

**Lecture 9:**

# **Intro to Ray-Tracing**

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**Computer Graphics and Imaging**  
**UC Berkeley CS184/284A**

# Towards Photorealistic Rendering



Credit: Bertrand Benoit. "Sweet Feast," 2009. [Blender /VRay]



# Discussion: What Do You See?

3 min, 3 people, 3 observations

- Look closely, curiously, and write down 3 visual features you want to know how to compute



"Sweet Feast," 2009. [Blender /VRay]  
Credit: Bertrand Benoit.

- <https://cgsociety.org/c/featured/6hgf/sweet-feast>



# Discussion: What Do You See?

## Your observations

- Different focal points — front of photo focused, back is blurred
- Transparency in the glass, tea
- Diffraction? Light spreading in the highlight, maybe around edges of shadows
- Refraction through drink in the back
- Light interacts with non-reflective materials, like the napkin — physical shape and attributes of the cloth
- Sponge cake — like a volume, so soft
- How light reflects light in the custard; bumps in the food
- The mug behind is reflecting the back of the mug in front that none of us can see
- Caramel — light diffusing through this; through the grapes too



# Course Roadmap

## Rasterization Pipeline

### Core Concepts

- Sampling
- Antialiasing
- Transforms

## Geometric Modeling

### Core Concepts

- Splines, Bezier Curves
- Topological Mesh Representations
- Subdivision, Geometry Processing

## Lighting & Materials

### Core Concepts

- Measuring Light
- Unbiased Integral Estimation
- Light Transport & Materials

## Cameras & Imaging

- Rasterization
- Transforms & Projection
- Texture Mapping
- Visibility, Shading, Overall Pipeline
- Intro to Geometry
- Curves and Surfaces
- Geometry Processing
- Ray-Tracing & Acceleration **Today**
- Radiometry & Photometry
- Monte Carlo Integration
- Global Illumination & Path Tracing
- Material Modeling





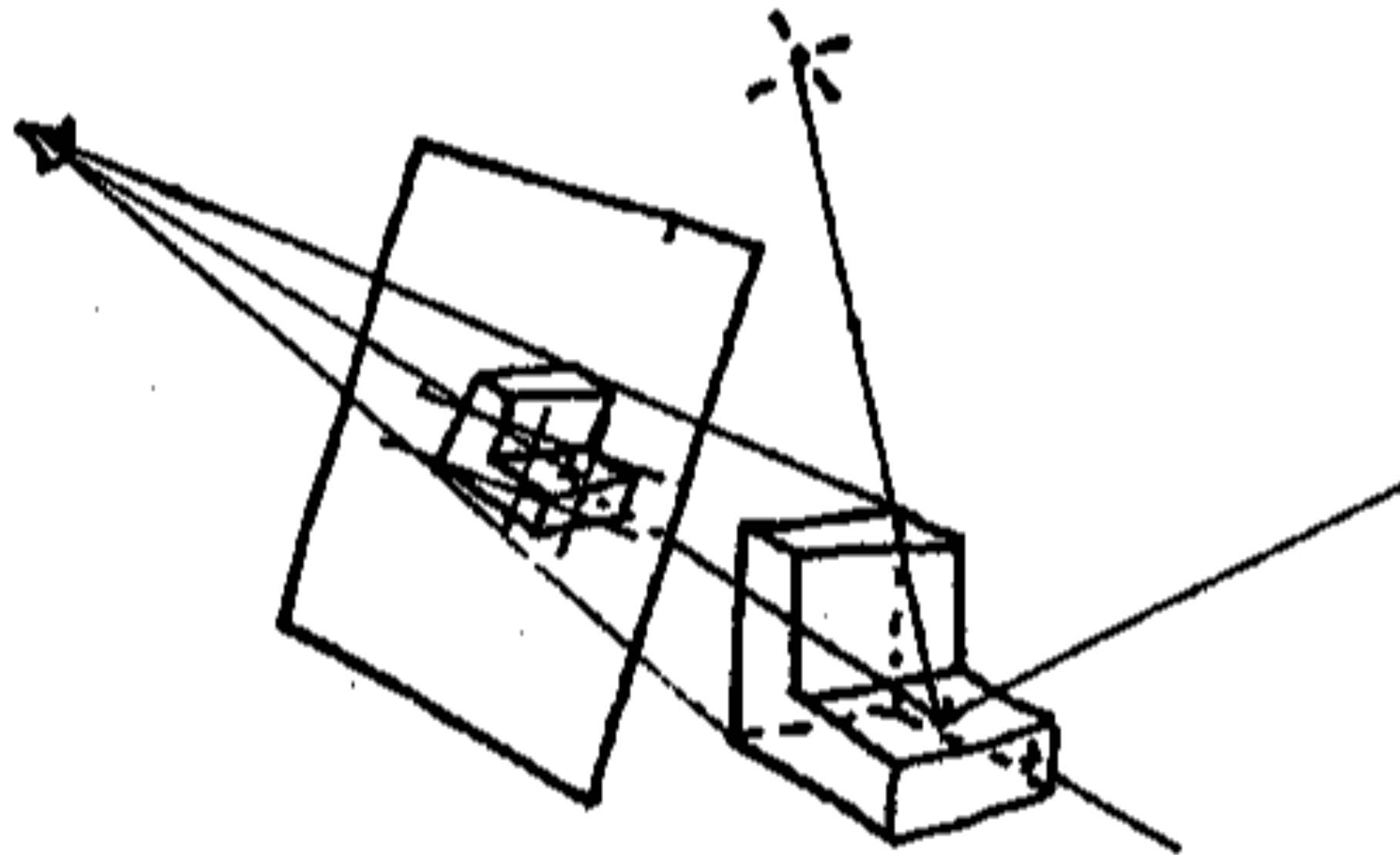
# **Basic Ray-Tracing Algorithm**



# Ray Casting

## Appel 1968 - Ray casting

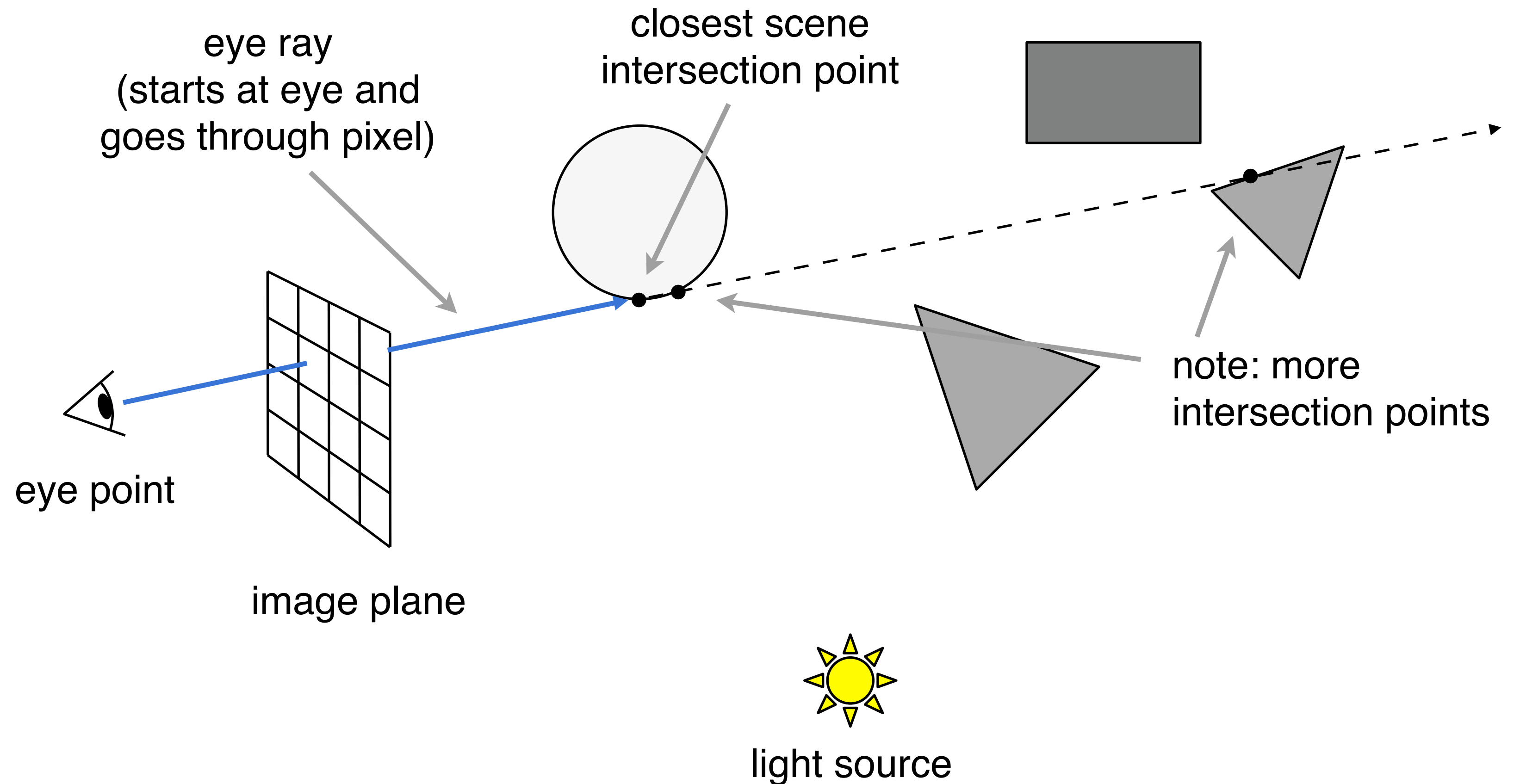
1. Generate an image by casting one ray per pixel
2. Check for shadows by sending a ray to the light





