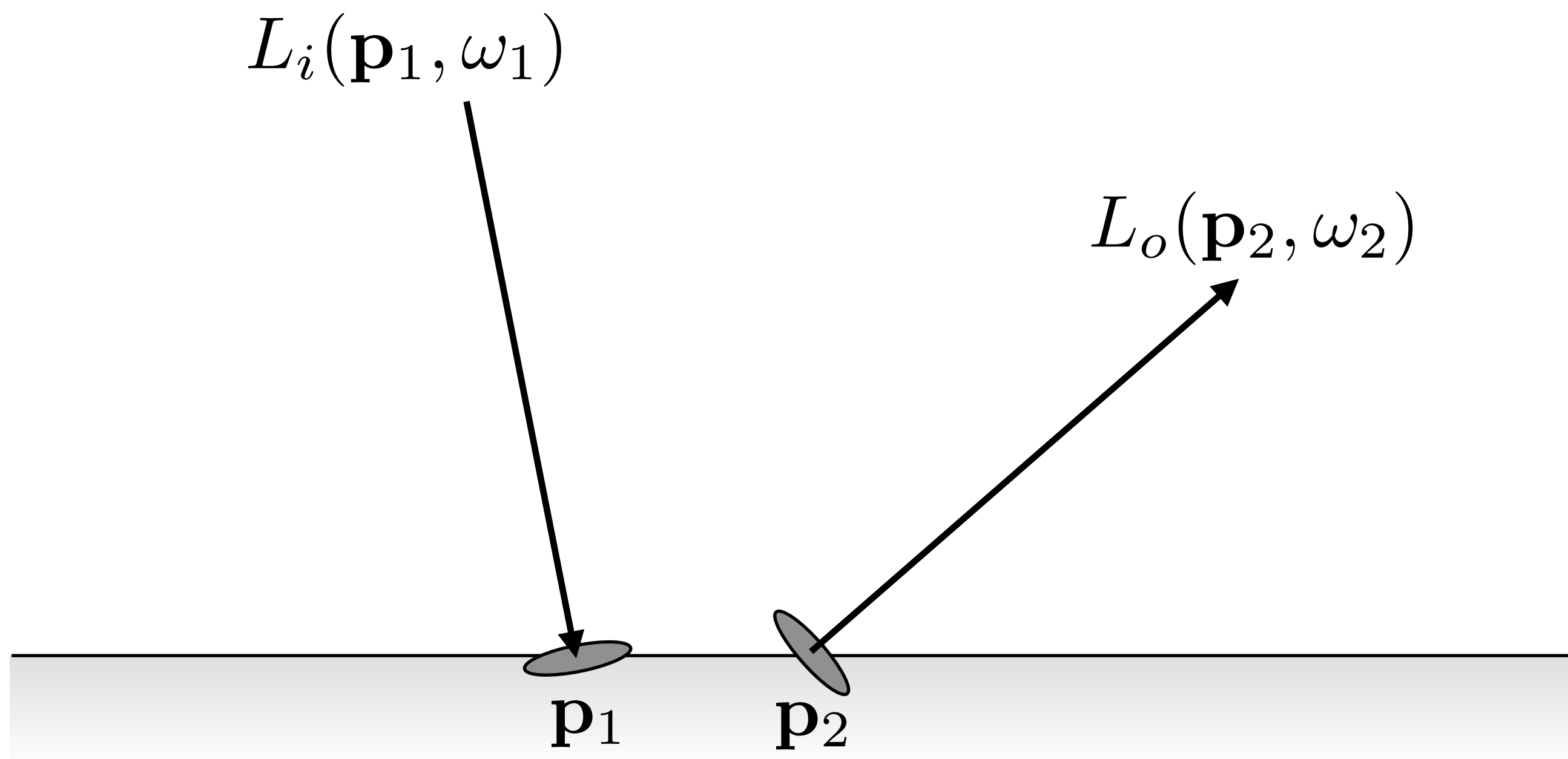


$$L(p, \omega) = \frac{dI(p, \omega)}{dA \cos \theta}$$

e.g. for an area light it is the light emitted along a given ray (point on surface and exit direction).

Incident & Exiting Surface Radiance Differ!

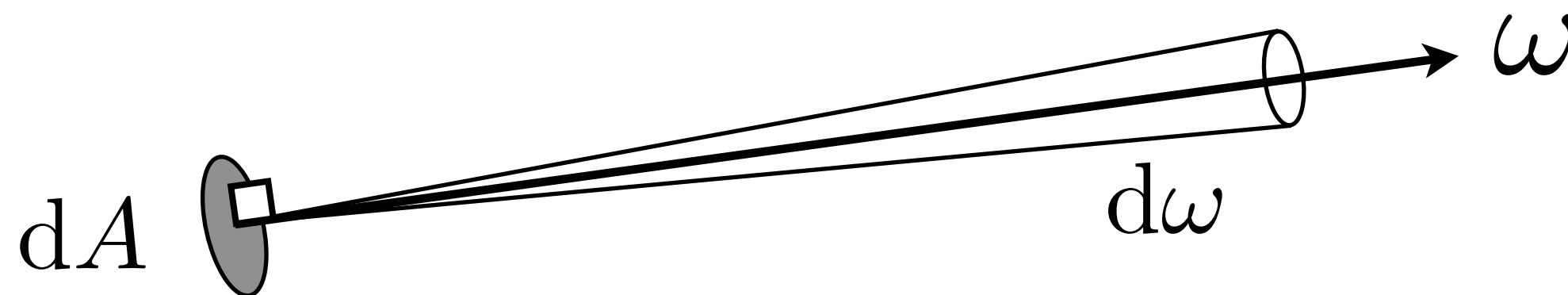
Need to distinguish between incident radiance and exitant radiance functions at a point on a surface



In general: $L_i(\mathbf{p}, \omega) \neq L_o(\mathbf{p}, \omega)$

Field Radiance or Light Field

Definition: The field radiance (luminance) at a point in space in a given direction is the power per unit solid angle per unit area perpendicular to the direction.



Typical Values of Luminance [cd/m²]

Surface of the sun	2,000,000,000 nits
Sunlight clouds	30,000
Clear sky	3,000
Cellphone display	500
Overcast sky	300
Scene at sunrise	30
Scene lit by moon	0.001
Threshold of vision	0.000001

Calculating with Radiance

Irradiance from the Environment

Computing flux per unit area on surface, due to incoming light from all directions.

