Lecture 21a:

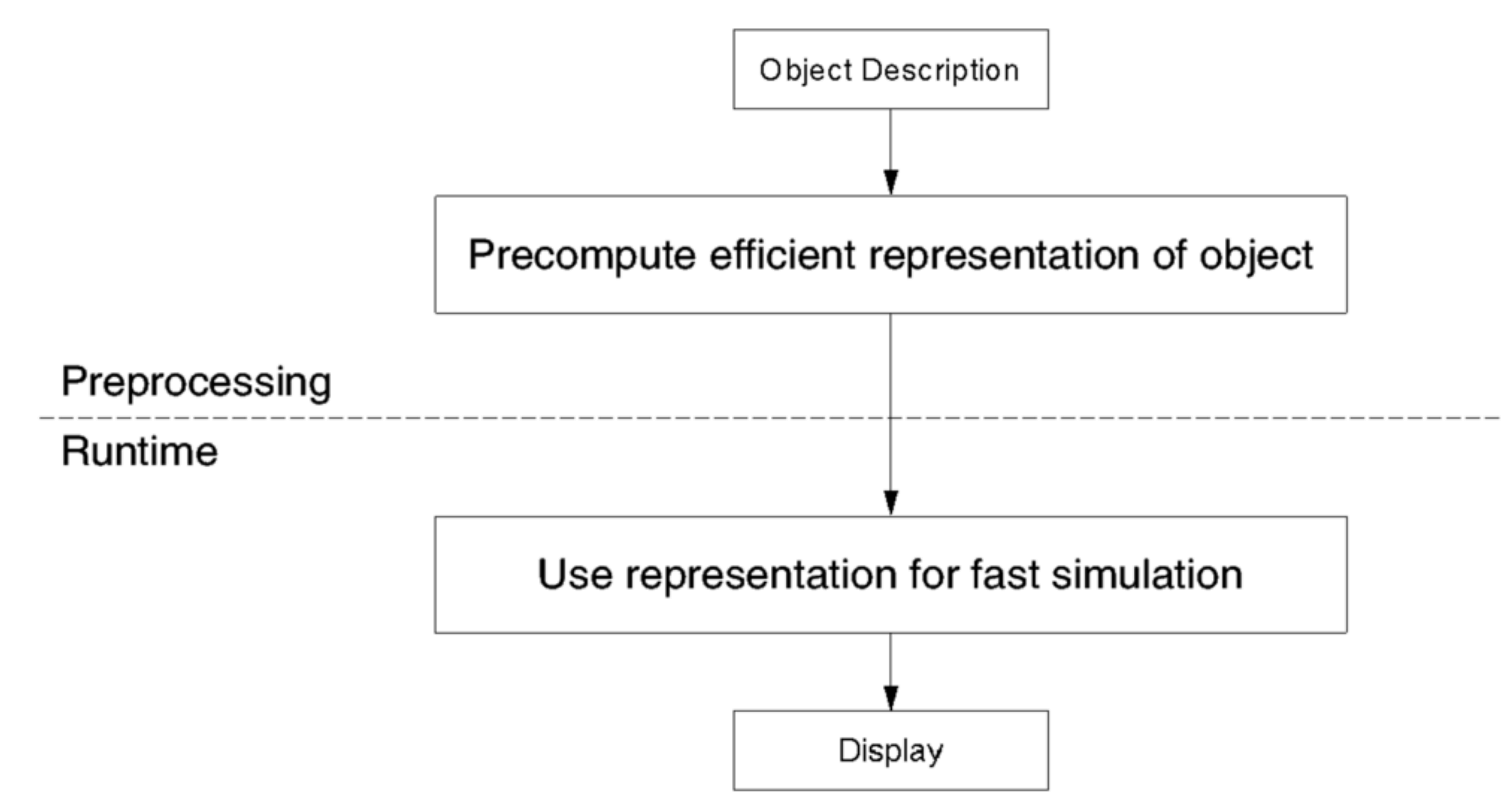
Modal Analysis

Computer Graphics and Imaging
UC Berkeley CS184/284A

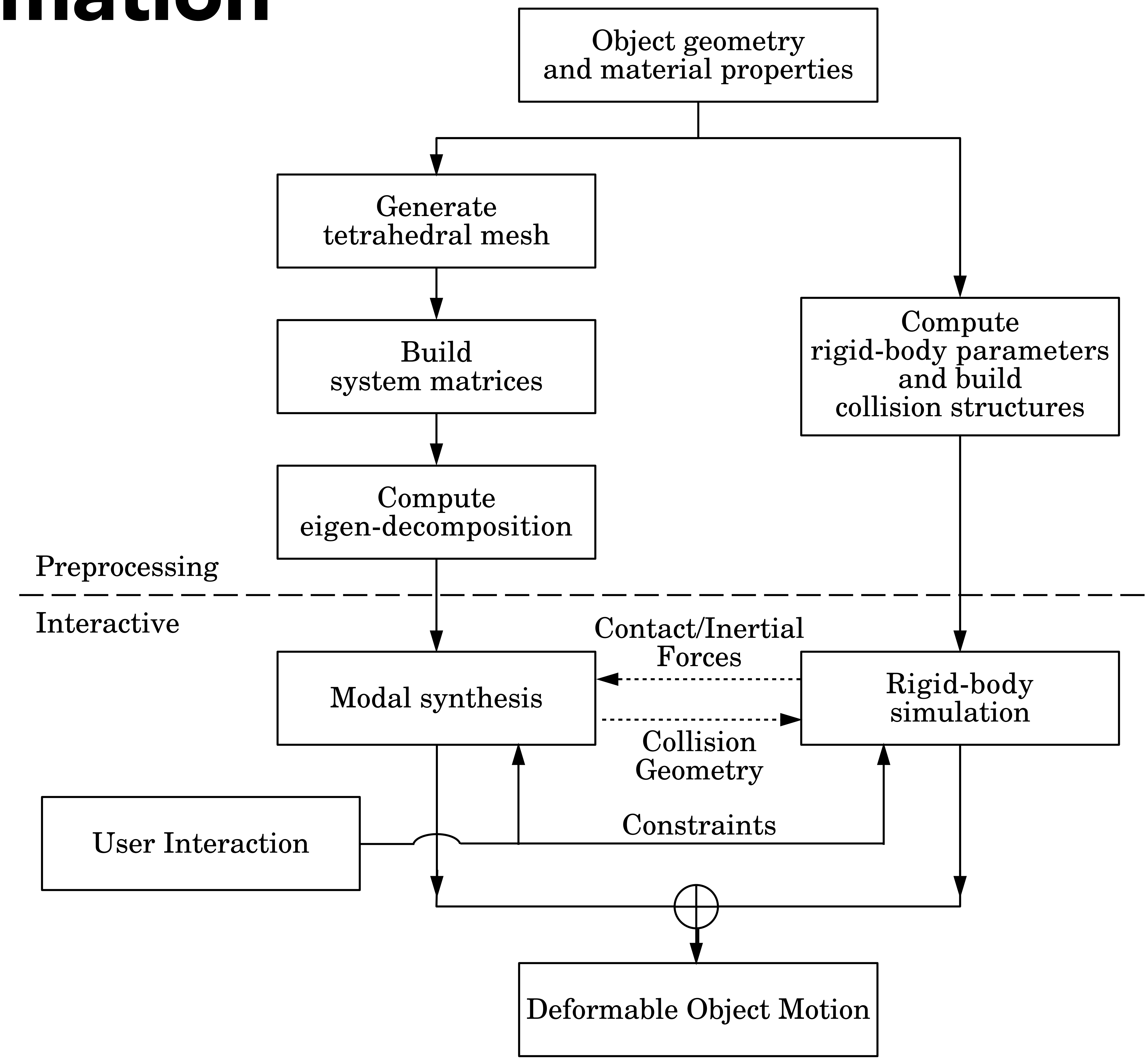
Note: The math on gray slides will not be on the exams.

