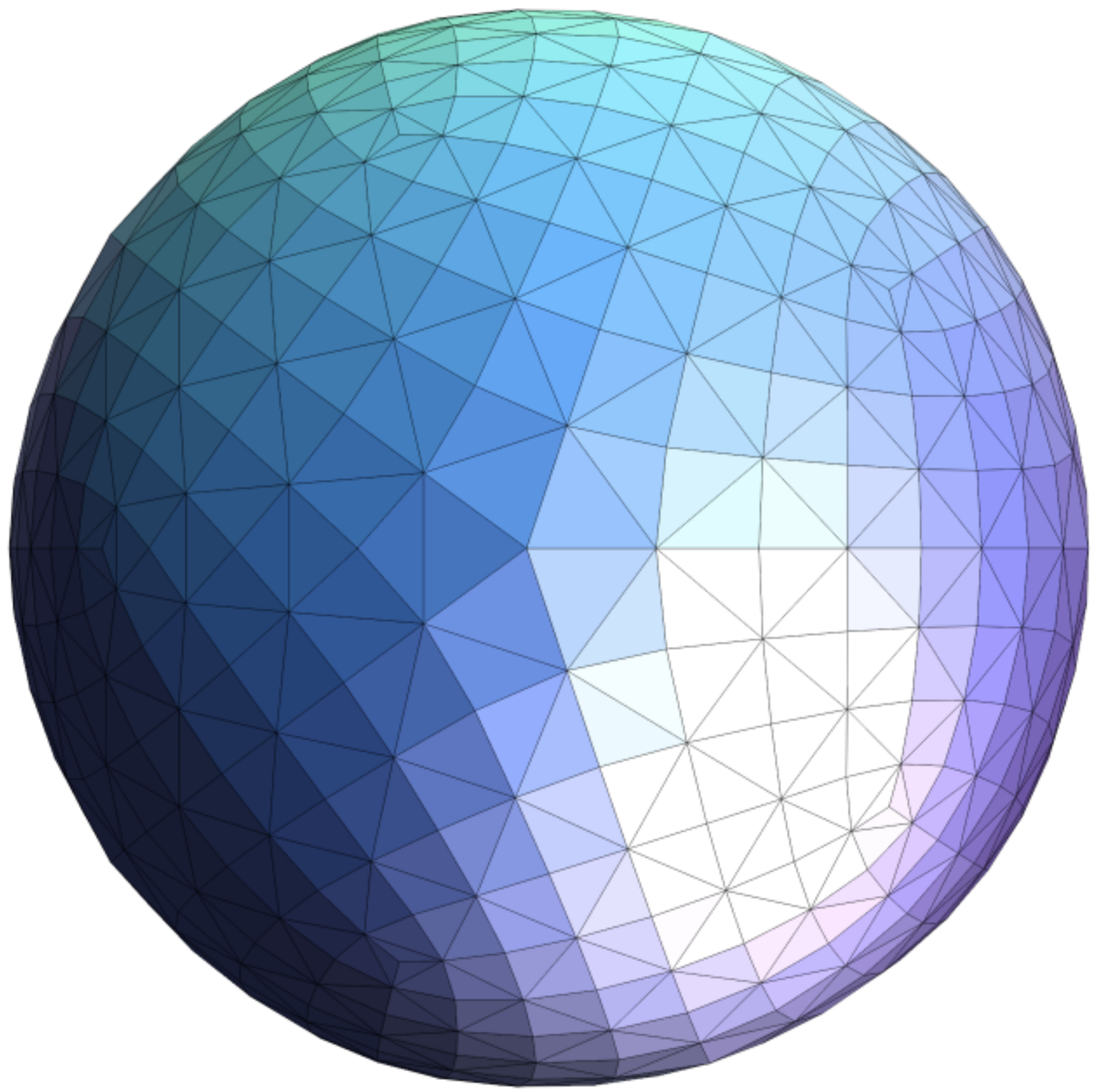
Lecture 2:

Digital Drawing

Computer Graphics and Imaging

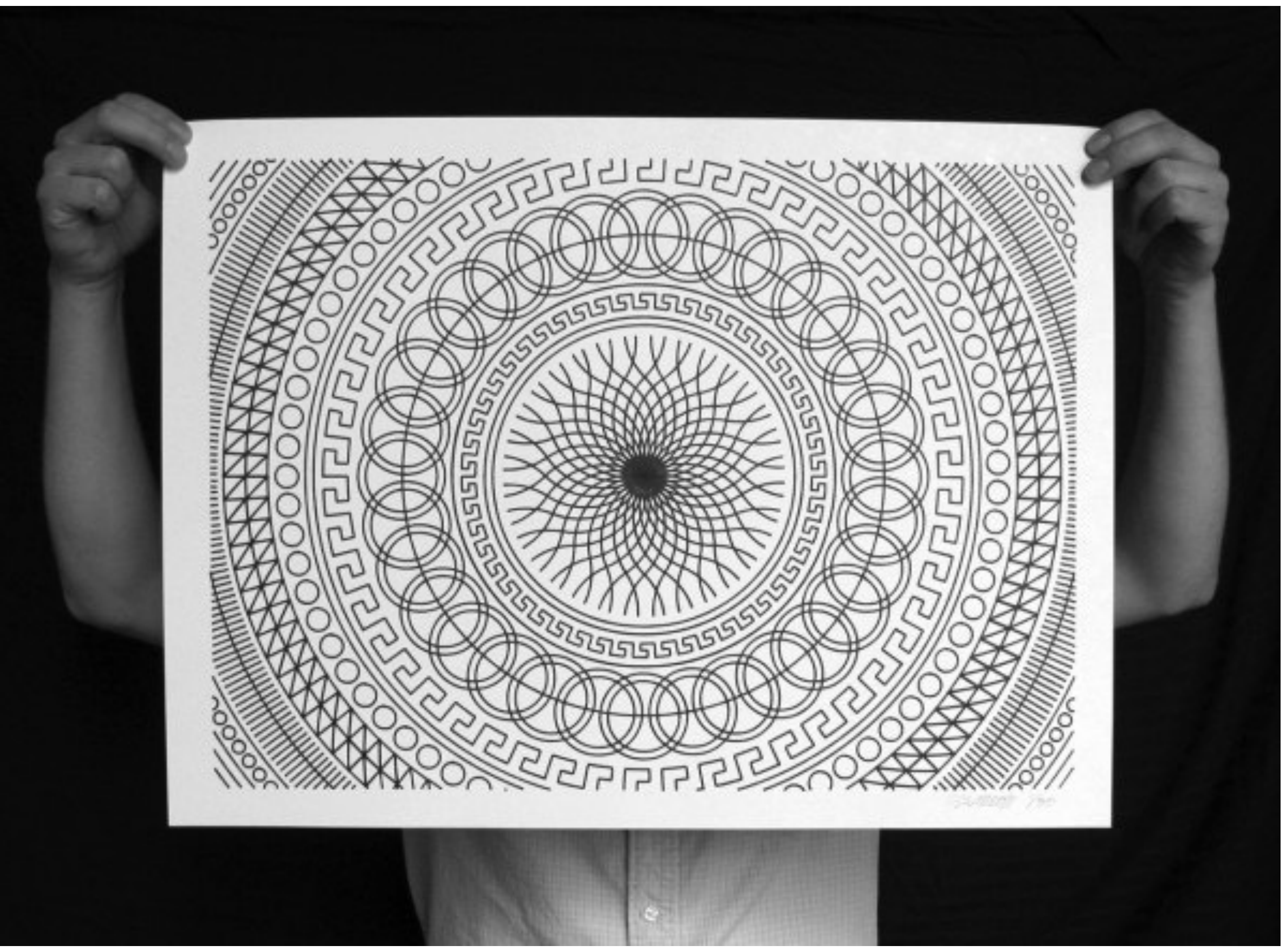
UC Berkeley CS184/284

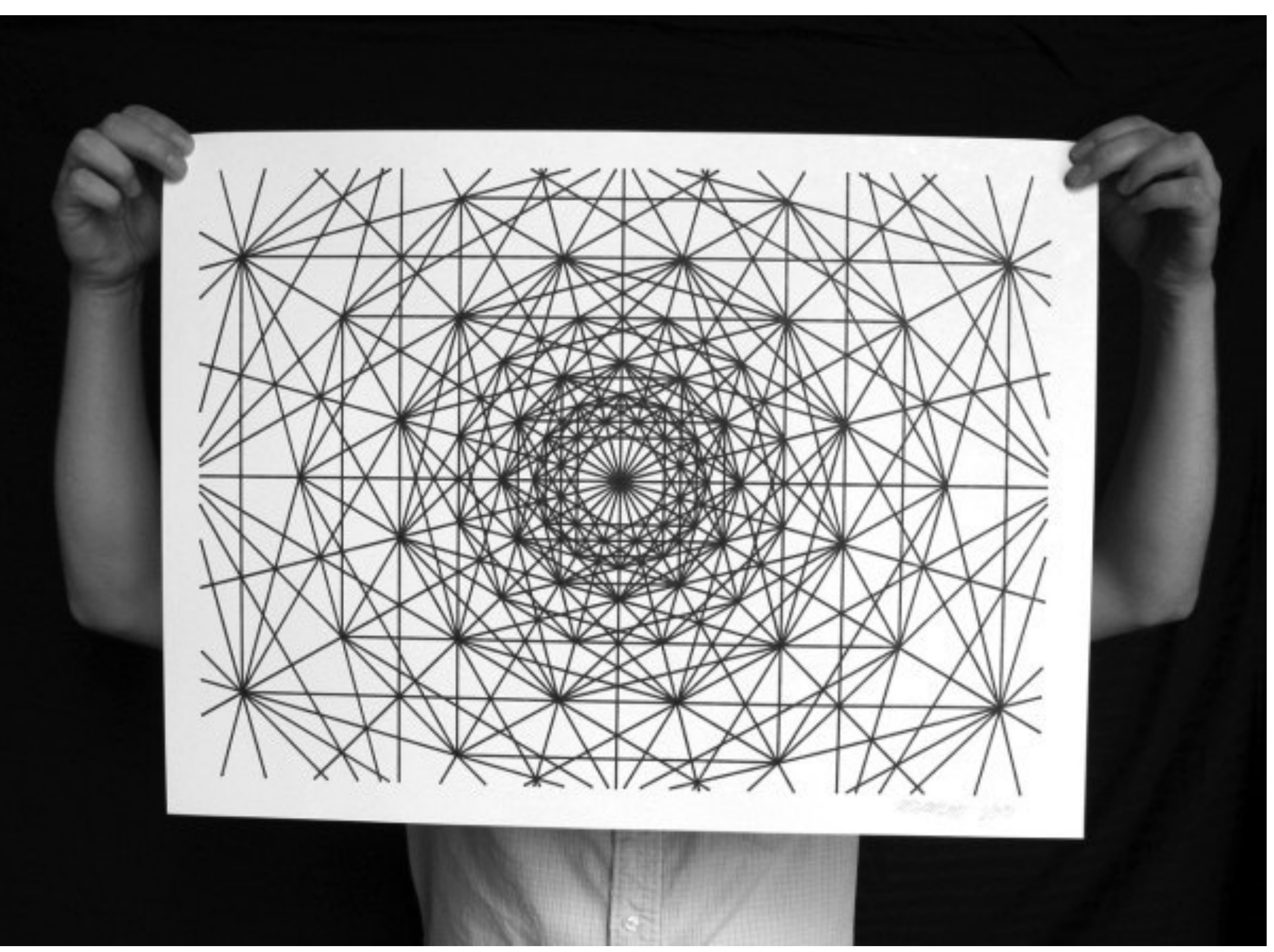
Today: Drawing Triangles to the Screen by Sampling



Drawing Machines

CNC Sharpie Drawing Machine



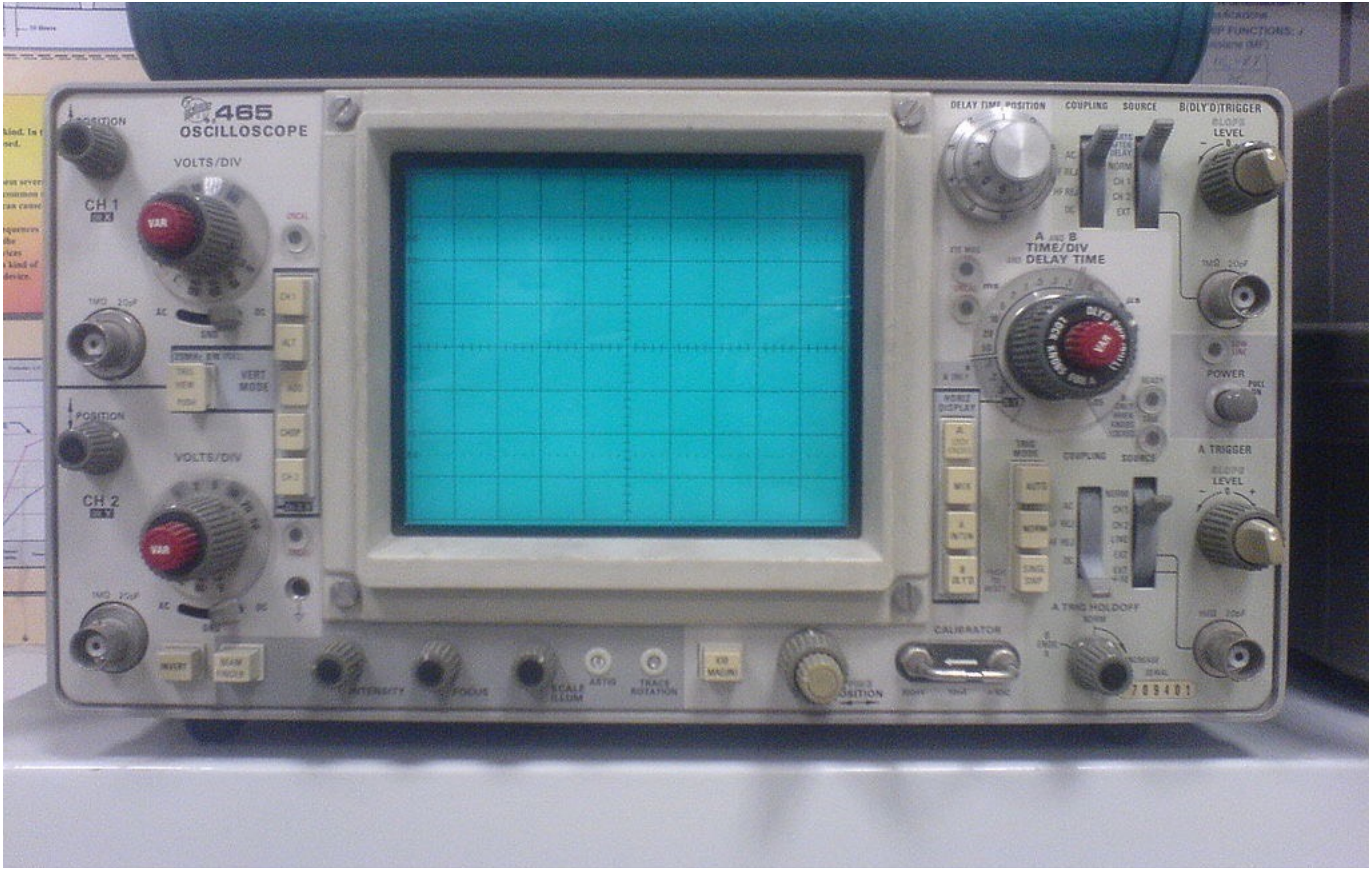


Aaron Panone with Matt W. Moore

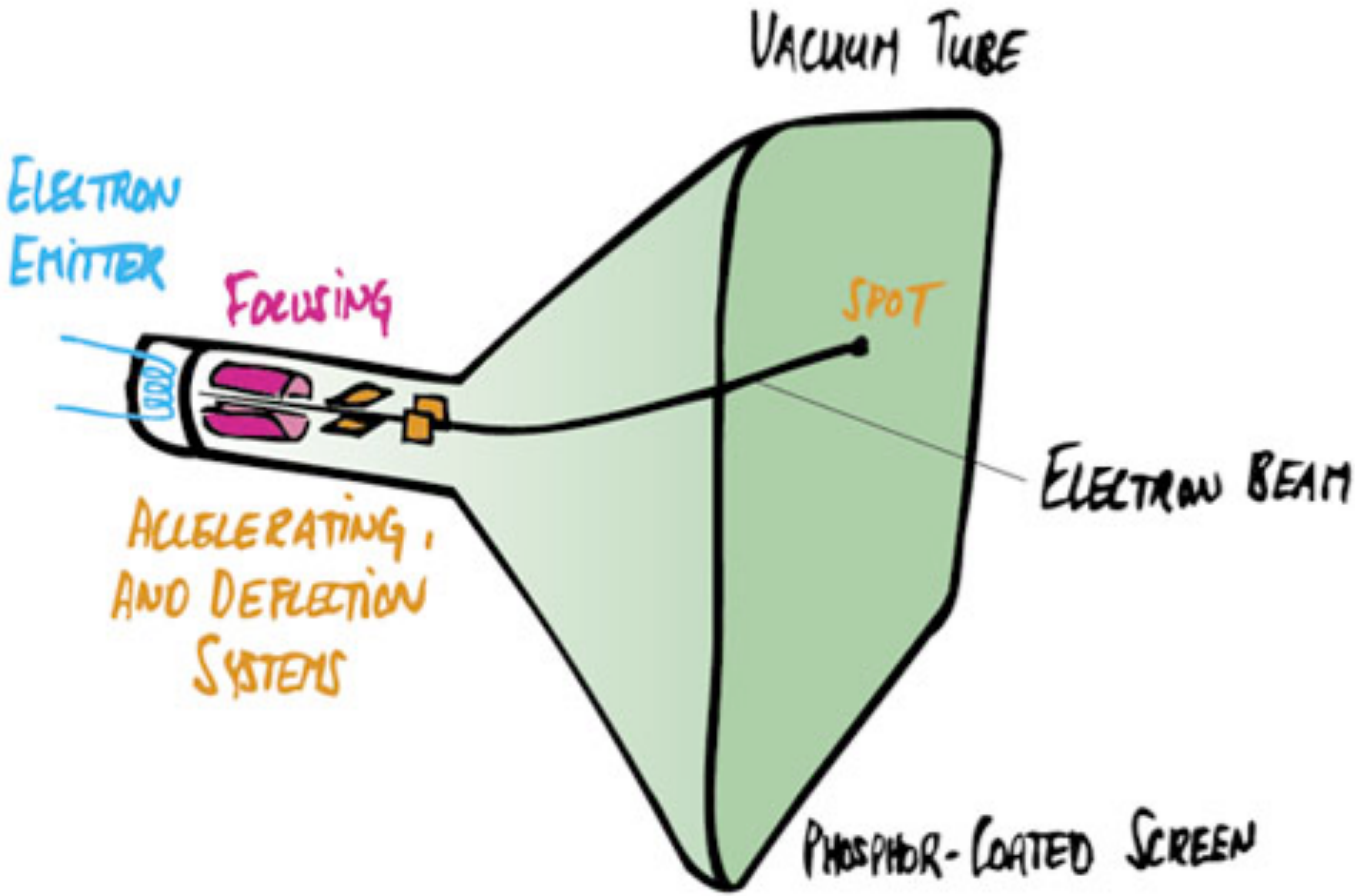
http://44rn.com/projects/numerically-controlled-poster-series-with-matt-w-moore/

Laser Cutters

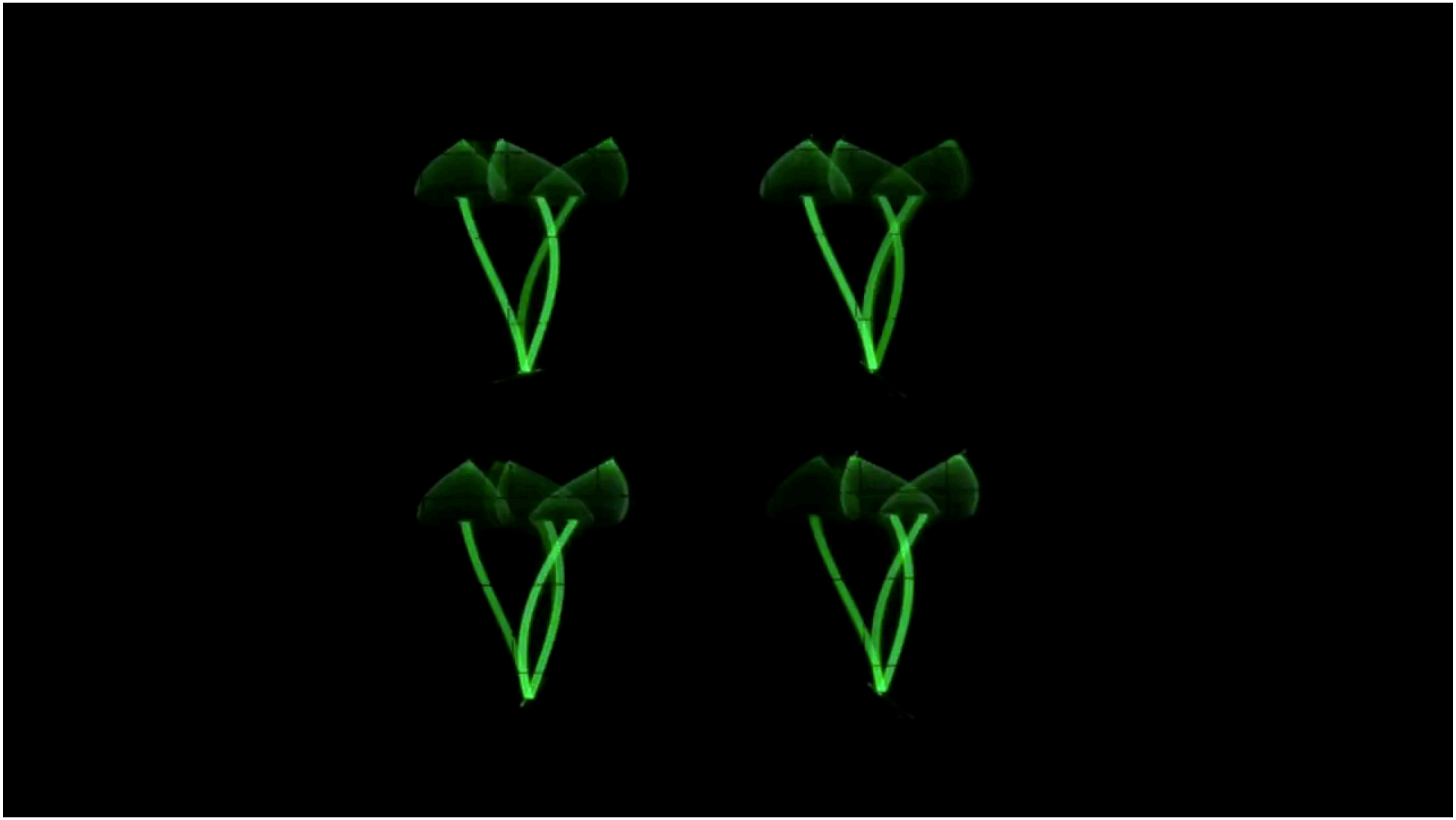
Oscilloscope



Cathode Ray Tube



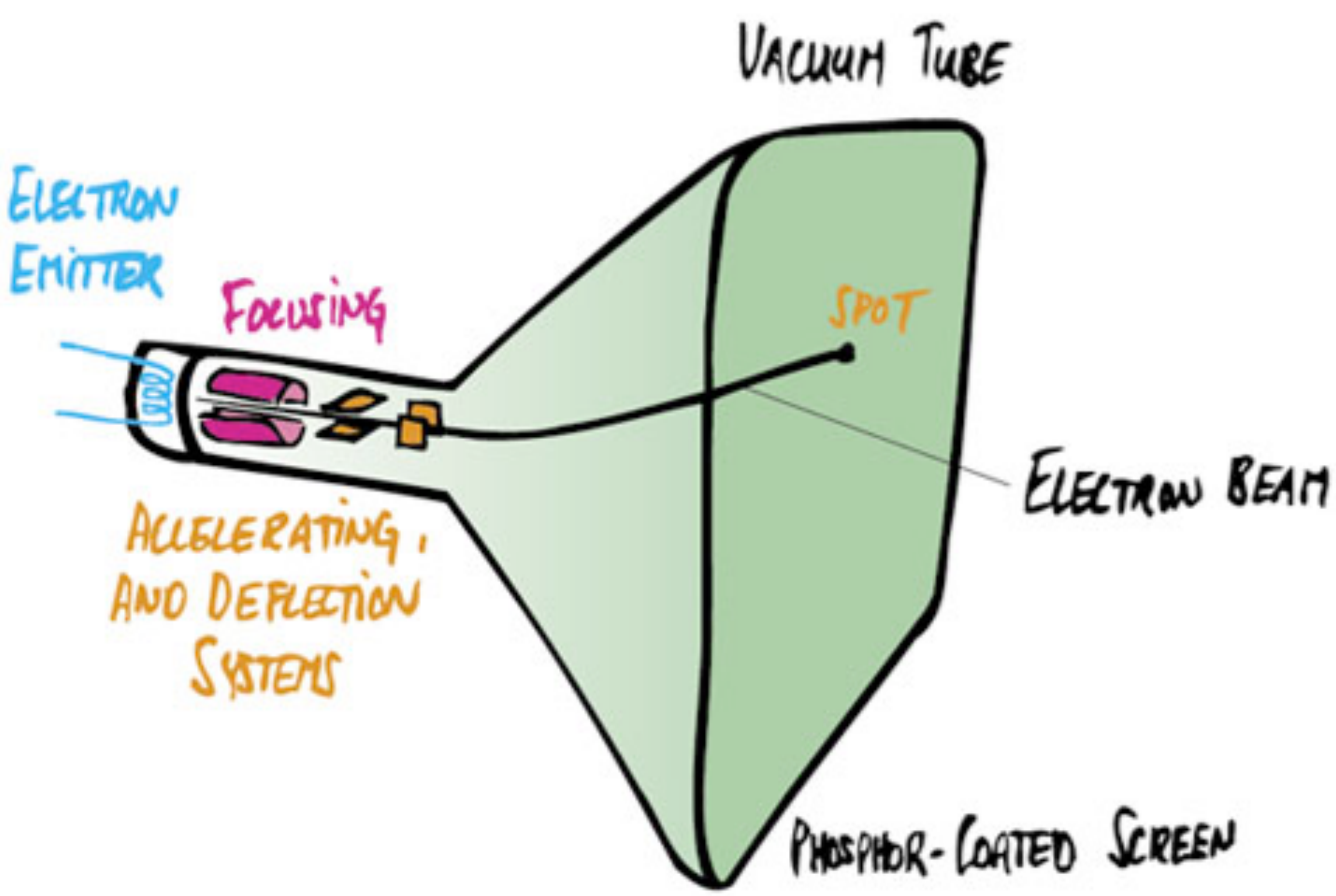
Oscilloscope Art



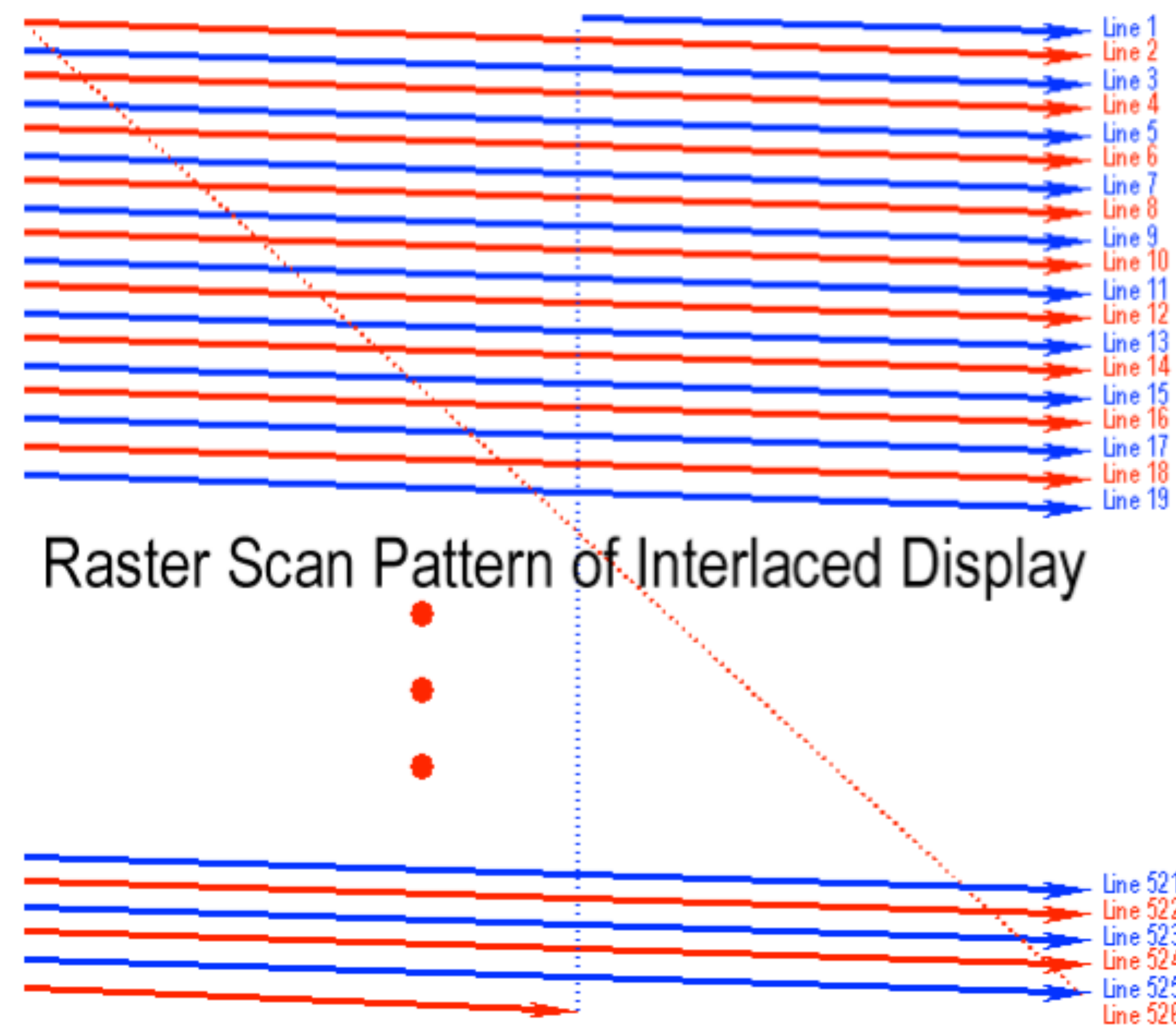
Jerobeam Fenderson

https://www.youtube.com/watch?v=rtR63-ecUNo

Television - Raster Display CRT



Cathode Ray Tube



Raster Scan
(modulate intensity)