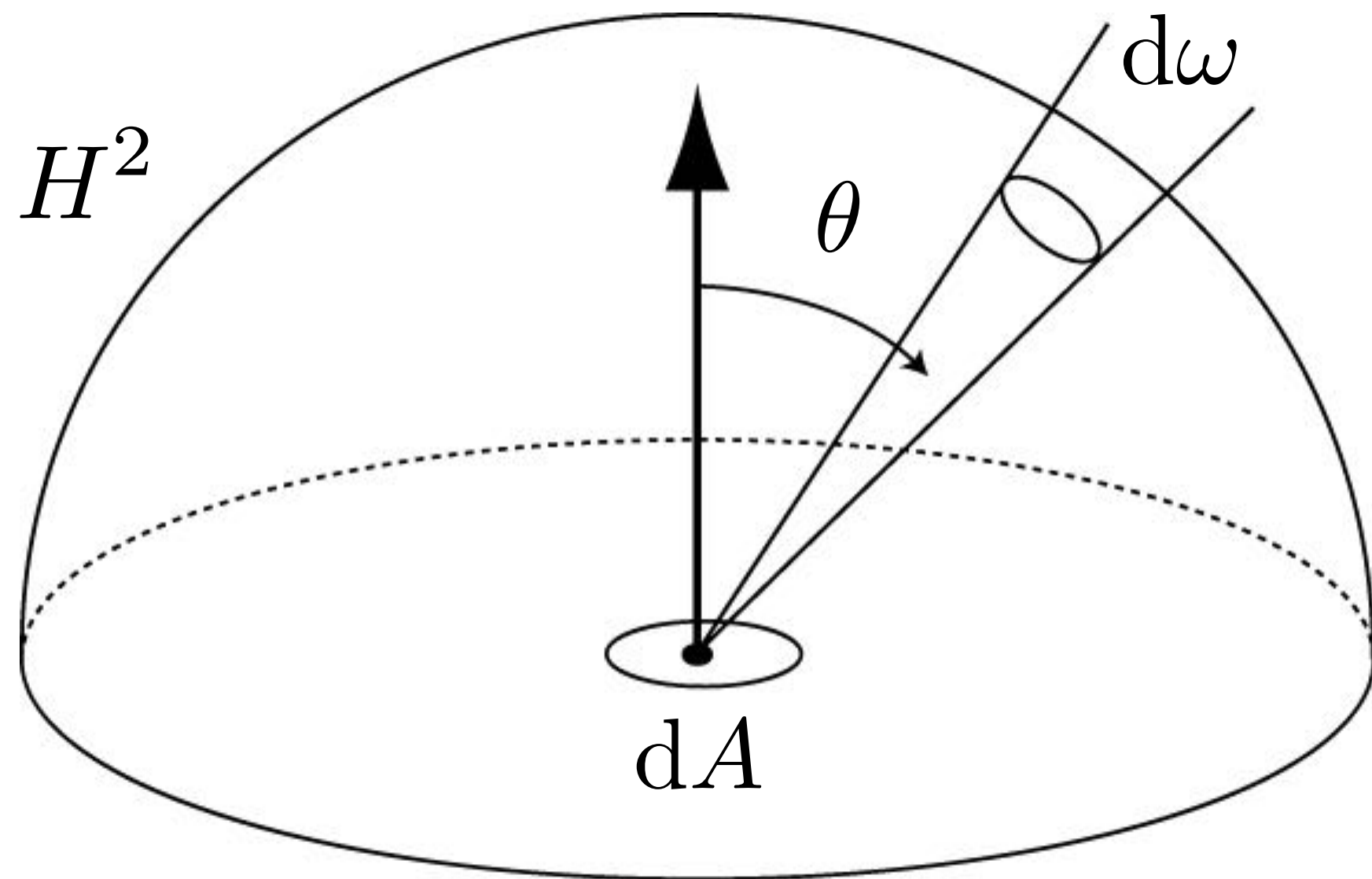
$$dE(p, \omega) = L_i(p, \omega) \cos \theta d\omega \quad \leftarrow \text{Contribution to irradiance from light arriving from direction } \omega$$

$$E(p) = \int_{H^2} L_i(p, \omega) \cos \theta d\omega$$



Light meter

Hemisphere: H^2

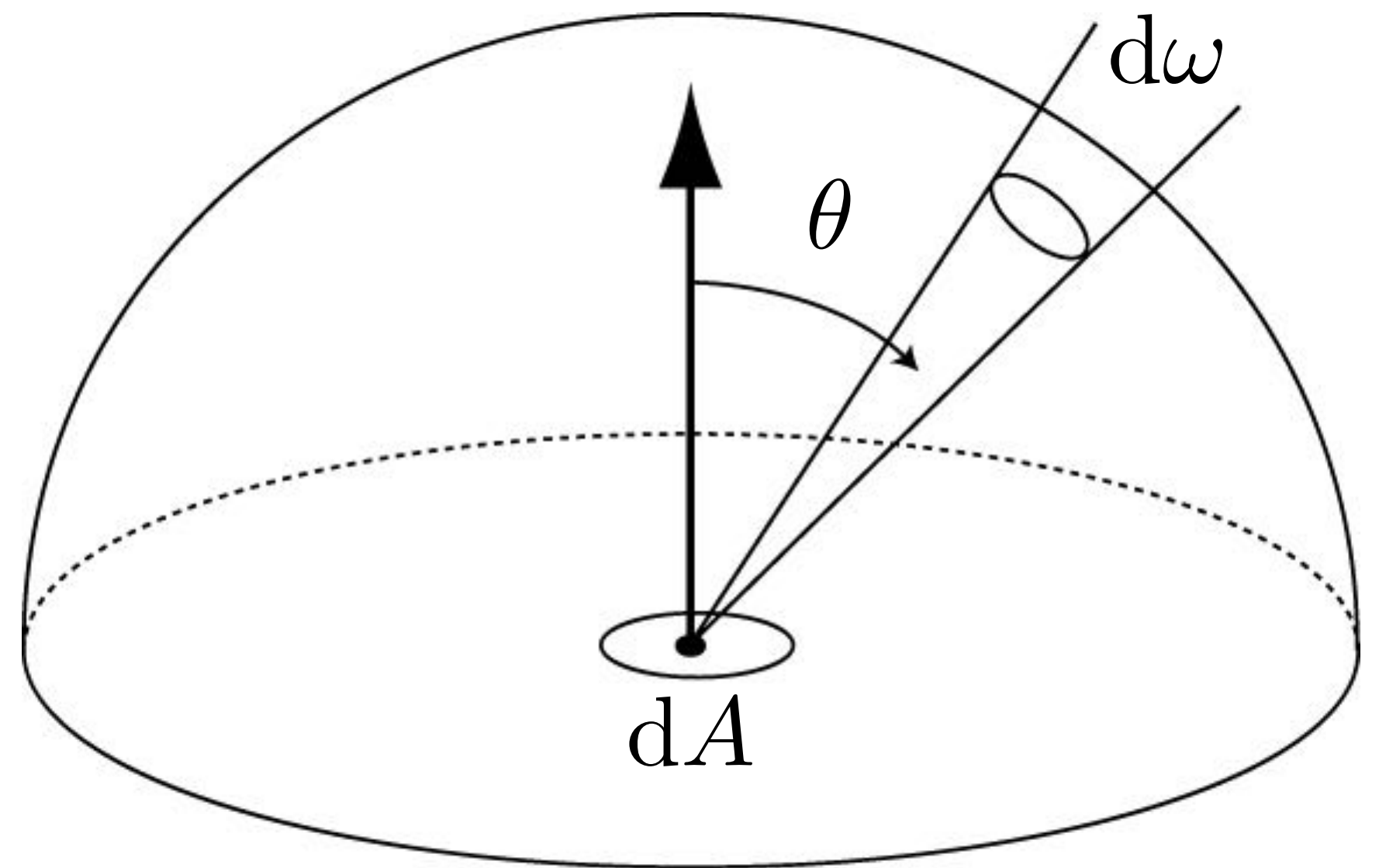


Irradiance from Uniform Hemispherical Light


$$\begin{aligned} E(p) &= \int_{H^2} L \cos \theta \, d\omega \\ &= L \int_0^{2\pi} \int_0^{\frac{\pi}{2}} \cos \theta \sin \theta \, d\theta \, d\phi \\ &= L \pi \end{aligned}$$

Two vertical arrows point from the text below to the constants π and $\cos \theta$ in the equations above.

Note: integral of cosine over hemisphere is only 1/2 the area of the hemisphere.