Ren Ng, James O’Brien

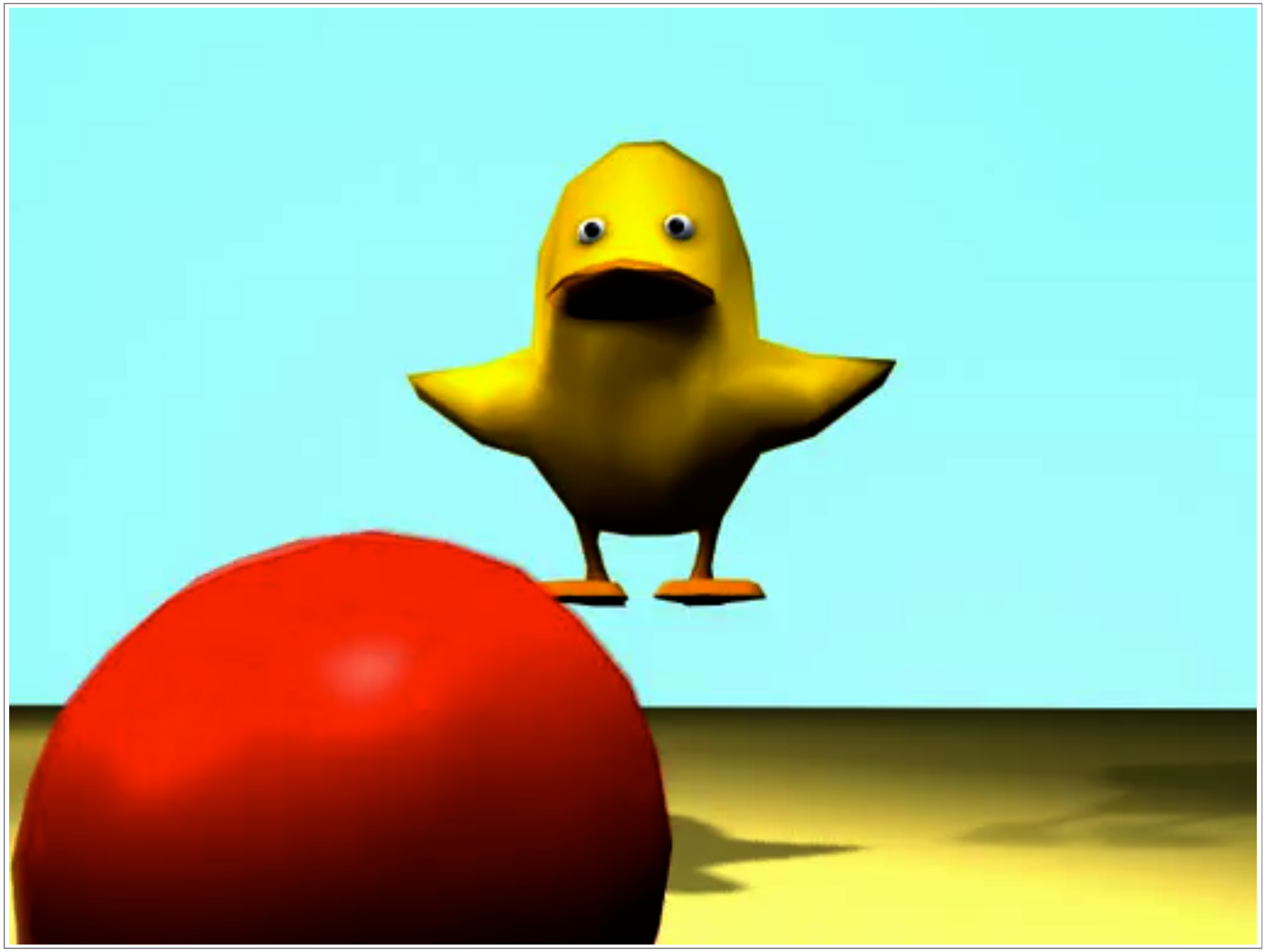
Modal Analysis



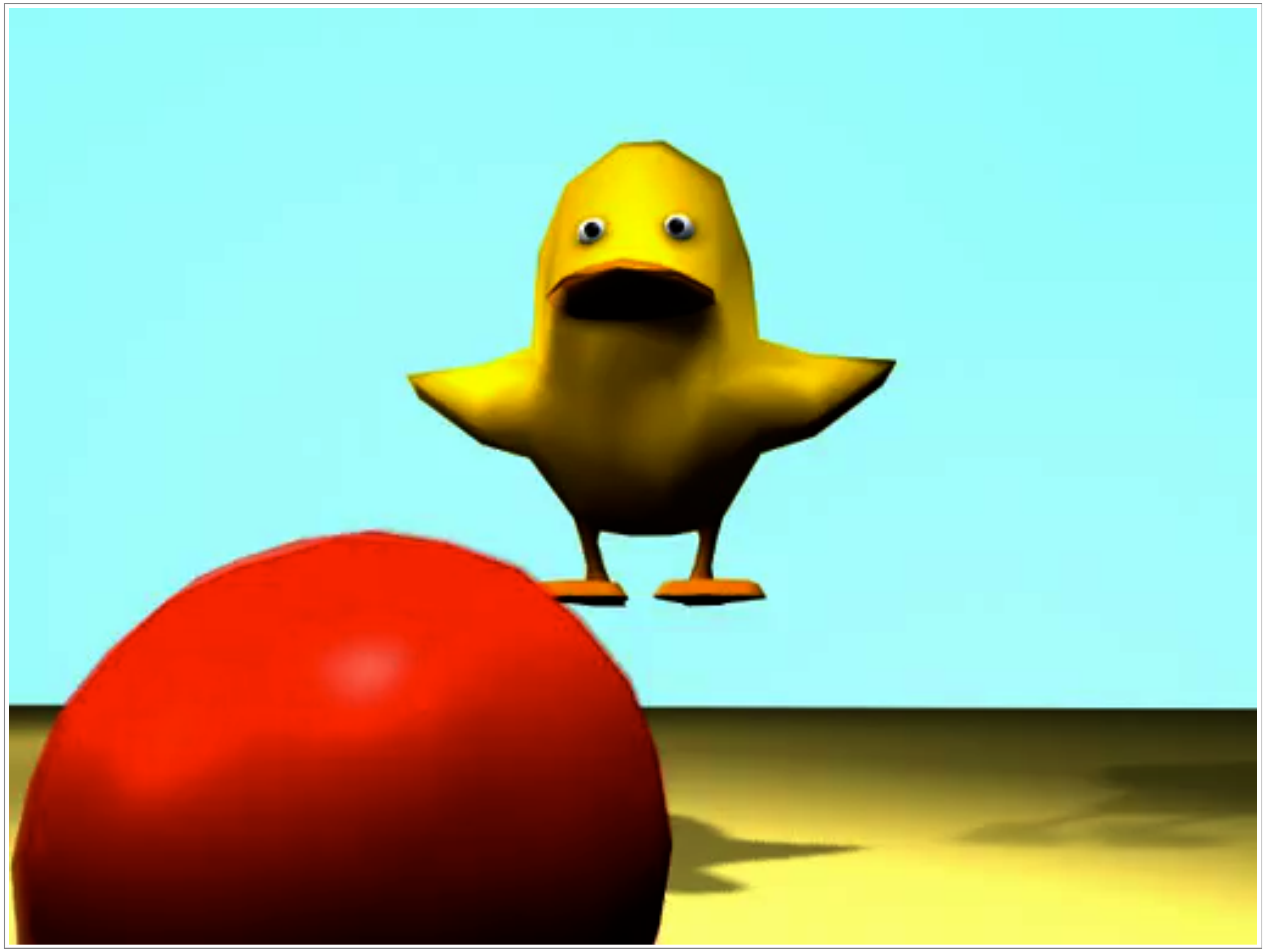
Deformation

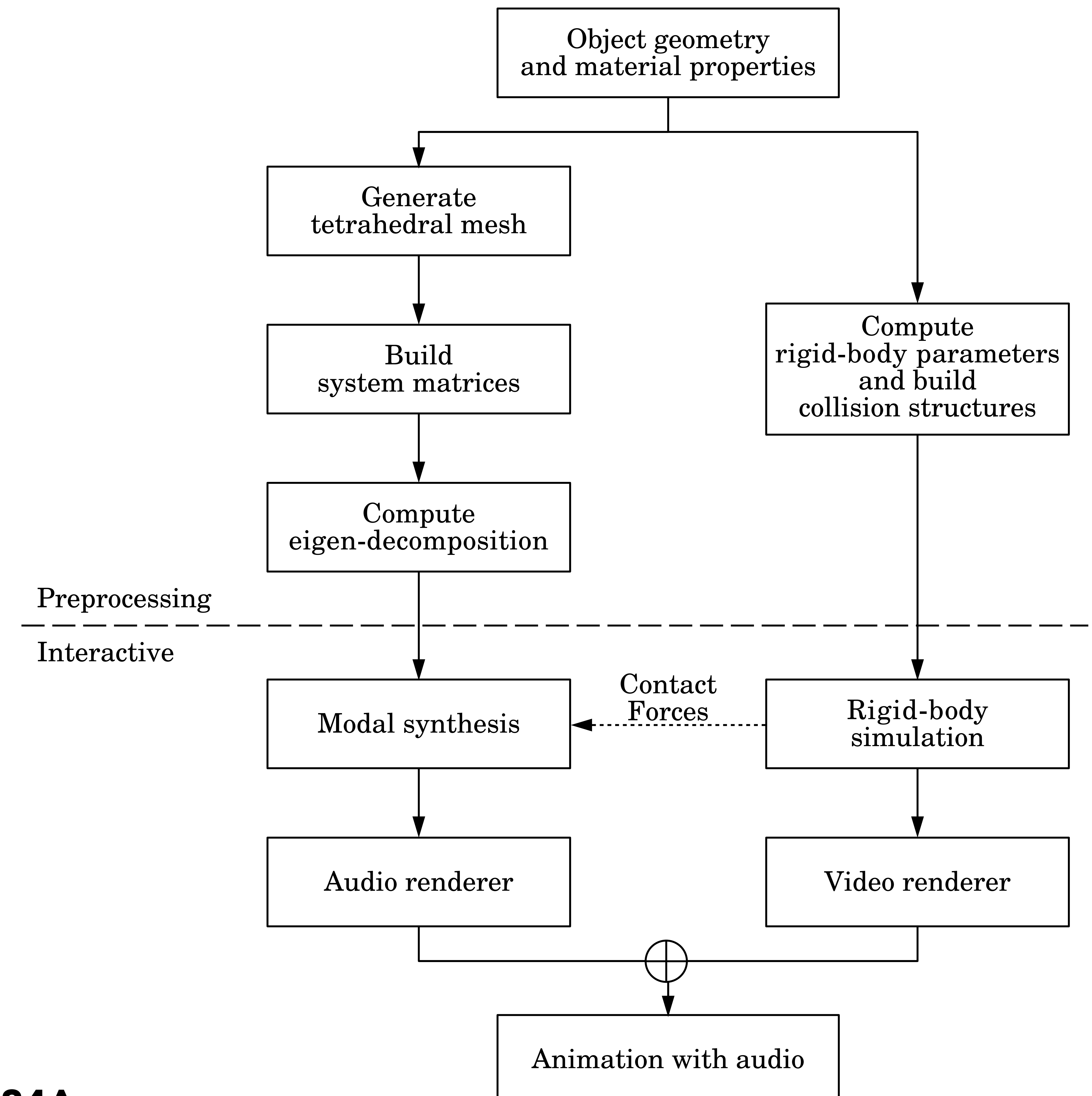


Example