Frame Buffer: Memory for a Raster Display

DAC =
Digital to Analog Convertors

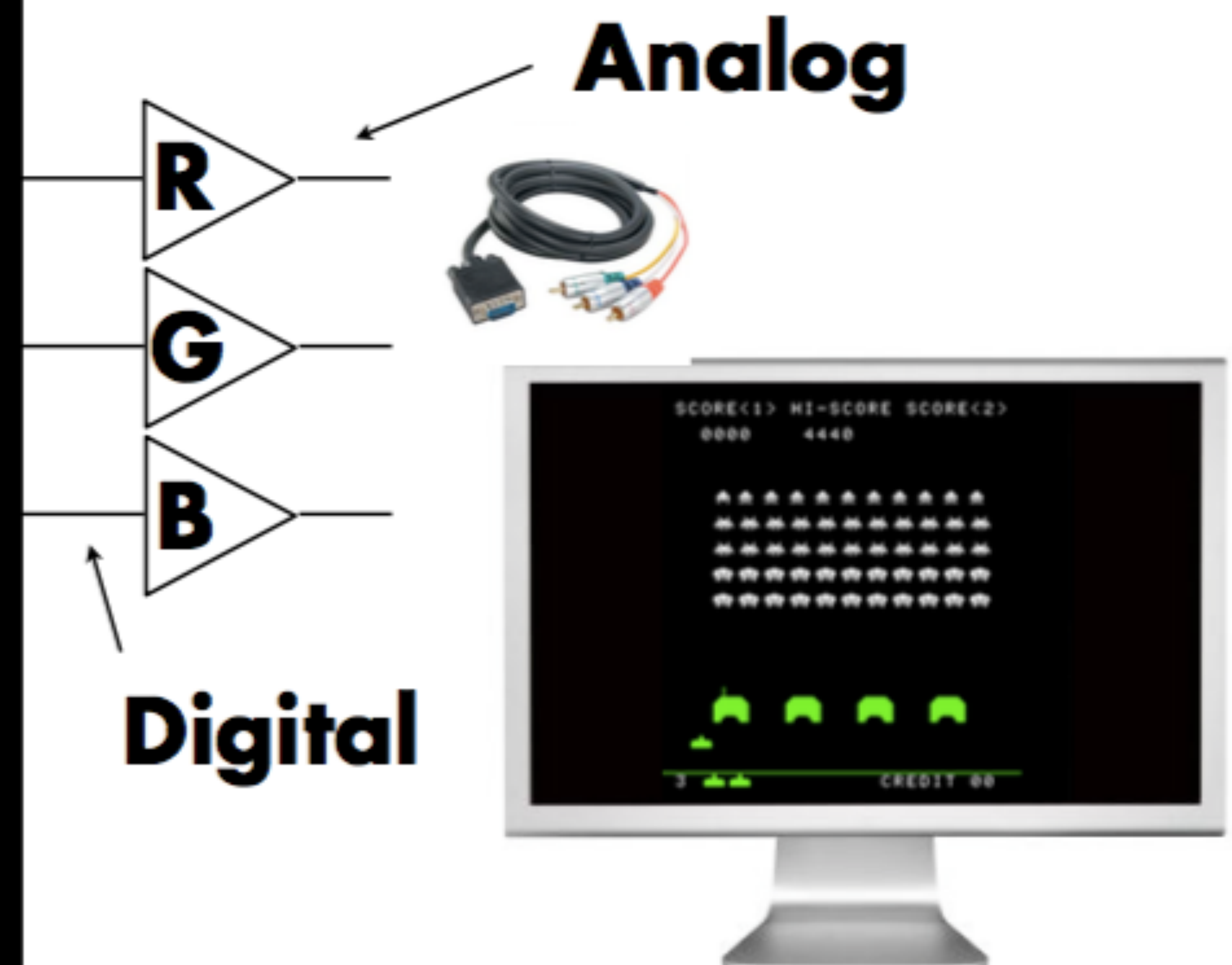


Image = 2D array of colors

A Sampling of Different Raster Displays

Flat Panel Displays

Low-Res LCD Display

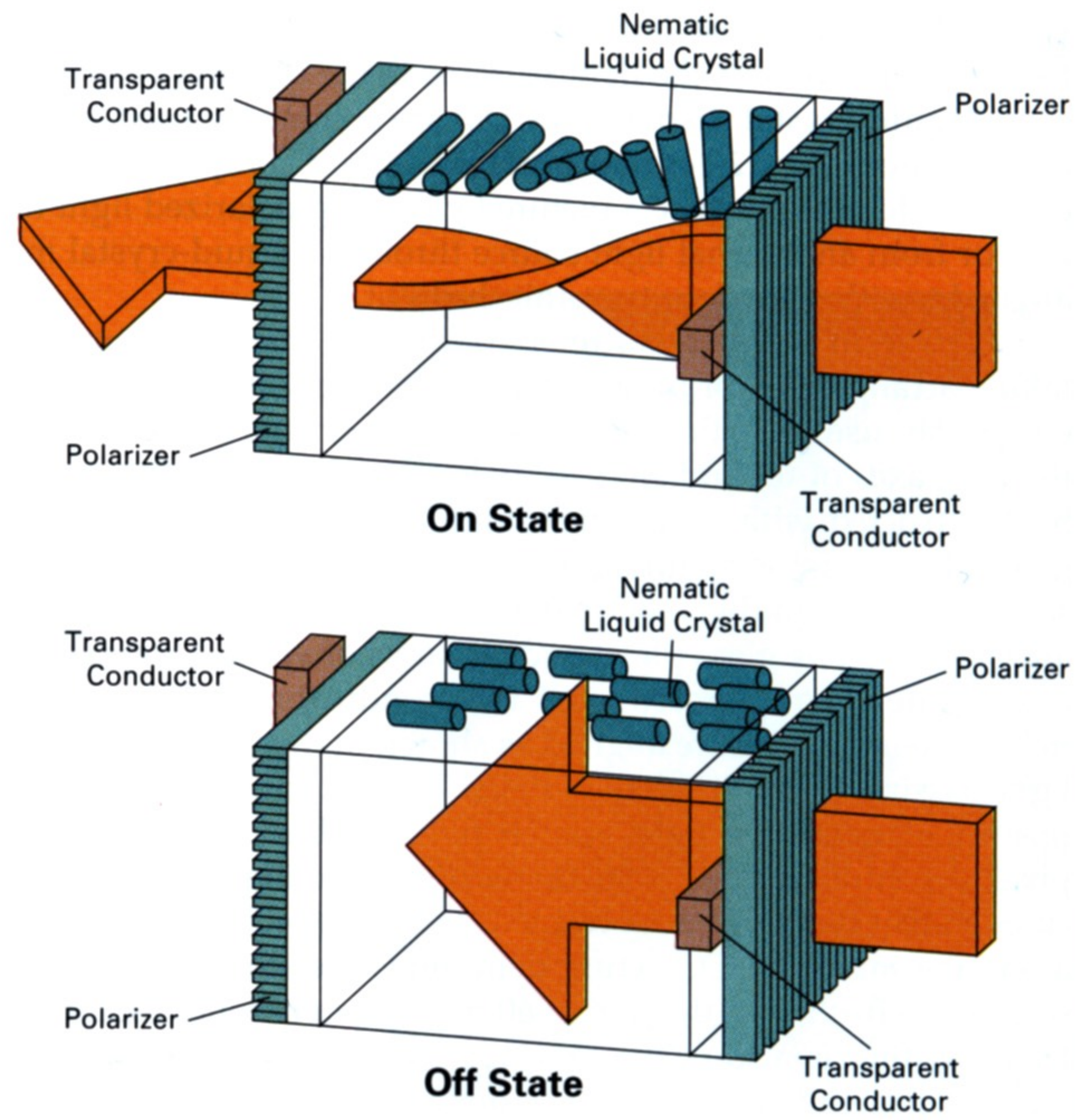
Color LCD, OLED, …

LCD (Liquid Crystal Display) Pixel

Principle: block or transmit light by twisting polarization

Illumination from backlight (e.g. fluorescent or LED)

Intermediate intensity levels by partial twist



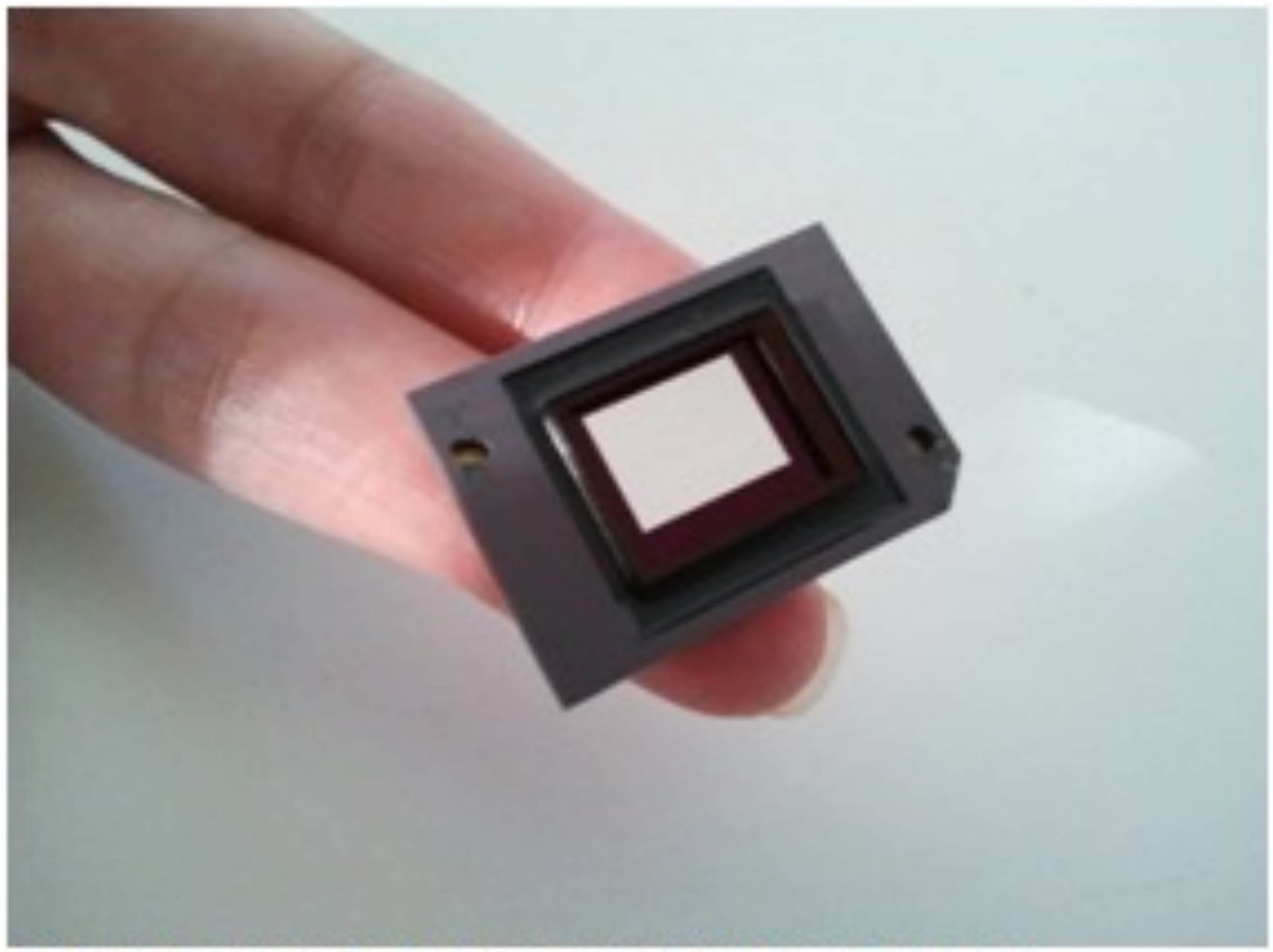
[H&B fig. 2-16]

LED Array Display

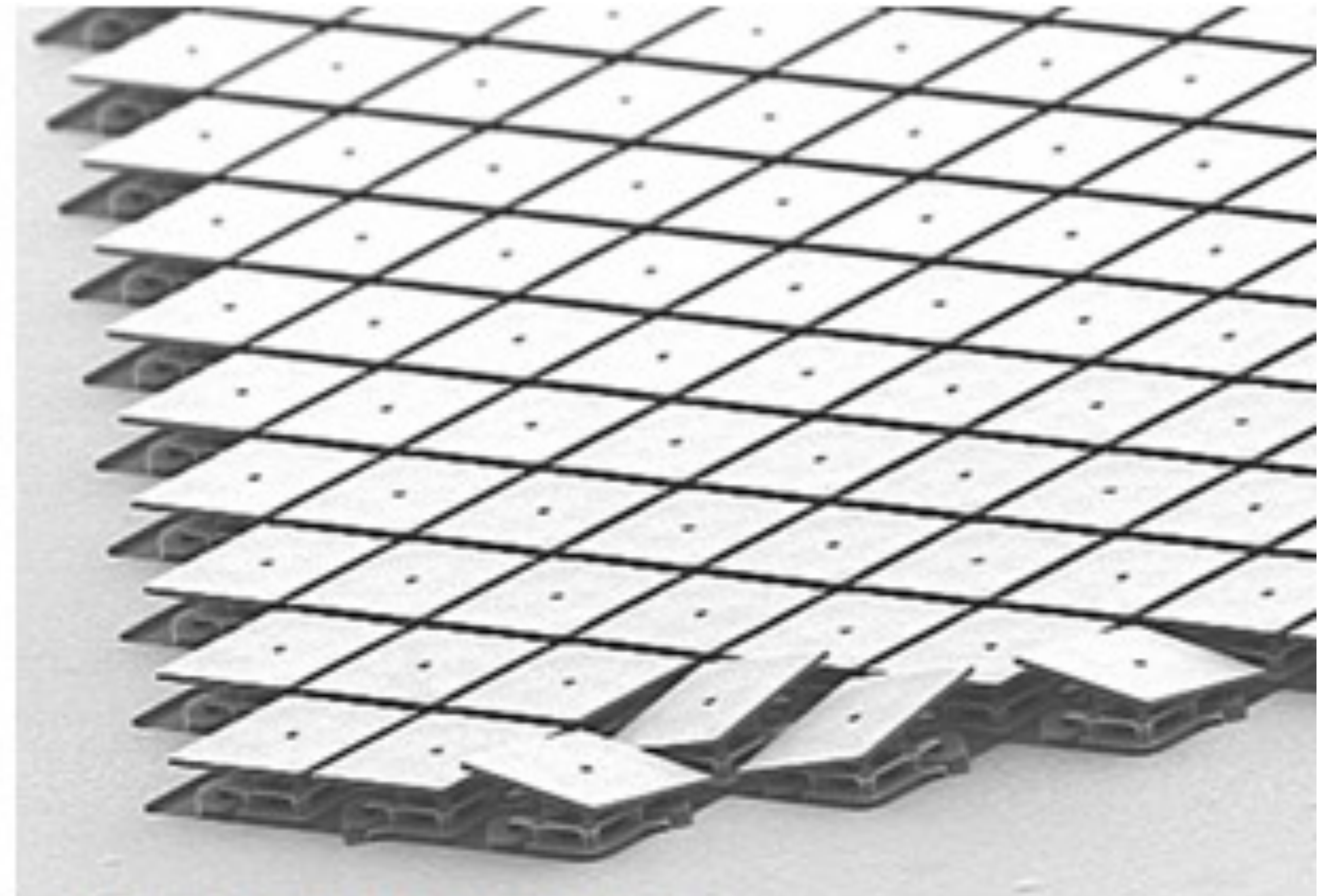
Light emitting diode array

BAMPFA: LED Array Display

DMD Projection Display



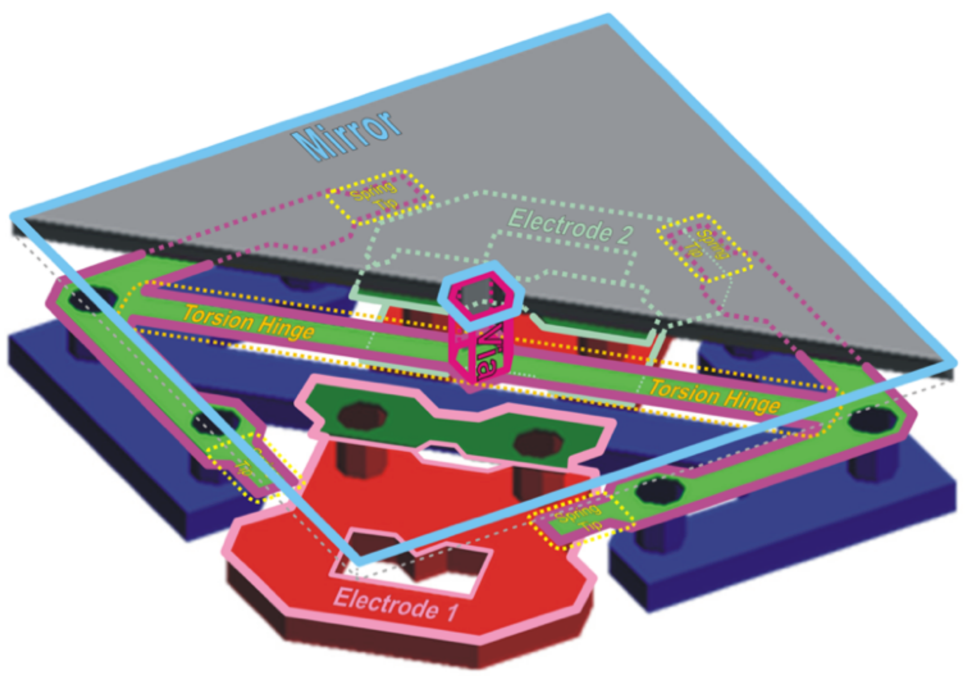
DIGITAL MICRO MIRROR DEVICE **(DMD)**
(**SLM** - Spatial Light Modulator)

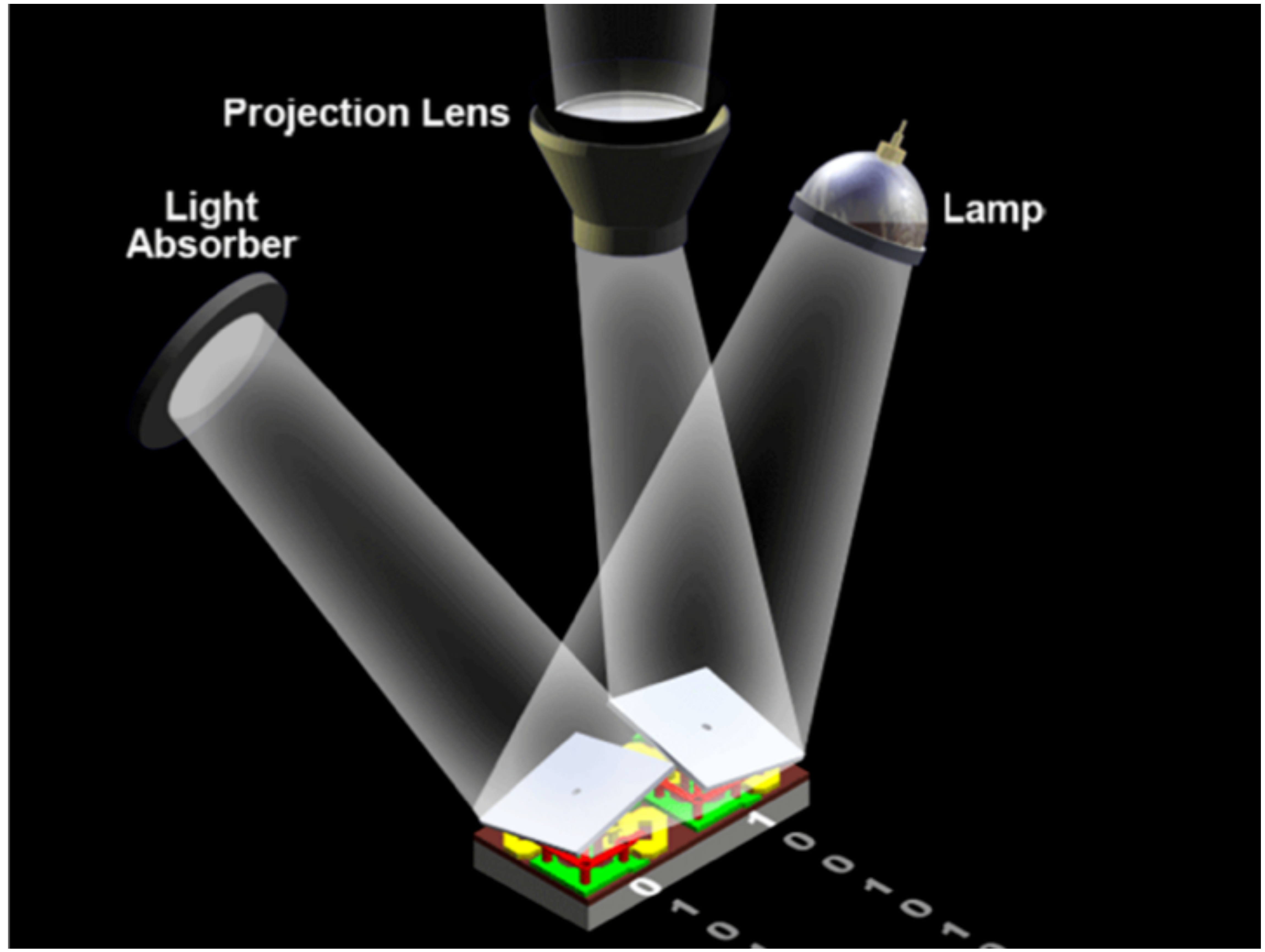


MICRO MIRRORS CLOSE UP

[Y.K. Rabinowitz; EKB Technologies

DMD Projection Display



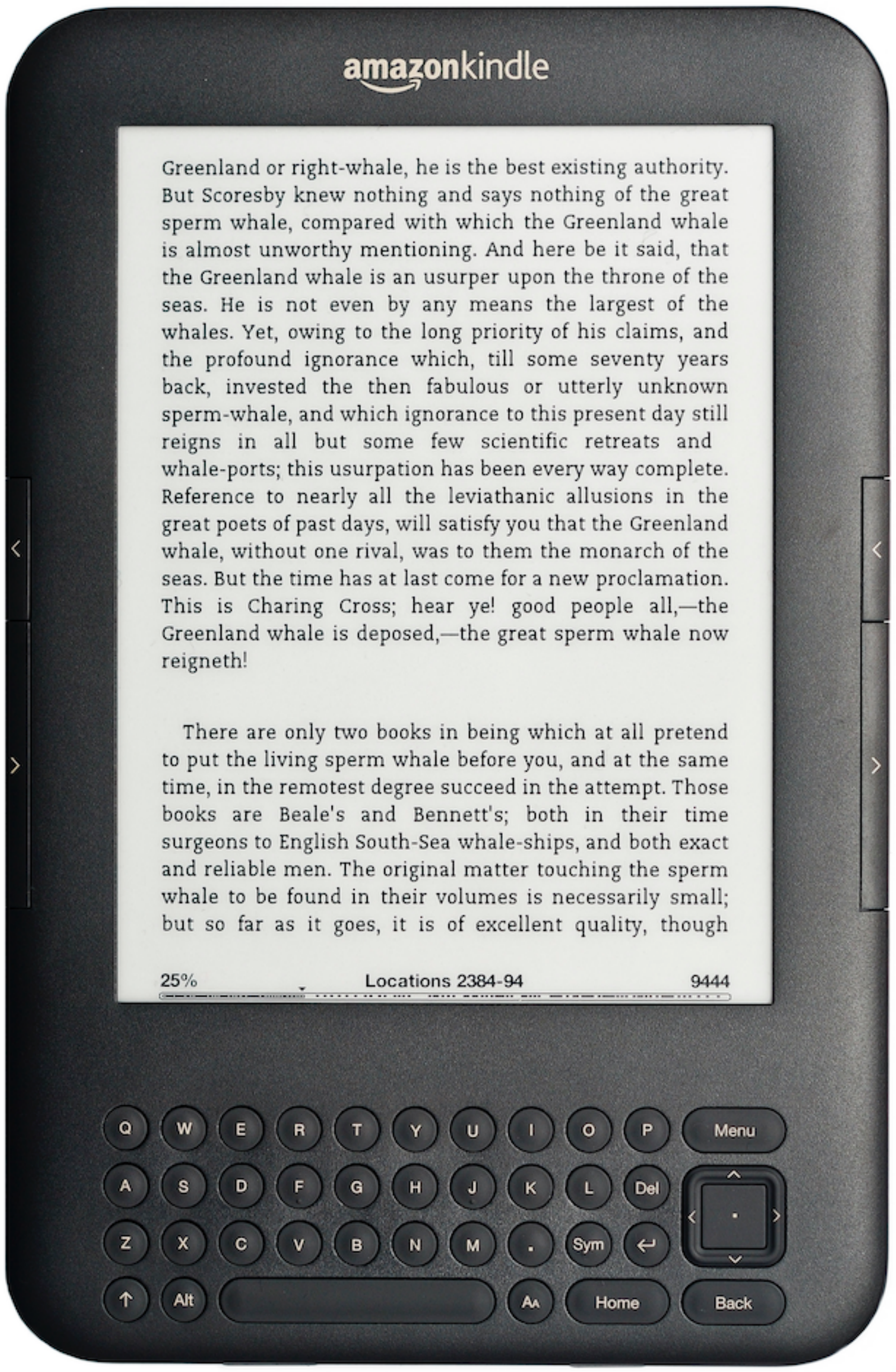


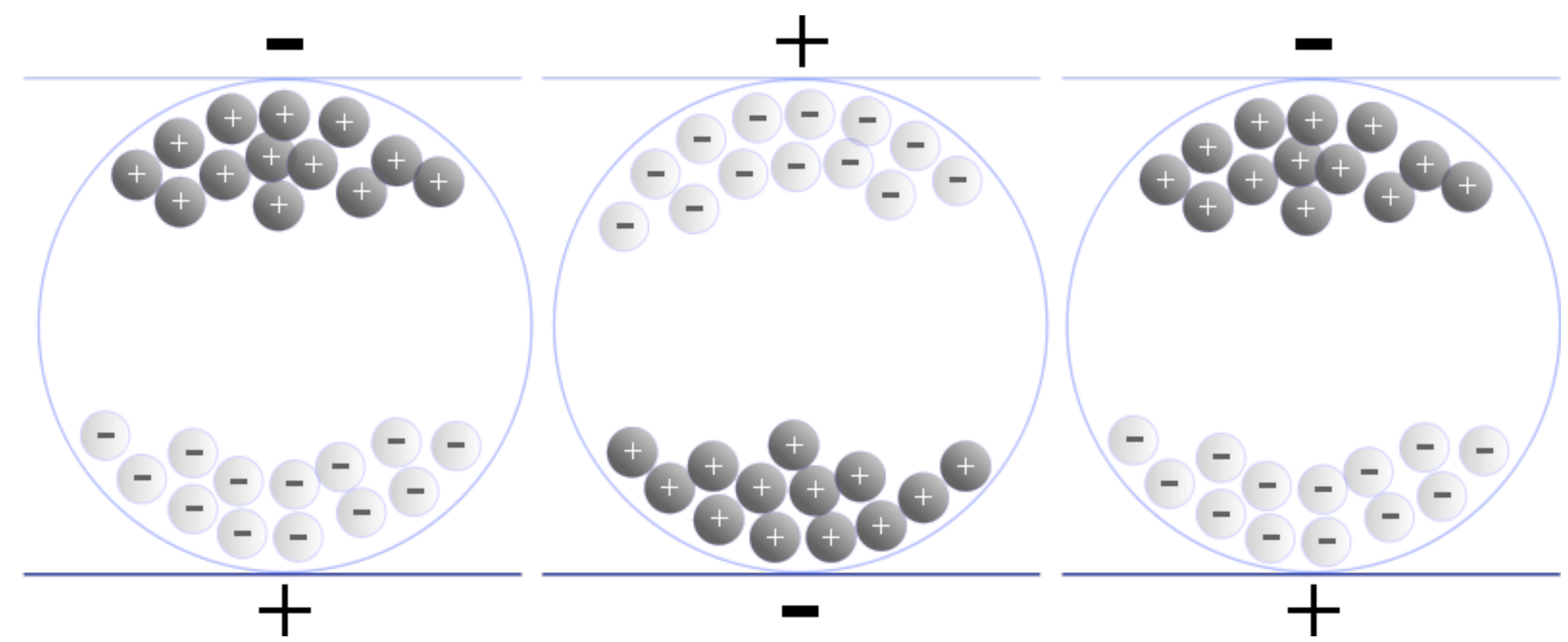
[Texas Instruments]