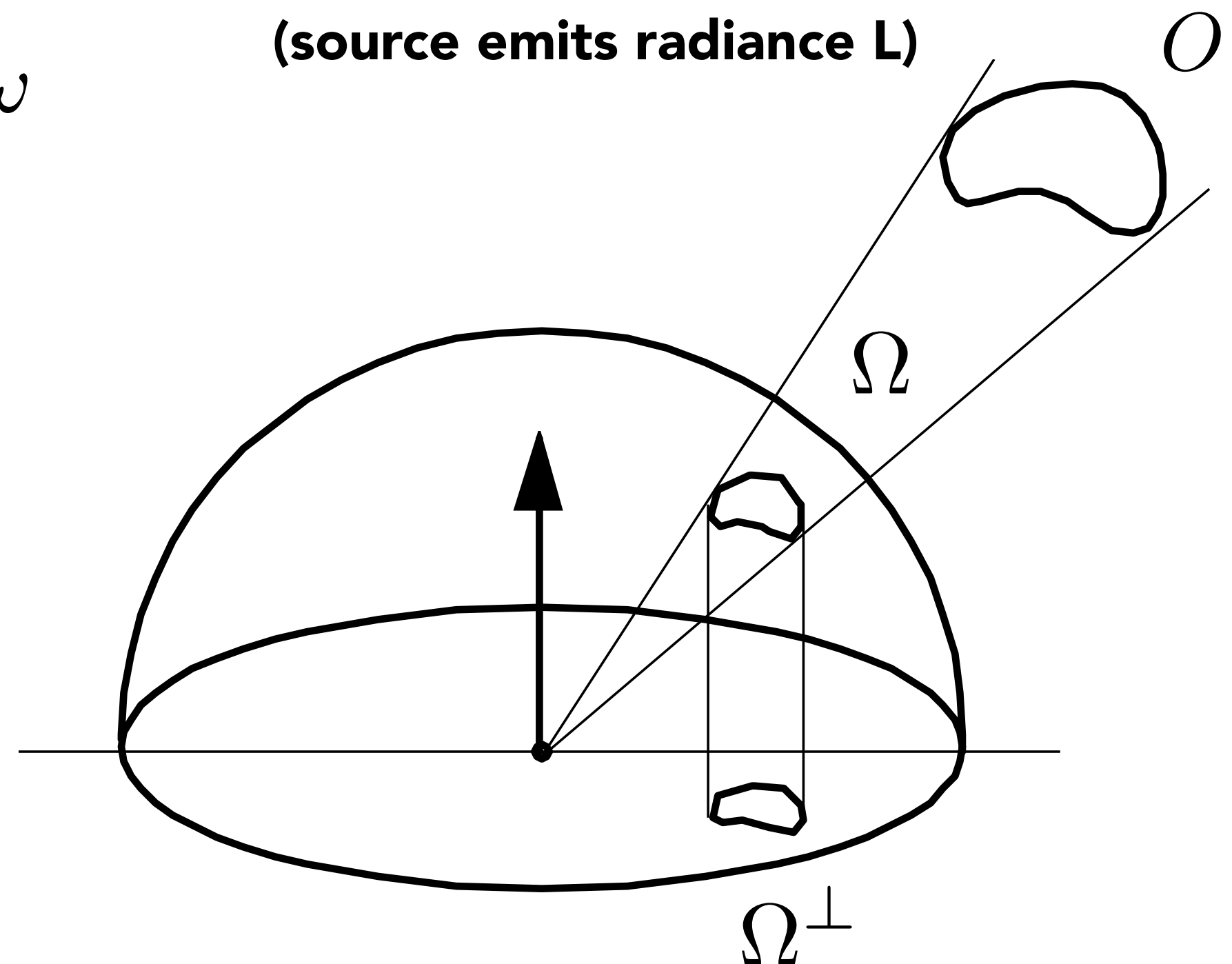


Irradiance from a Uniform Area Source

$$\begin{aligned} E(p) &= \int_{H^2} L(p, \omega) \cos \theta \, d\omega \\ &= L \int_{\Omega} \cos \theta \, d\omega \\ &= L \Omega^{\perp} \end{aligned}$$


Projected solid angle:

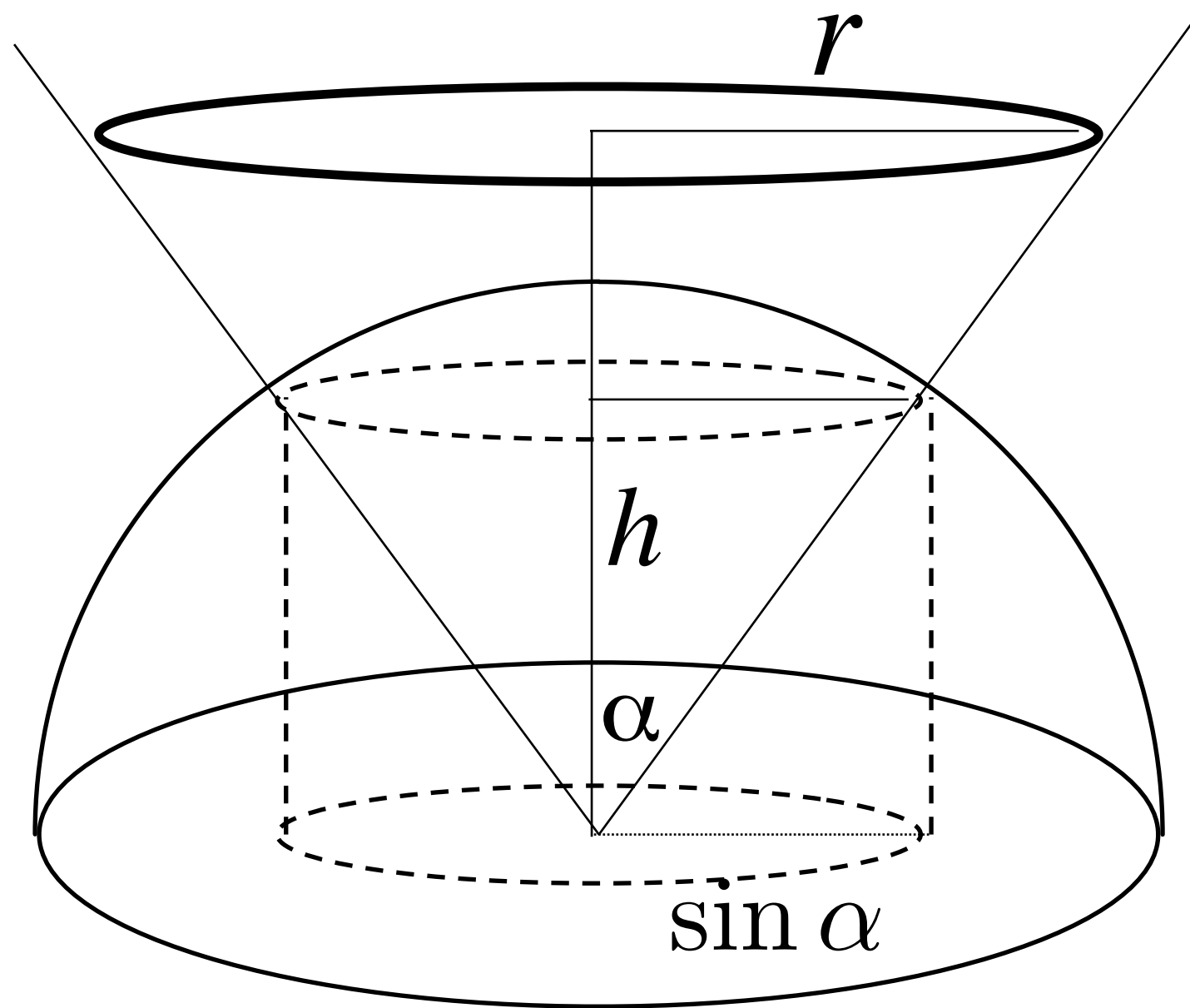
- **Cosine-weighted solid angle**
- **Area of object O projected onto unit sphere, then projected onto plane**