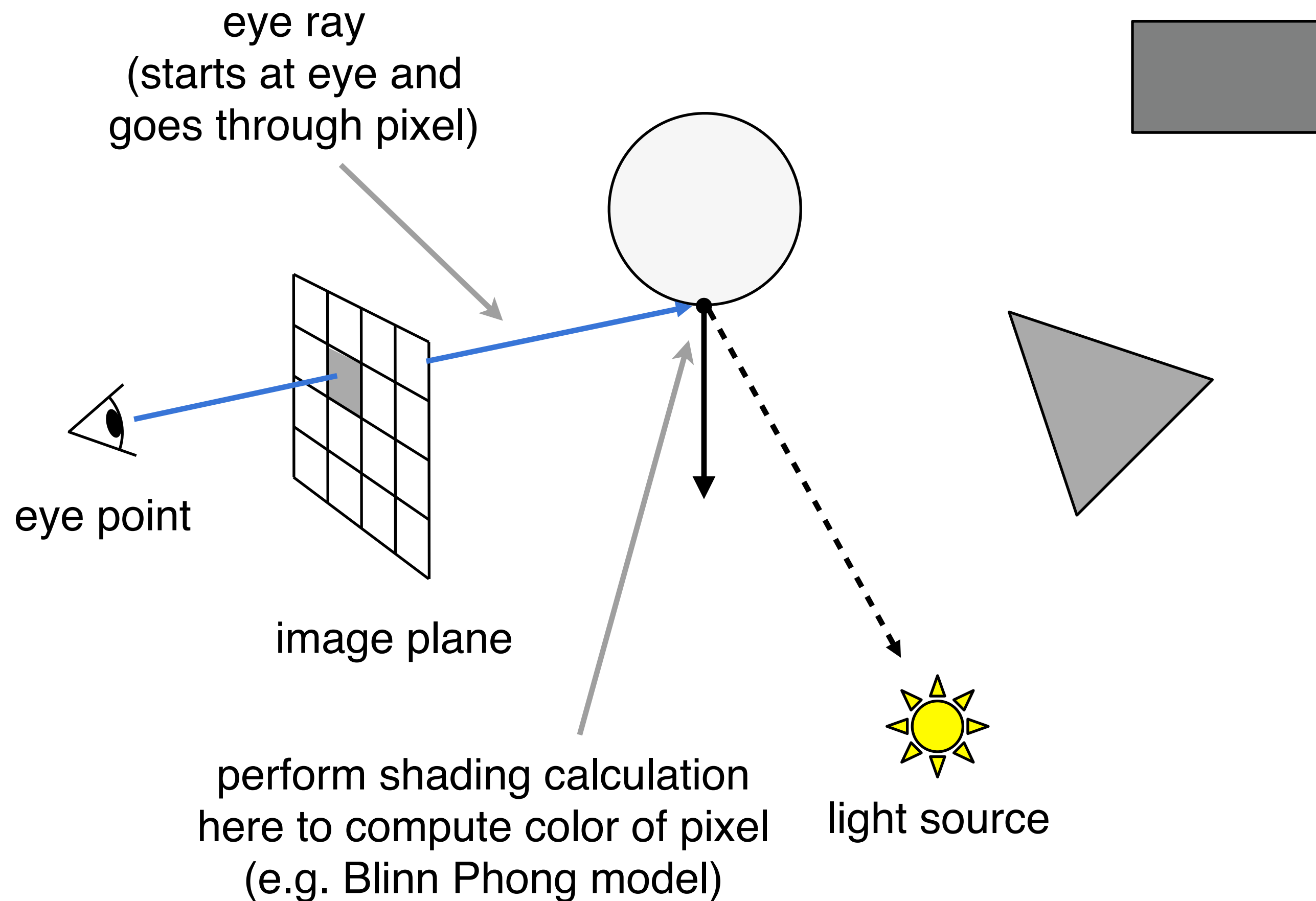
# Ray Casting - Generating Eye Rays

## Pinhole Camera Model



# Ray Casting - Shading Pixels (Local Only)

## Pinhole Camera Model



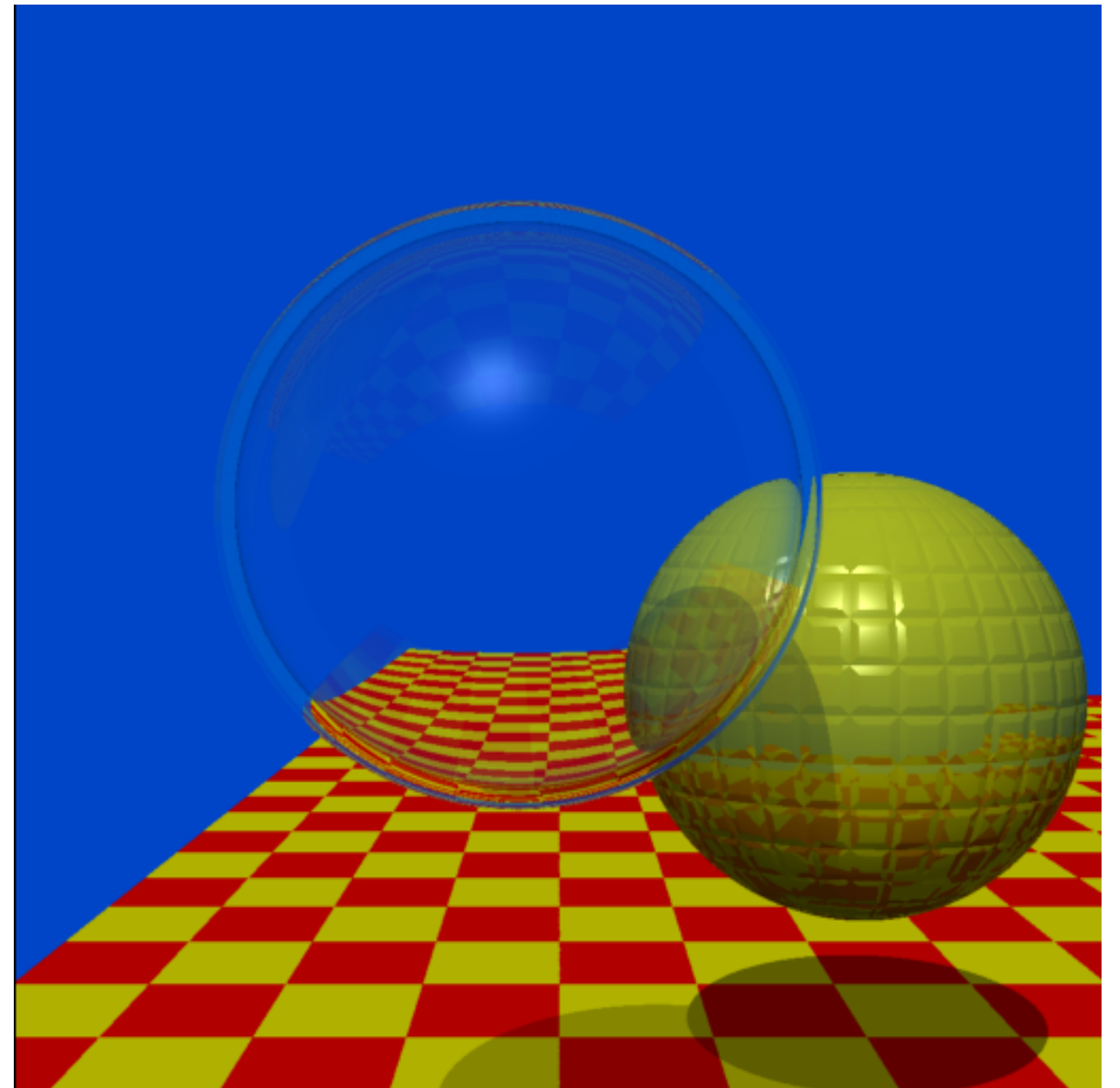


# Recursive Ray Tracing

“An improved Illumination model for shaded display”  
T. Whitted, CACM 1980

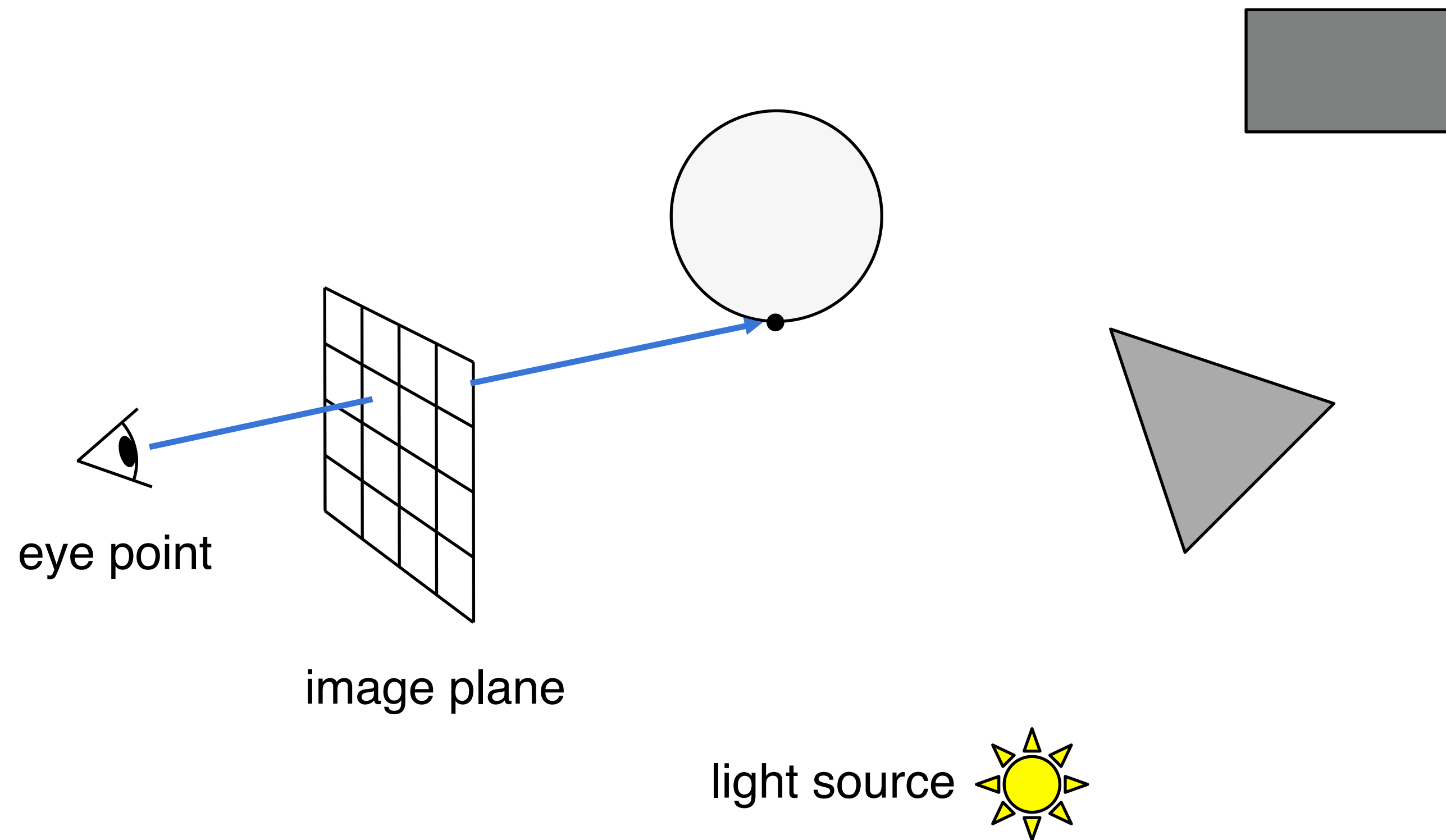
Time:

- VAX 11/780 (1979) 74m
- PC (2006) 6s
- GPU (2012) 1/30s



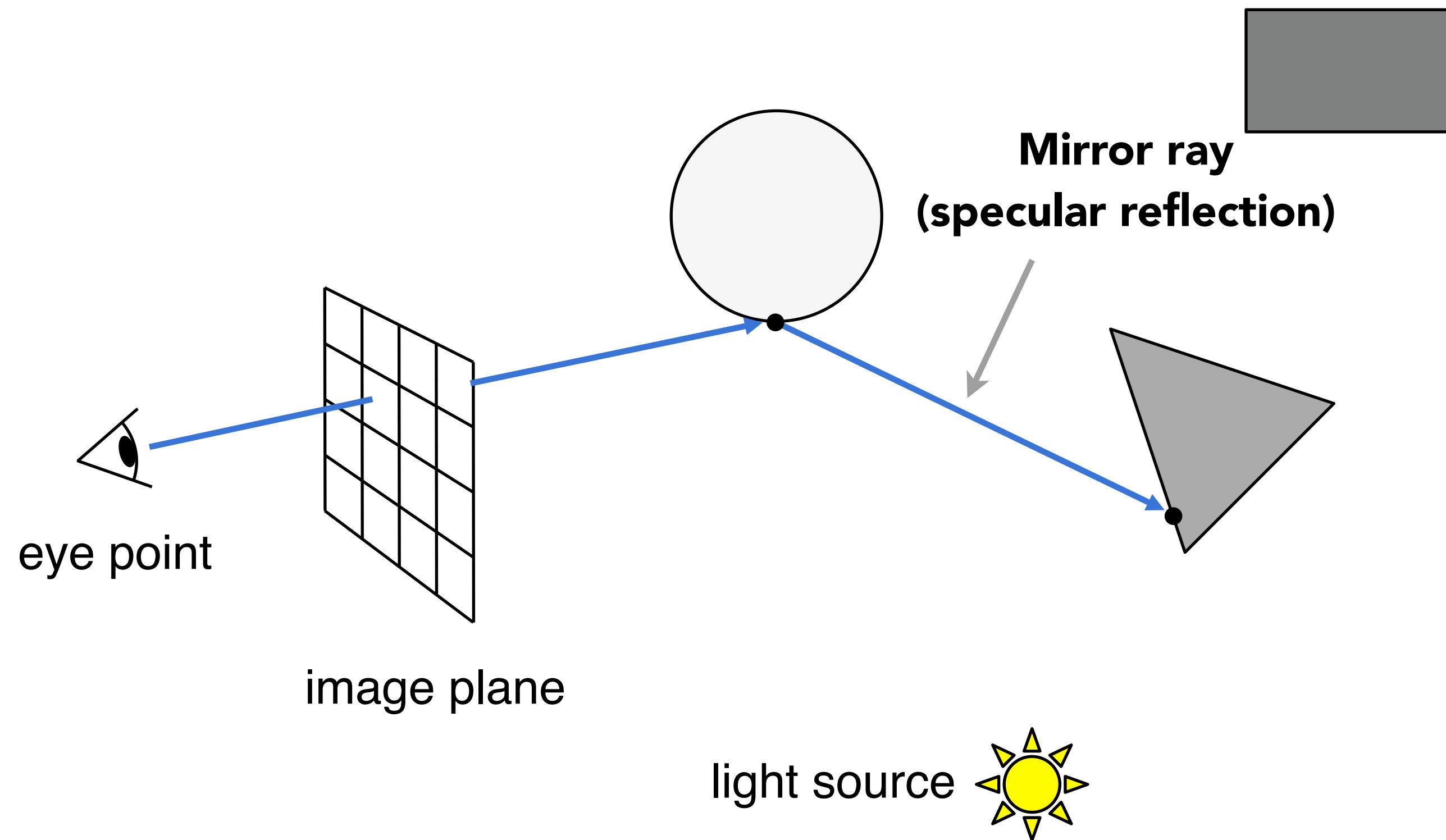
Spheres and Checkerboard, T. Whitted, 1979