Array of micro-mirror pixels

DMD = Digital Micromirror Device

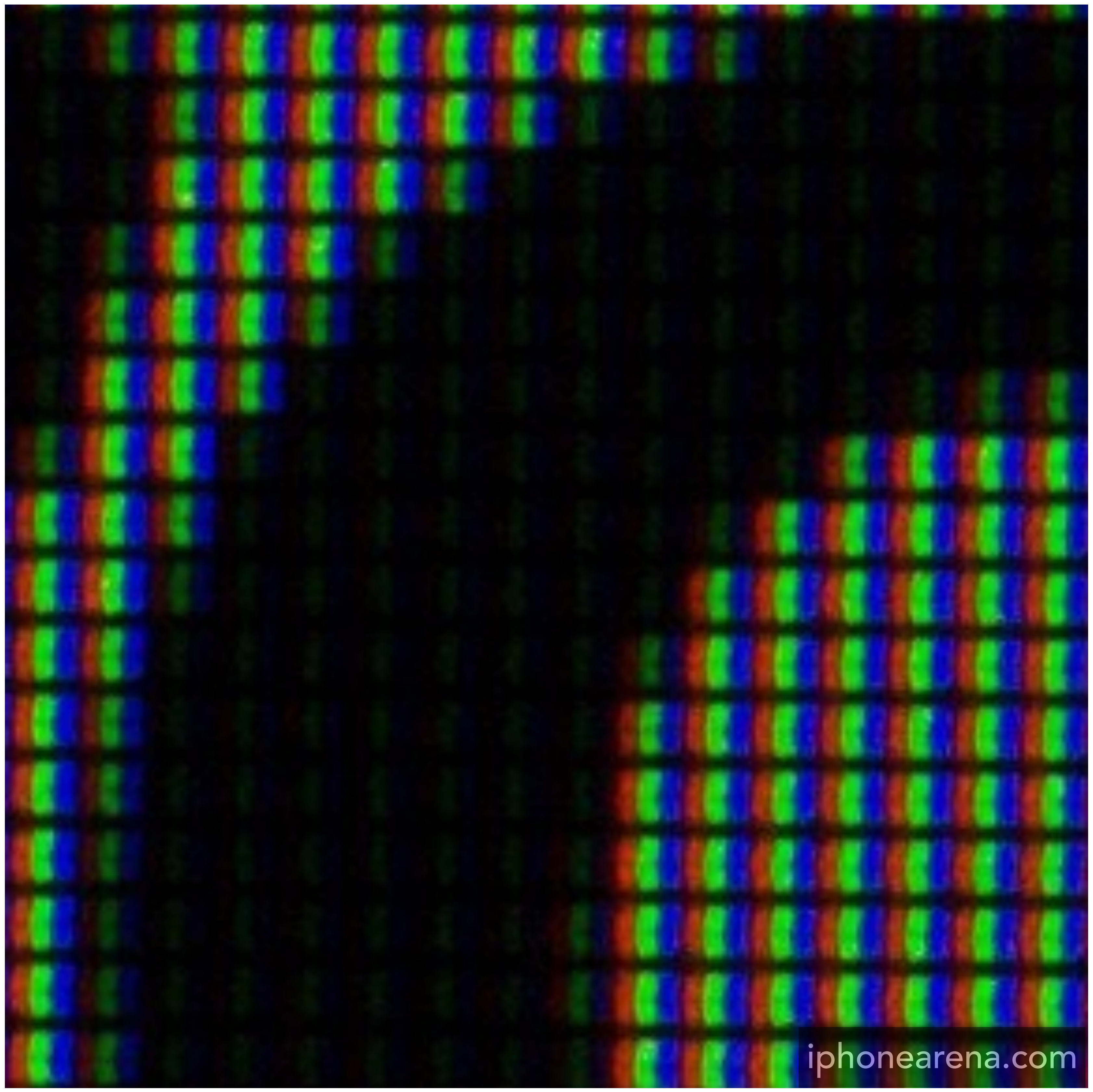
Electrophoretic (Electronic Ink) Display



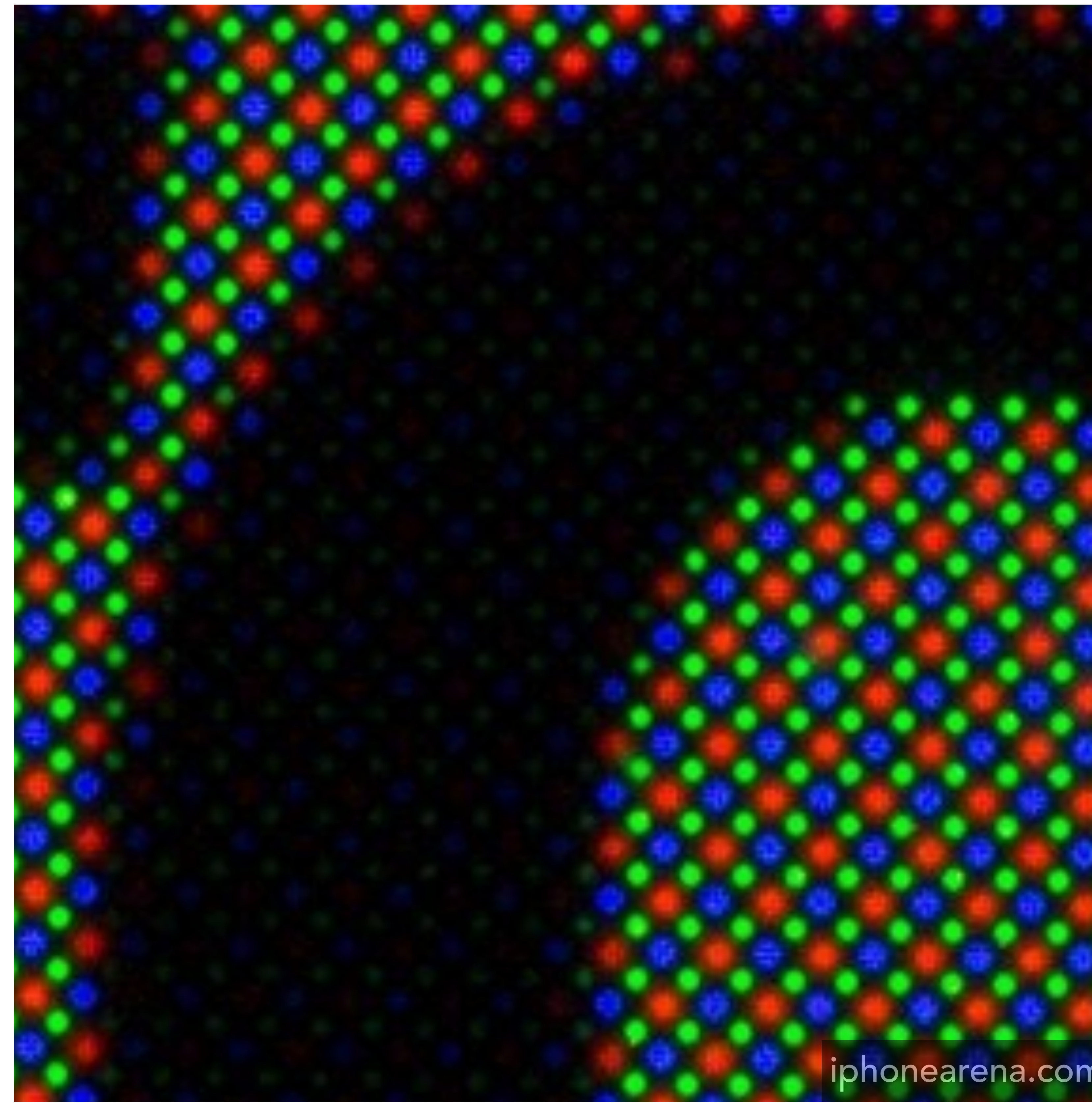


[Wikimedia Commons —Senarclens]

Smartphone Screen Pixels (Closeup)



iPhone 6S

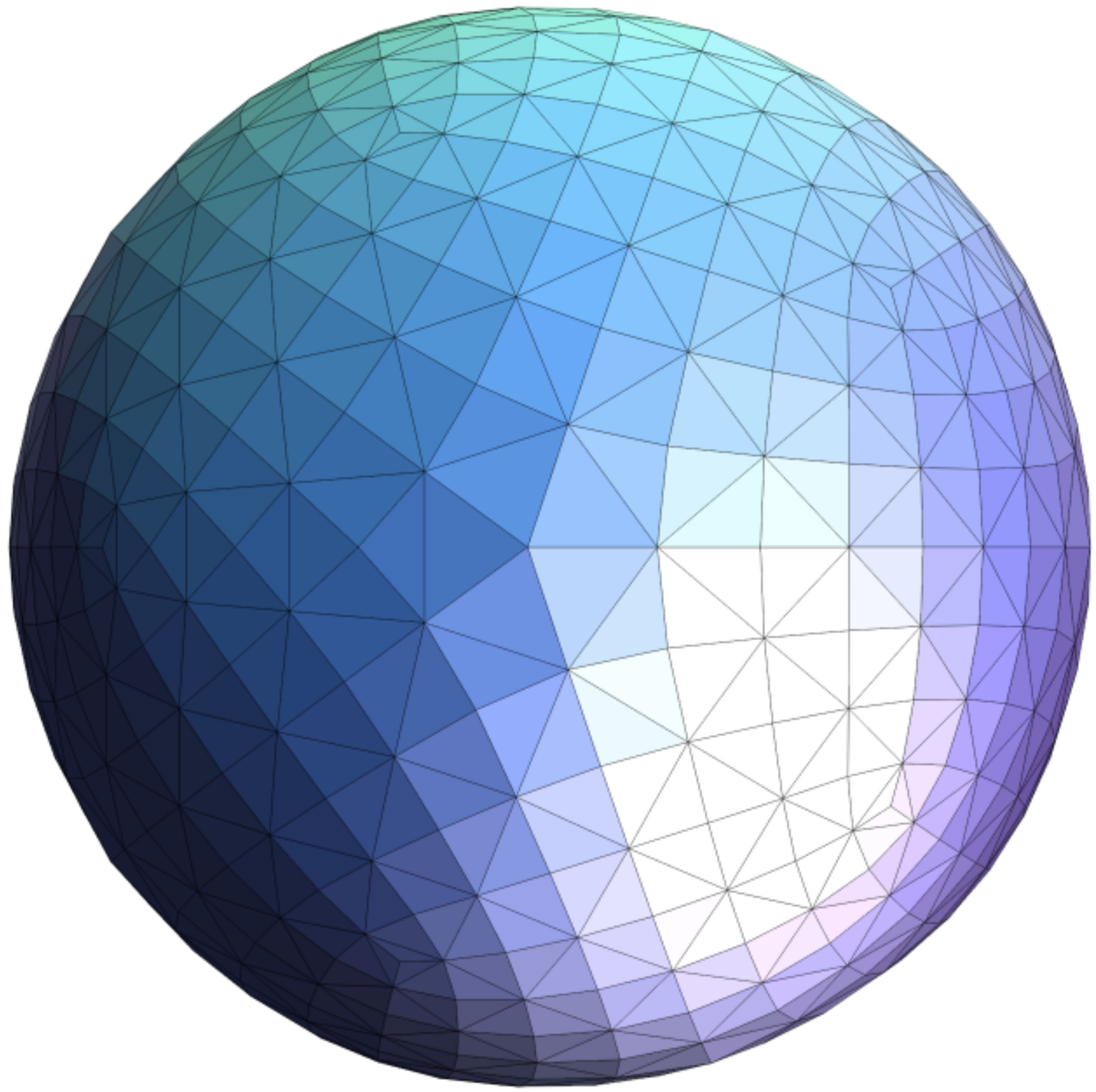


Galaxy S5

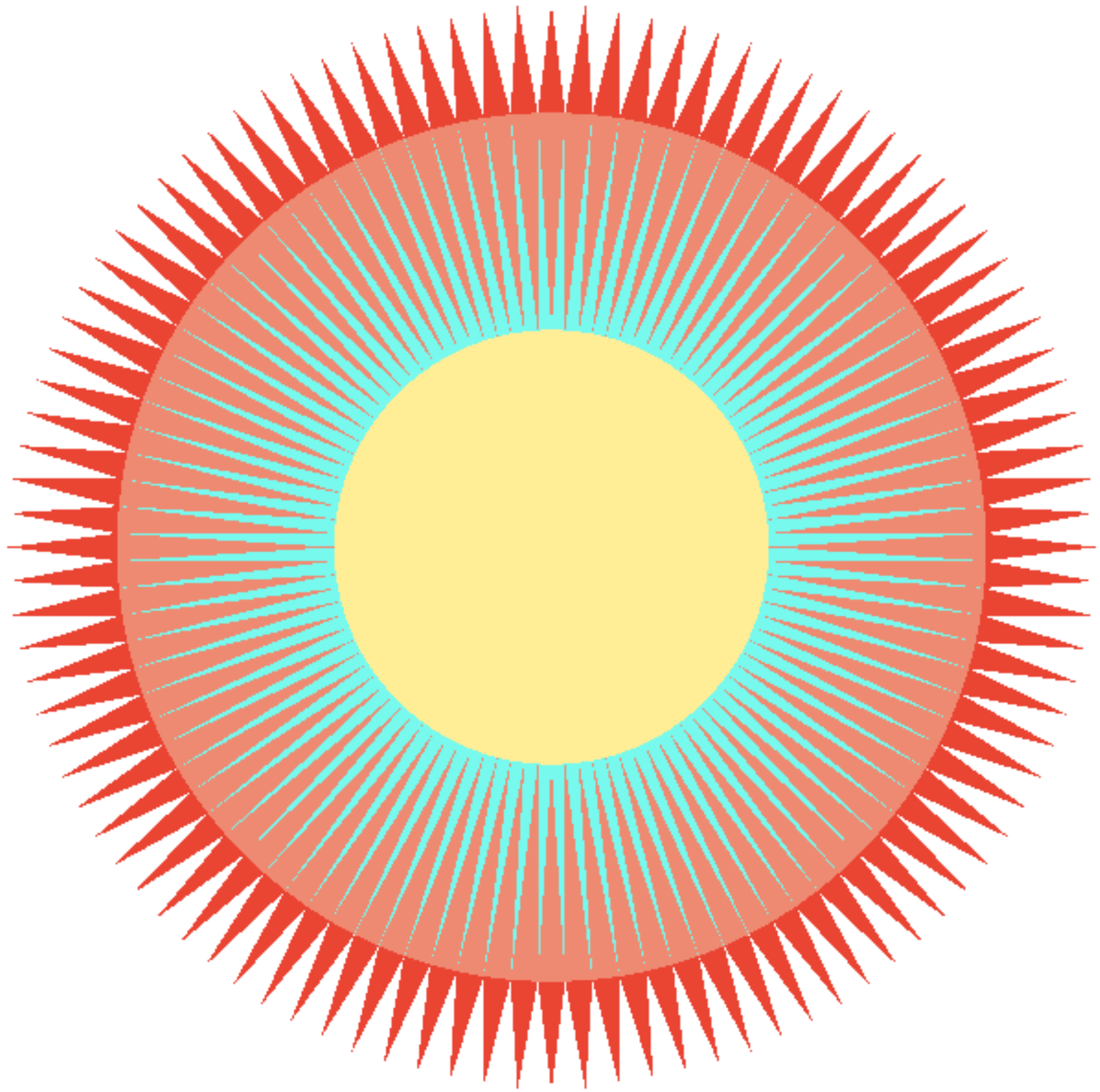
Drawing to Raster Displays

Polygon Meshes

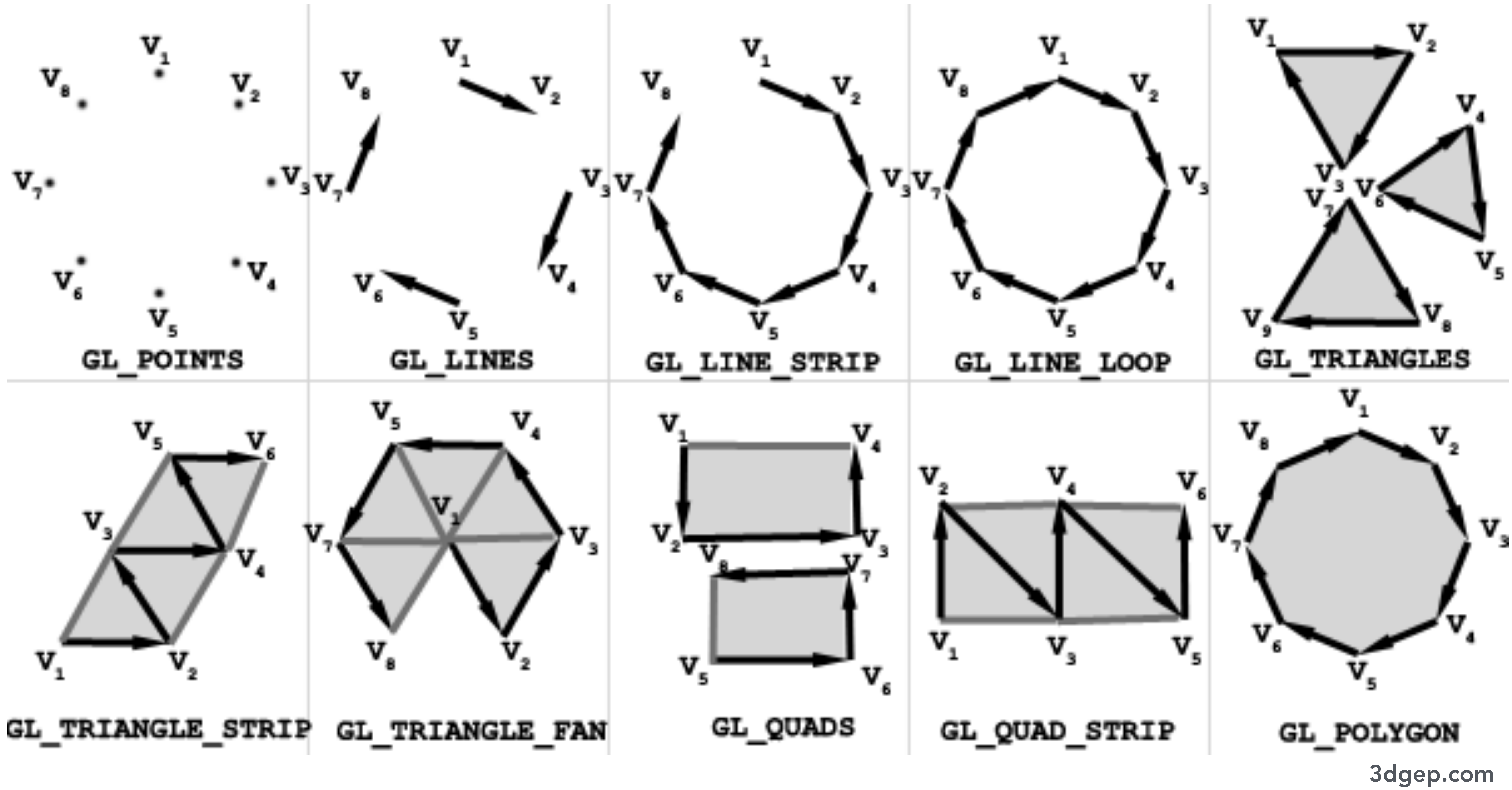
Triangle Meshes



Triangle Meshes

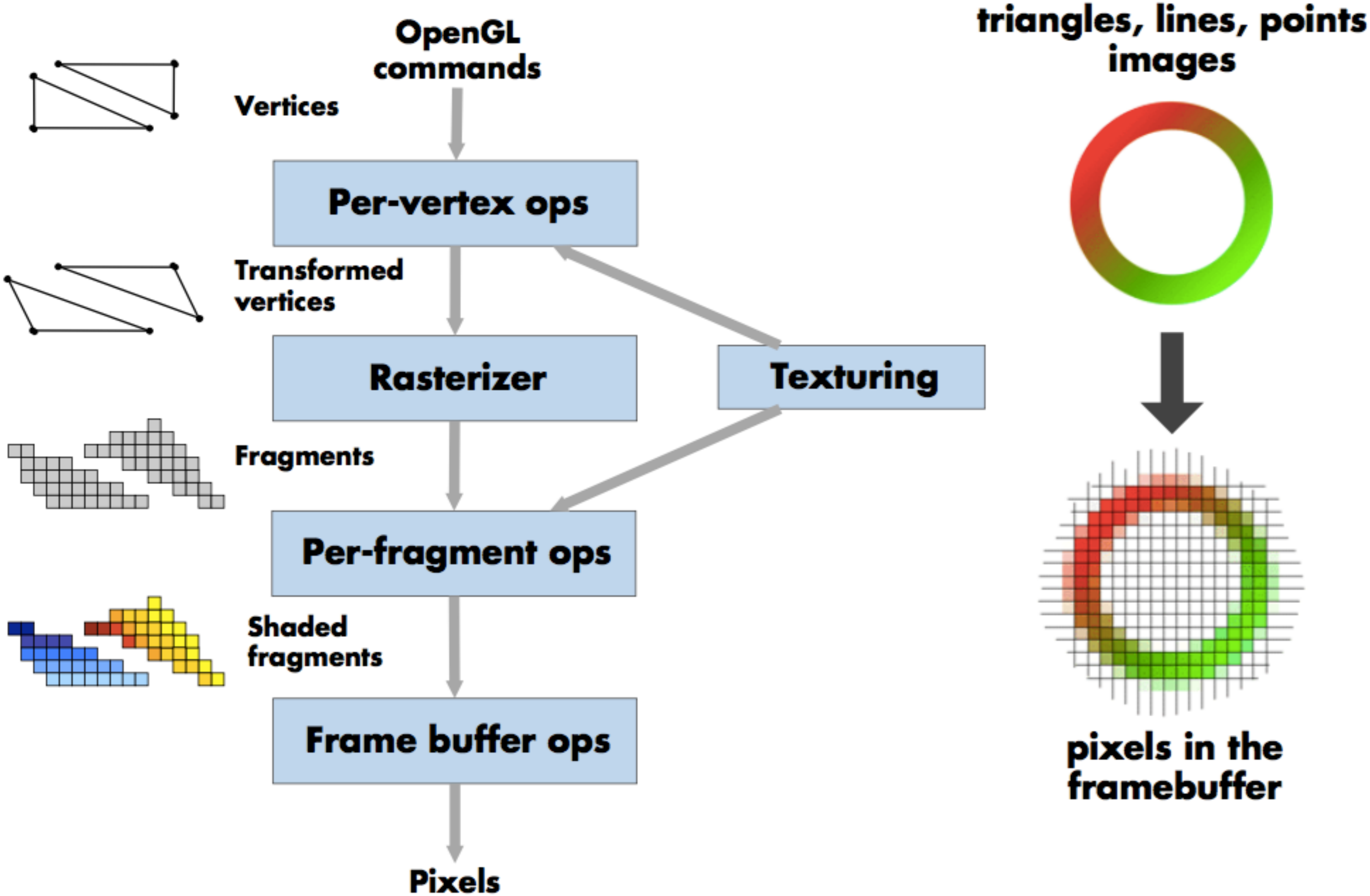


Shape Primitives



Example shape primitives (OpenGL)

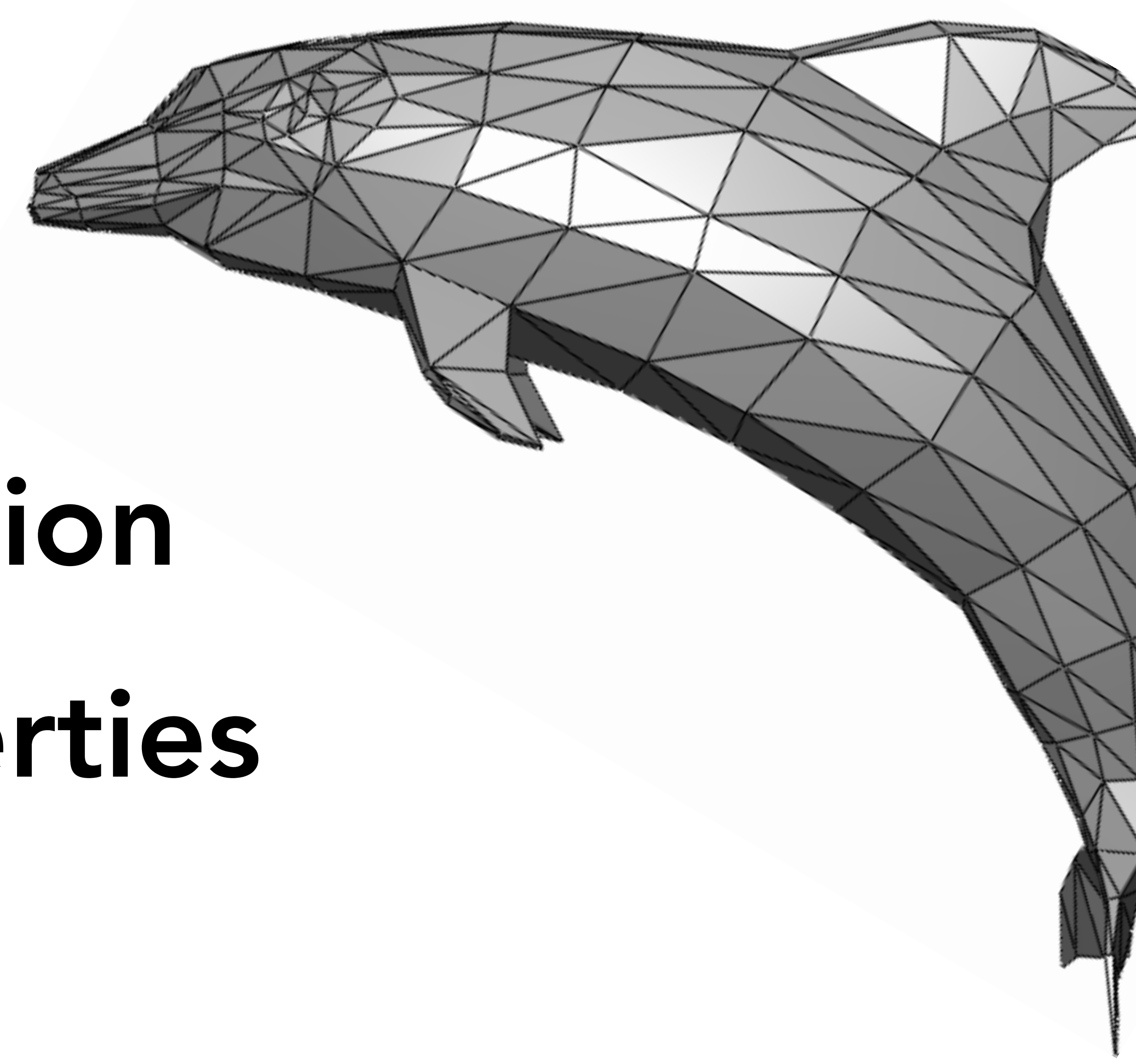
Graphics Pipeline = Abstract Drawing Machine



Triangles - Fundamental Area Primitive

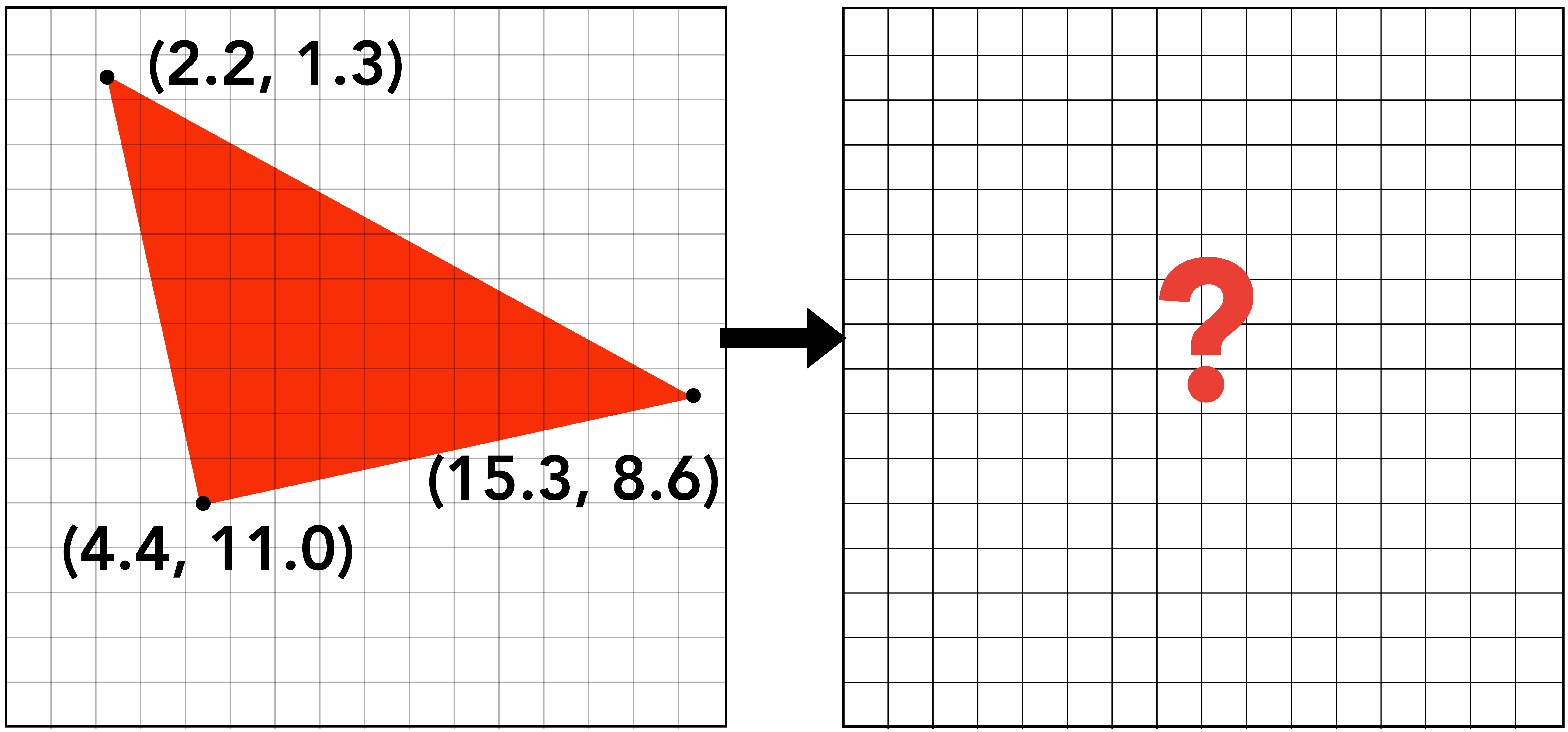
Why triangles?