$$d\omega^{\perp} = |\cos \theta| \, d\omega$$

Uniform Disk Source Overhead

Geometric Derivation



$$\Omega^\perp = \pi \sin^2 \alpha$$

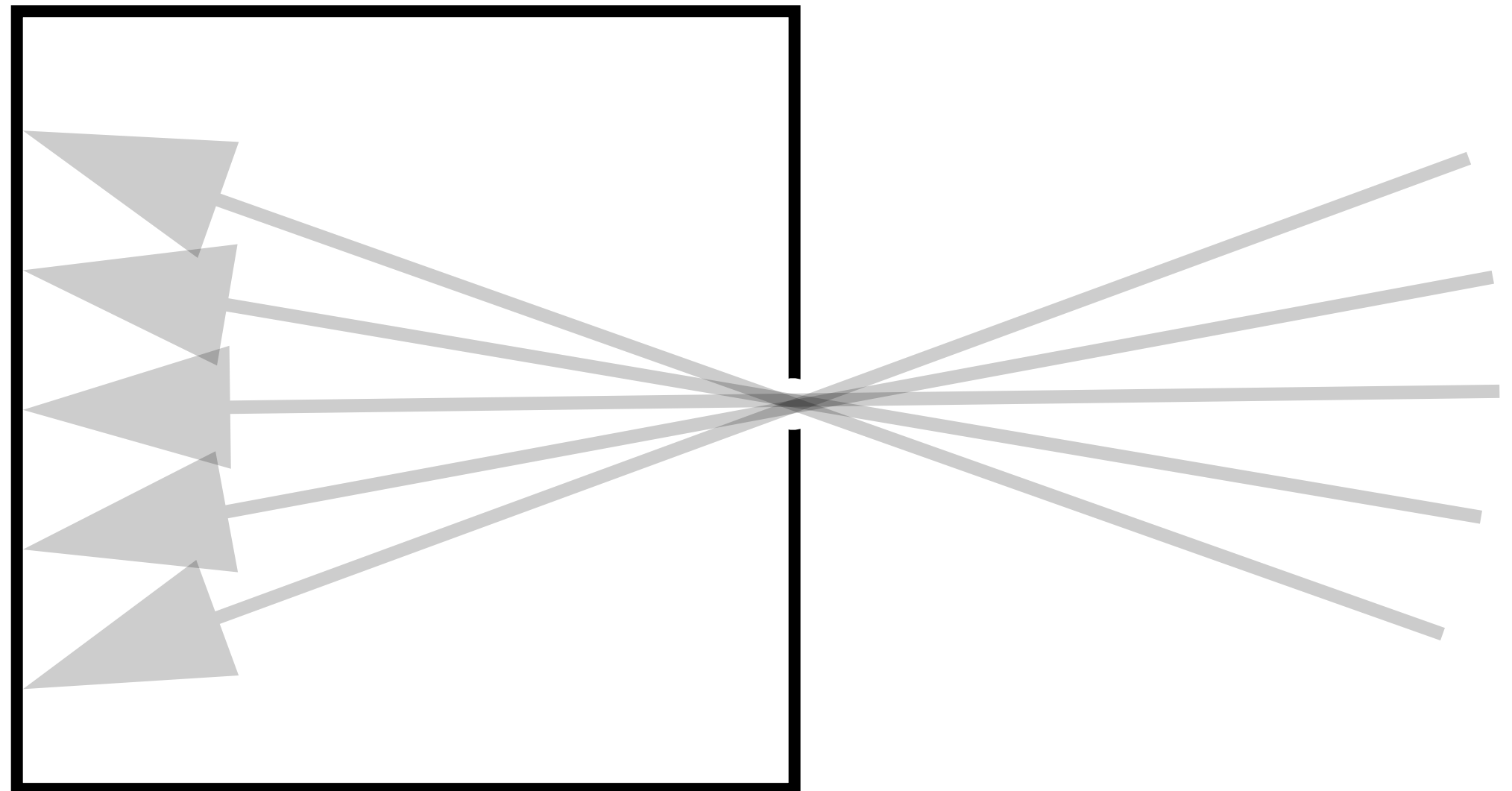
Algebraic Derivation

$$\begin{aligned}\Omega^\perp &= \int_0^{2\pi} \int_0^\alpha \cos \theta \sin \theta \, d\theta \, d\phi \\ &= 2\pi \left. \frac{\sin^2 \theta}{2} \right|_0^\alpha \\ &= \pi \sin^2 \alpha\end{aligned}$$

Measuring Radiance

A Pinhole Camera Samples Radiance

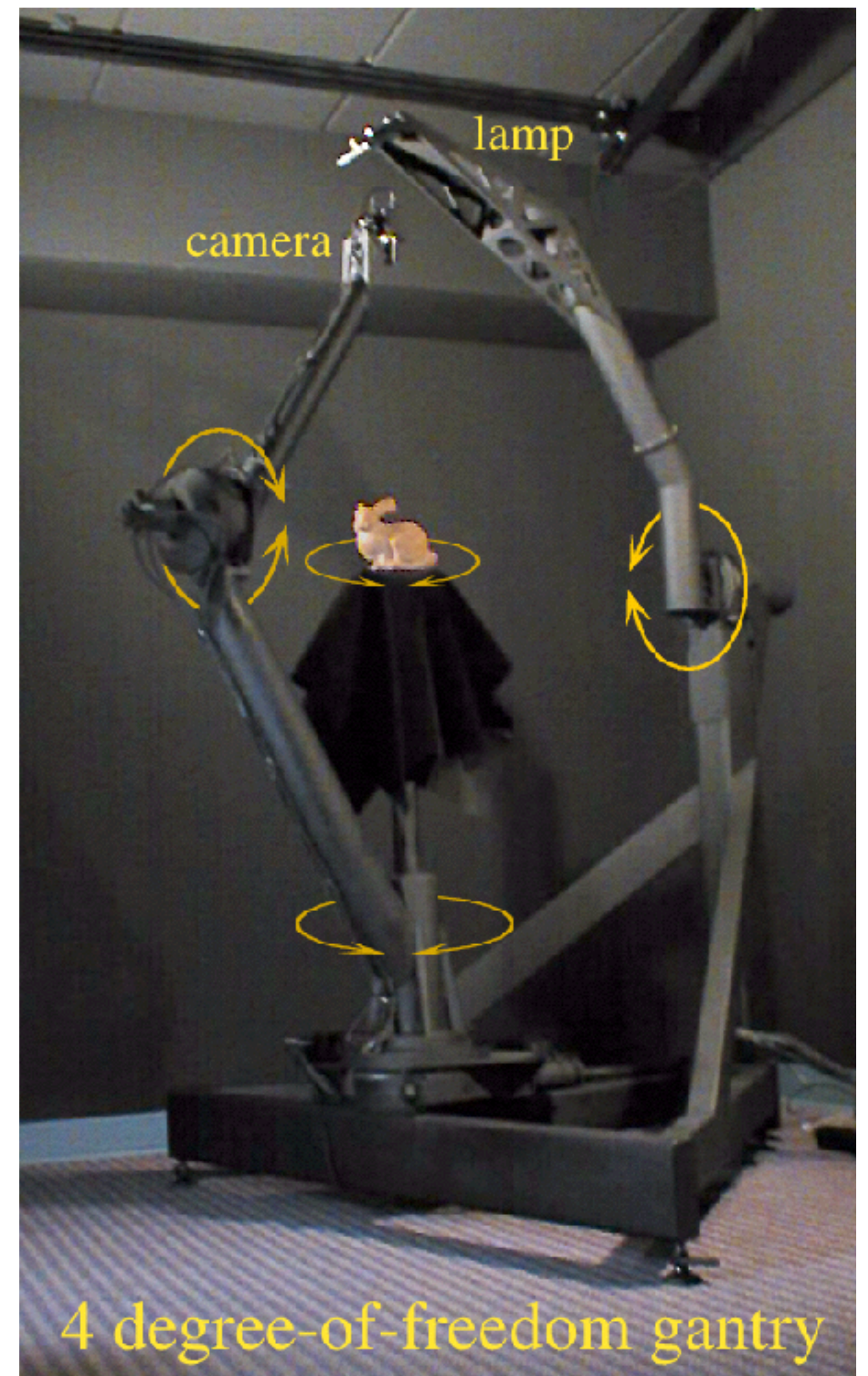
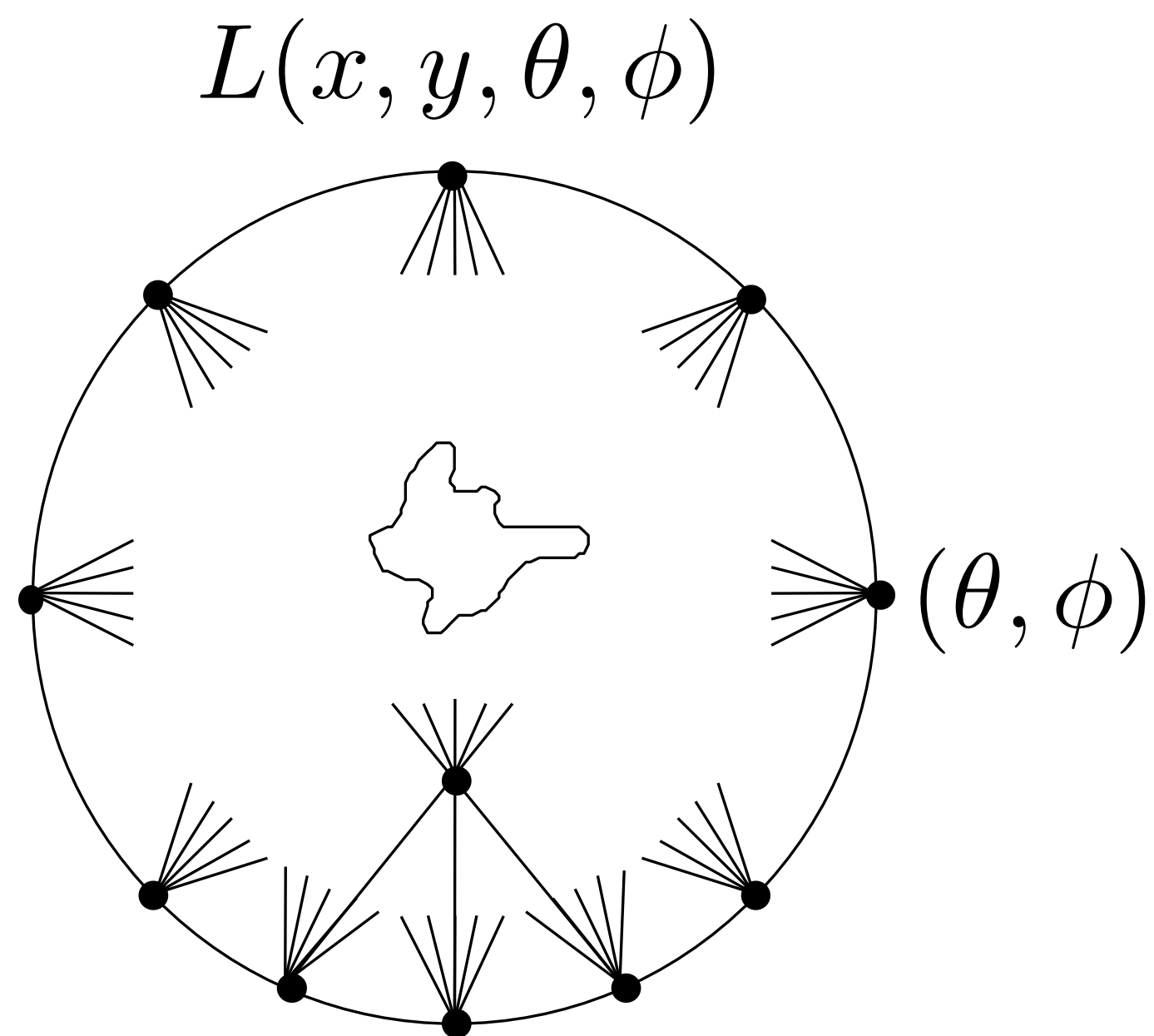
Photograph pixels
measure radiance
for rays passing
through pinhole in
different directions