# Recursive Ray Tracing

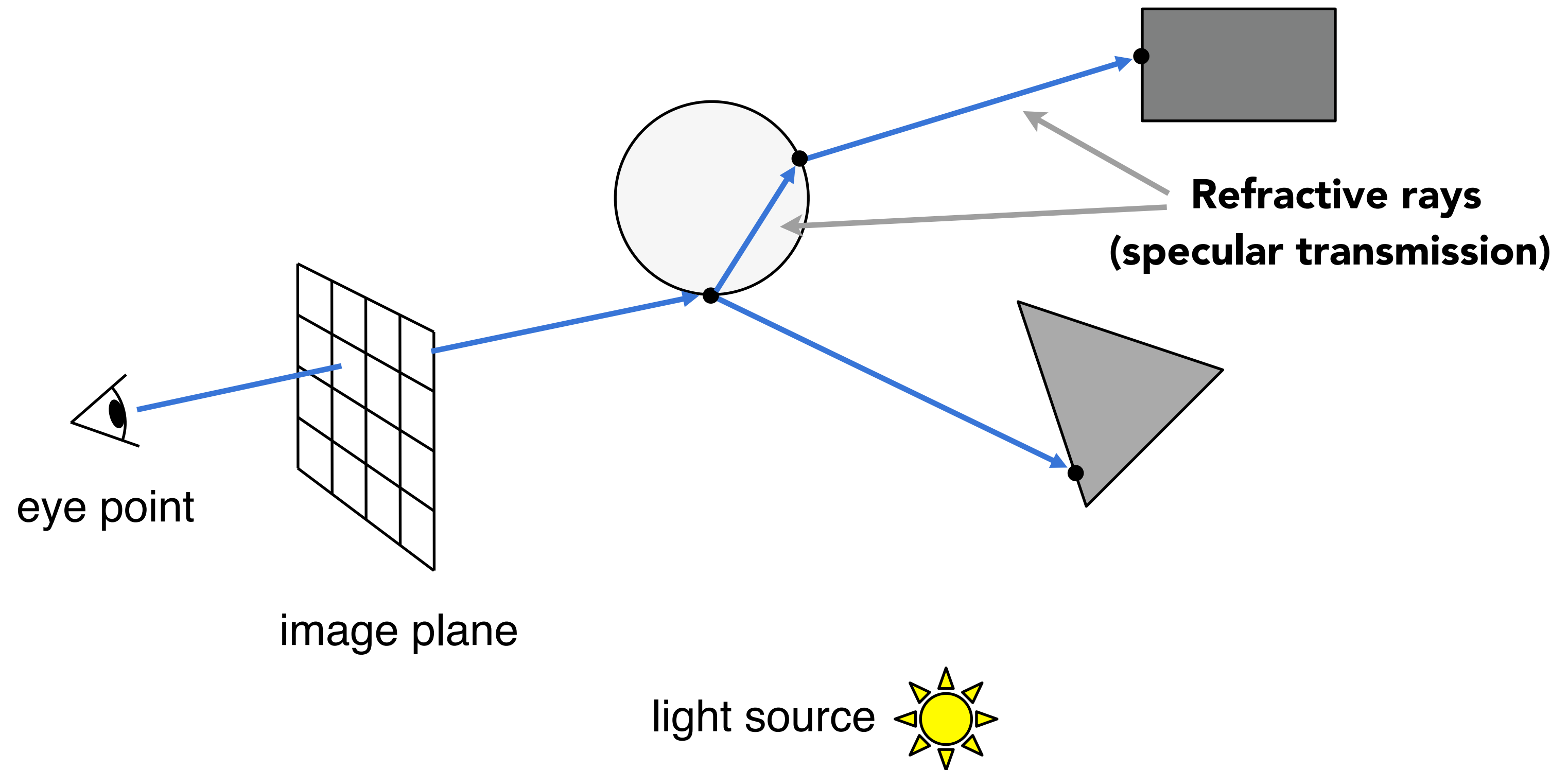




# Recursive Ray Tracing

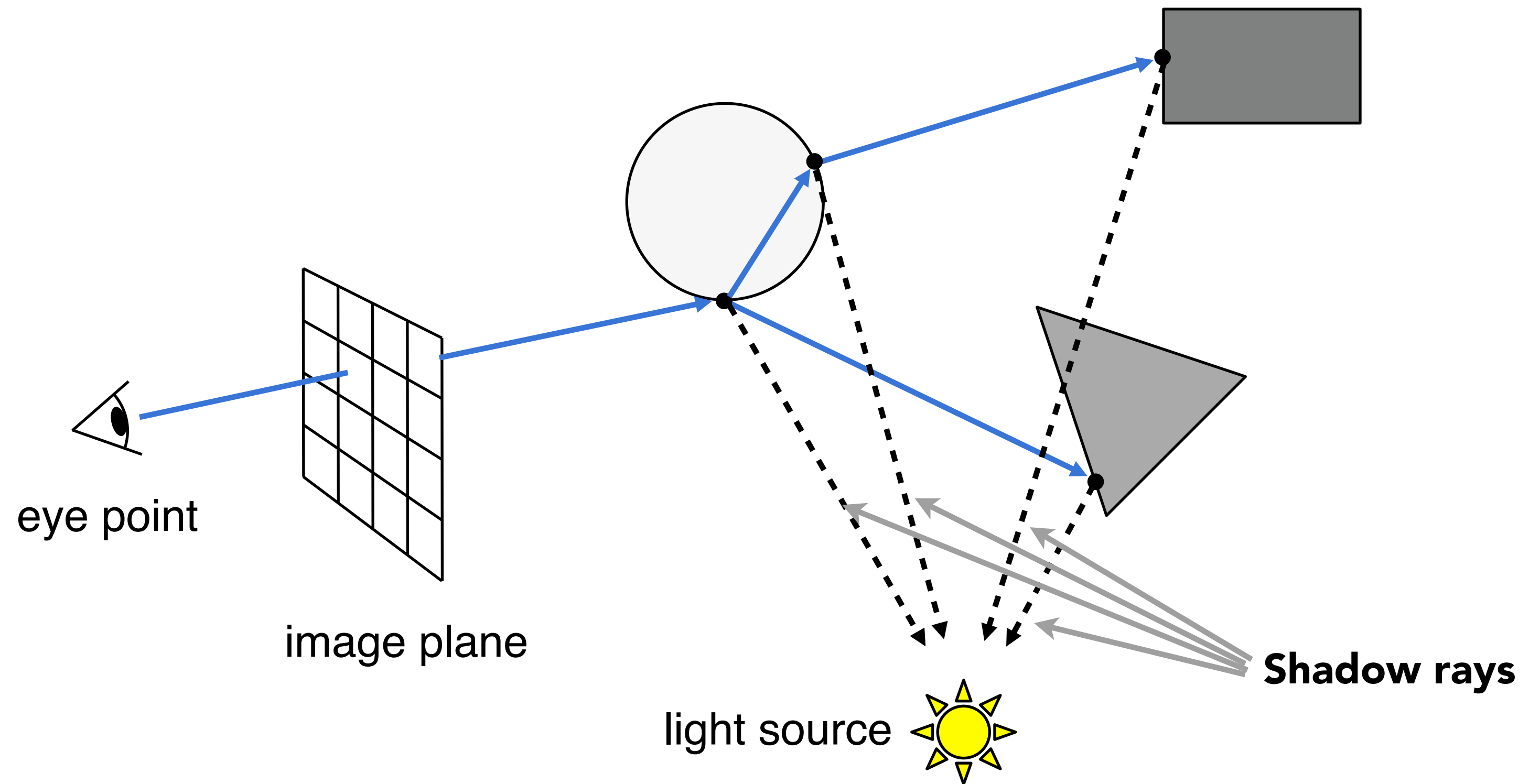


# Recursive Ray Tracing

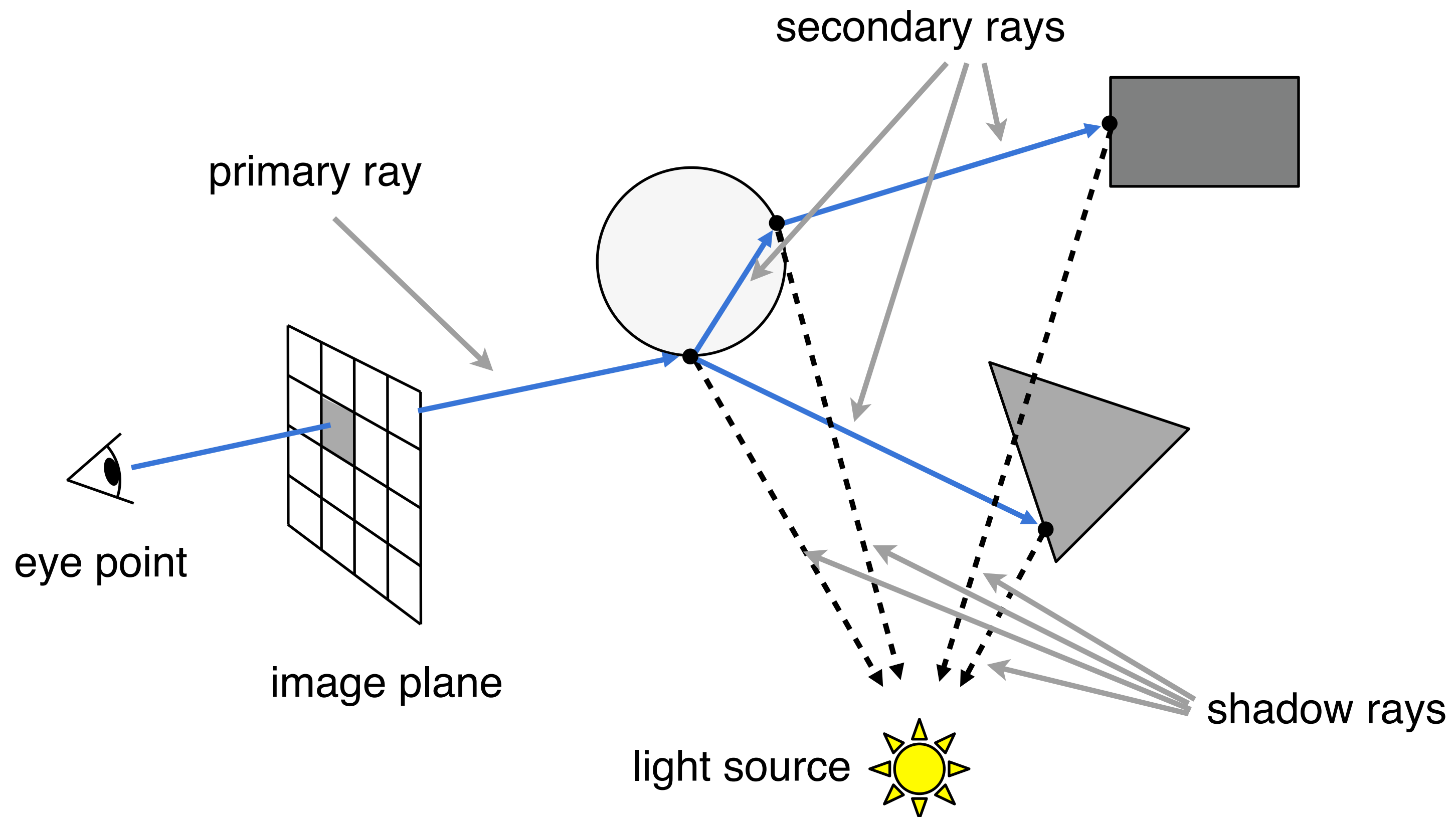




# Recursive Ray Tracing



# Recursive Ray Tracing



- Trace secondary rays recursively until hit a non-specular surface (or max desired levels of recursion)
- At each hit point, trace shadow rays to test light visibility (no contribution if blocked)
- Final pixel color is weighted sum of contributions along rays, as shown
- Gives more sophisticated effects (e.g. specular reflection, refraction, shadows), but we will go much further to derive a physically-based illumination model

# Recursive Ray Tracing





# **Ray-Surface Intersection**

# Ray Intersection With Triangle Mesh

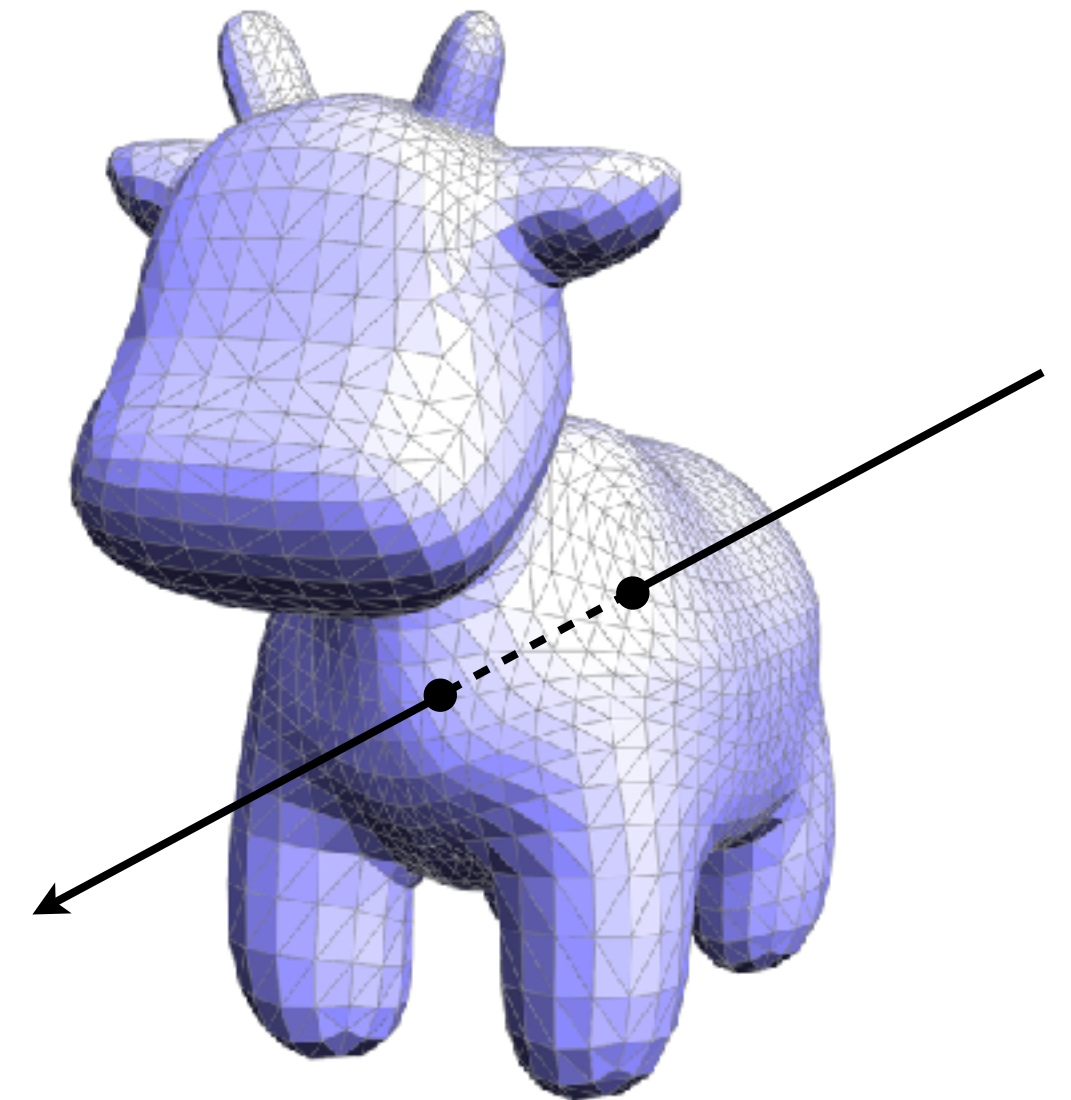
## Why?