- Most basic polygon
  - Break up other polygons
  - Optimize one implementation
- Triangles have unique properties
  - Guaranteed to be planar
  - Well-defined interior
  - Well-defined method for interpolating values at vertices over triangle (barycentric interpolation)



Drawing a Triangle
To The Framebuffer
(“Rasterization”)

What Pixel Values Approximate a Triangle?



Input: position of triangle vertices projected on screen

Output: set of pixel values approximating triangle

Today, Let's Start With A Simple Approach: Sampling

Sampling a Function

Evaluating a function at a point is sampling.

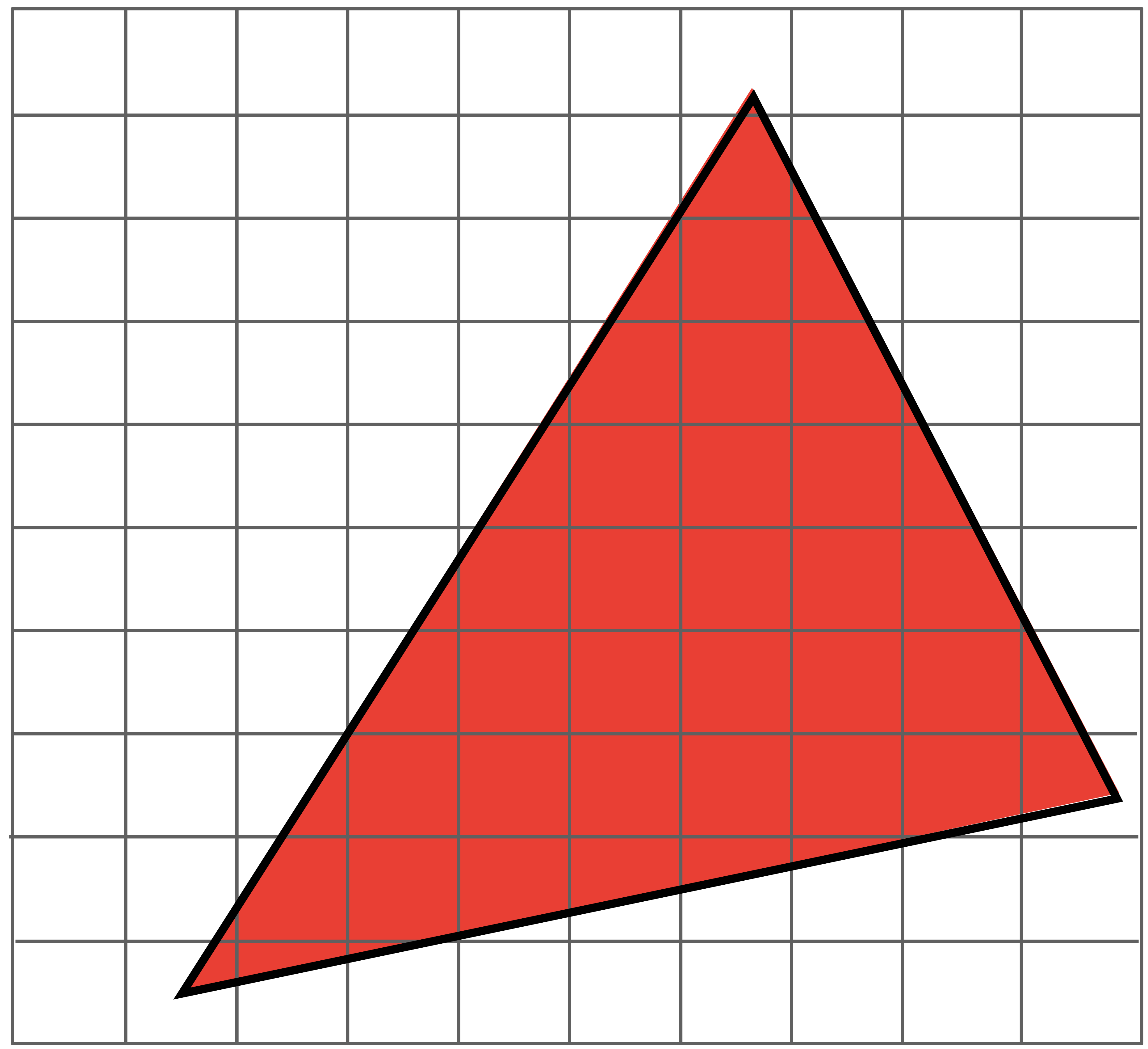
We can discretize a function by periodic sampling.

```
for( int x = 0; x < xmax; x++ )
    output[x] = f(x);
```

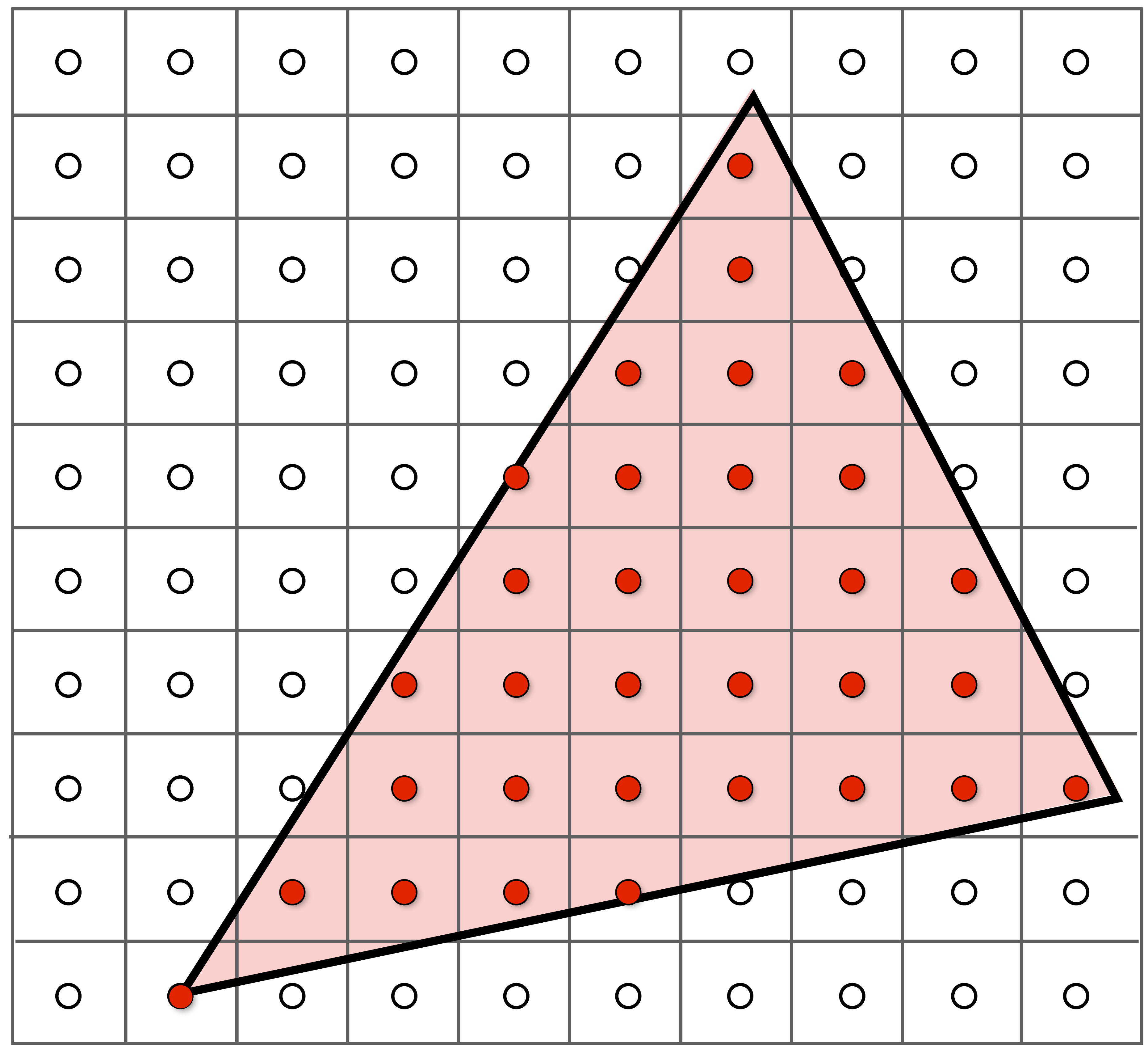
Sampling is a core idea in graphics. We'll sample time (1D), area (2D), angle (2D), volume (3D) …

We'll sample N-dimensional functions, even infinite dimensional functions.

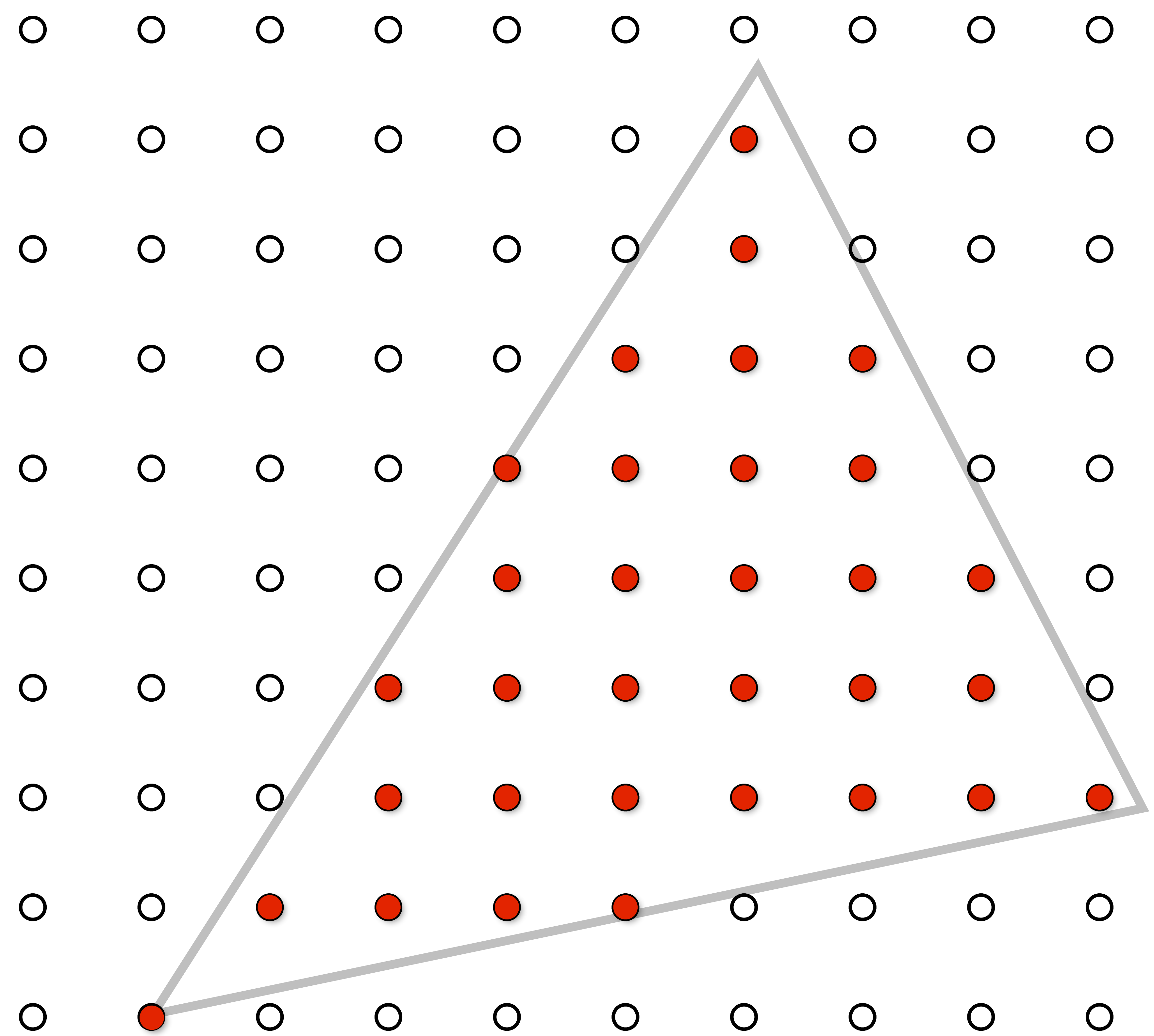
Let’s Try Rasterization As 2D Sampling



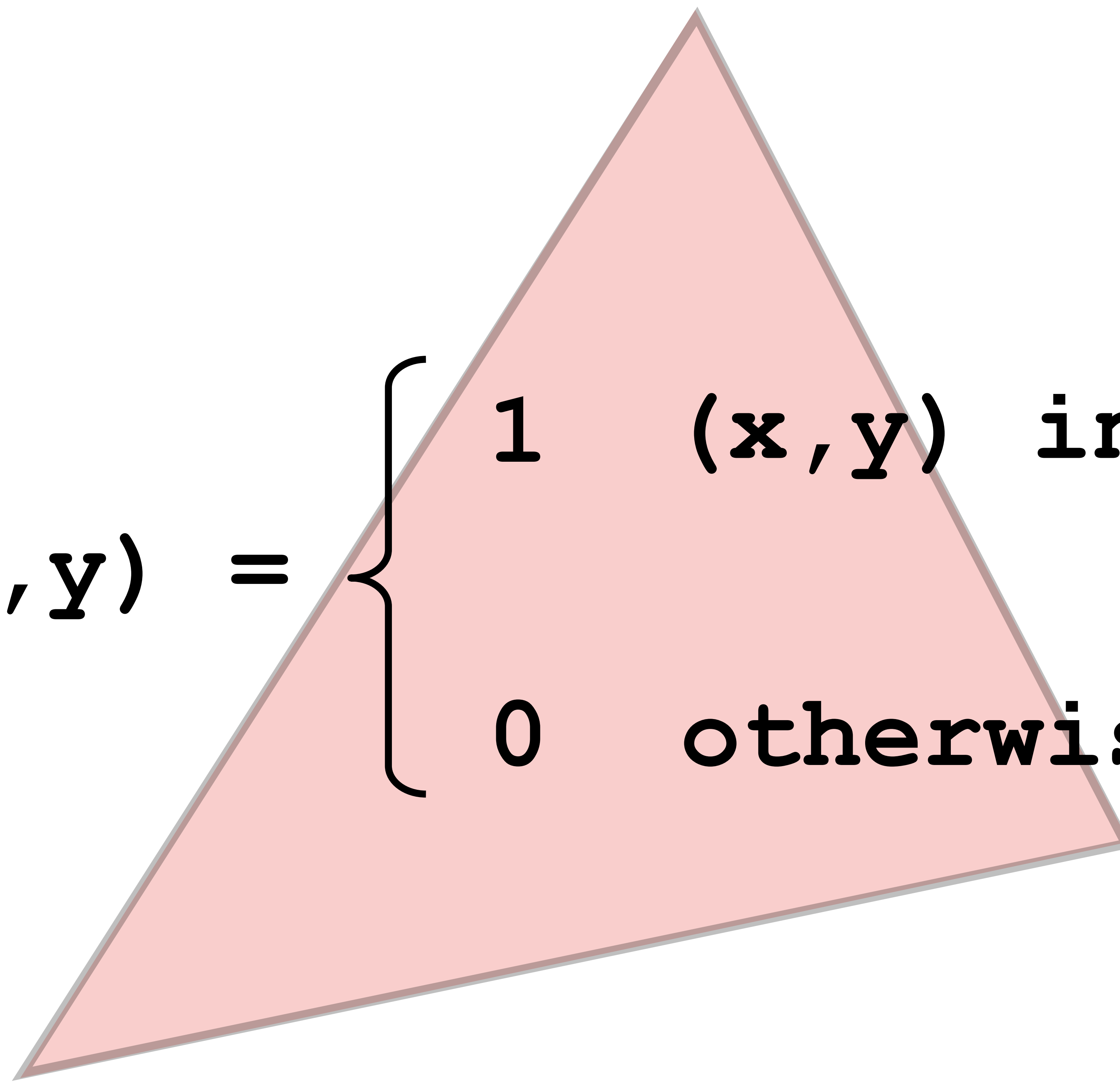
Sample If Each Pixel Center Is Inside Triangle



Sample If Each Pixel Center Is Inside Triangle



Define Binary Function: `inside(tri,x,y)`



$$\texttt{inside(t,x,y)} = \begin{cases} 1 & \texttt{(x,y) in triangle t} \\ 0 & \texttt{otherwise} \end{cases}$$

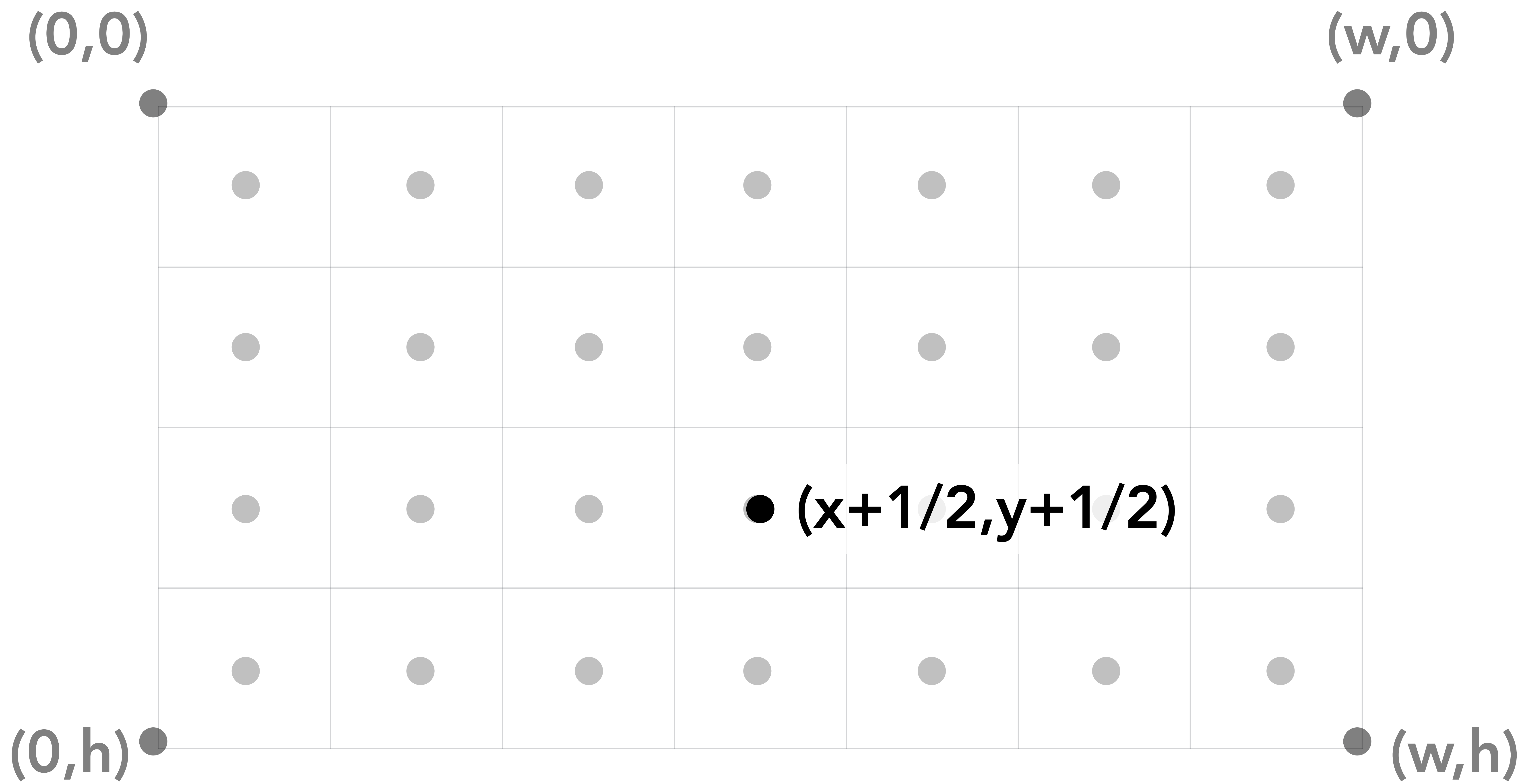
Rasterization = Sampling A 2D Indicator Function

```
for( int x = 0; x < xmax; x++ )
  for( int y = 0; y < ymax; y++ )
    Image[x][y] = f(x + 0.5, y + 0.5);
```

Rasterize triangle `tri` by sampling the function

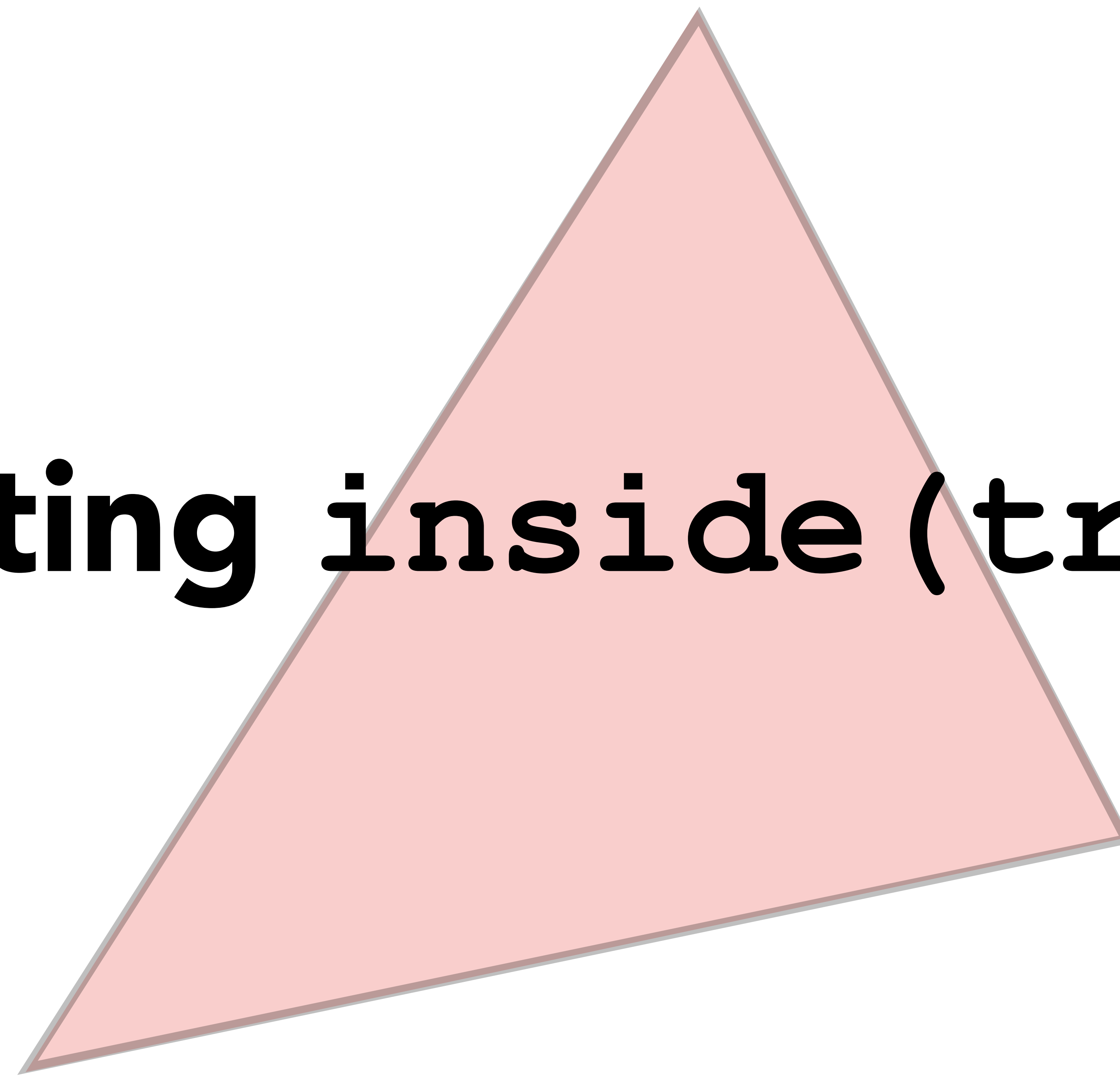
`f(x,y) = inside(tri,x,y)`

Implementation Detail: Sample Locations

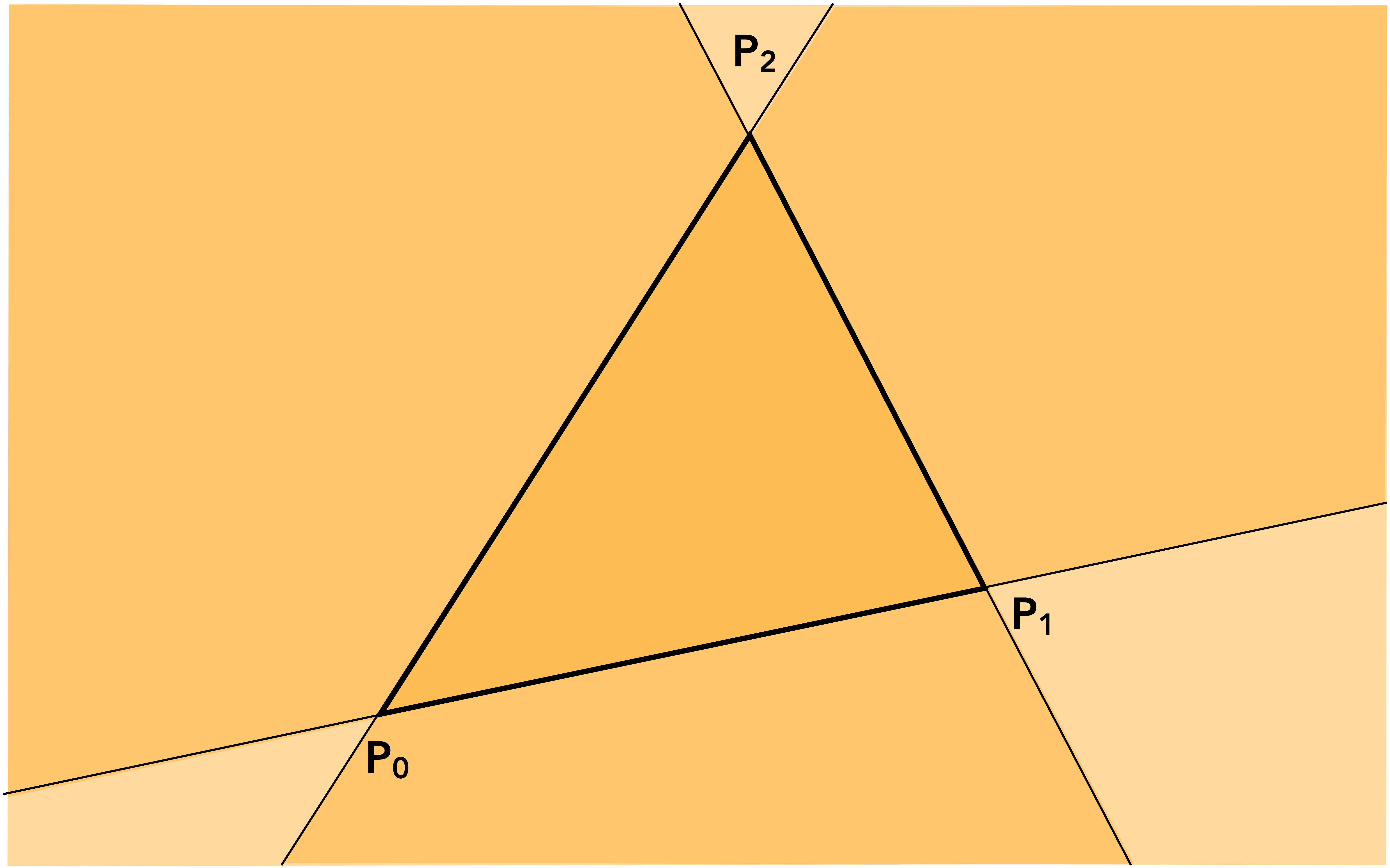


Sample location for pixel (x,y)