Spherical Gantry \Rightarrow 4D Light Field

Take photographs of an object from all points on an enclosing sphere

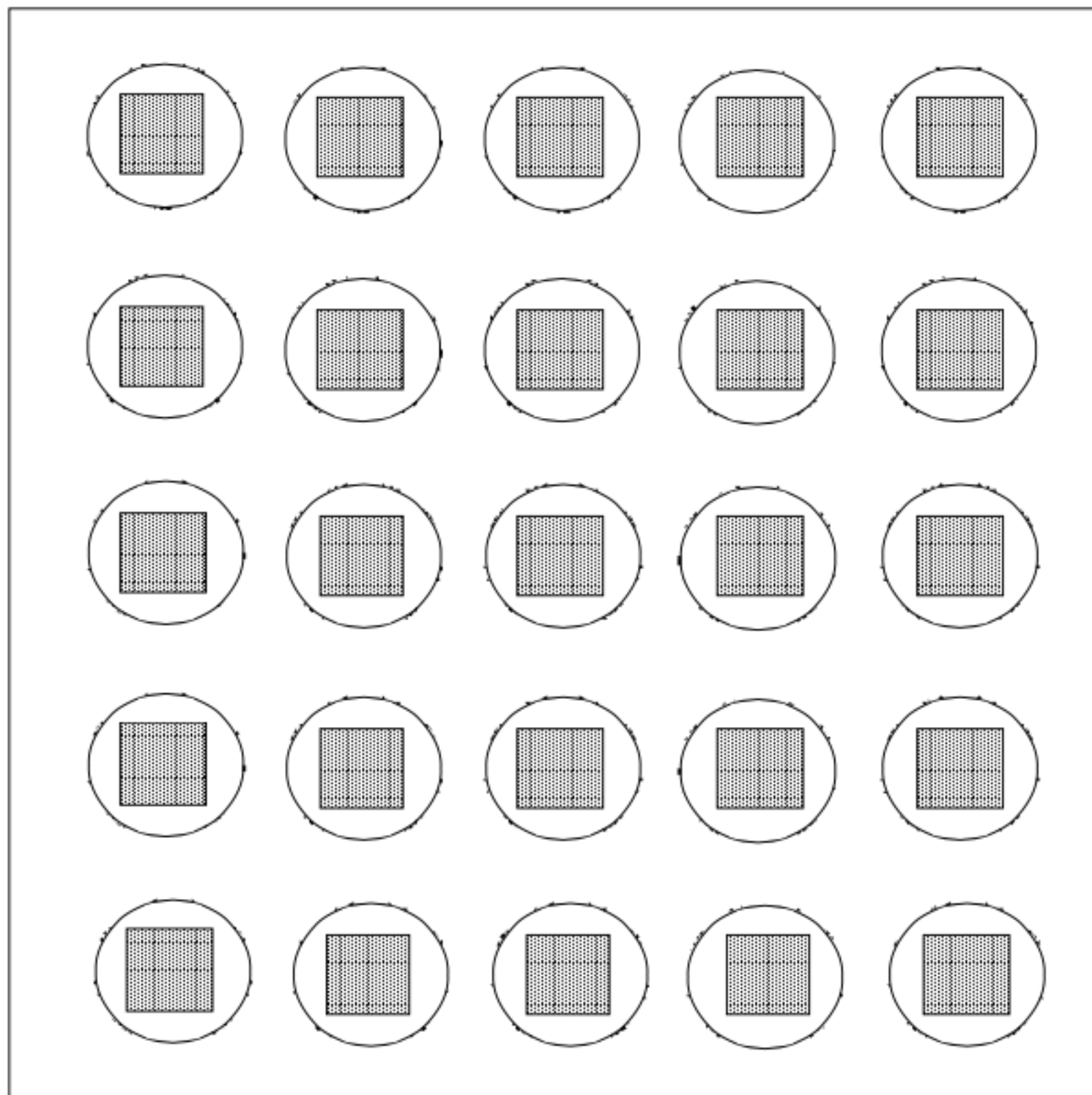
Captures all light leaving an object – like a hologram



Multi-Camera Array \Rightarrow 4D Light Field



Two-Plane Light Field



2D Array of Cameras



2D Array of Images

Radiometry & Photometry

Terms & Units

Radiometric & Photometric Terms & Units

Physics		Radiometry		Photometry	
			Units		Units
Energy	Q	Radiant Energy	Joules (W·sec)	Luminous Energy	Lumen·sec
Flux (Power)	Φ	Radiant Power	W	Luminous Power	Lumen (Candela sr)
Angular Flux Density	I	Radiant Intensity	W/sr	Luminous Intensity	Candela (Lumen/sr)
Spatial Flux Density	E	Irradiance (in) Radiosity (out)	W/m ²	Illuminance (in) Luminosity (out)	Lux (Lumen/m ²)
Spatio-Angular Flux Density	L	Radiance	W/m ² /sr	Luminance	Nit (Candela/m ²)

“Thus one nit is one lux per steradian is one candela per square meter is one lumen per square meter per steradian. Got it?” — *James Kajiya*

Things to Remember

Radiometry vs photometry: physics vs human response

Spatial measures of light:

- **Flux, intensity, irradiance, radiance**
- **Pinhole cameras and light field cameras**

Lighting calculations

- **Integration on sphere / hemisphere**
- **Cosine weight: project from hemisphere onto disk**
- **Photon counting**

BRDF: 4D function for material reflection at a point

Acknowledgments

Many thanks to Kayvon Fatahalian, Matt Pharr, Pat Hanrahan, and Steve Marschner for presentation resources.