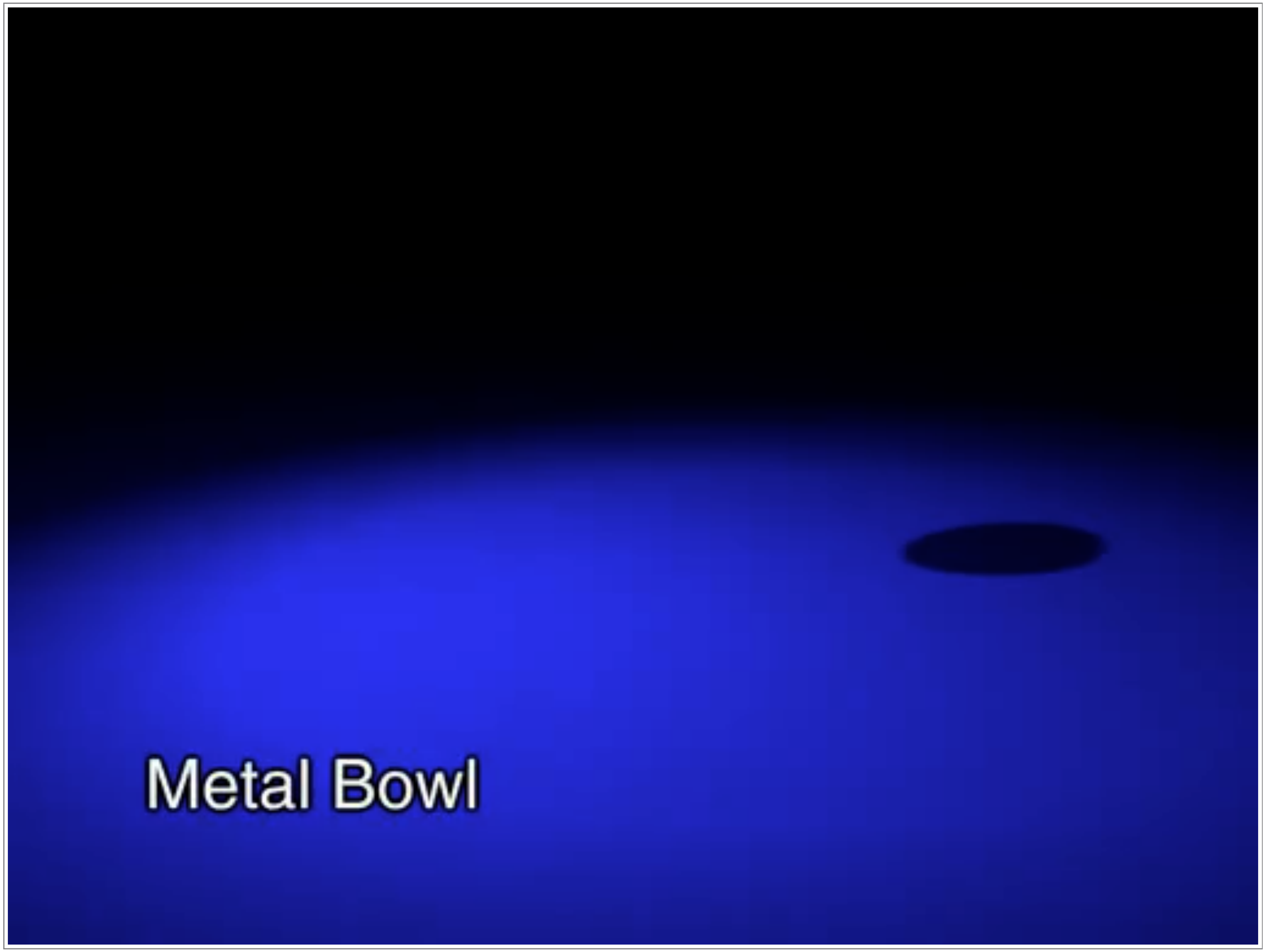


Example

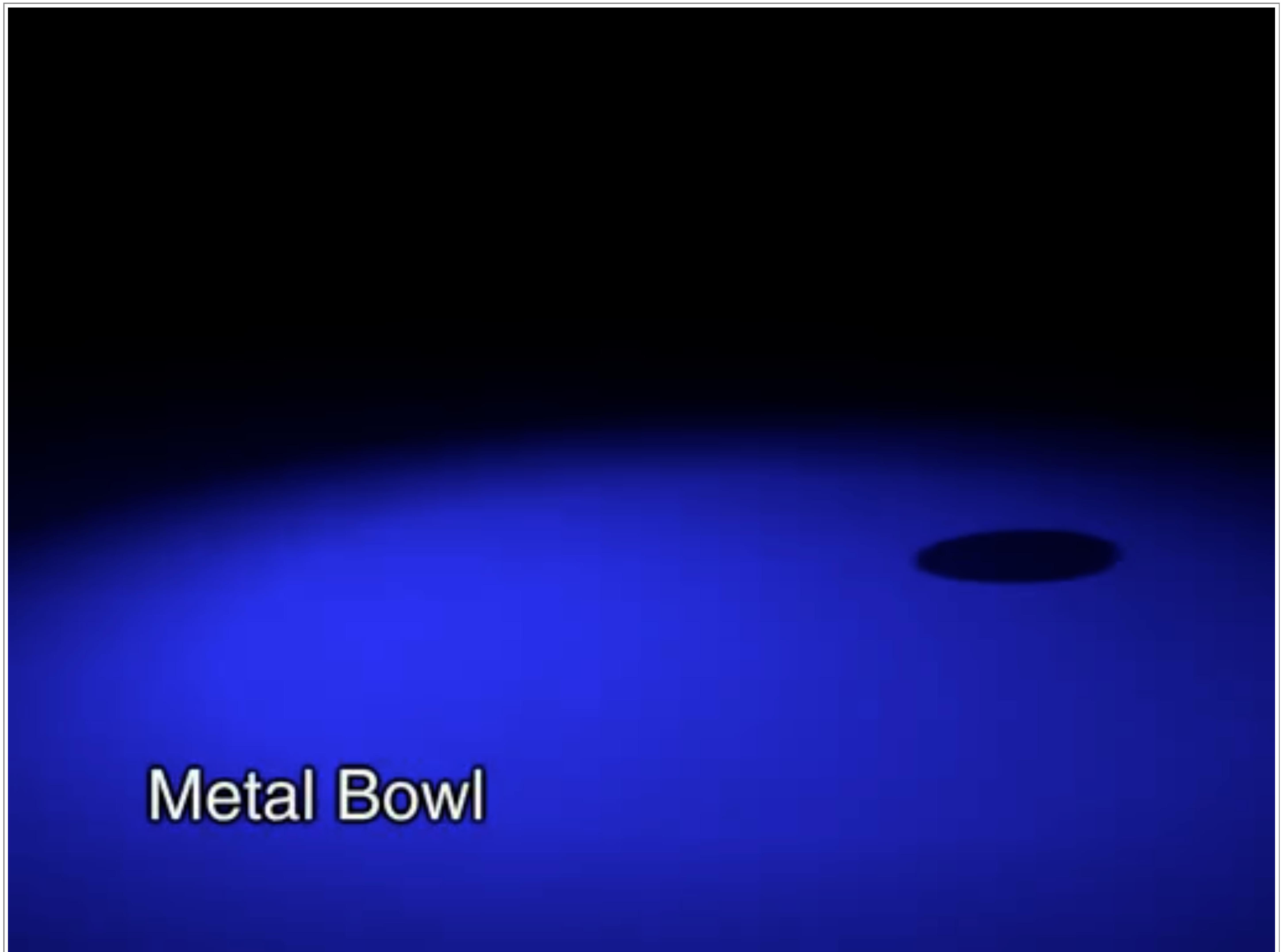



Object geometry and material properties
Generate tetrahedral mesh
Build system matrices
Compute eigen-decomposition
Compute rigid-body parameters and build collision structures
Preprocessing
Interactive
Contact Forces
Modal synthesis
Rigid-body simulation
Audio renderer
Video renderer
Animation with audio