Evaluating `inside(tri,x,y)`



Triangle = Intersection of Three Half Planes

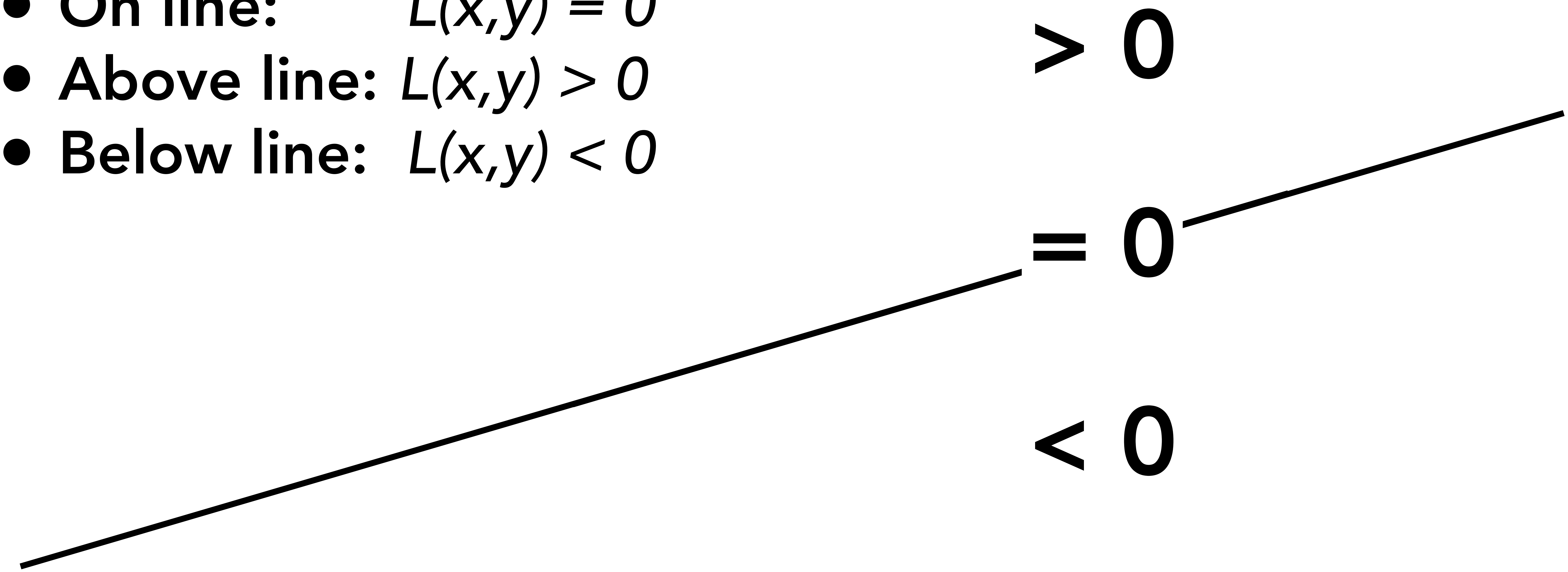


Each Line Defines Two Half-Planes

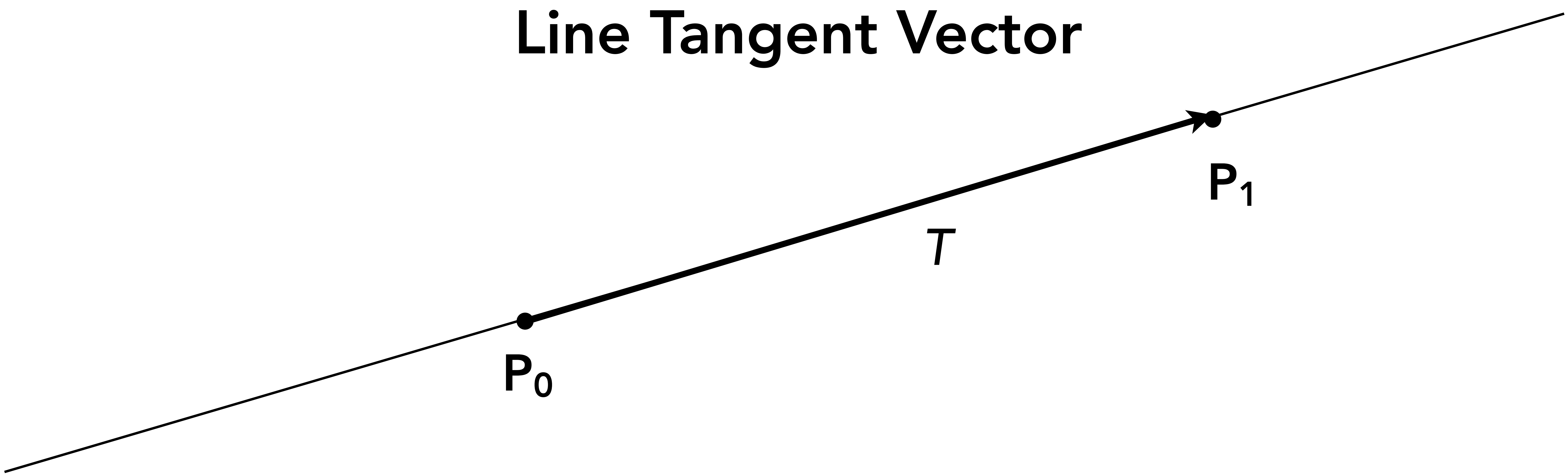
Implicit line equation

- $L(x,y) = Ax + By + C$

- On line: $L(x,y) = 0$
- Above line: $L(x,y) > 0$
- Below line: $L(x,y) < 0$

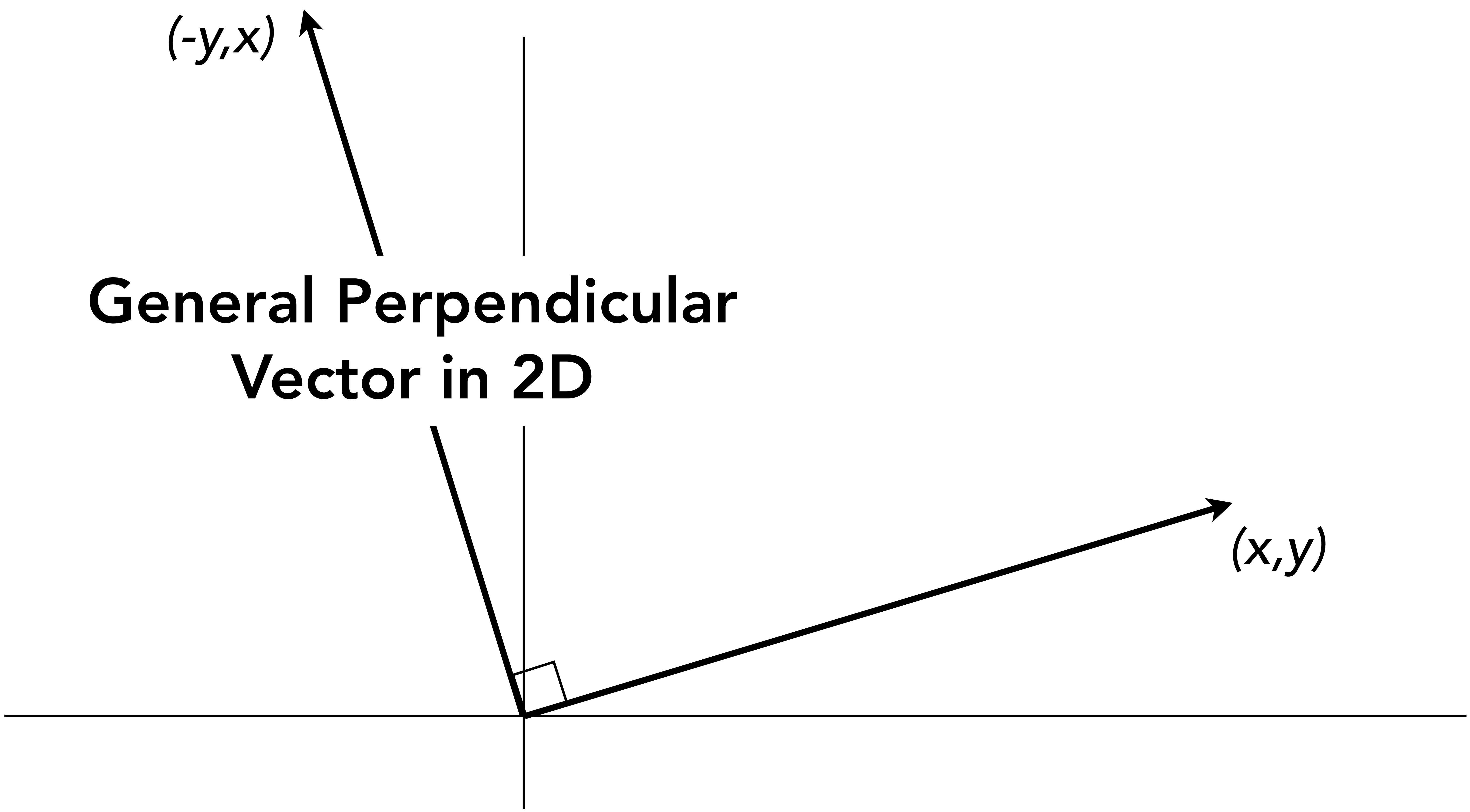


Line Equation Derivation



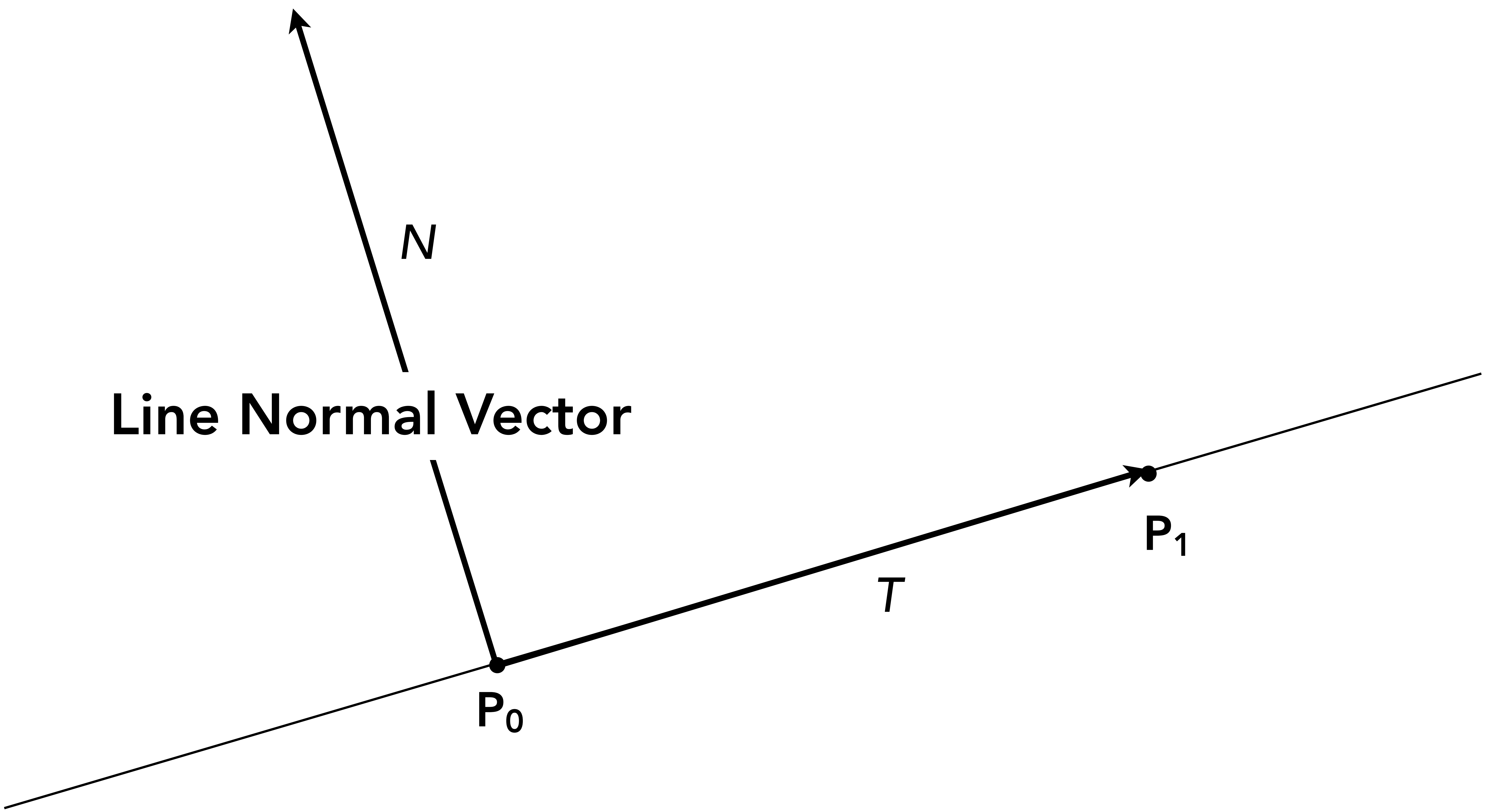
$$T = P_1 - P_0 = (x_1 - x_0, y_1 - y_0)$$

Line Equation Derivation



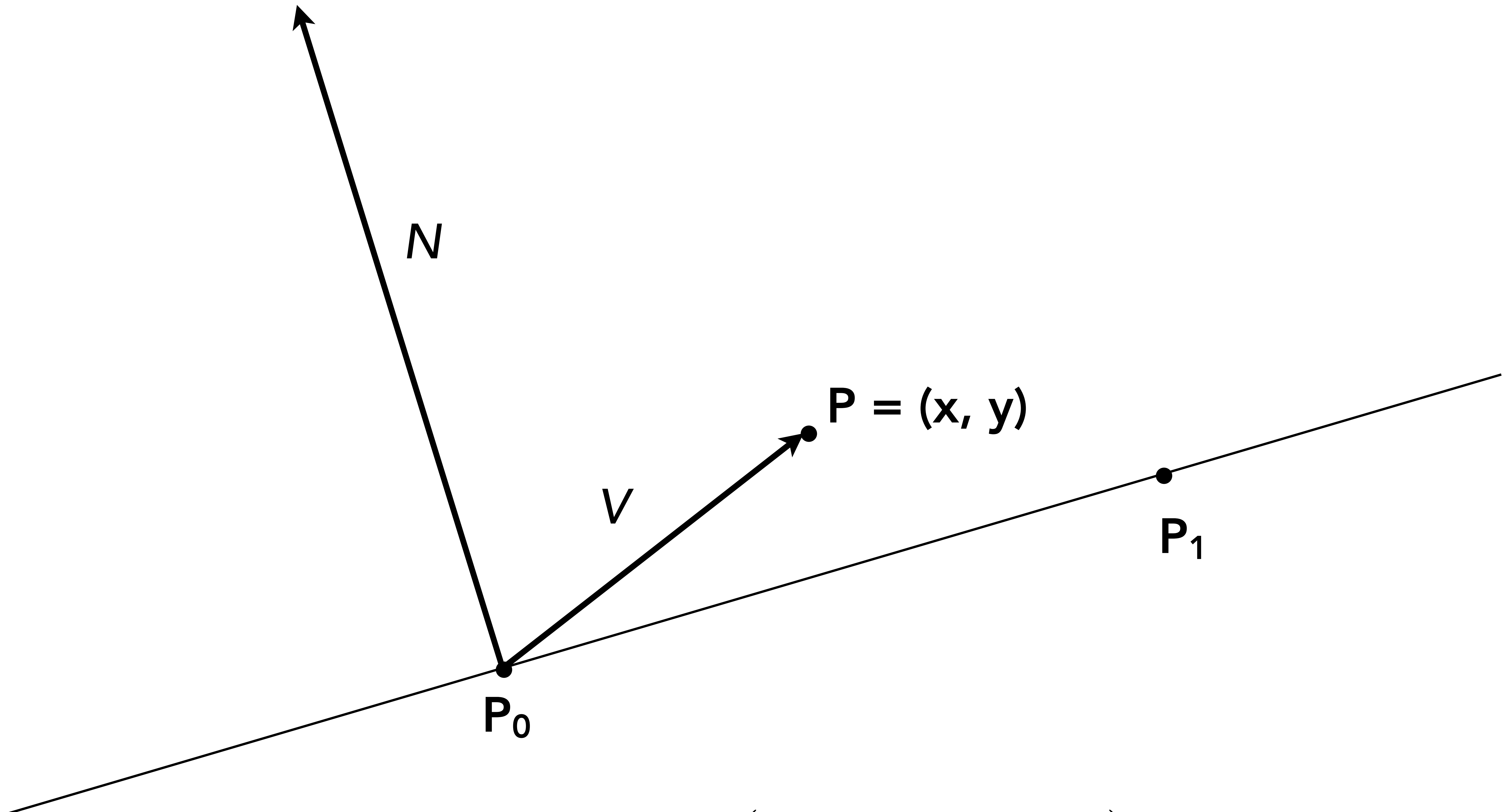
$$\text{Perp}(x, y) = (-y, x)$$

Line Equation Derivation



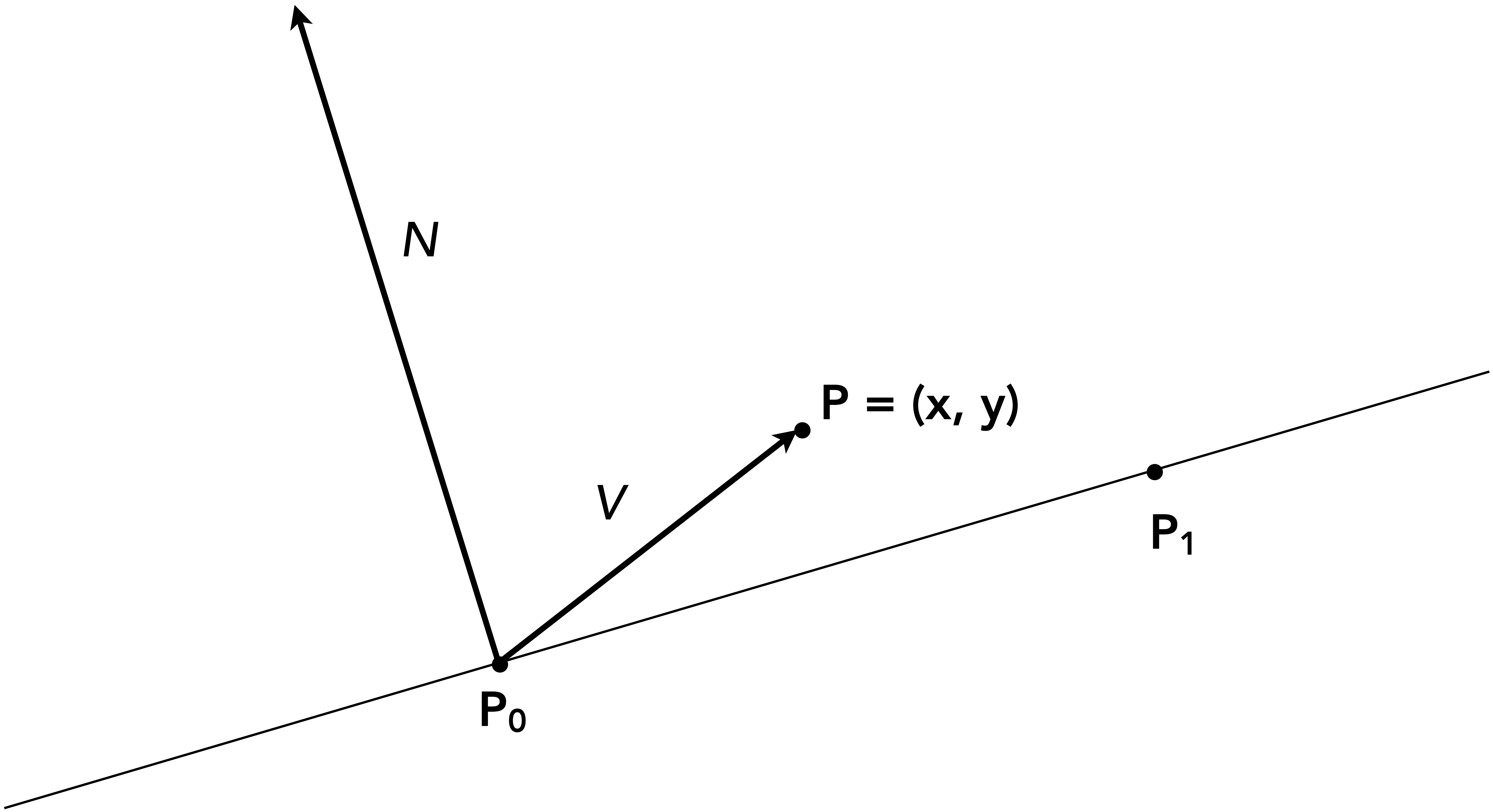
$$N = \mathrm{Perp}(T) = (-(y_1 - y_0), x_1 - x_0)$$

Line Equation Derivation



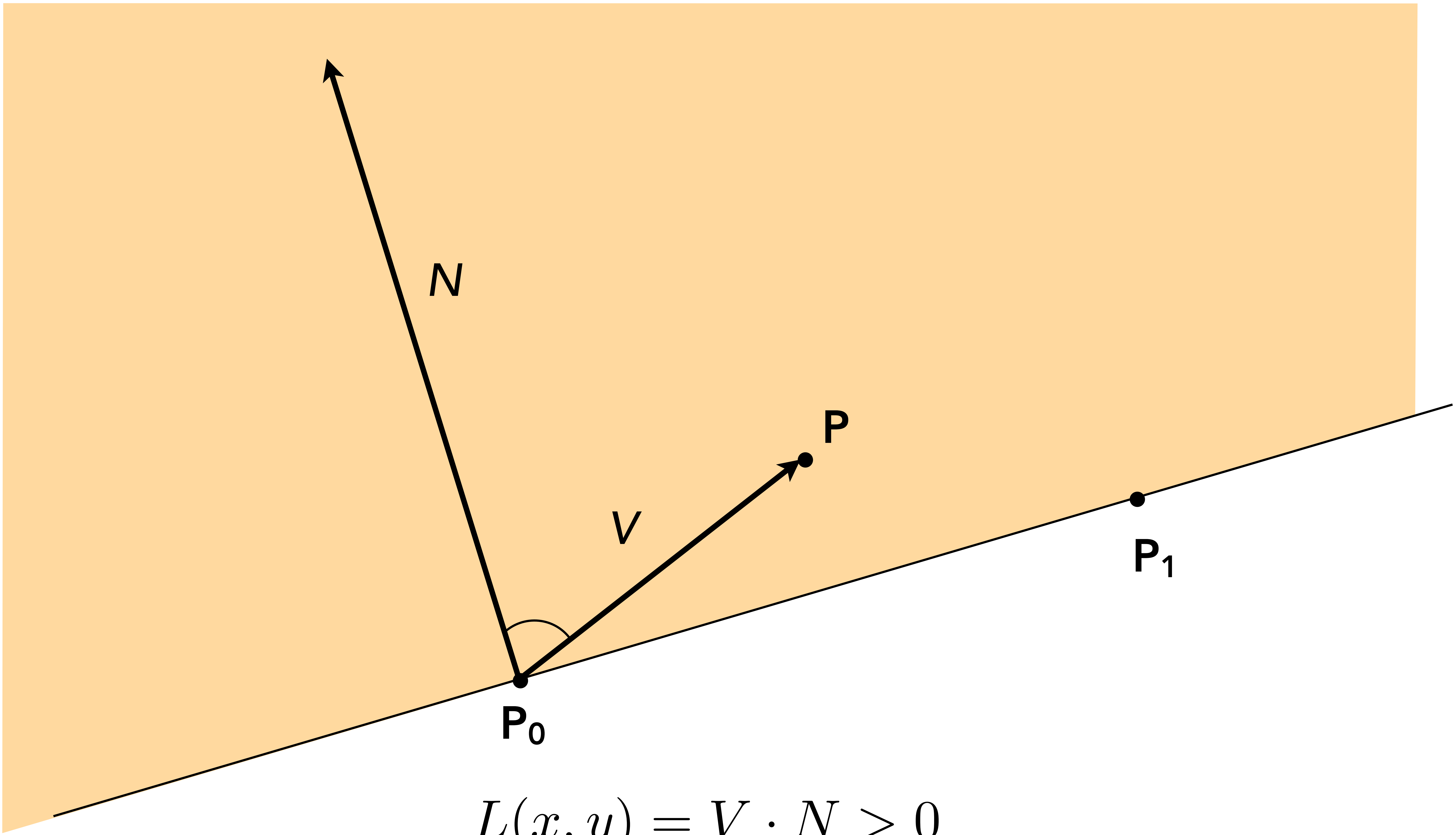
$$V = P - P_0 = (x - x_0, y - y_0)$$

Line Equation



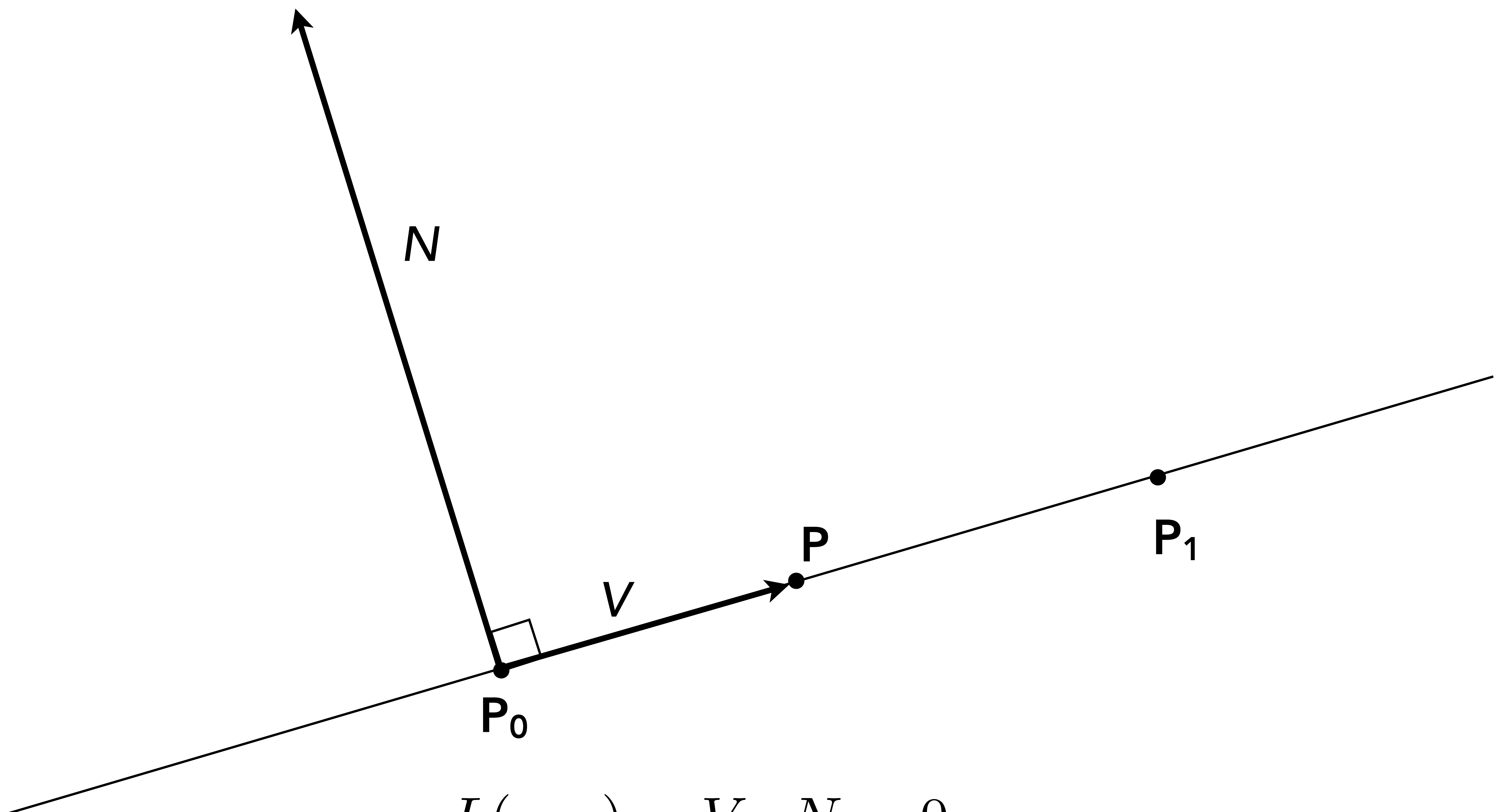
$$L(x, y) = V \cdot N = -(x - x_0)(y_1 - y_0) + (y - y_0)(x_1 - x_0)$$