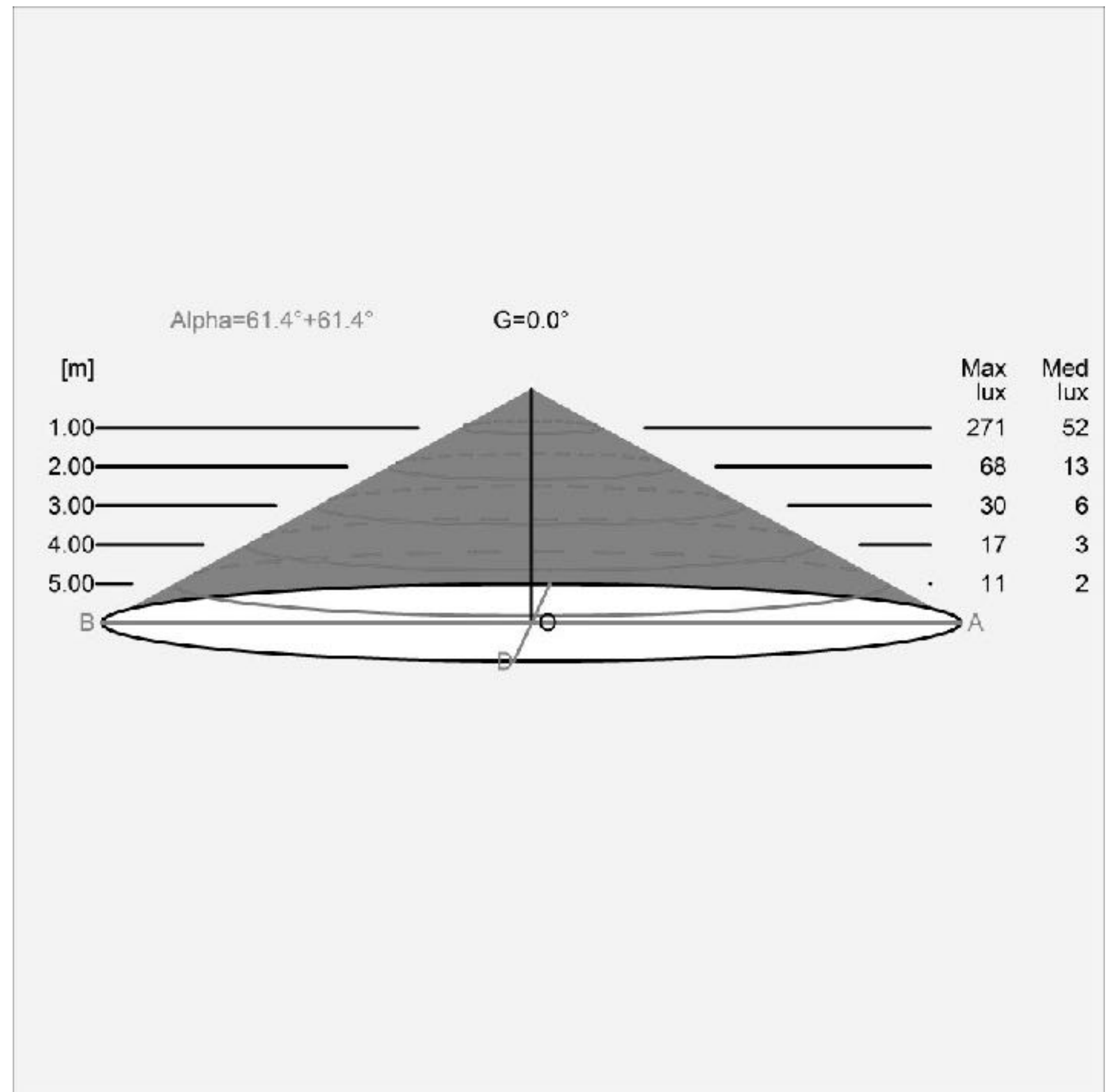
Extra

Light Fixture Measurements



Poul Henningsen's Artichoke Lamp

CS184/284A

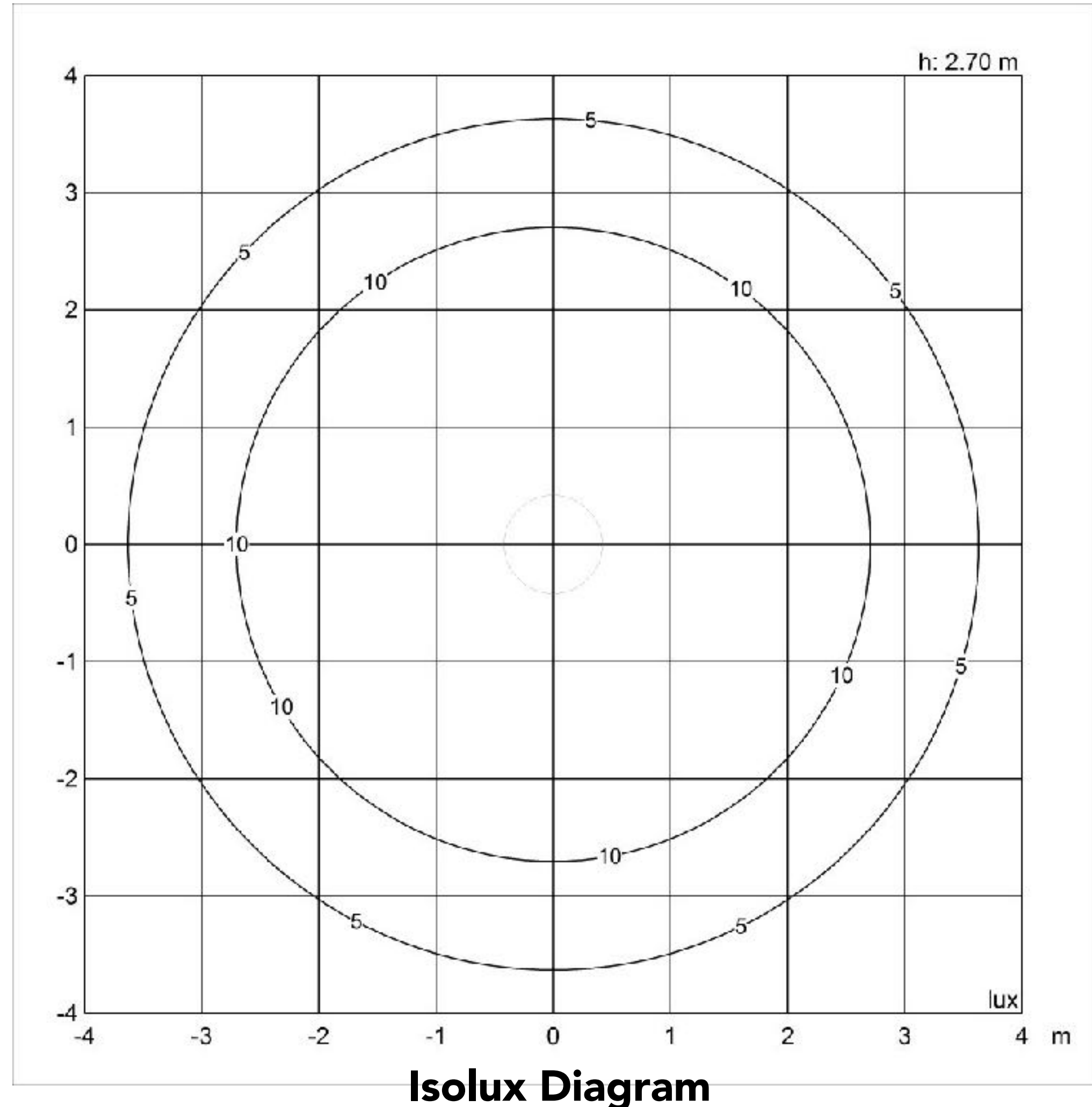


Cartesian Diagram

<http://www.louispuolsen.com/>

Ren Ng

Light Fixture Measurements



Poul Henningsen's Artichoke Lamp

CS184/284A

<http://www.louispuolsen.com/>

Ren Ng

Quantitative Photometry

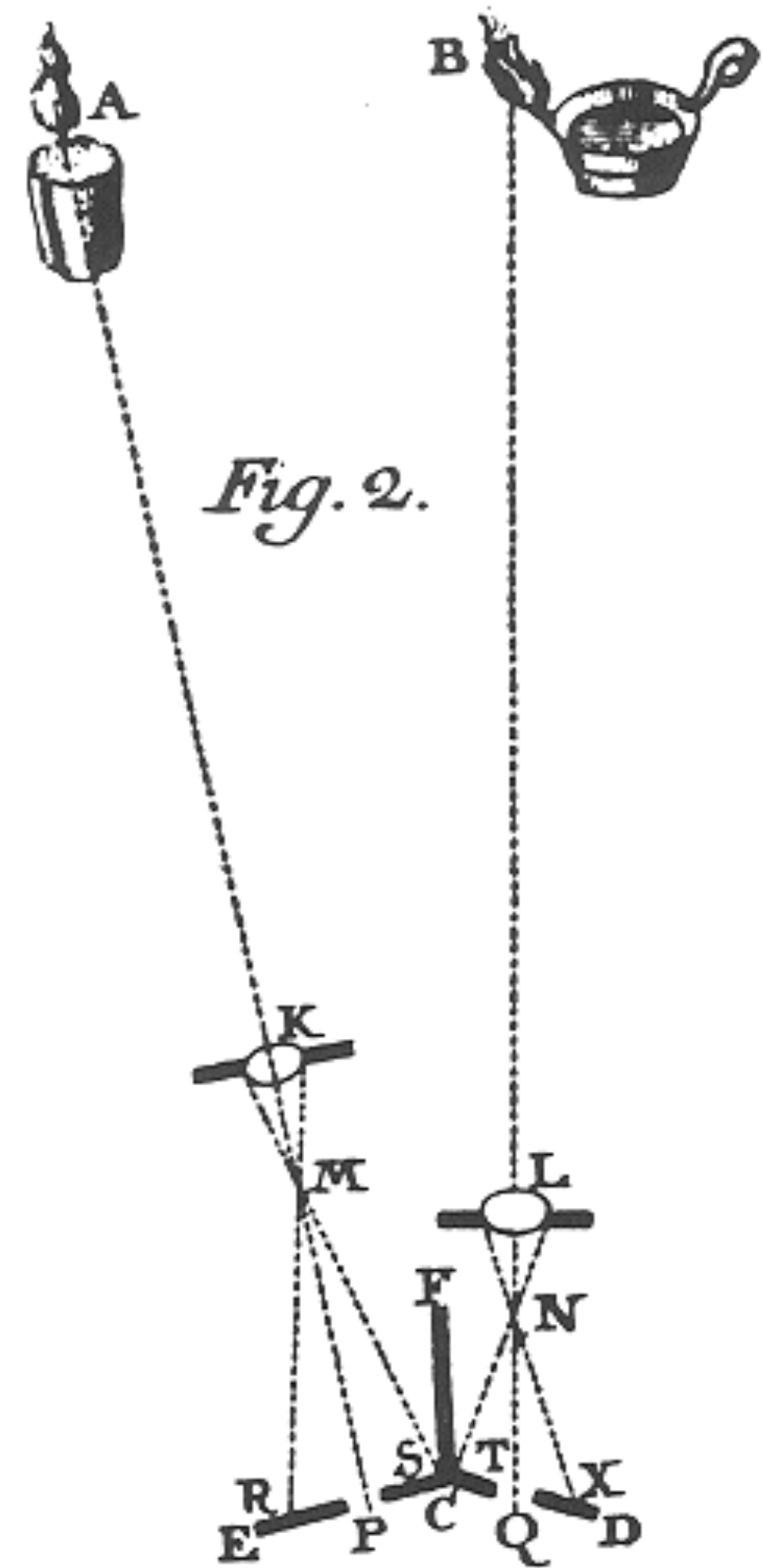
The Invention of Photometry

Bouguer's classic experiment

- Compare a light source and a candle
- Move until appear equally bright
- Intensity is proportional to ratio of distances squared

Definition of a candela

- Originally a "standard" candle
- Currently 555 nm laser with power $1/683 \text{ W/sr}$
- One of seven SI base units



Counting Photons

Given a sensor/light, we can count how many photons it receives/emits