- **Rendering: visibility, shadows, lighting ...**
- **Geometry: inside/outside test**

## How to compute?

## Let's break this down:

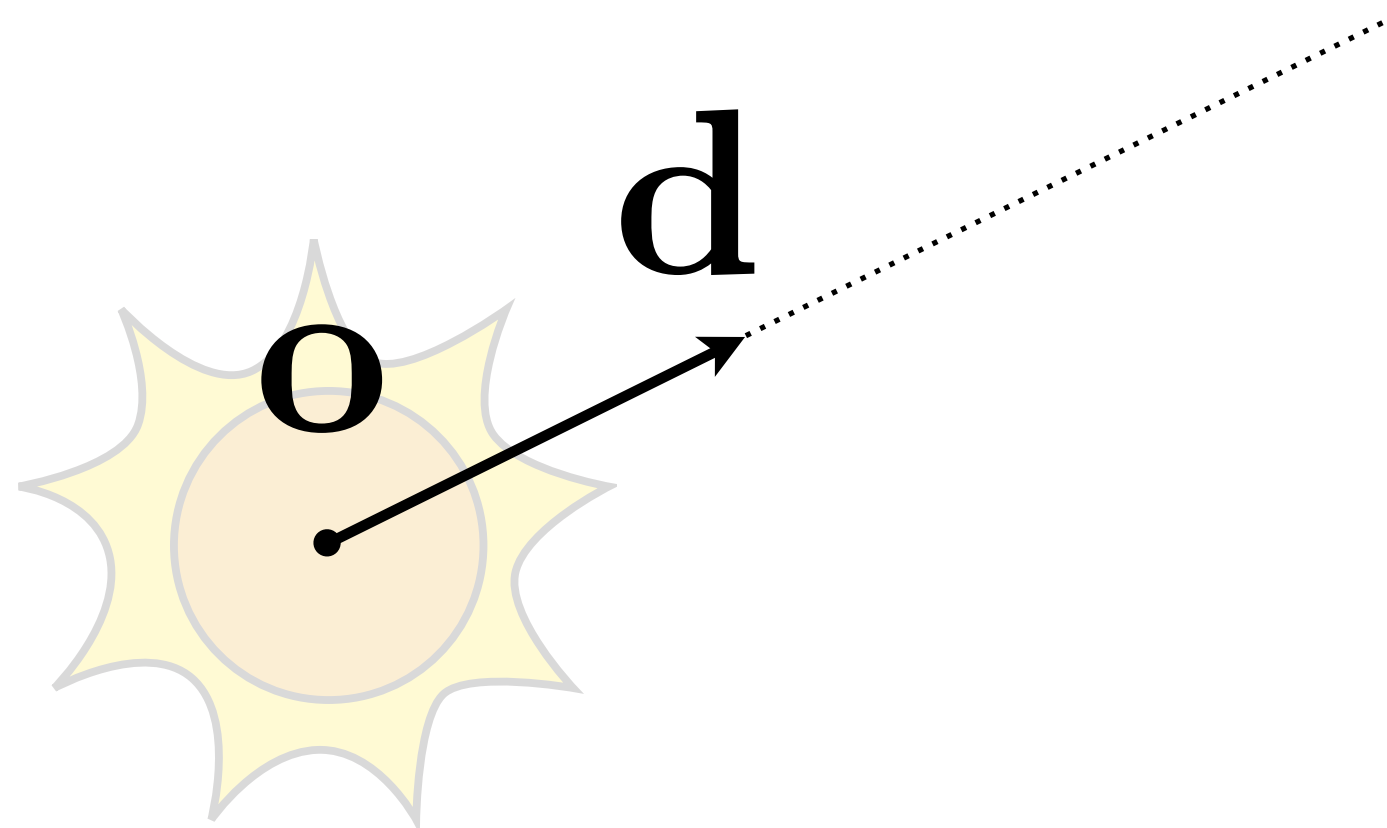
- **Simple idea: just intersect ray with each triangle**
- **Simple, but slow (accelerate next time)**
- **Note: can have 0, 1 or multiple intersections**



# Ray Equation

Ray is defined by its origin and a direction vector

Example:



Ray equation:

$$\mathbf{r}(t) = \mathbf{o} + t\mathbf{d} \quad 0 \leq t < \infty$$

↑ ↑  
point along ray "time"

↑  
origin

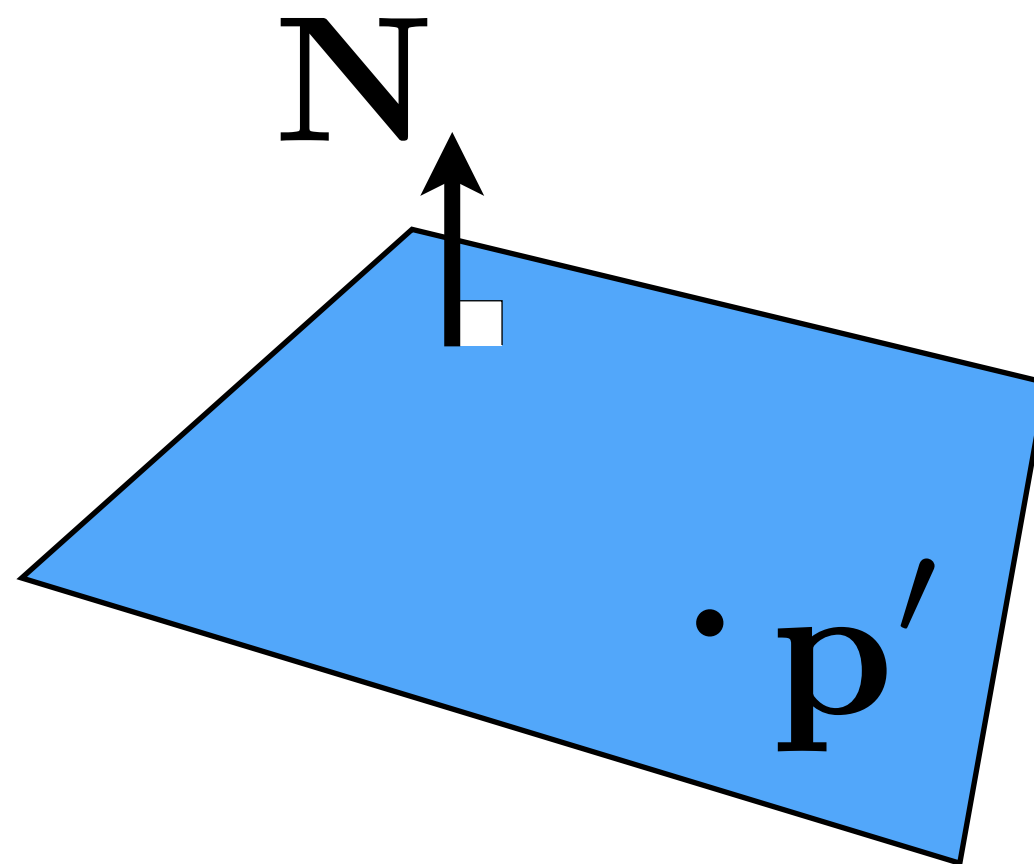
↑  
unit direction



# Plane Equation

Plane is defined by normal vector and a point on plane

Example:



Plane Equation:

$$\mathbf{p} : (\mathbf{p} - \mathbf{p}') \cdot \mathbf{N} = 0$$

↑  
all points on plane

↑  
any point

↑  
normal vector

$$ax + by + cz + d = 0$$

# Ray Intersection With Plane

Ray equation:

$$\mathbf{r}(t) = \mathbf{o} + t \mathbf{d}, \quad 0 \leq t < \infty$$

Plane equation:

$$\mathbf{p} : (\mathbf{p} - \mathbf{p}') \cdot \mathbf{N} = 0$$

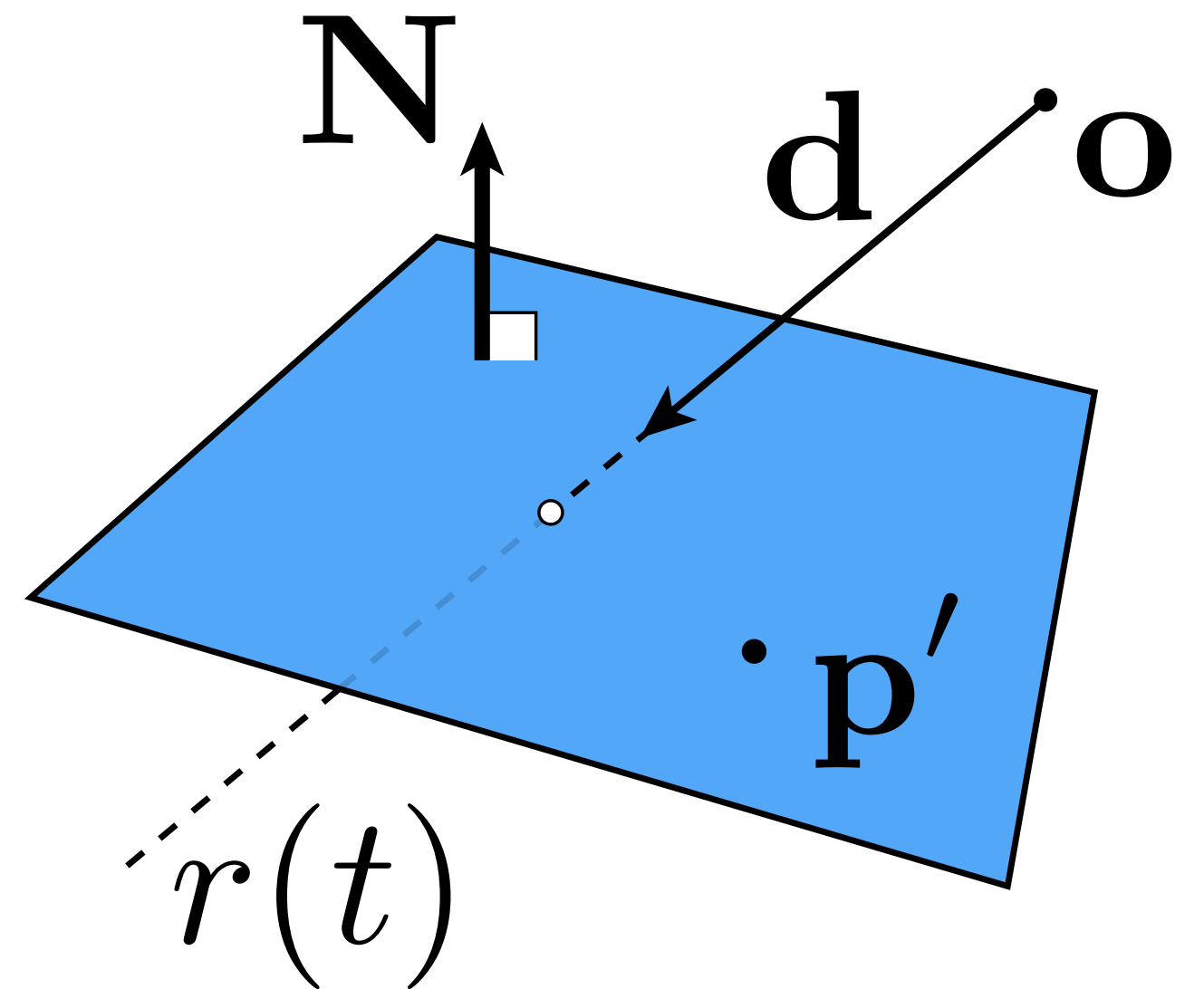
Solve for intersection

Set  $\mathbf{p} = \mathbf{r}(t)$  and solve for  $t$

$$(\mathbf{p} - \mathbf{p}') \cdot \mathbf{N} = (\mathbf{o} + t \mathbf{d} - \mathbf{p}') \cdot \mathbf{N} = 0$$

$$t = \frac{(\mathbf{p}' - \mathbf{o}) \cdot \mathbf{N}}{\mathbf{d} \cdot \mathbf{N}}$$

Check:  $0 \leq t < \infty$

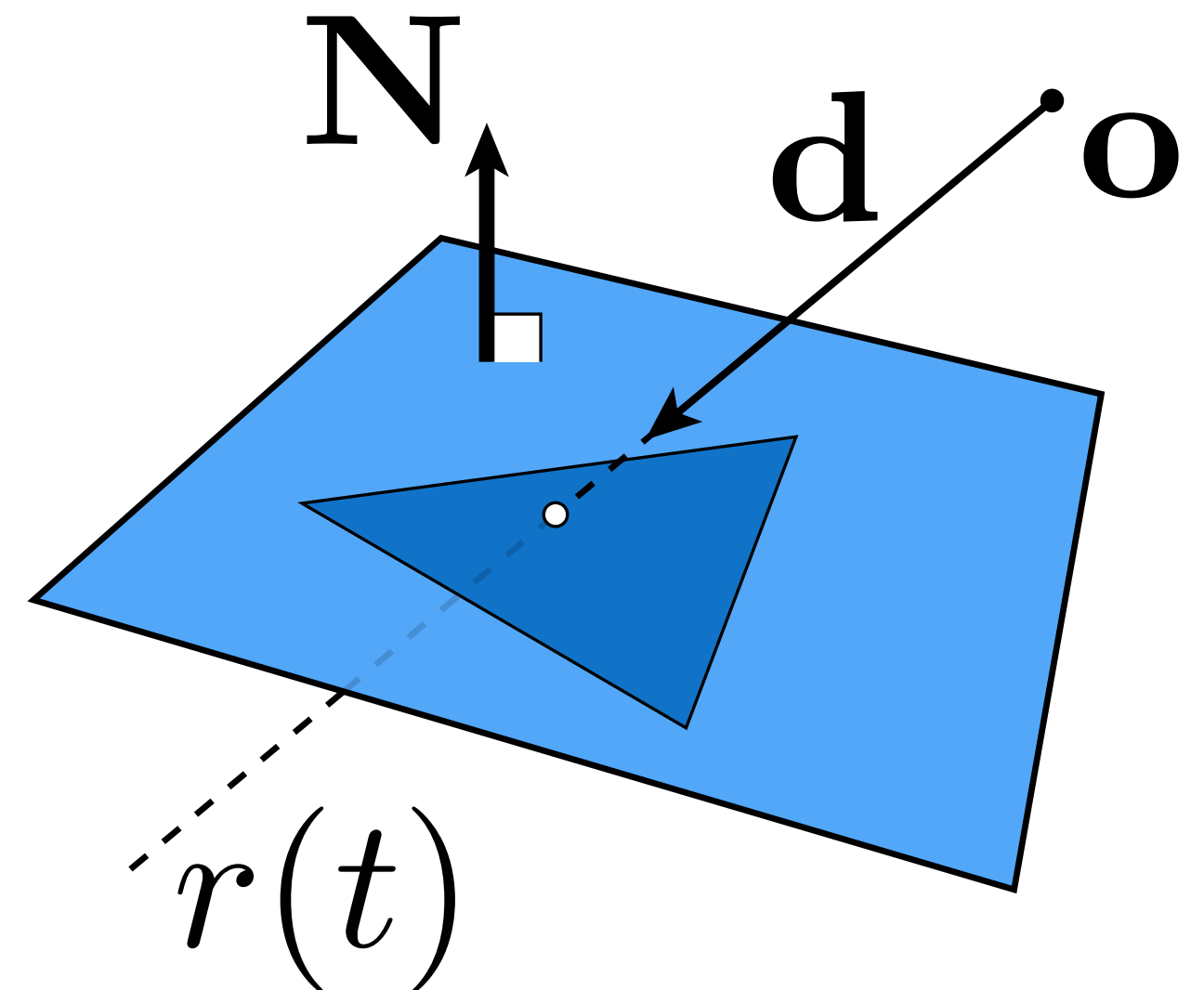


# Ray Intersection With Triangle

Triangle is in a plane

- Ray-plane intersection
- Test if hit point is inside triangle (Assignment 1!)

Many ways to optimize...





# Can Optimize: e.g. Möller Trumbore Algorithm

$$\vec{\mathbf{O}} + t\vec{\mathbf{D}} = (1 - b_1 - b_2)\vec{\mathbf{P}}_0 + b_1\vec{\mathbf{P}}_1 + b_2\vec{\mathbf{P}}_2$$

**Where:**

$$\begin{bmatrix} t \\ b_1 \\ b_2 \end{bmatrix} = \frac{1}{\vec{\mathbf{S}}_1 \cdot \vec{\mathbf{E}}_1} \begin{bmatrix} \vec{\mathbf{S}}_2 \cdot \vec{\mathbf{E}}_2 \\ \vec{\mathbf{S}}_1 \cdot \vec{\mathbf{S}} \\ \vec{\mathbf{S}}_2 \cdot \vec{\mathbf{D}} \end{bmatrix}$$

$$\vec{\mathbf{E}}_1 = \vec{\mathbf{P}}_1 - \vec{\mathbf{P}}_0$$

$$\vec{\mathbf{E}}_2 = \vec{\mathbf{P}}_2 - \vec{\mathbf{P}}_0$$

$$\vec{\mathbf{S}} = \vec{\mathbf{O}} - \vec{\mathbf{P}}_0$$

$$\vec{\mathbf{S}}_1 = \vec{\mathbf{D}} \times \vec{\mathbf{E}}_2$$

$$\vec{\mathbf{S}}_2 = \vec{\mathbf{S}} \times \vec{\mathbf{E}}_1$$

**Cost = (1 div, 27 mul, 17 add)**

# Ray Intersection With Sphere

**Ray:**  $\mathbf{r}(t) = \mathbf{o} + t \mathbf{d}$ ,  $0 \leq t < \infty$

**Sphere:**  $\mathbf{p} : (\mathbf{p} - \mathbf{c})^2 - R^2 = 0$

**Solve for intersection:**

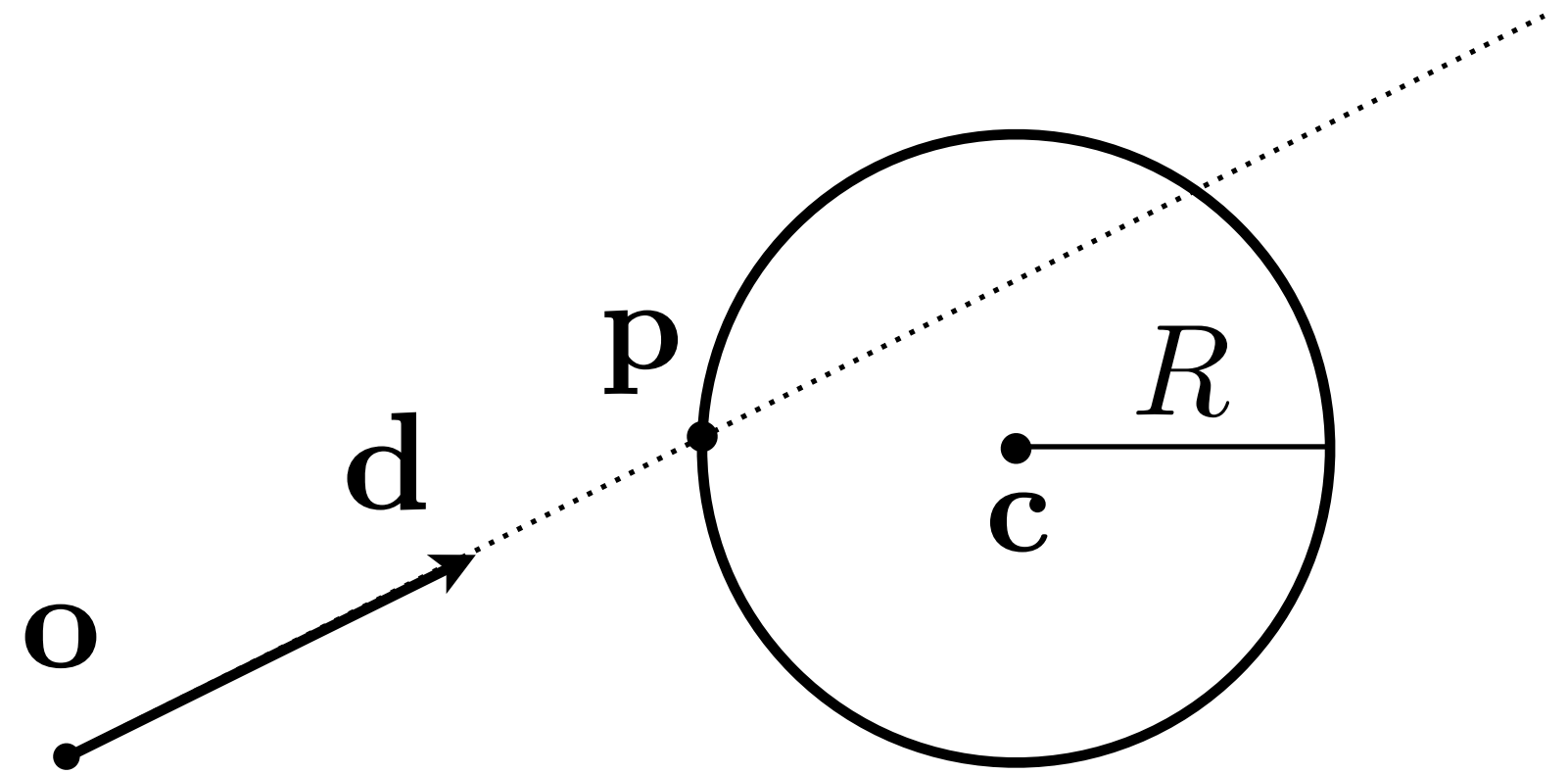
$$(\mathbf{o} + t \mathbf{d} - \mathbf{c})^2 - R^2 = 0$$