Example



Example



Modal Decomposition

Linearize non-linear system

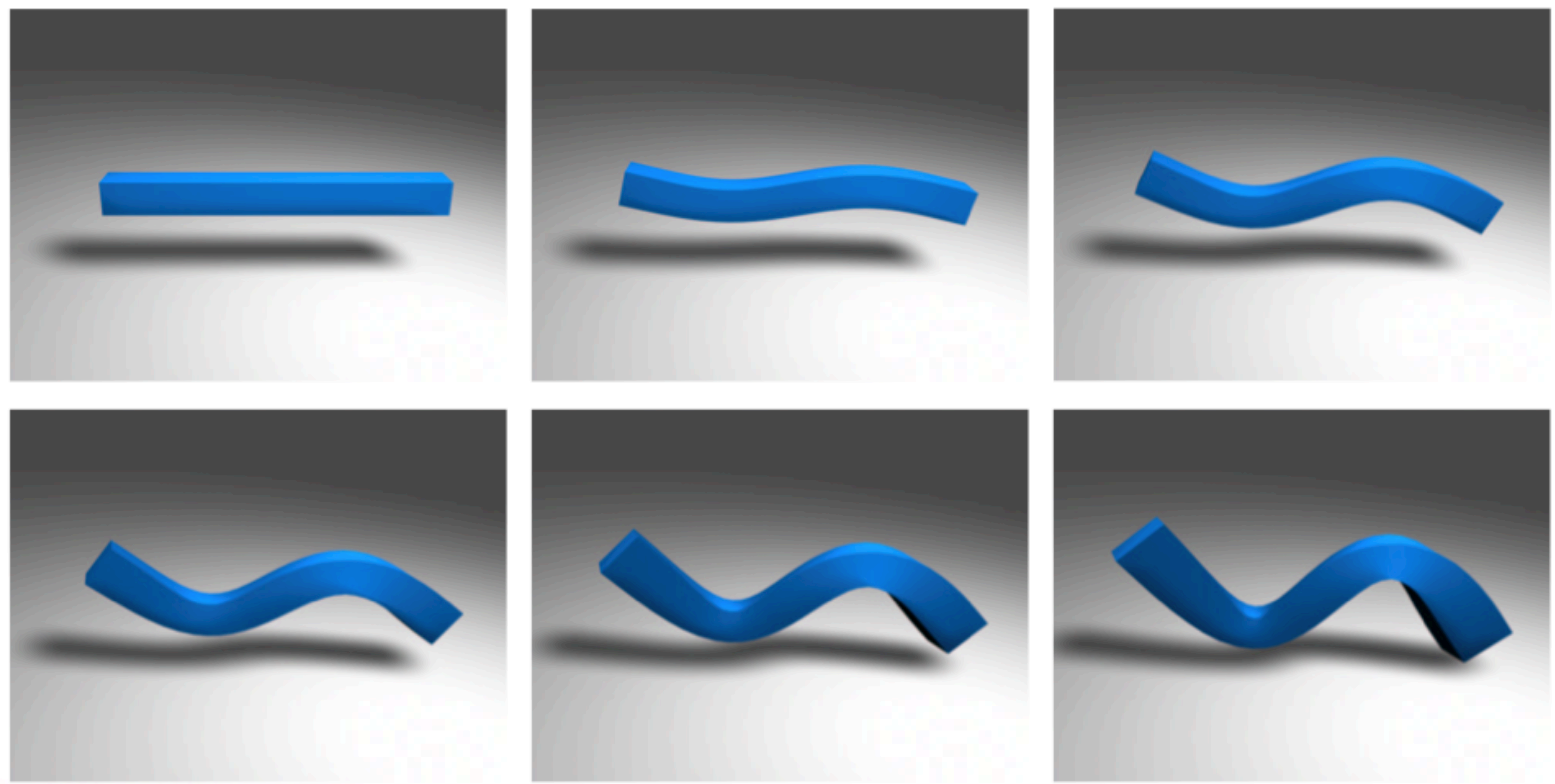
$$\boldsymbol{\mathcal{K}}(\boldsymbol{d}) + \boldsymbol{\mathcal{C}}(\boldsymbol{d}, \dot{\boldsymbol{d}}) + \boldsymbol{\mathcal{M}}(\ddot{\boldsymbol{d}}) = \boldsymbol{f}$$

$$\boldsymbol{K}\boldsymbol{d} + \boldsymbol{C}\dot{\boldsymbol{d}} + \boldsymbol{M}\ddot{\boldsymbol{d}} = \boldsymbol{f}$$

Modal Decomposition

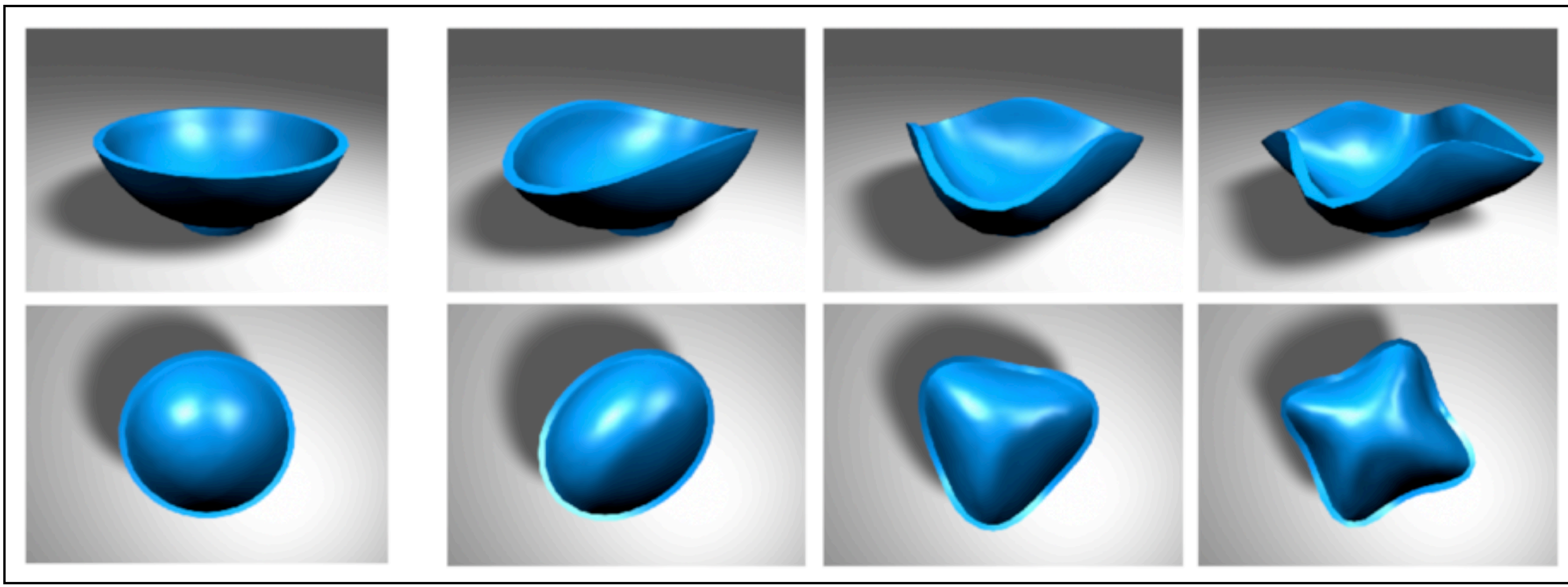
Consequences of linearization

- No local rotations



Modal Decomposition

$$\boldsymbol{\Lambda z} + (\alpha_1 \boldsymbol{\Lambda} + \alpha_2 \boldsymbol{I})\dot{\boldsymbol{z}} + \ddot{\boldsymbol{z}} = \boldsymbol{g}$$



Step 1: Linearization

$$\mathcal{K}(\, d \,) + \mathcal{C}(\, \dot{d} \,) + \mathcal{M}(\, \ddot{d} \,) = f$$

$$\boldsymbol{K} \boldsymbol{d} + \boldsymbol{C} \dot{\boldsymbol{d}} + \boldsymbol{M} \ddot{\boldsymbol{d}} = \boldsymbol{f}$$

Assume: $\boldsymbol{C} = \alpha_1 \boldsymbol{K} + \alpha_2 \boldsymbol{M}$

$$\boldsymbol{K}(\boldsymbol{d} + \alpha_1 \dot{\boldsymbol{d}}) + \boldsymbol{M}(\alpha_2 \dot{\boldsymbol{d}} + \ddot{\boldsymbol{d}}) = \boldsymbol{f}$$

Step 2: Normalize Mass

Normalize for mass by change of coordinates:

- Cholesky decompositions: $\mathbf{M} = \mathbf{L}\mathbf{L}^\mathsf{T}$
- Change of variables: $\mathbf{y} = \mathbf{L}^\mathsf{T}\mathbf{d} \qquad \mathbf{d} = \mathbf{L}^{-\mathsf{T}}\mathbf{y}$

$$\boldsymbol{K}(\boldsymbol{d} + \alpha_1 \dot{\boldsymbol{d}}) + \boldsymbol{M}(\alpha_2 \dot{\boldsymbol{d}} + \ddot{\boldsymbol{d}}) = \boldsymbol{f}$$

$$\Downarrow$$

$$\boldsymbol{L}^{-1}\boldsymbol{K}\boldsymbol{L}^{-\mathsf{T}}(\boldsymbol{y} + \alpha_1 \dot{\boldsymbol{y}}) + (\alpha_2 \dot{\boldsymbol{y}} + \ddot{\boldsymbol{y}}) = \boldsymbol{L}^{-1}\boldsymbol{f}$$

Step 3: Diagonalize

Diagonalize with second change of coordinates:

- Eigen decompositions: $\mathbf{L}^{-1}\mathbf{K}\mathbf{L}^{-\mathsf{T}} = \mathbf{V}\boldsymbol{\Lambda}\mathbf{V}^{\mathsf{T}}$
- Change of variables: $\mathbf{z} = \mathbf{V}^{\mathsf{T}}\mathbf{y}$ $\quad \mathbf{y} = \mathbf{V}\mathbf{z}$

$$\boldsymbol{L}^{-1}\boldsymbol{K}\boldsymbol{L}^{-\mathsf{T}}(\boldsymbol{y} + \alpha_1\dot{\boldsymbol{y}}) + (\alpha_2\dot{\boldsymbol{y}} + \ddot{\boldsymbol{y}}) = \boldsymbol{L}^{-1}\boldsymbol{f}$$

$$\Downarrow$$

$$\boldsymbol{\Lambda}(\boldsymbol{z} + \alpha_1\dot{\boldsymbol{z}}) + (\alpha_2\dot{\boldsymbol{z}} + \ddot{\boldsymbol{z}}) = \boldsymbol{V}^{\mathsf{T}}\boldsymbol{L}^{-1}\boldsymbol{f}$$

$$\Downarrow$$

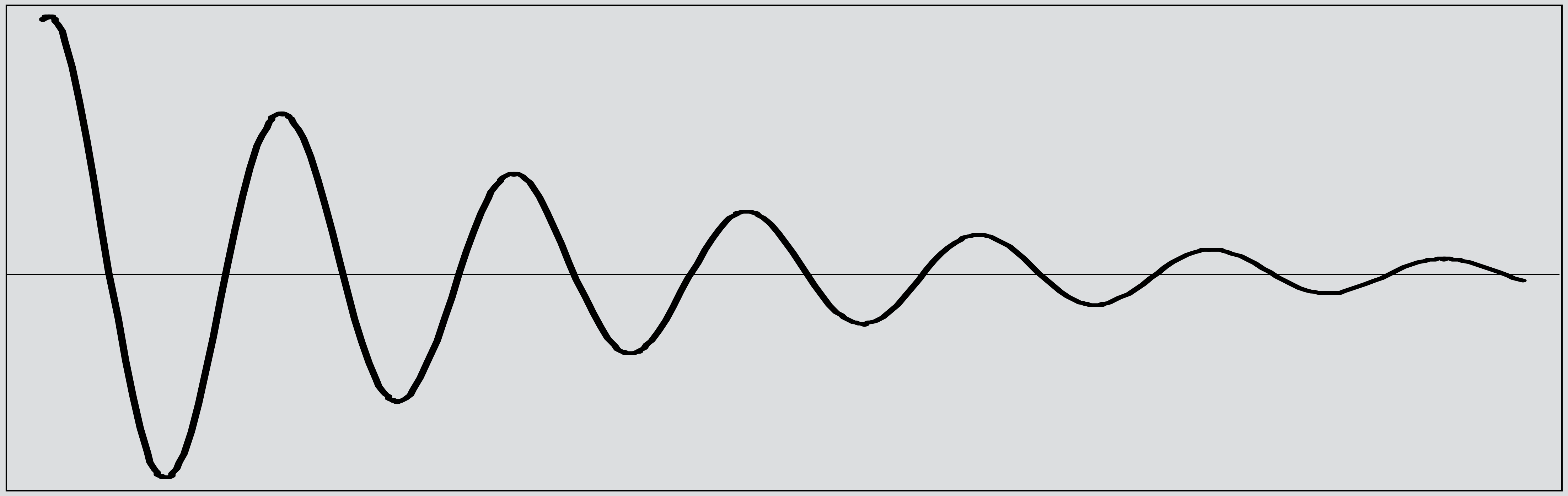
$$\boldsymbol{\Lambda}\boldsymbol{z} + (\alpha_1\boldsymbol{\Lambda} + \alpha_2\boldsymbol{I})\dot{\boldsymbol{z}} + \ddot{\boldsymbol{z}} = \boldsymbol{g}$$

Result: Individual Modes

$$\lambda_i z_i + (\alpha_1 \lambda_i + \alpha_2)\dot{z}_i + \ddot{z}_i = g_i$$

$$z_i = c_1 e^{t\omega_i^+} + c_2 e^{t\omega_i^-}$$

$$\omega_i^{\pm} = \frac{-(\alpha_1\lambda_i+\alpha_2) \pm \sqrt{(\alpha_1\lambda_i+\alpha_2)^2 - 4\lambda_i}}{2}$$