Line Equation Tests



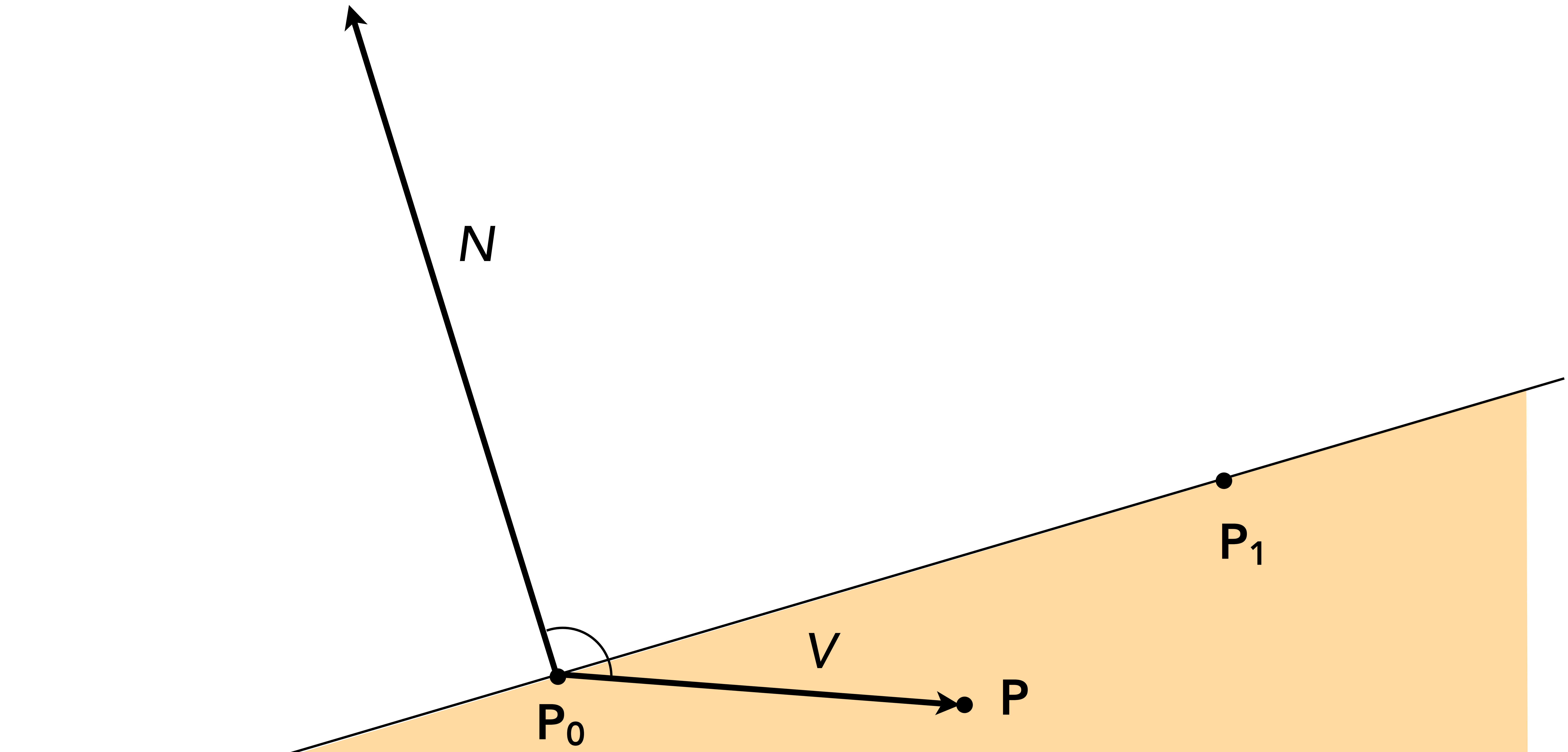
$$L(x, y) = V \cdot N > 0$$

Line Equation Tests



$$L(x, y) = V \cdot N = 0$$

Line Equation Tests



$$L(x, y) = V \cdot N < 0$$

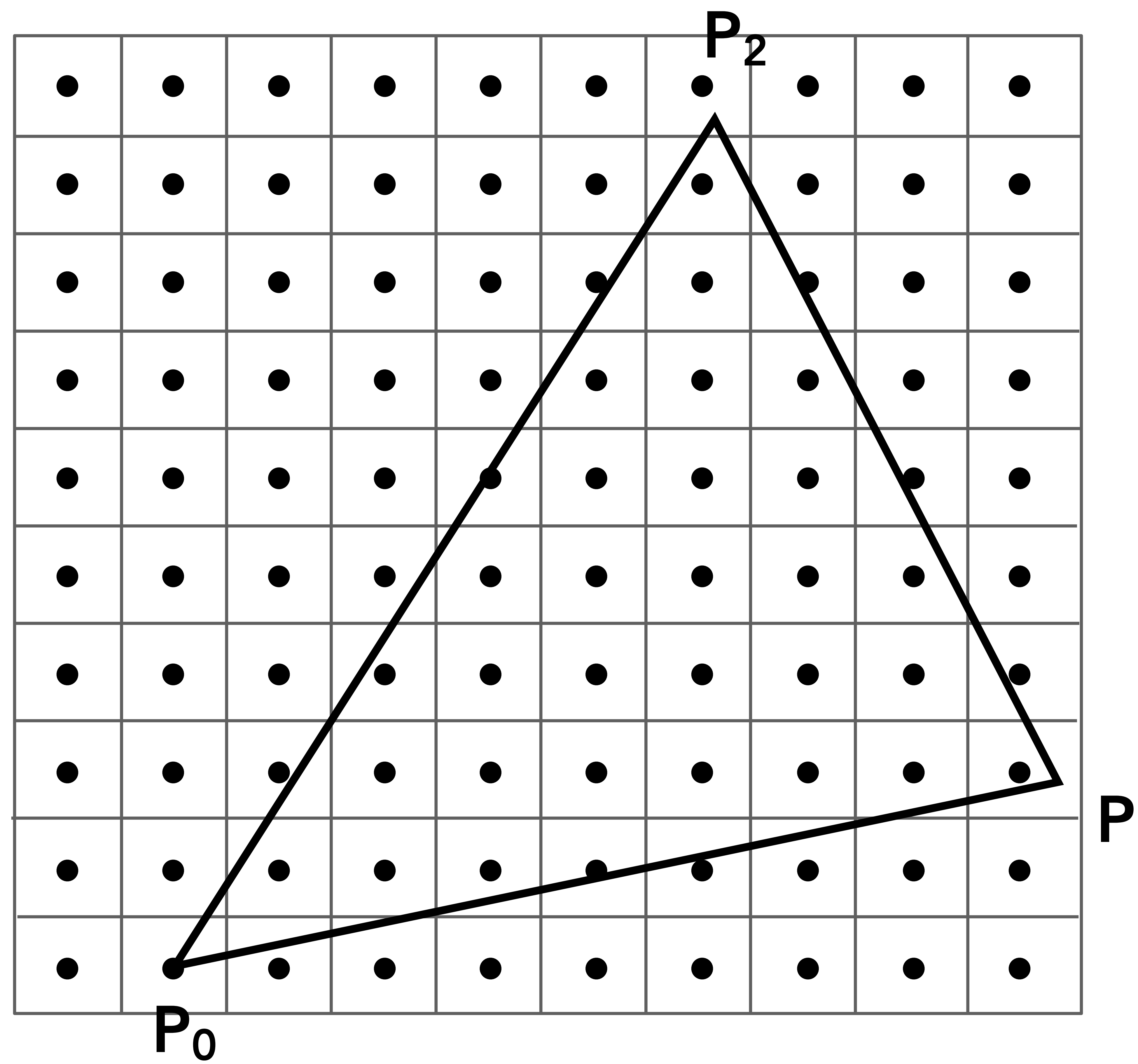
Point-in-Triangle Test: Three Line Tests

$P_i = (X_i, Y_i)$

$dX_i = X_{i+1} - X_i$
$dY_i = Y_{i+1} - Y_i$

$L_i(x, y) = -(x - X_i)\, dY_i + (y - Y_i)\, dX_i$
$= A_i\, x + B_i\, y + C_i$

$L_i(x, y) = 0$: point on edge
< 0 : outside edge
> 0 : inside edge



Compute line equations from pairs of vertices

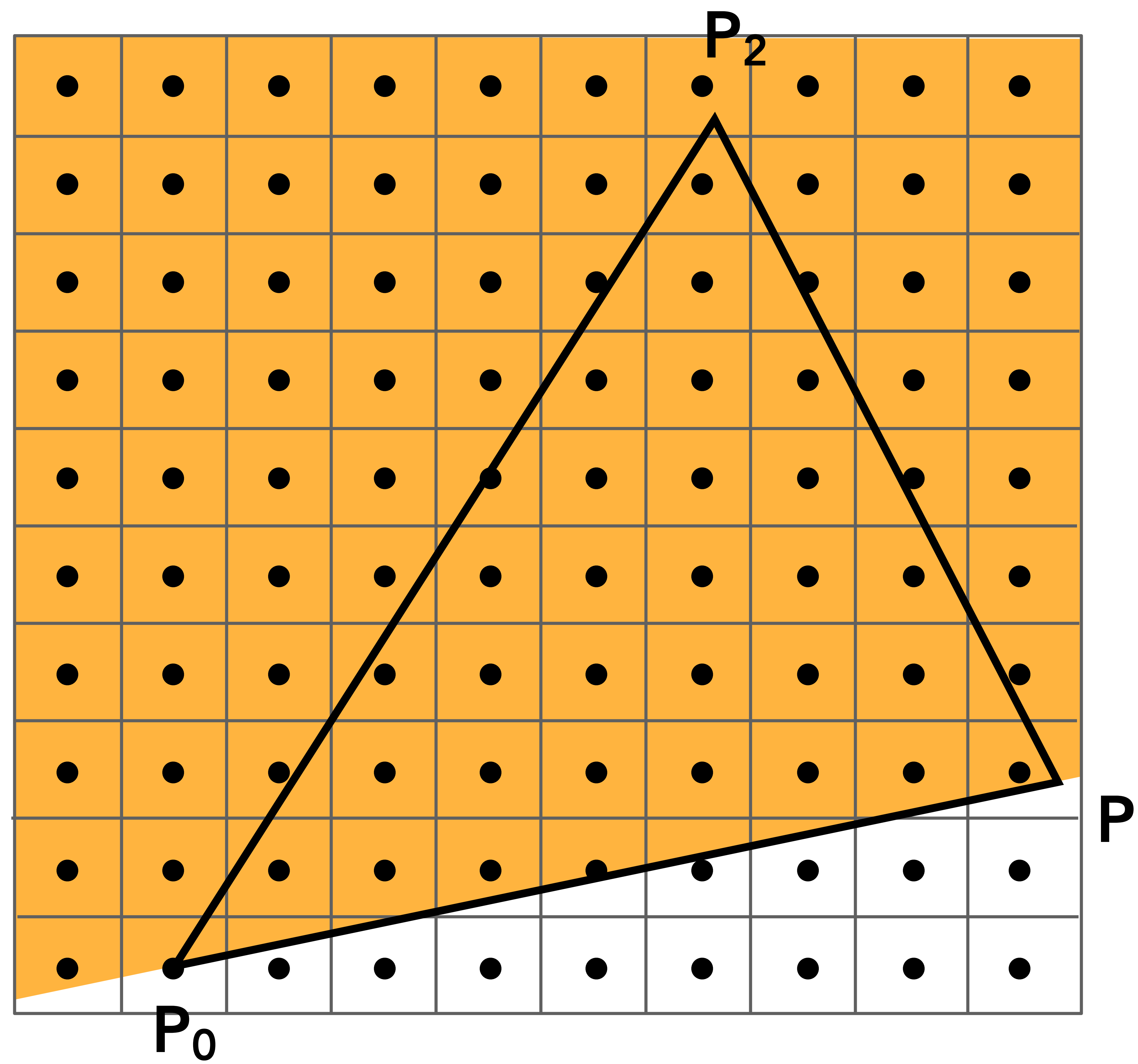
Point-in-Triangle Test: Three Line Tests

$P_i = (X_i, Y_i)$

$dX_i = X_{i+1} - X_i$
$dY_i = Y_{i+1} - Y_i$

$L_i(x, y) = -(x - X_i)\, dY_i + (y - Y_i)\, dX_i$
$= A_i x + B_i y + C_i$

$L_i(x, y) = 0$: point on edge
< 0 : outside edge
> 0 : inside edge



$L_0(x, y) > 0$

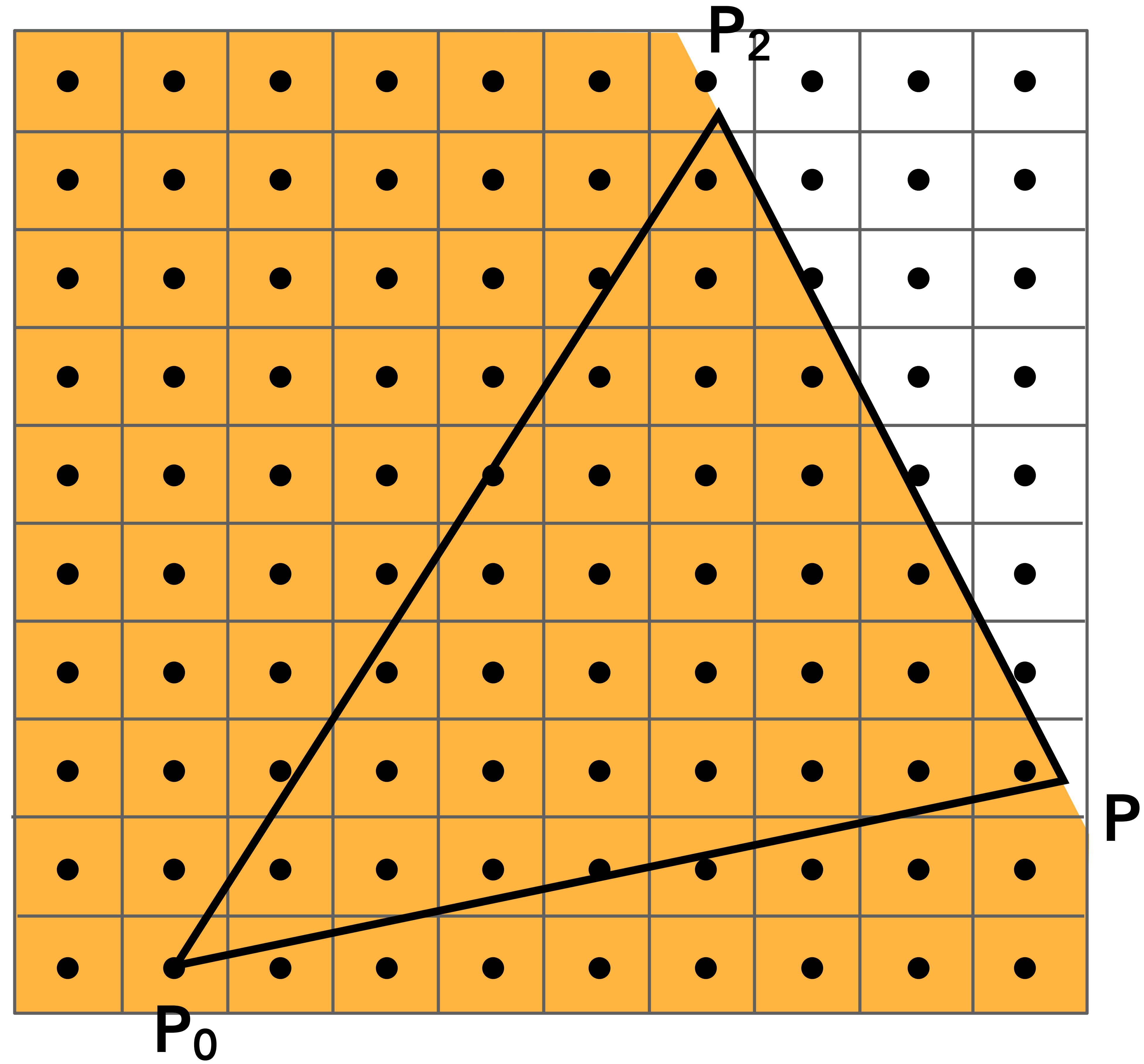
Point-in-Triangle Test: Three Line Tests

$P_i = (X_i, Y_i)$

$dX_i = X_{i+1} - X_i$
$dY_i = Y_{i+1} - Y_i$

$L_i(x, y) = -(x - X_i)\, dY_i + (y - Y_i)\, dX_i$
$= A_i x + B_i y + C_i$

$L_i(x, y) = 0$: point on edge
< 0 : outside edge
> 0 : inside edge



$L_1(x, y) > 0$

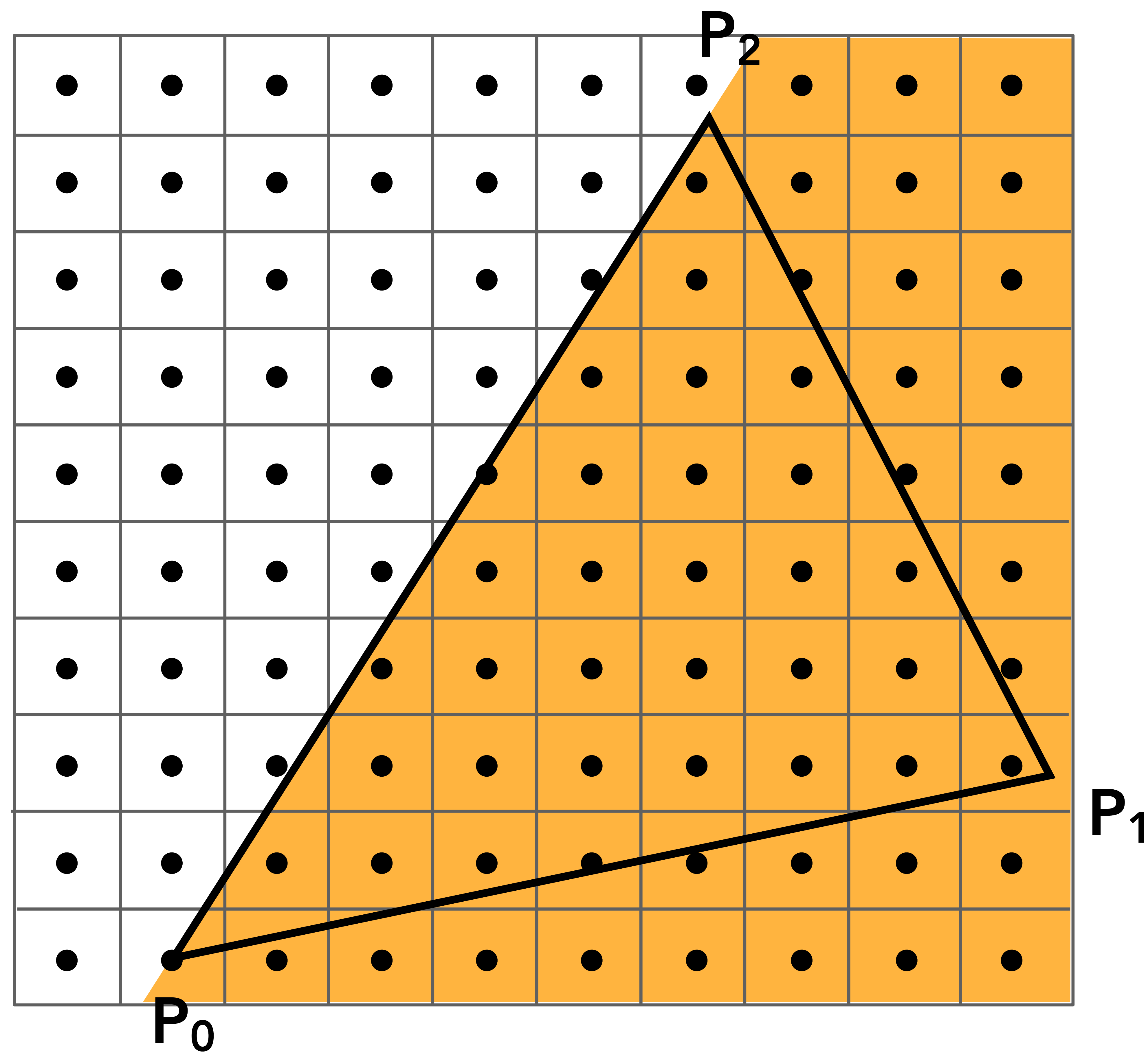
Point-in-Triangle Test: Three Line Tests

$P_i = (X_i, Y_i)$

$dX_i = X_{i+1} - X_i$
$dY_i = Y_{i+1} - Y_i$

$L_i(x, y) = -(x - X_i)\, dY_i + (y - Y_i)\, dX_i$
$= A_i x + B_i y + C_i$

$L_i(x, y) = 0$: point on edge
< 0 : outside edge
> 0 : inside edge

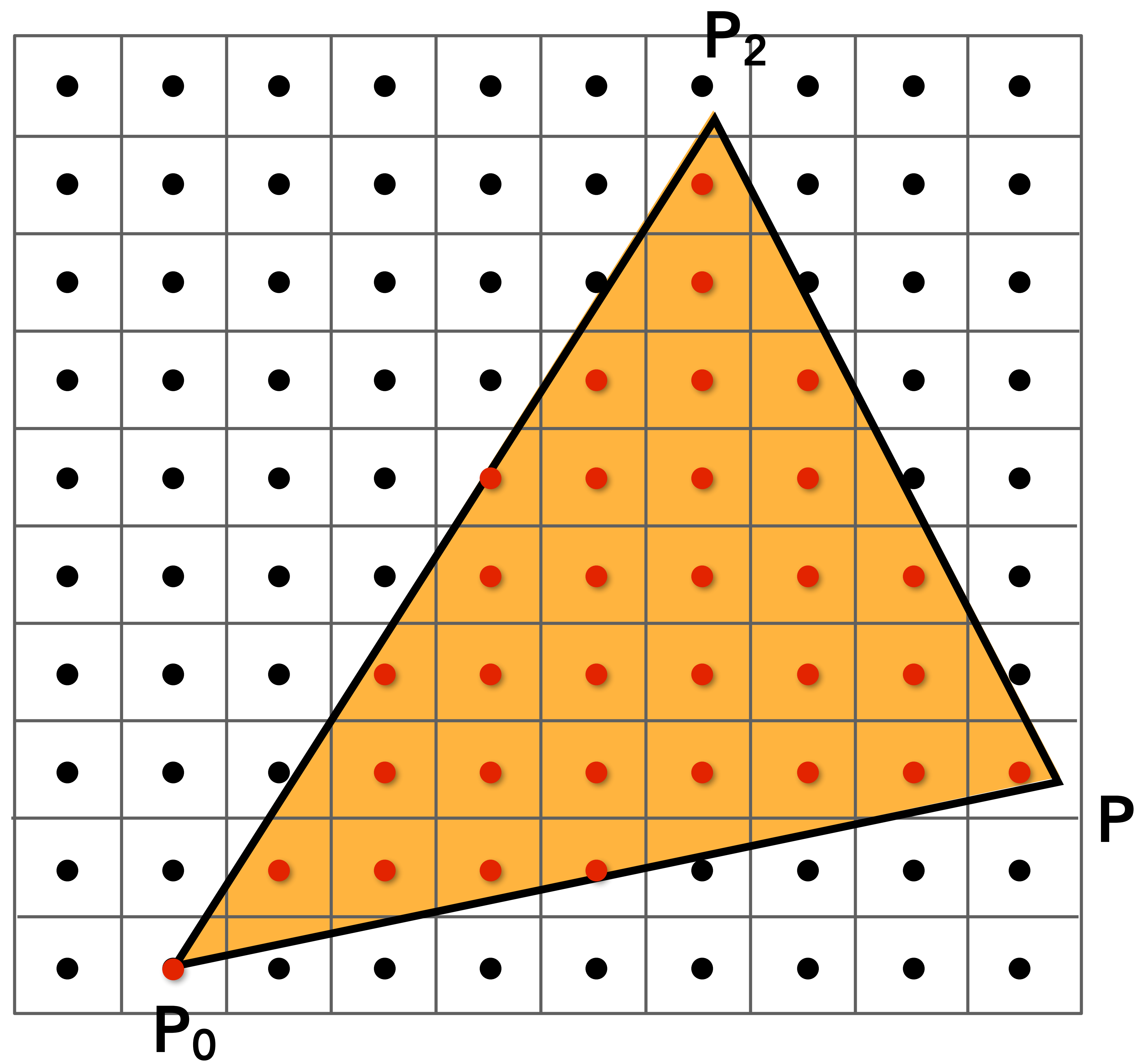


$L_2(x, y) > 0$

Point-in-Triangle Test: Three Line Tests

Sample point $s = (sx, sy)$ is inside the triangle if it is inside all three lines.

$$inside(sx, sy) =$$
$$L_0(sx, sy) > 0 \;\&\&$$
$$L_1(sx, sy) > 0 \;\&\&$$
$$L_2(sx, sy) > 0;$$