$$a t^2 + b t + c = 0, \text{ where}$$

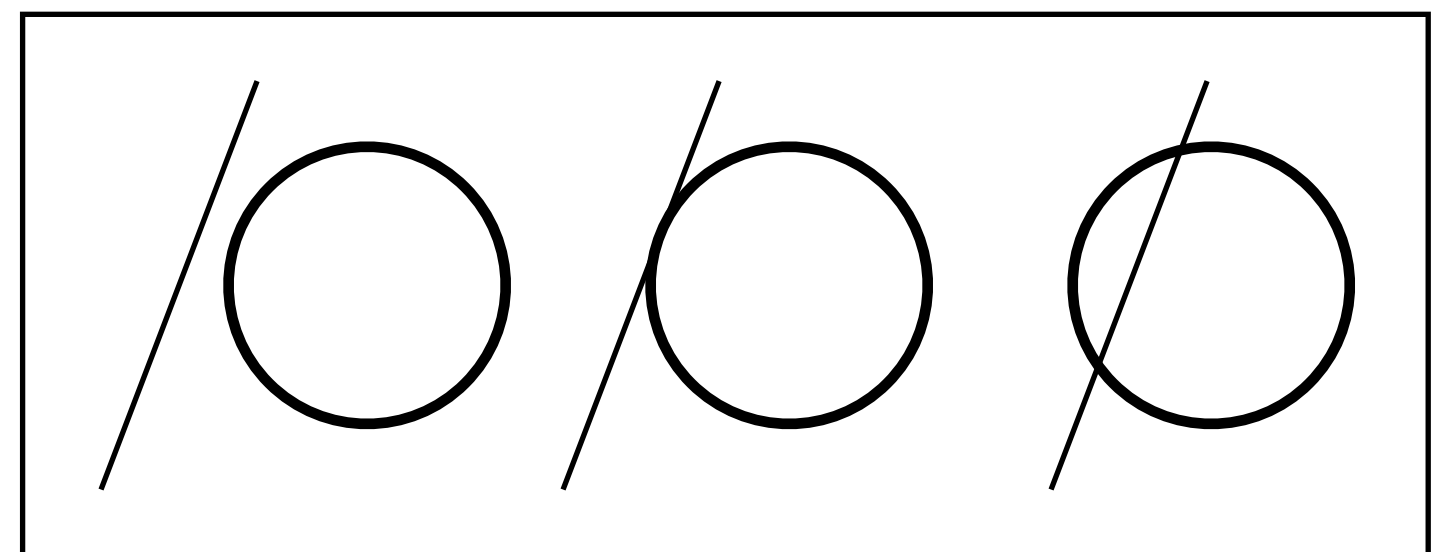
$$a = \mathbf{d} \cdot \mathbf{d}$$

$$b = 2(\mathbf{o} - \mathbf{c}) \cdot \mathbf{d}$$

$$c = (\mathbf{o} - \mathbf{c}) \cdot (\mathbf{o} - \mathbf{c}) - R^2$$



$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$



# Ray Intersection With Implicit Surface

Ray:  $\mathbf{r}(t) = \mathbf{o} + t \mathbf{d}$ ,  $0 \leq t < \infty$

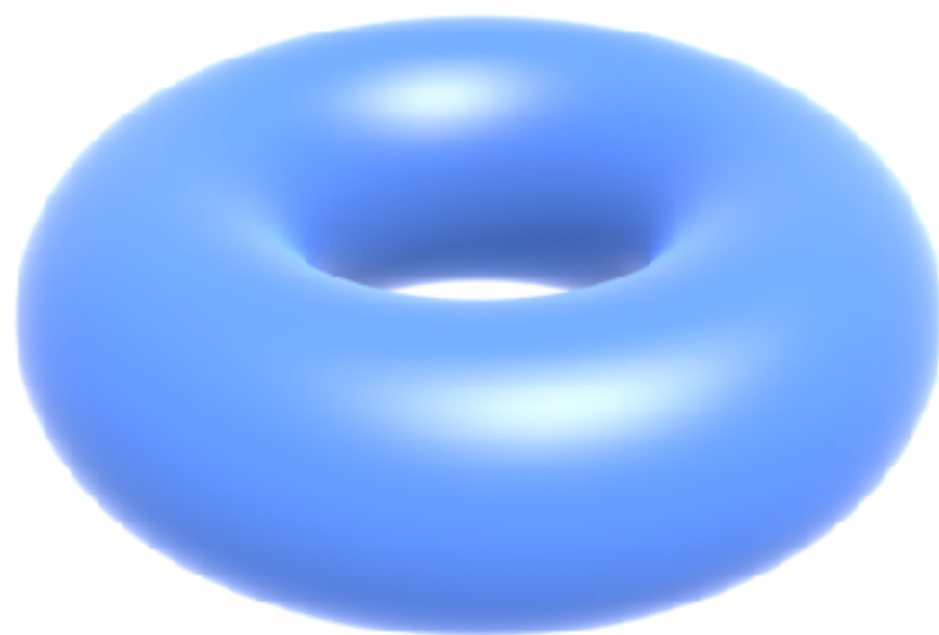
General implicit surface:  $\mathbf{p} : f(\mathbf{p}) = 0$

Substitute ray equation:  $f(\mathbf{o} + t \mathbf{d}) = 0$

Solve for real, positive roots



$$x^2 + y^2 + z^2 = 1$$



$$(R - \sqrt{x^2 + y^2})^2 + z^2 = r^2$$



$$\left(x^2 + \frac{9y^2}{4} + z^2 - 1\right)^3 = x^2 z^3 + \frac{9y^2 z^3}{80}$$



# **Accelerating Ray-Surface Intersection**

# Ray Tracing – Performance Challenges

## Simple ray-scene intersection

- Exhaustively test ray-intersection with every object

## Problem:

- Exhaustive algorithm =  $\#pixels \times \#objects$
- Very slow!



# Ray Tracing – Performance Challenges



Jun Yan, Tracy Renderer

**San Miguel Scene, 10.7M triangles**



# Ray Tracing – Performance Challenges



Deussen et al; Pharr & Humphreys, PBRT

**Plant Ecosystem, 20M triangles**



# Discussion: Accelerating Ray-Scene Intersection

~1 million pixels, ~20 million triangles



Deussen et al; Pharr & Humphreys, PBRT

**In pairs, brainstorm accelerations, small or big ideas.**

**Write down 3-4 ideas.**



# Discussion: Accelerating Ray-Scene Intersection

Brainstorm 3 or 4 accelerations, small or big ideas.

- Heuristics for which triangle to look at first
- Ignore triangles behind the camera
- Get lazy with small triangles and blur faraway regions
- Copy and paste computation if many similar parts of trees
- Inspired by quick sort, try to bound these are the only triangles in this area — and maybe do so recursively
- If image is sparse, apply compressive sensing somehow?
- Quad tree to hierarchically organize the scene
- “Parenting” objects - idea of hierarchical representation of scene
- Stop recursion after a few steps for non-detailed areas
- Reverse the ray — start from the camera — why?
- Perform blocking to improve cache performance
- Selective ray-tracing — optimize for specific visual effects trace only the relevant rays from those surfaces



# **Bounding Volumes**

# Bounding Volumes

Quick way to avoid intersections: bound complex object with a simple volume

- Object is fully contained in the volume
- If it doesn't hit the volume, it doesn't hit the object
- So test bvol first, then test object if it hits