- Over a period of time, gives the energy Q and flux (power) Φ received/emitted by the sensor/light

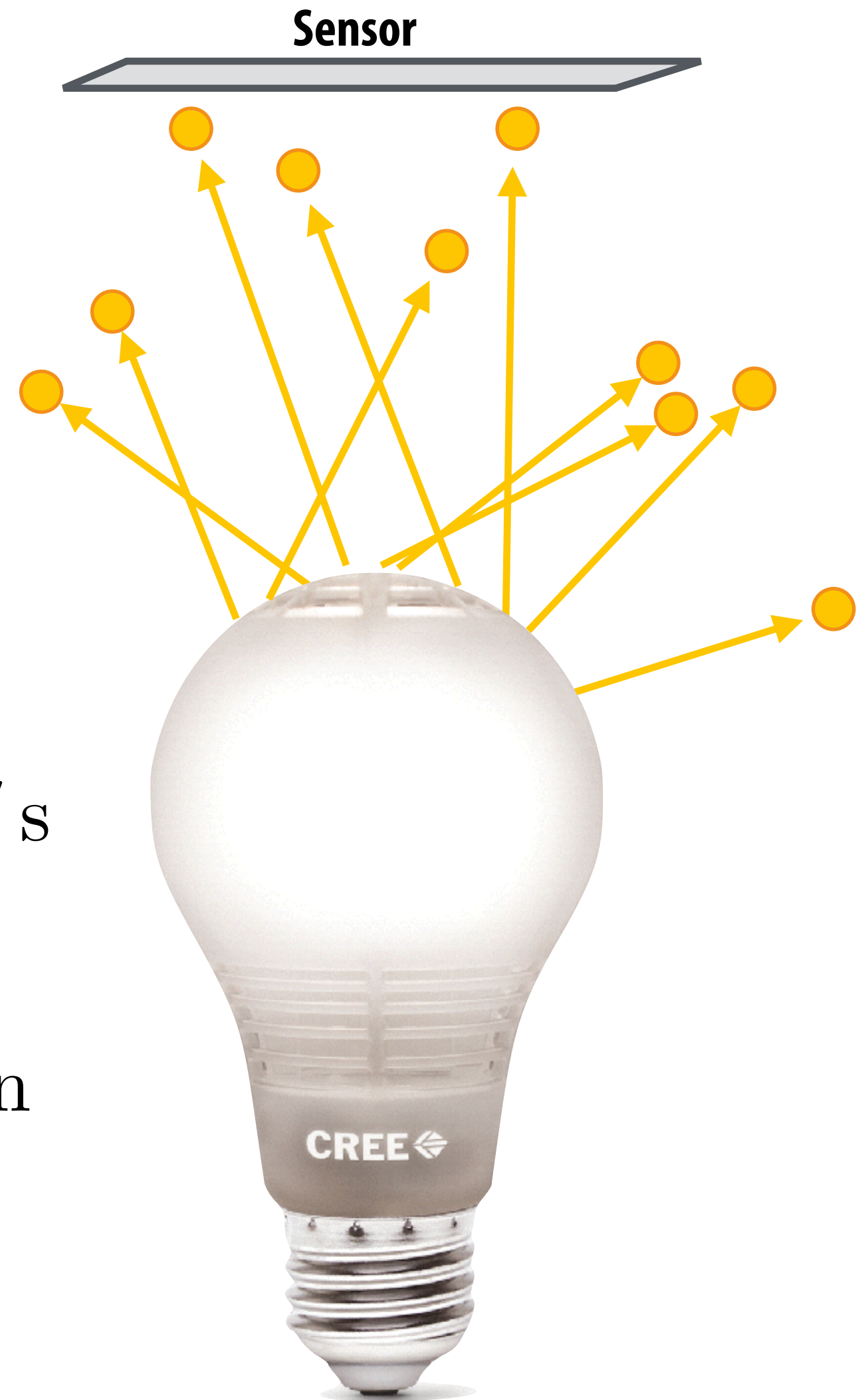
- Energy carried by a photon:

$$Q = \frac{hc}{\lambda}, \text{ where } h \approx 6.626 \times 10^{-34} \text{ m}^2 \text{ kg} / \text{s}$$

$$c = 299,792,458 \text{ m/s}$$

λ = wavelength of photon

- ~ 3.6 E-19J for a 555nm green photon
- ~ 2.8 E18 green photons for 1W of radiant energy



Modern LED Light: Estimate Efficiency?

Input power: 11 W

Output: 815 lumens
(~80 lumens / Watt)

Incandescent bulb?

Input power: 60W

Output: ~700 lumens
(~12 lumens / Watt)



Modern LED Light: Estimate Efficiency?

Input power: 11 W

If all power into light with
555nm average wavelength,
get $3.1\text{E}19$ photons/s

Intensity rating is 815
lumens, equivalent to
555nm laser at 815/683W.
If average wavelength is
555nm, get $3.3\text{E}18$
photons/s.