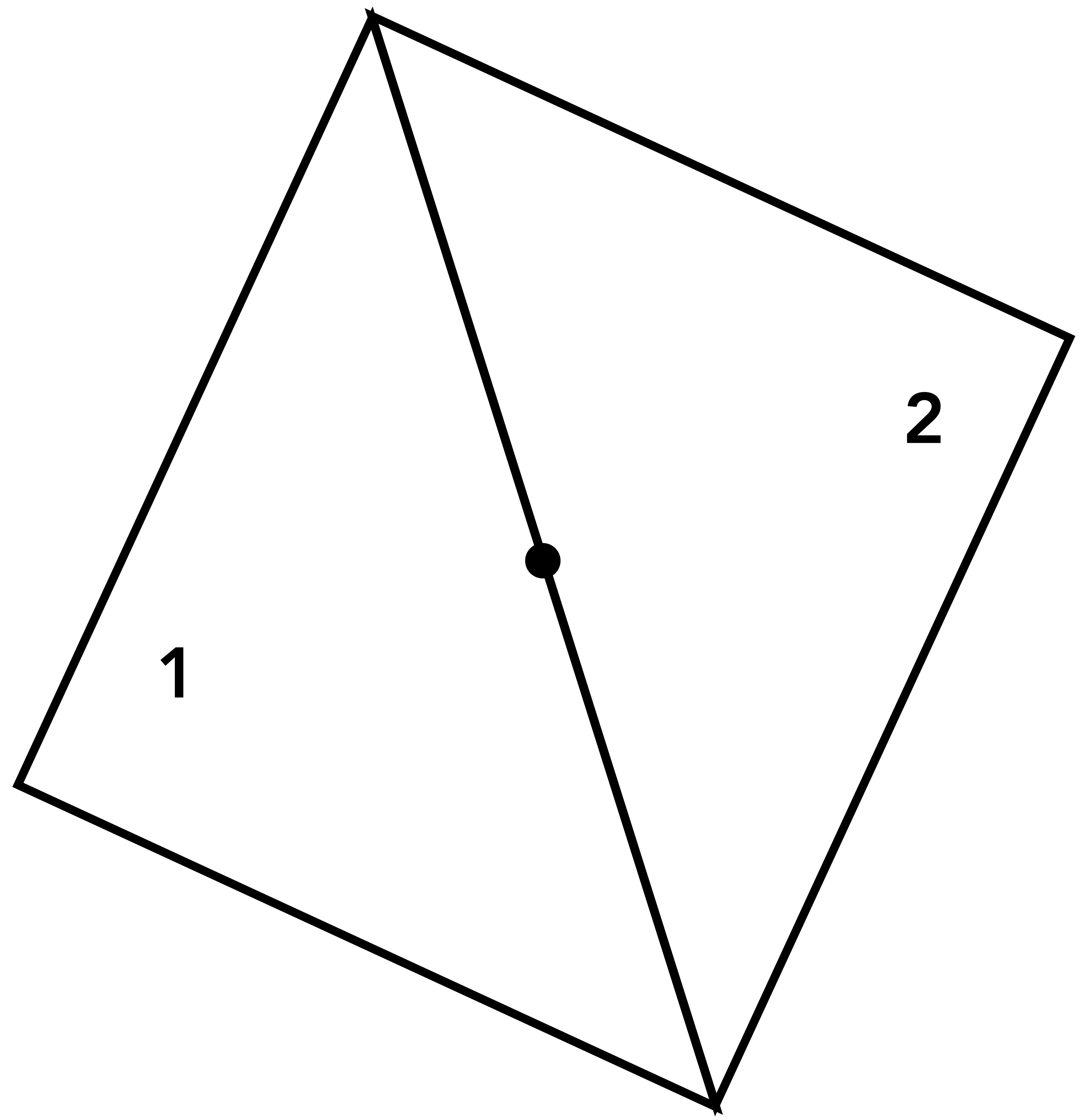


Note: actual implementation of $inside(sx,sy)$ involves ≤ checks based on edge rules

Some Details

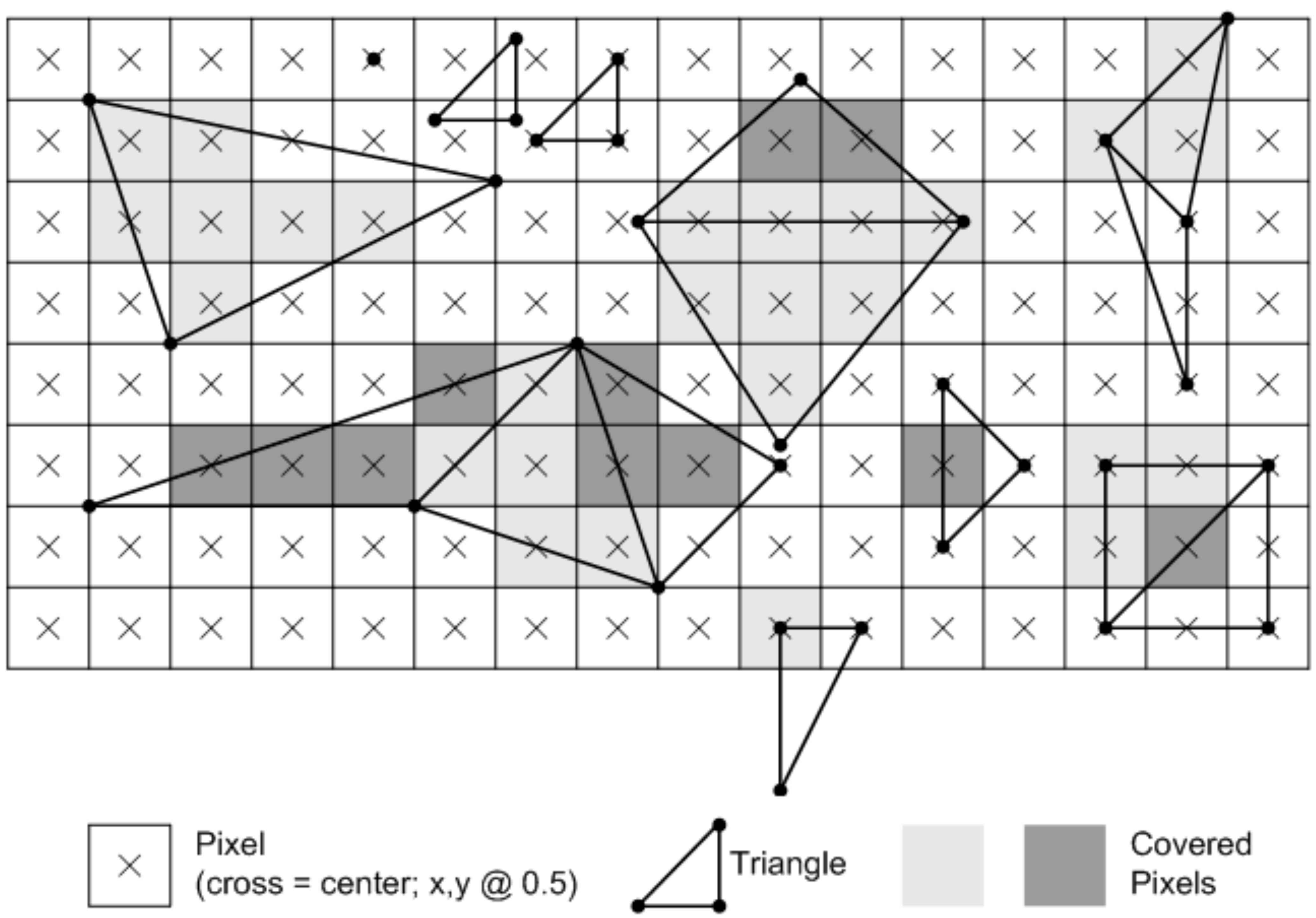
Edge Cases (Literally)

Is this sample point covered by triangle 1, triangle 2, or both?



OpenGL/Direct3D Edge Rules

When sample point falls on an edge, the sample is classified as within triangle if the edge is a “top edge” or “left edge”

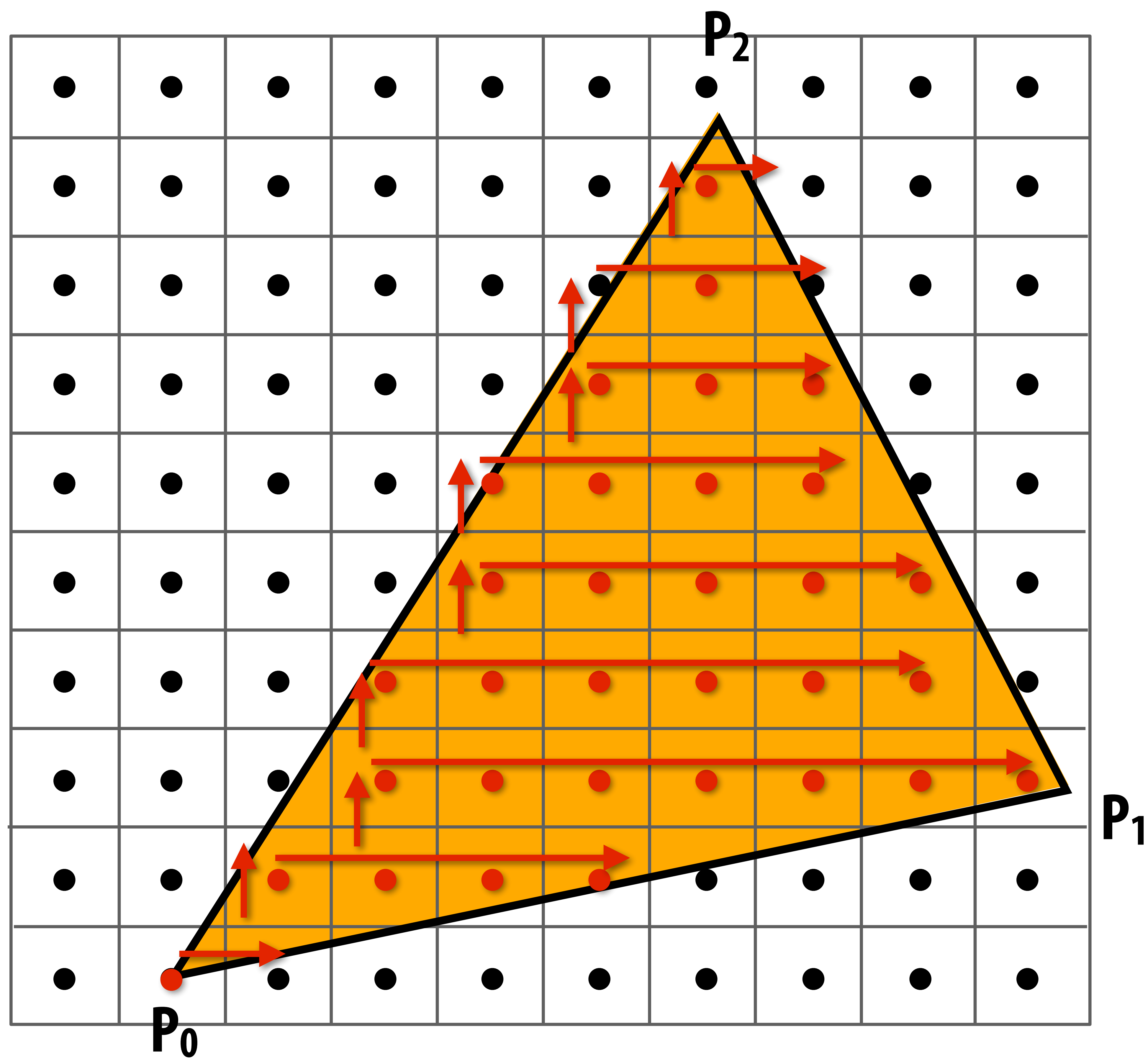


Top edge: horizontal edge that is above all other edges

Left edge: an edge that is not exactly horizontal and is on the left side of the triangle. (triangle can have one or two left edges)

Source: Direct3D Programming Guide, Microsoft

Incremental Triangle Traversal (Faster?)



Modern Approach: Tiled Triangle Traversal

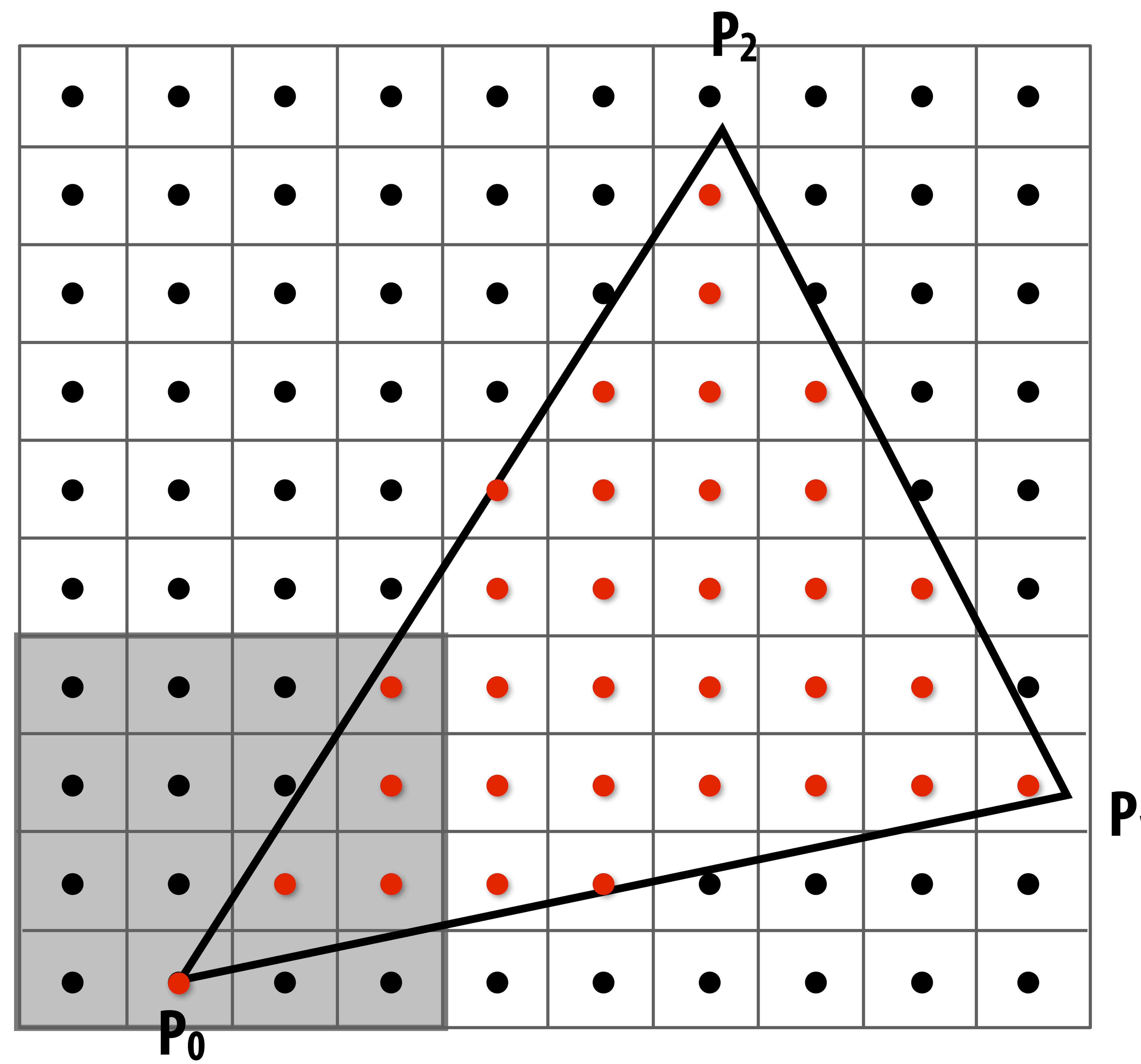
Traverse triangle in blocks

Test all samples in block in parallel

Advantages:

- Simplicity of wide parallel execution overcomes cost of extra point-in-triangle tests (most triangles cover many samples, especially when super-sampling)

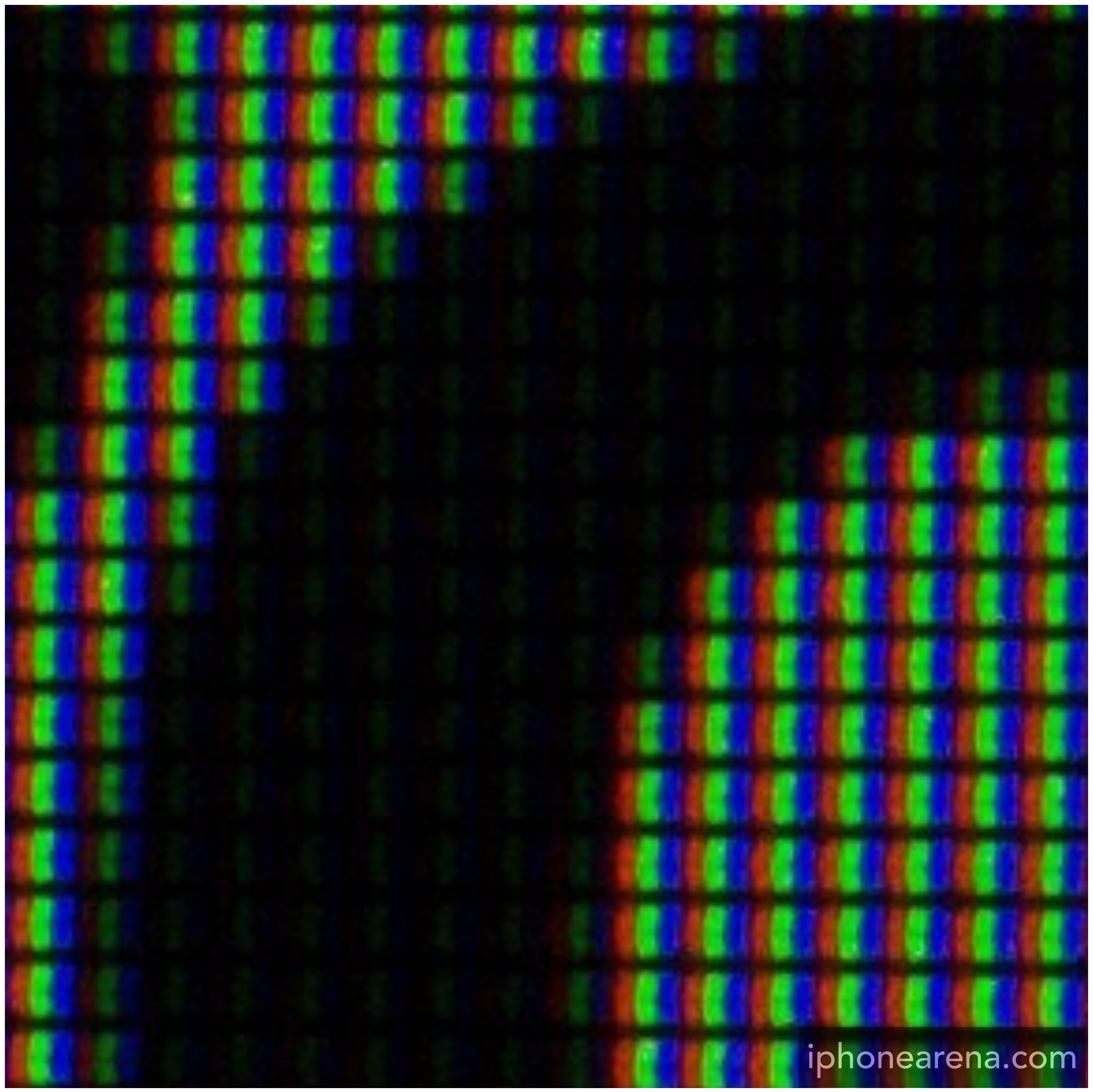
- Can skip sample testing work: entire block not in triangle ("early out"), entire block entirely within triangle ("early in")



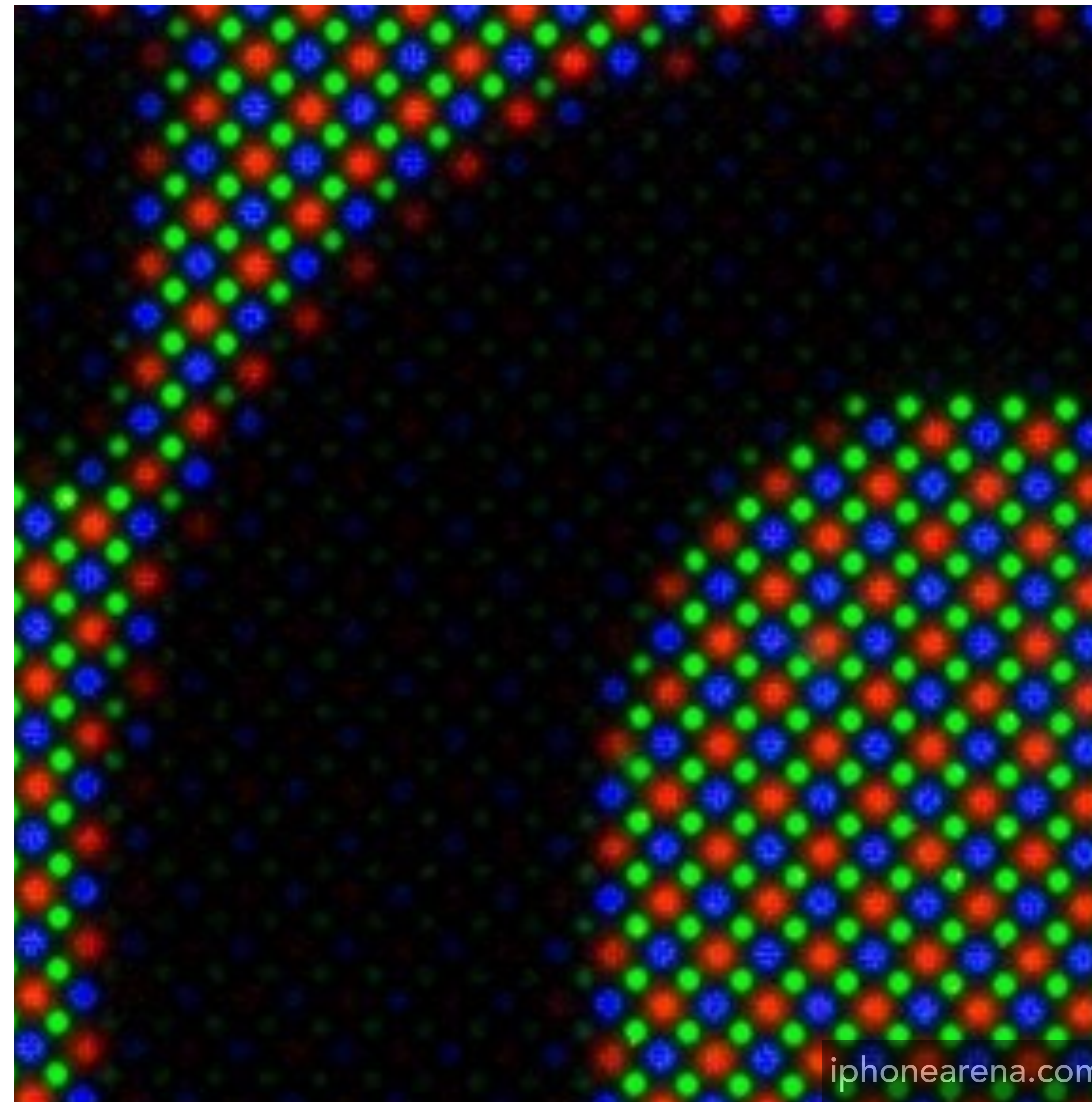
All modern GPUs have special-purpose hardware for efficient point-in-triangle tests

Signal Reconstruction on Real Displays

Real LCD Screen Pixels (Closeup)



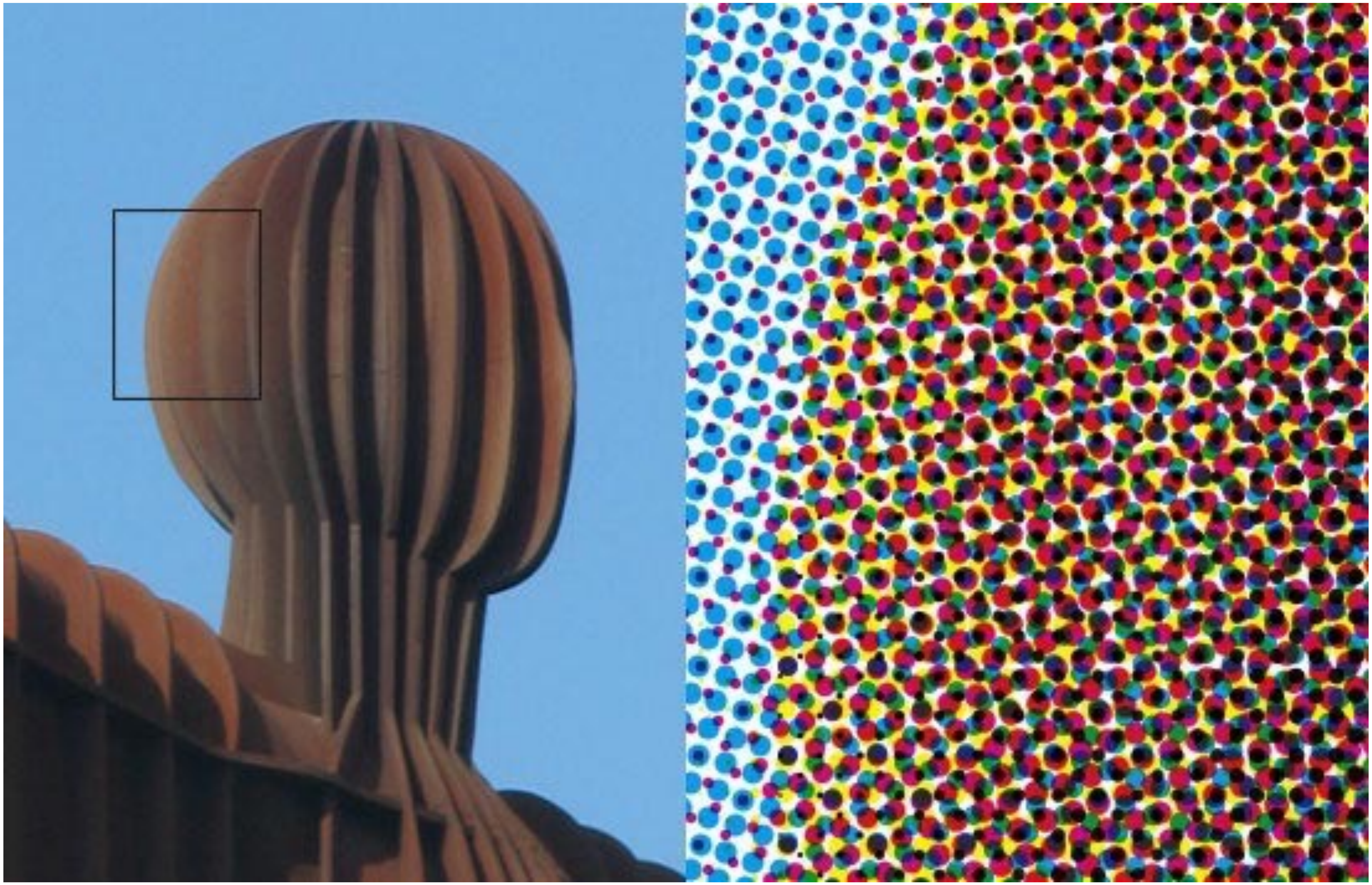
iPhone 6S



Galaxy S5

Notice R,G,B pixel geometry! But in this class, we will assume a colored square full-color pixel.

Aside: What About Other Display Methods?