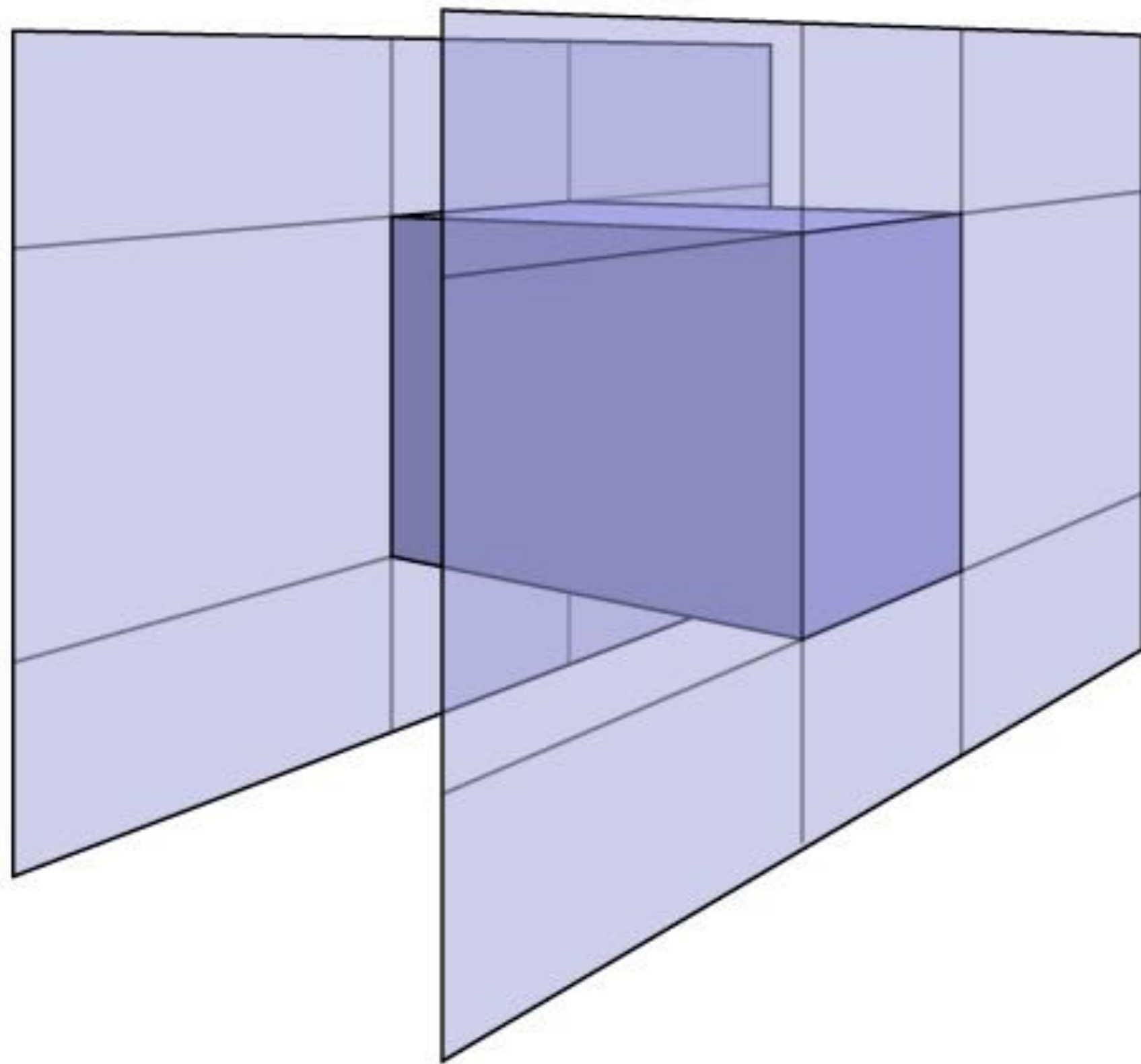




# Ray-Intersection With Box

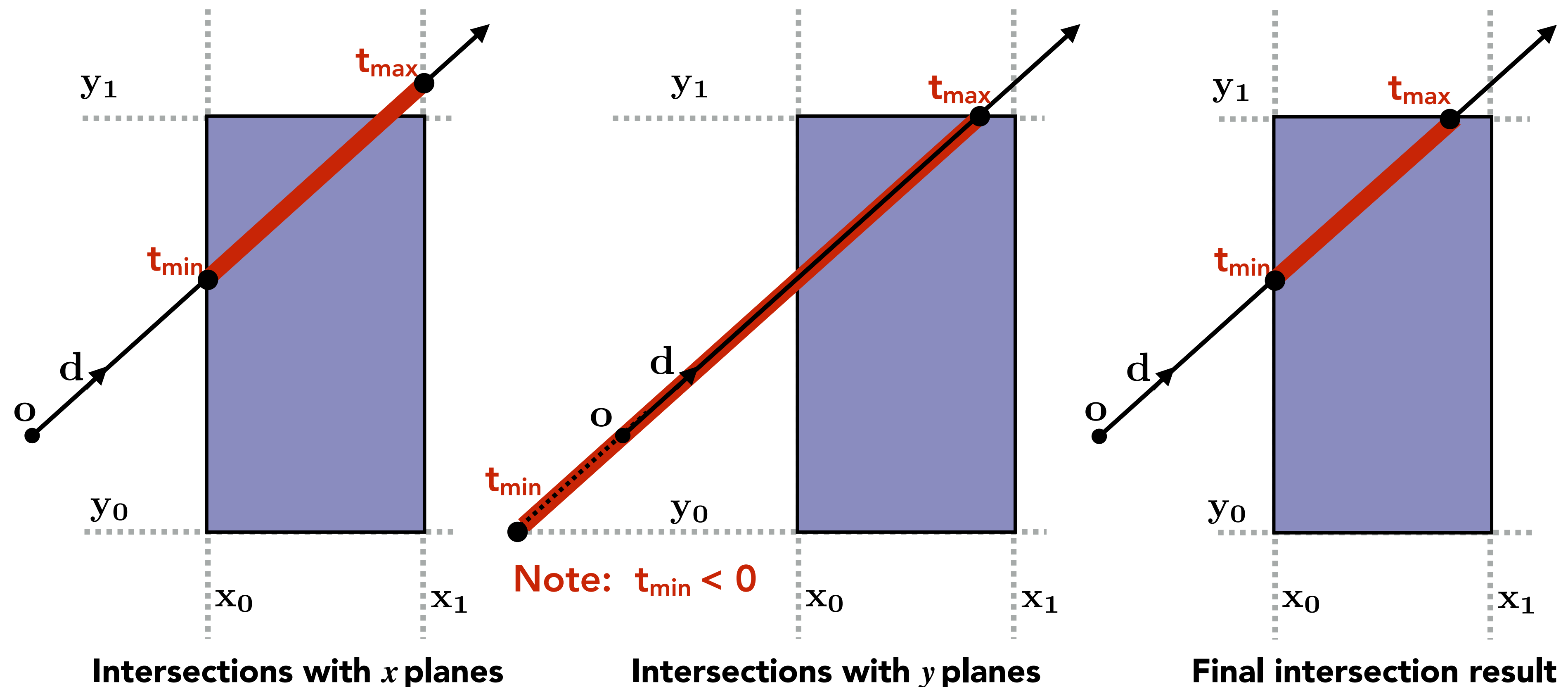
Could intersect with 6 faces individually

Better way: box is the intersection of 3 slabs



# Ray Intersection with Axis-Aligned Box

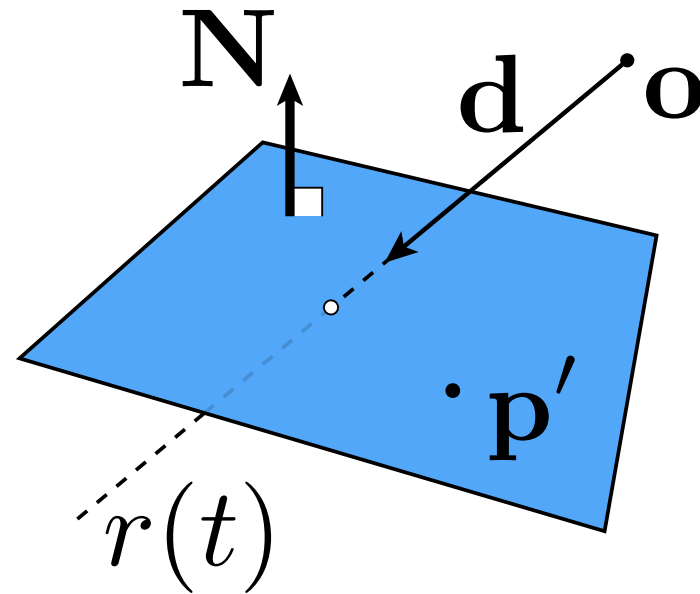
2D example; 3D is the same! Compute intersections with slabs and take intersection of  $t_{\min}/t_{\max}$  intervals



How do we know when the ray misses the box?

# Optimize Ray-Plane Intersection For Axis-Aligned Planes?

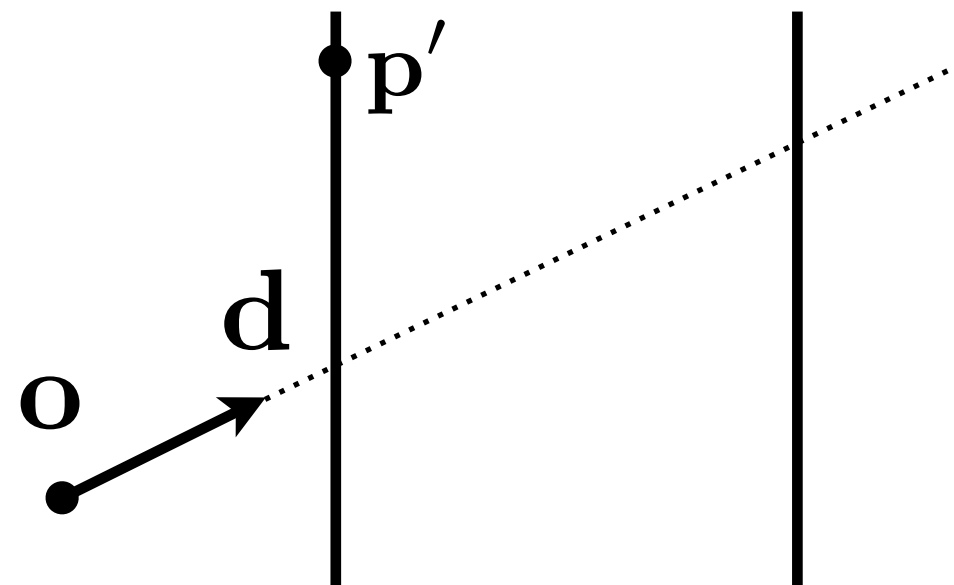
General



$$t = \frac{(\mathbf{p}' - \mathbf{o}) \cdot \mathbf{N}}{\mathbf{d} \cdot \mathbf{N}}$$

3 subtractions, 6 multiplies, 1 division

Perpendicular  
to x-axis



$$t = \frac{p'_x - o_x}{d_x}$$

1 subtraction, 1 division

**To Be Continued**



# Acknowledgments

Thanks to Pat Hanrahan, Kayvon Fatahalian, Mark Pauly and Steve Marschner for lecture resources.