Efficiency*:

$$3.3\text{E}18 / 3.1\text{E}19 = 11\%$$



Art Competition #1 Results

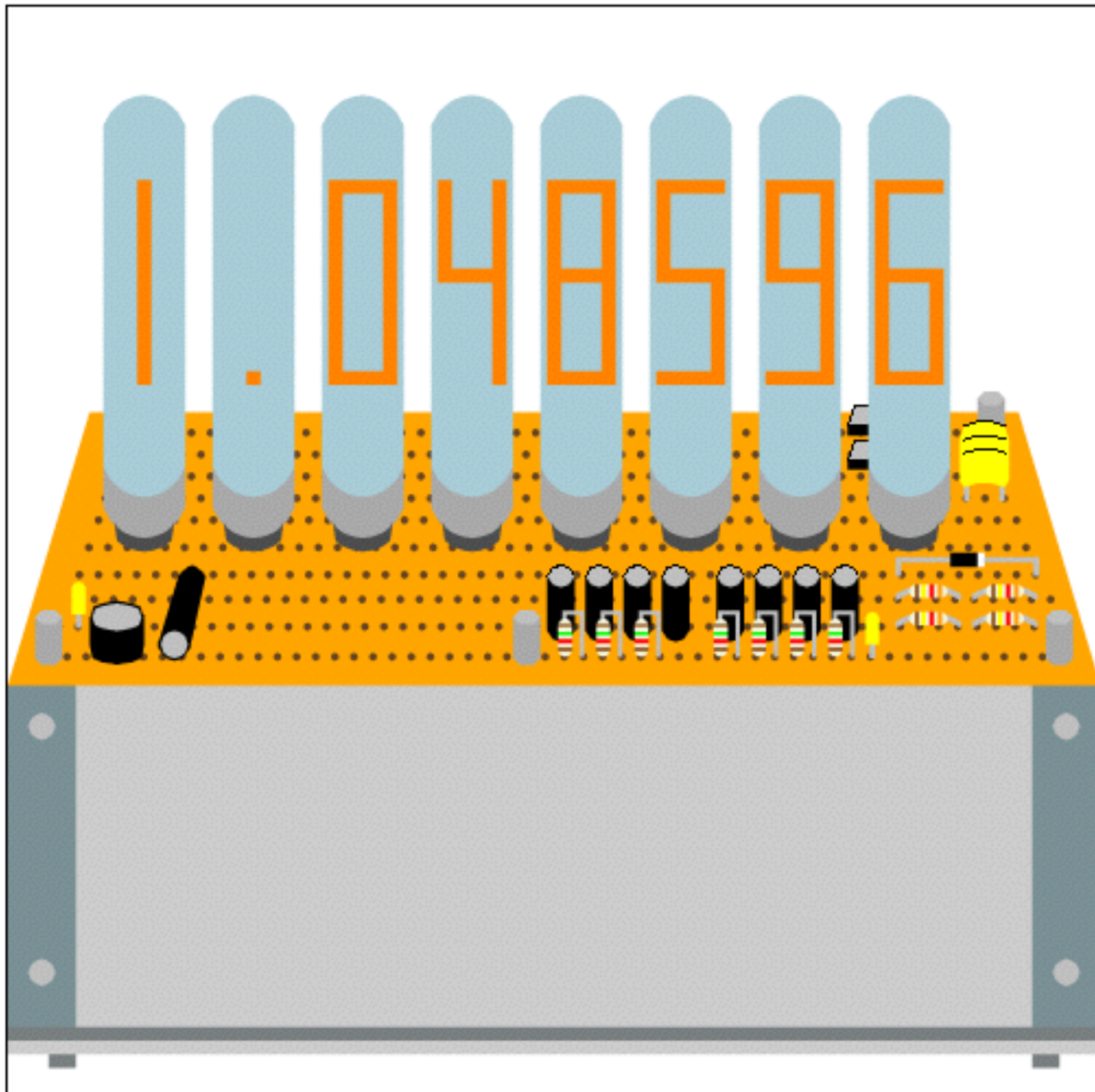
Art Competition #1 – 3rd Place Winner

Shannon Hu

It's a bunch of Nixie tubes sitting on a perforated circuit board with some other electronic components (resistors, capacitors, etc.).

Approach:

I manually wrote the SVG code for it, using a Google Image as reference for the dimensions. I used mostly polygons and polylines.



Art Competition #1 – 2nd Place Winner

Jason Qiao

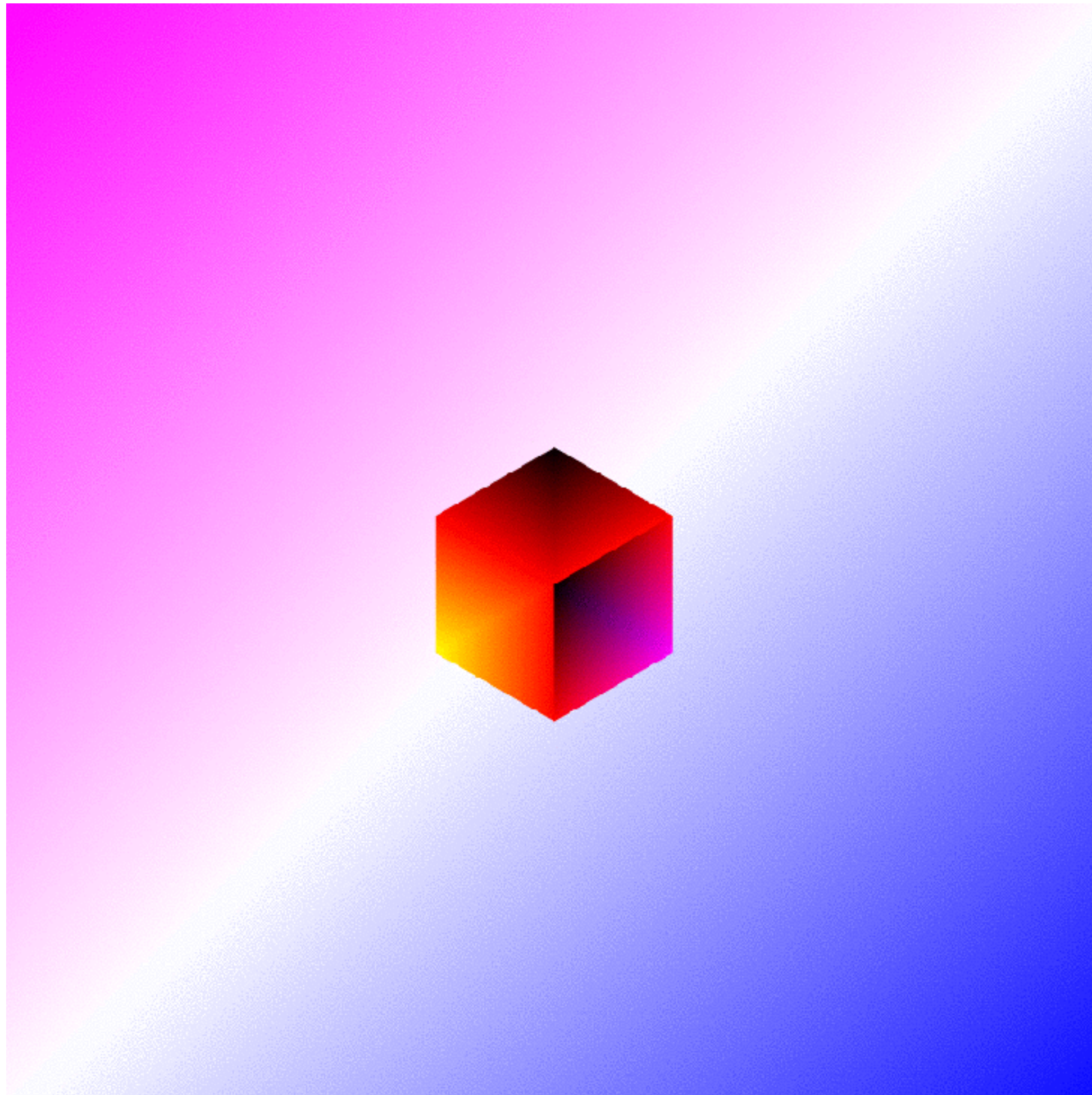
The happiest sad moment in film history.

Approach:

I created a triangle image generator in Python. It will iteratively place triangles and "jitter" them to find a local optimum, comparing our new image with the source image using RMSE. The triangles get smaller as more are placed so we can capture the details.



Art Competition #1 – 1st Place Winner



Ariel Hirschberg

"Cubeman's city is falling apart! Oh wait it's rebuilding itself. Perpetual motion?"

Approach:

- Python script outputs 74 SVG files describing a decent number of triangles.
- Project code (with a modified DrawRend class to match the desired 800x800 viewport) outputs 74 png files.
- Ezgif online stitches the pngs together into the final GIF.