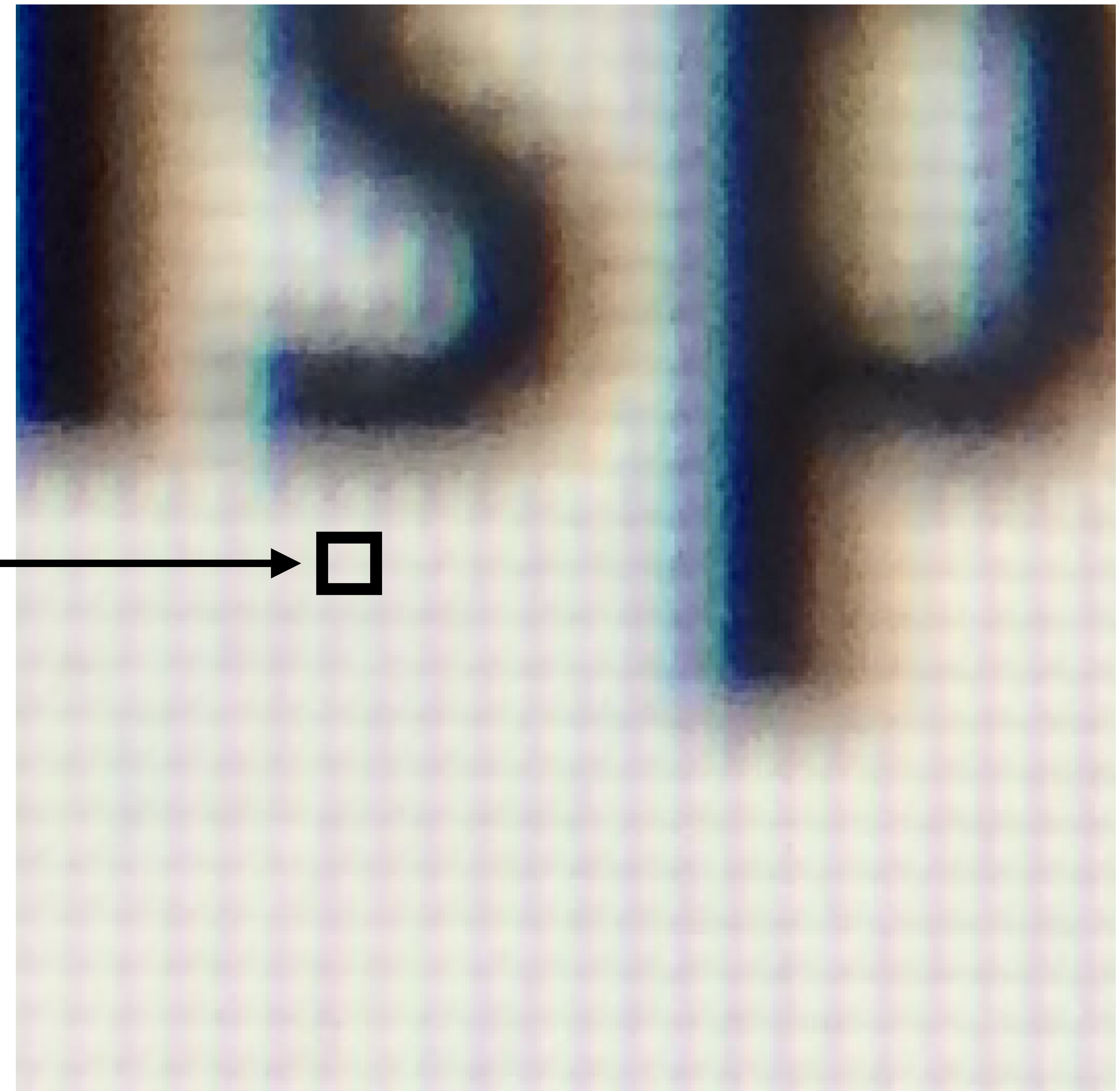


Color print: observe half-tone pattern

Assume Display Pixels Emit Square of Light

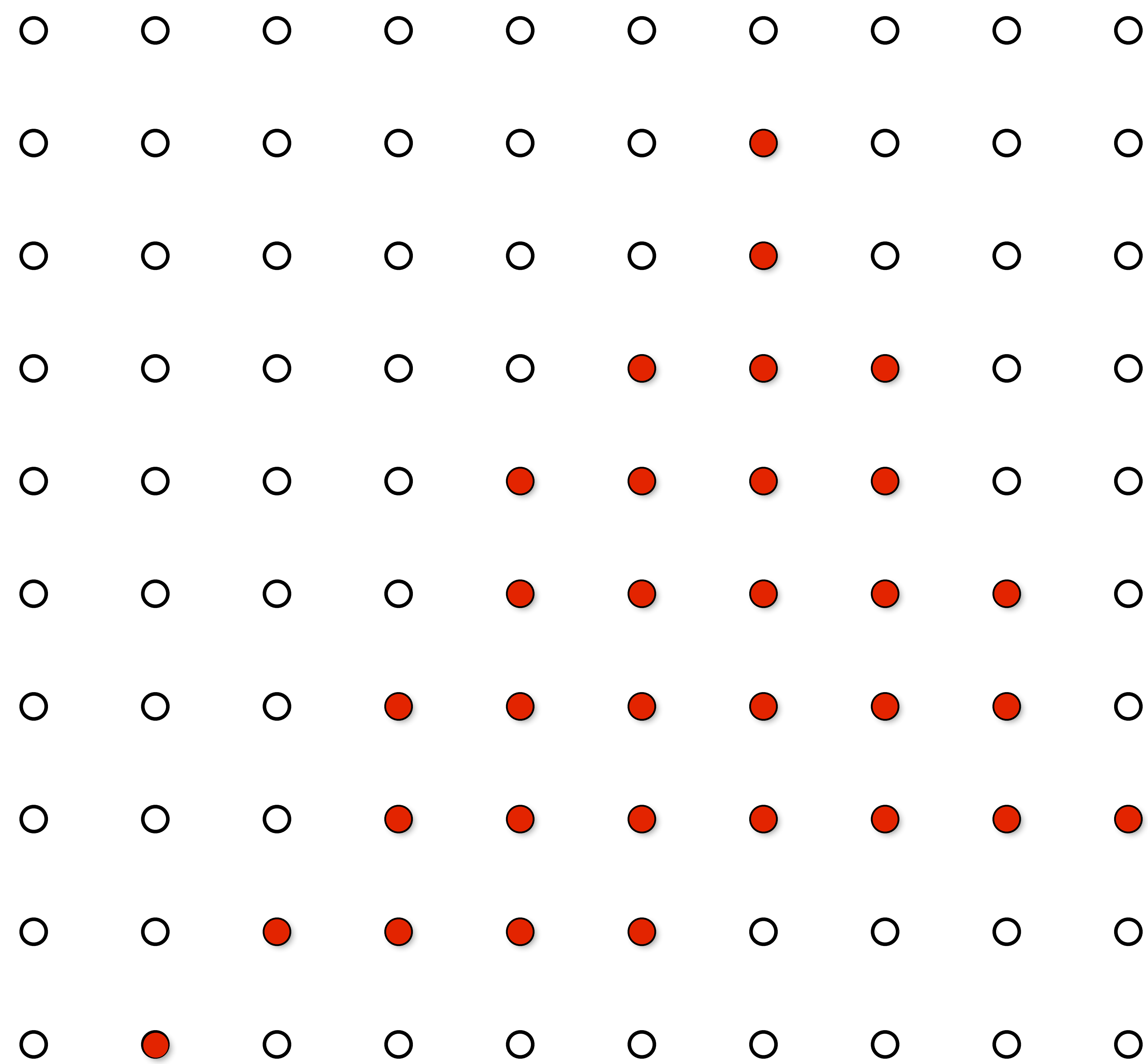
Each image sample sent to the display is converted into a little square of light of the appropriate color: (a pixel = picture element)

LCD pixel on laptop

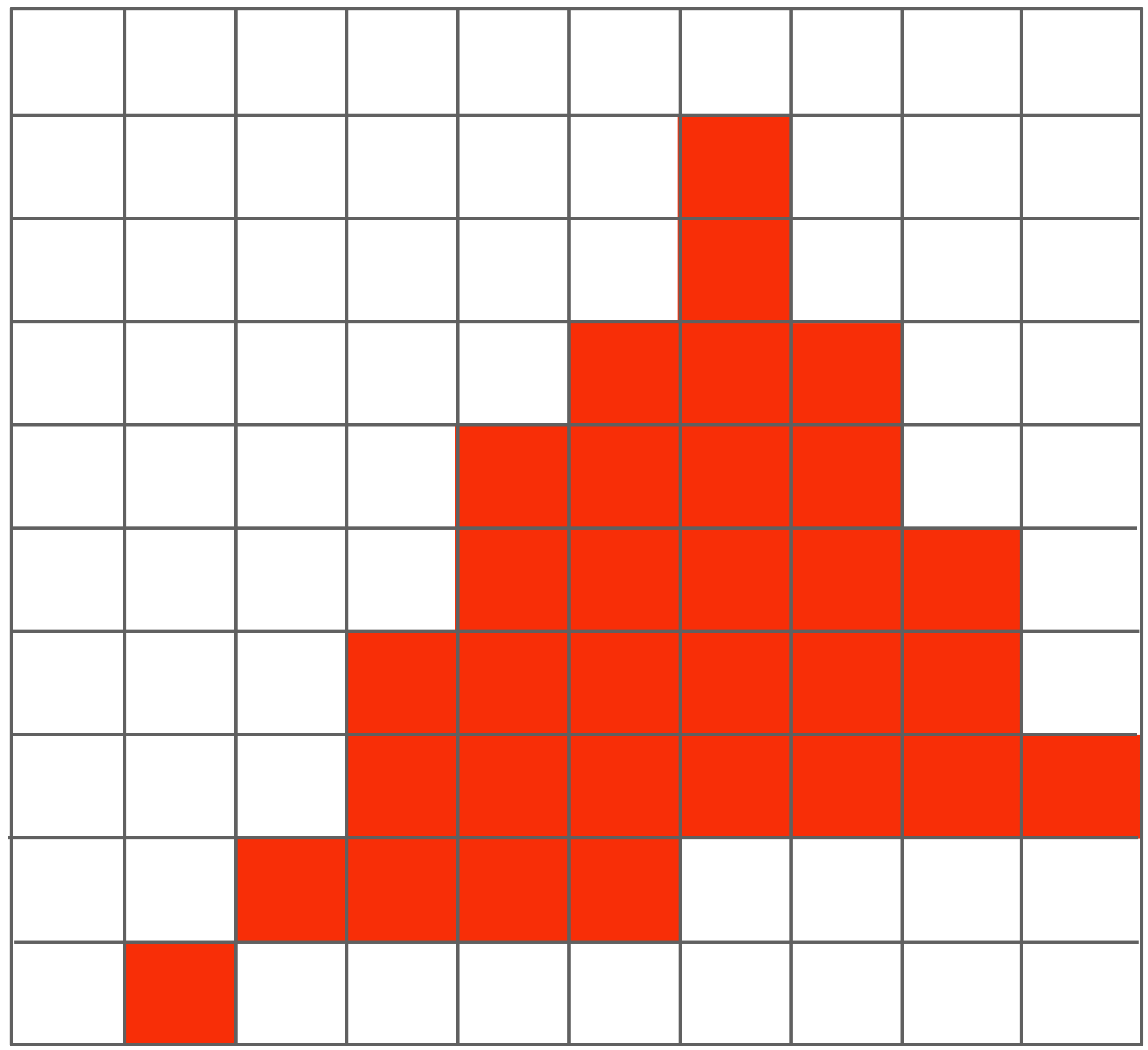


* LCD pixels do not actually emit light in a square of uniform color, but this approximation suffices for our current discussion

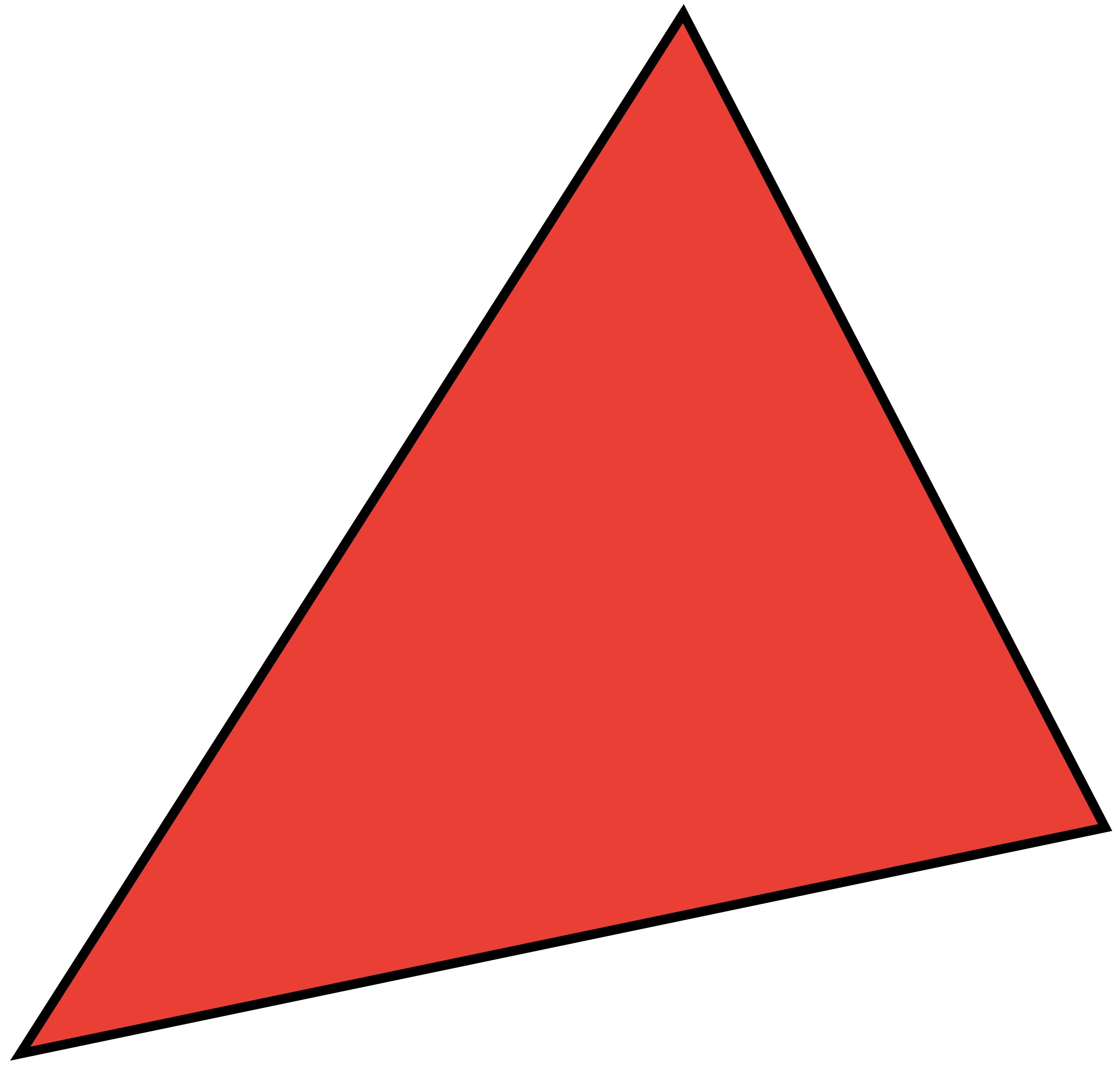
So, If We Send The Display This Sampled Signal



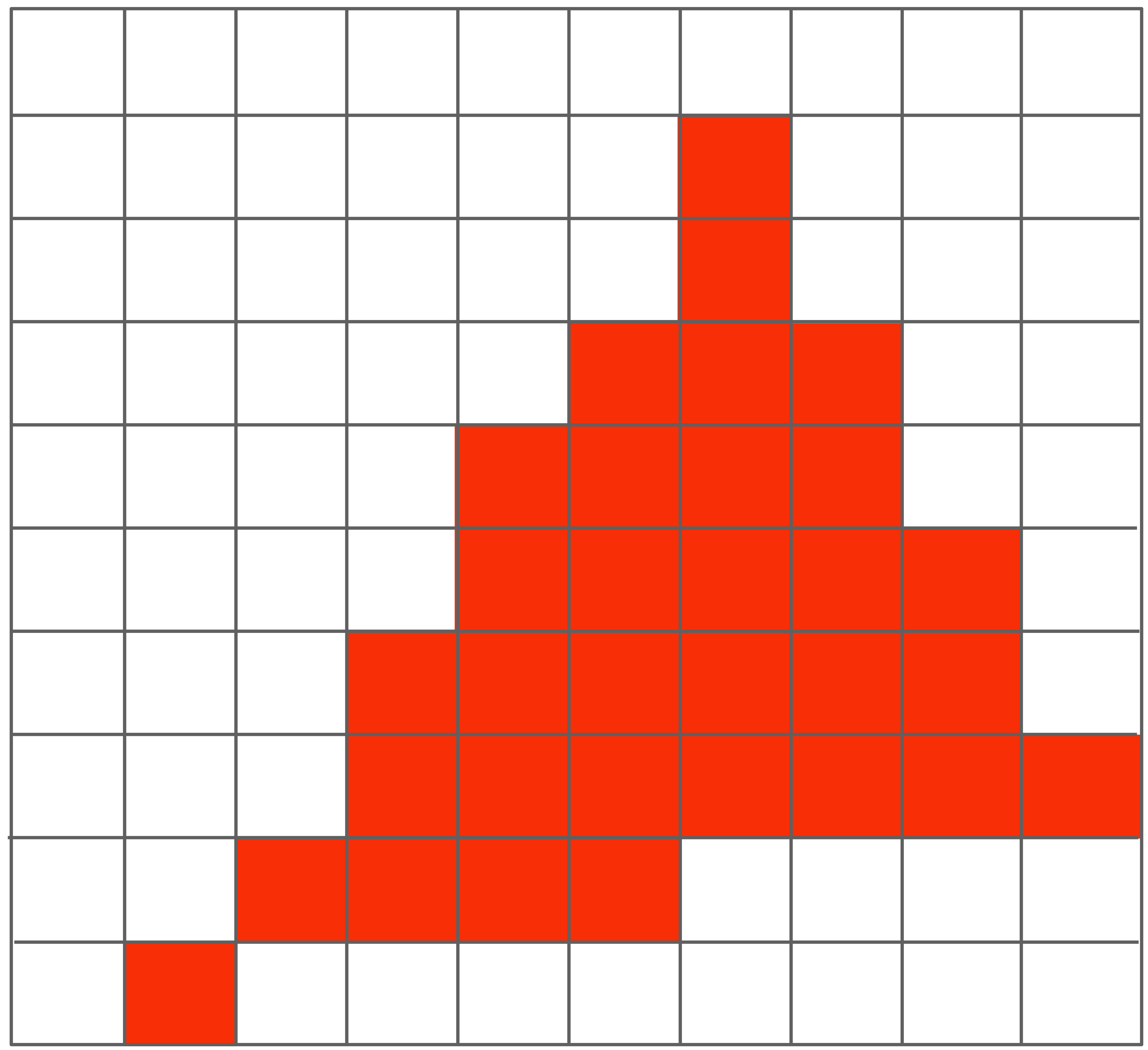
The Display Physically Emits This Signal



Compare: The Continuous Triangle Function

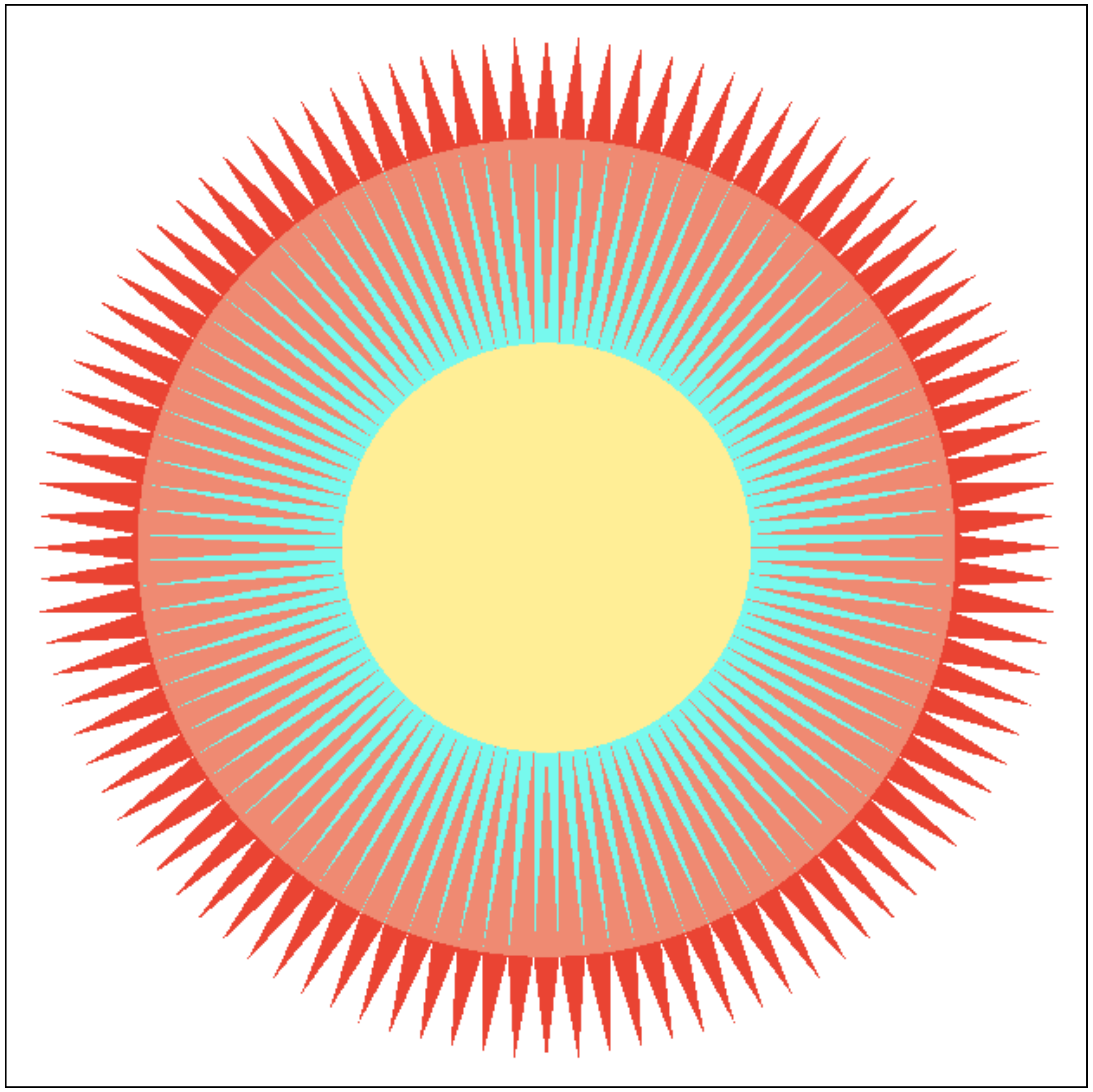


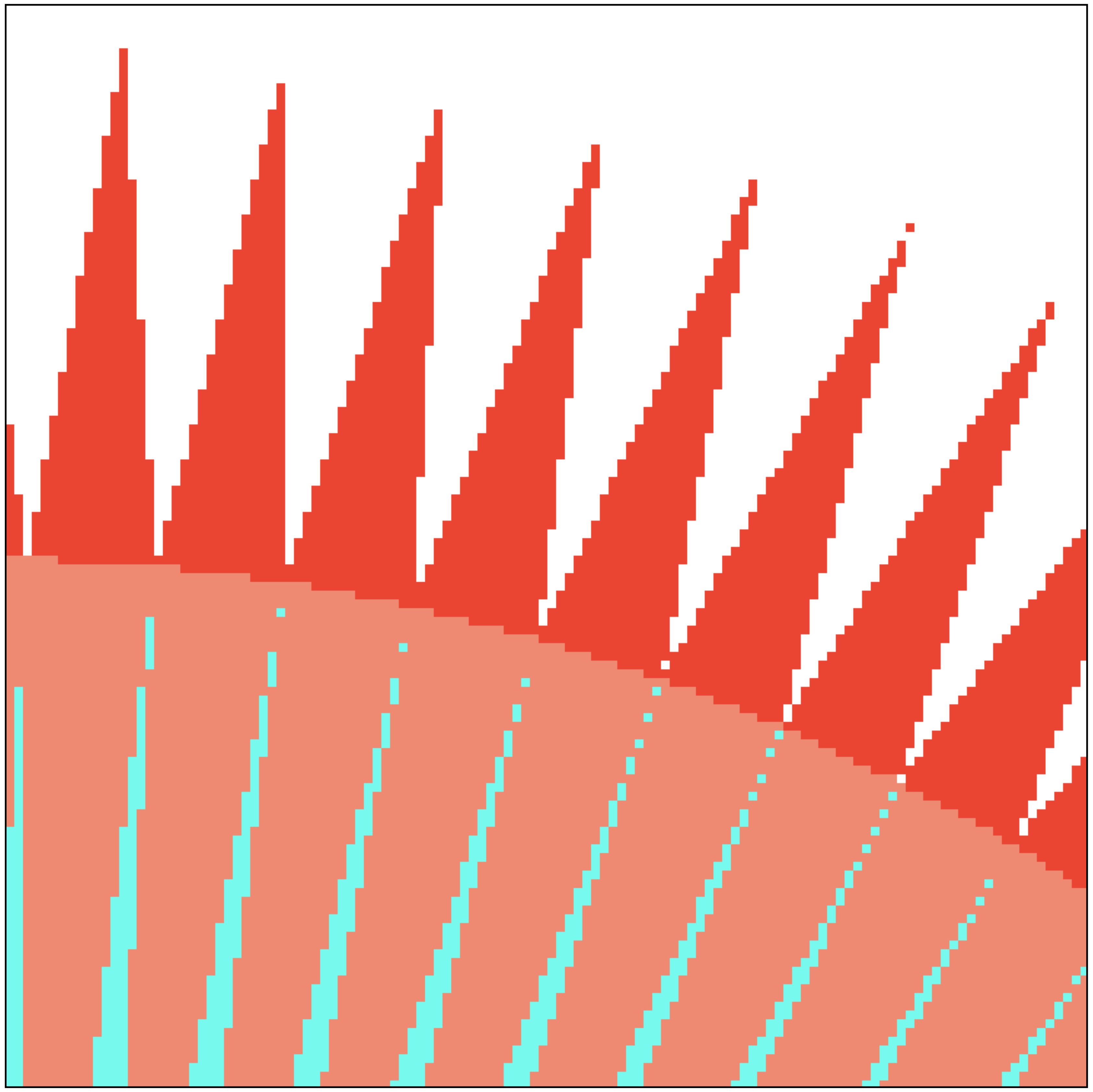
What's Wrong With This Picture?



Jaggies!

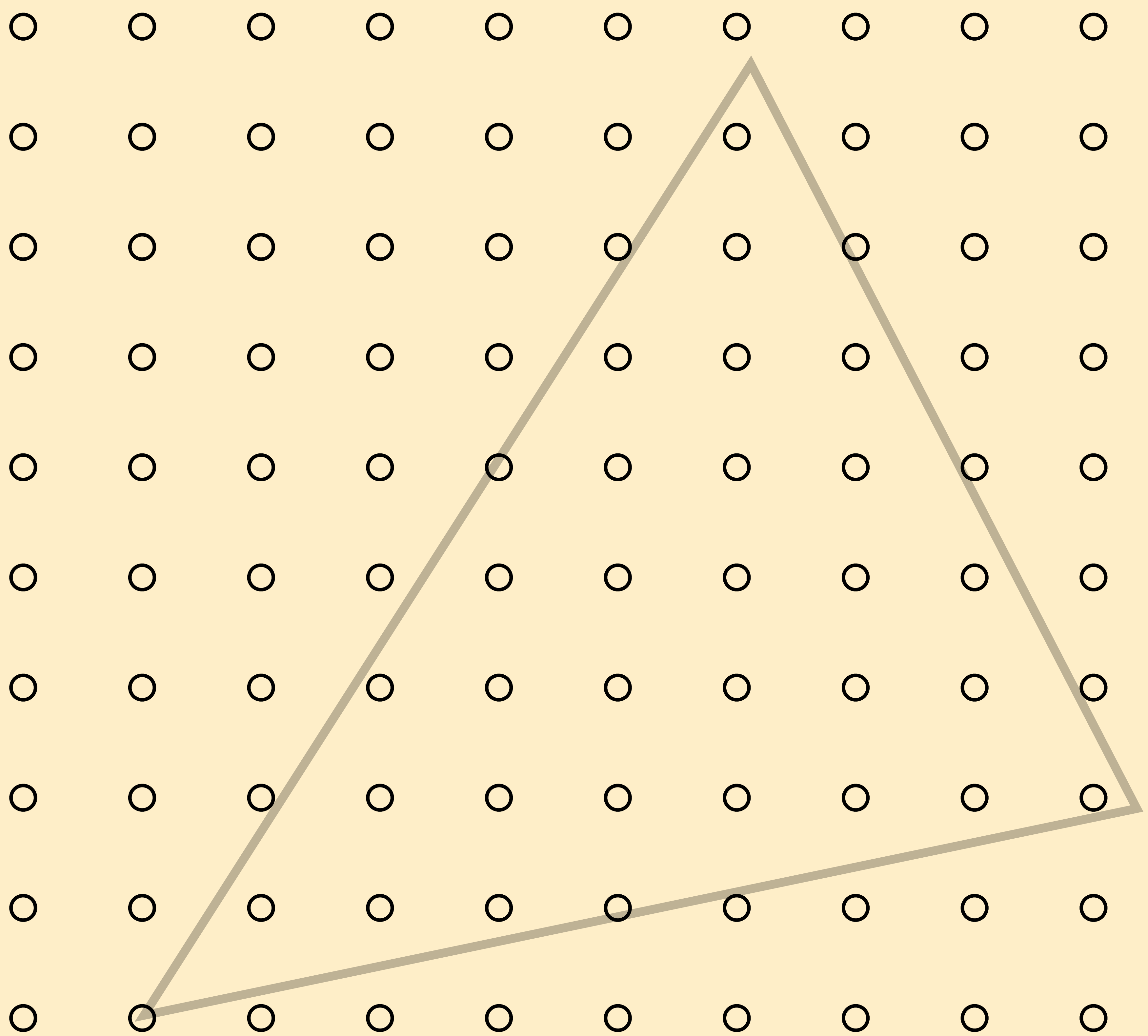
Jaggies (Staircase Pattern)





Is this the best we can do?

Discussion: What Value Should a Pixel Have?



Potential topics for your pair discussion:

- Ideas for "higher quality" pixel formula?
- What are all the relevant factors?
- What's right/wrong about point sampling?
- Why do jaggies look "wrong"?

Things to Remember

Drawing machines

- Many possibilities
- Why framebuffers and raster displays?
- Why triangles?

We posed rasterization as a 2D sampling process

- Test a binary function `inside(triangle,x,y)`
- Evaluate triangle coverage by 3 point-in-edge tests
- Finite sampling rate causes “jaggies” artifact
  (next time we will analyze in more detail)

Acknowledgments

Thanks to Kayvon Fatahalian, Pat Hanrahan, Mark Pauly and Steve Marschner for slide resources.