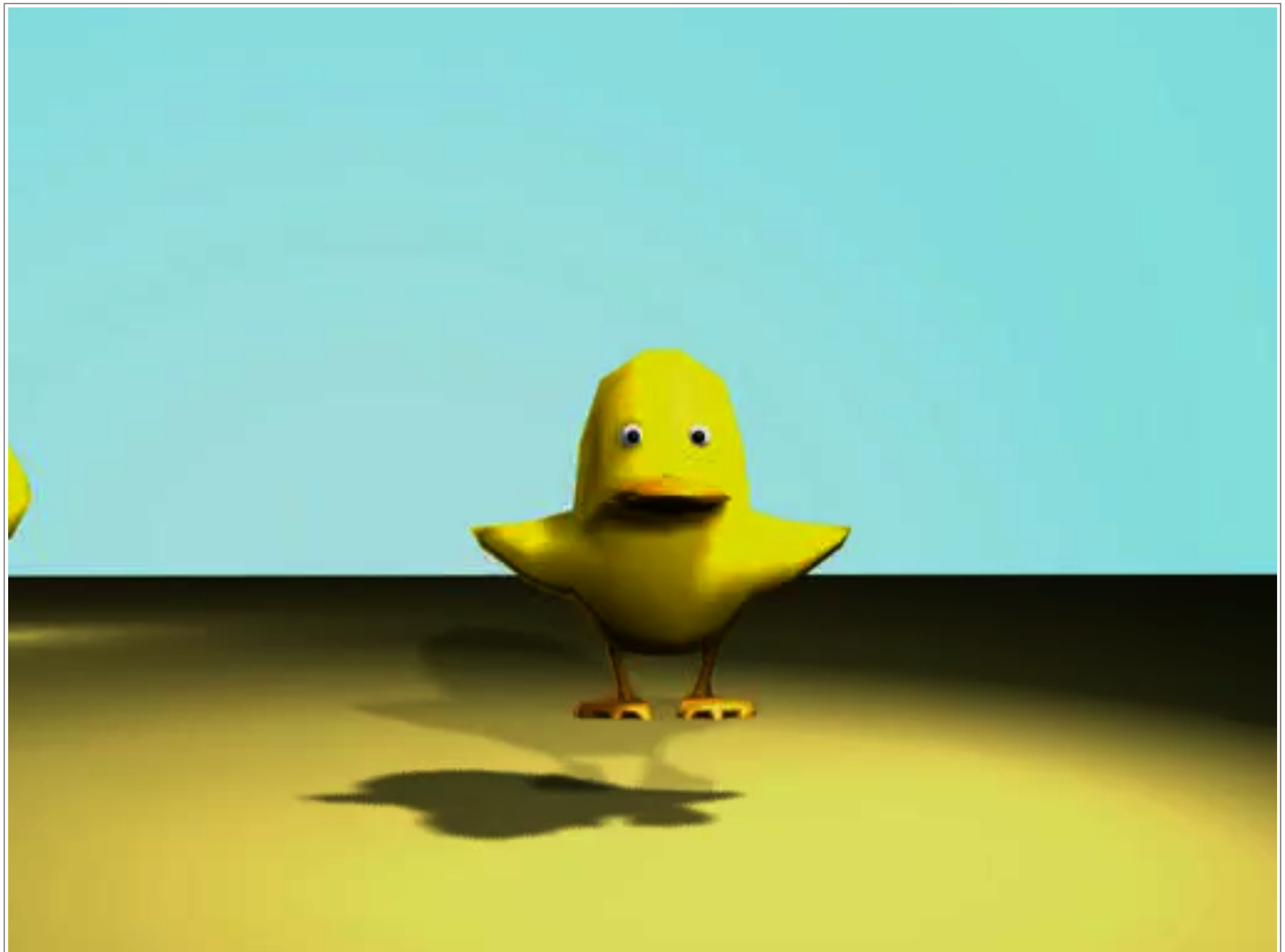
Fast Simulation

Only a pair of complex multiplies per time step

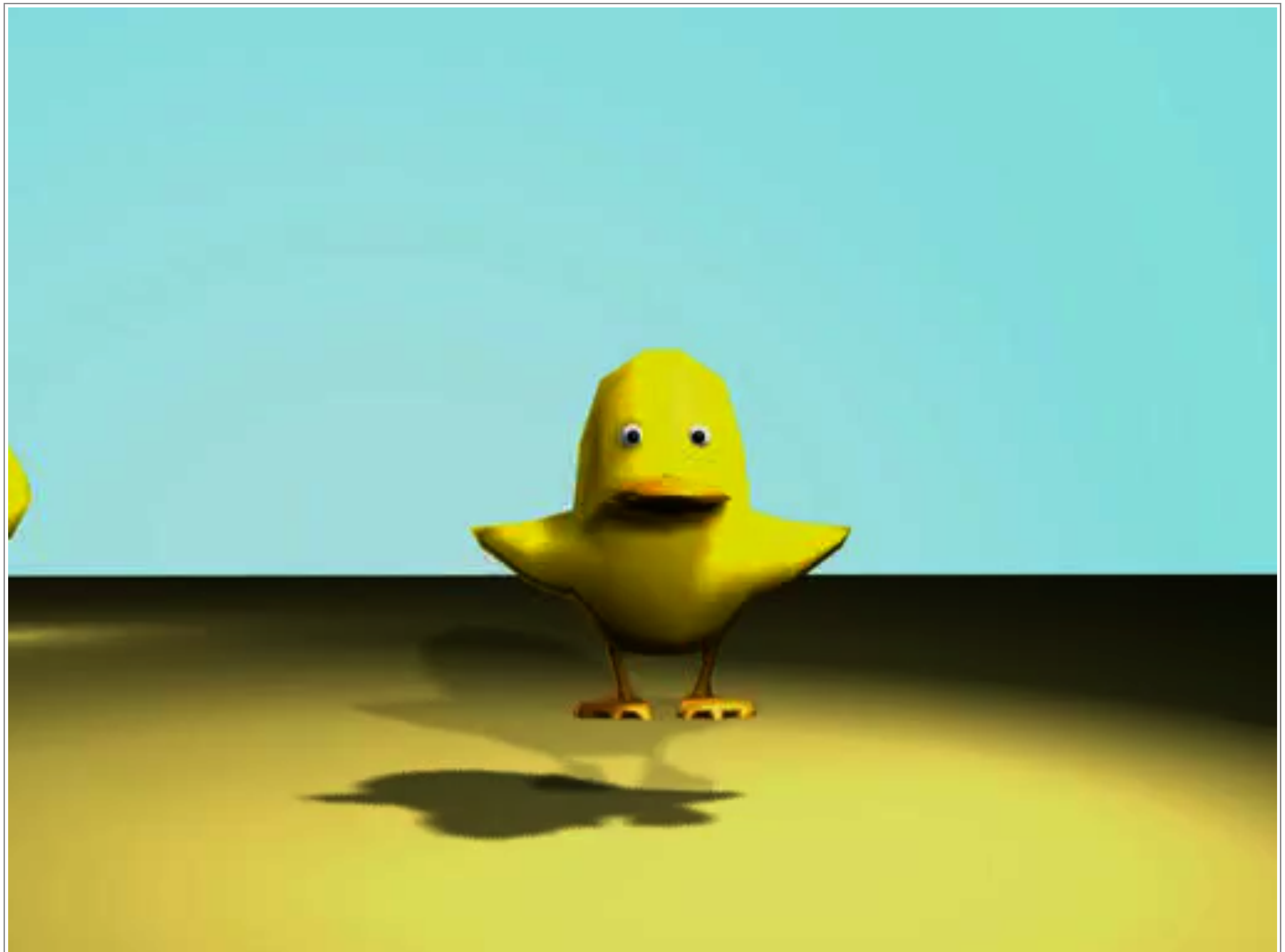
$$e^{\omega(t+\Delta t)} = e^{\omega(t)} e^{\omega(\Delta t)}$$

- No stability limit on step size
- Jump to arbitrary point in time
- Only keep useful modes

Examples



Examples



Examples

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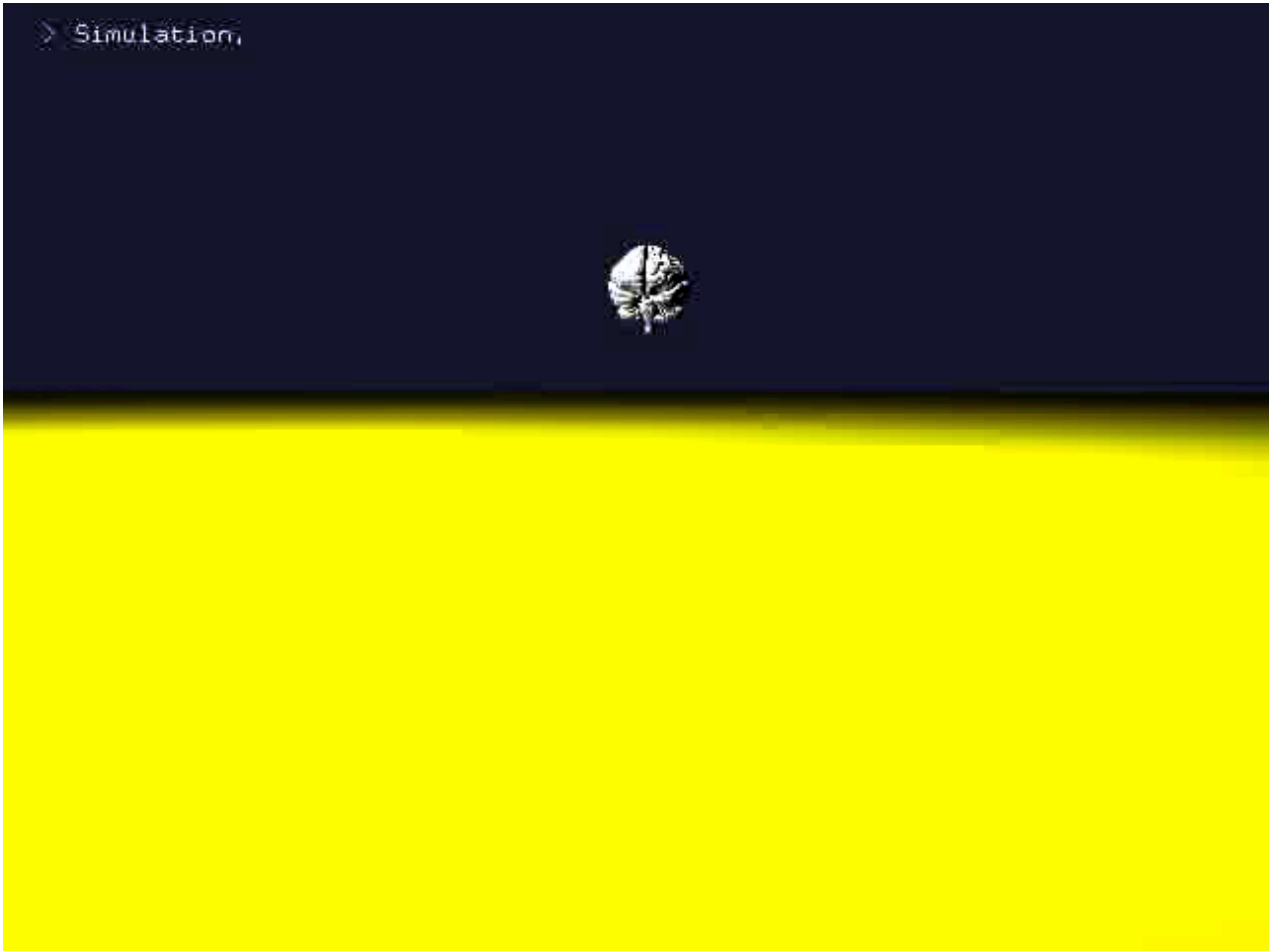
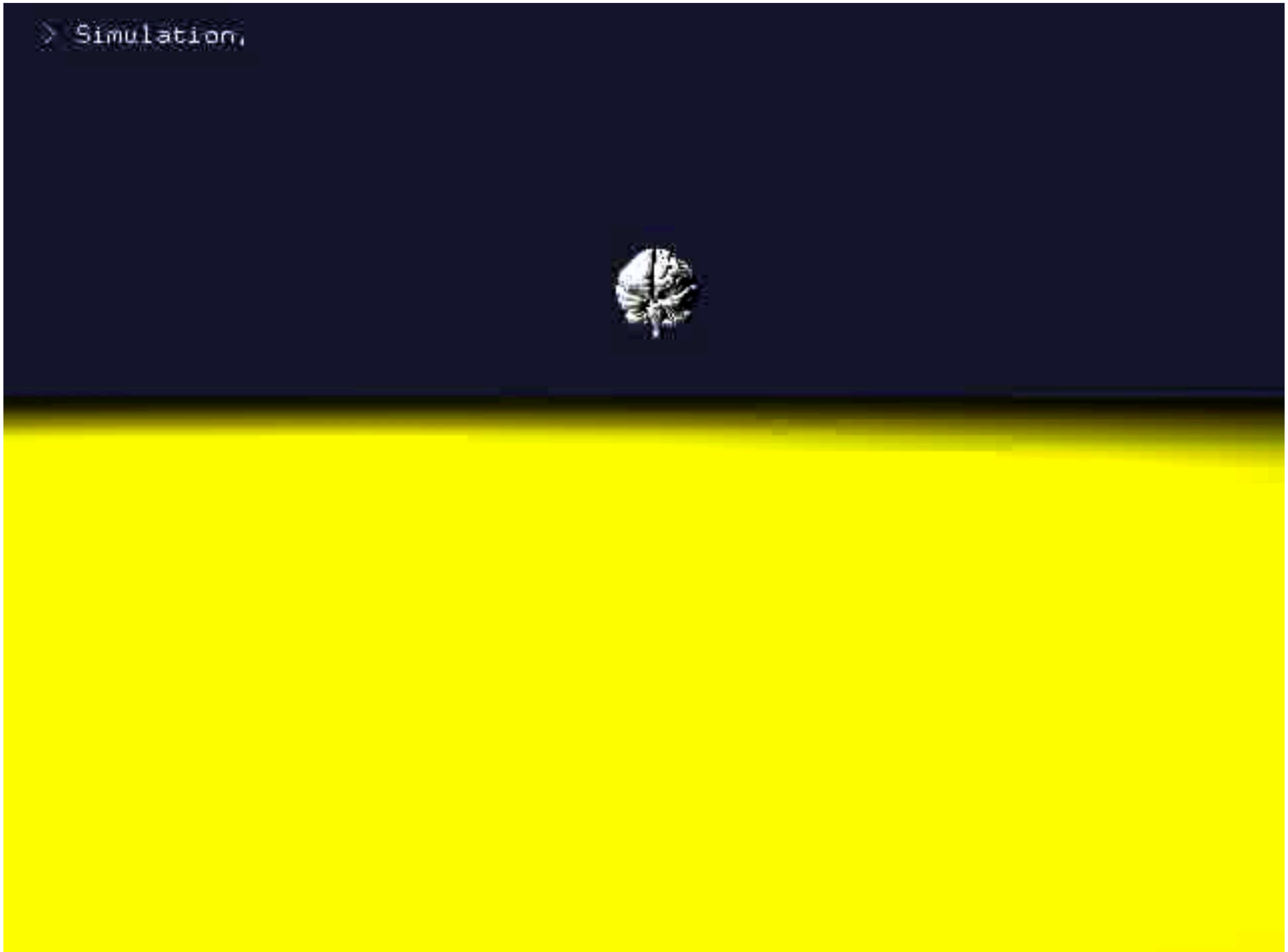


Examples



Examples


Synthesizing Sounds from
Rigid–Body Simulation
James F. O'Brien
Chen Shen
Christine M. Gatchalian
University of California, Berkeley
ACM SIGGRAPH Symposium on Computer Animation 2002

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High-order Differentiable Autoencoder for Nonlinear Model Reduction

SIGGRAPH 2021, Shen, *et. al*

High-order Differentiable Autoencoder for Nonlinear Model Reduction

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Backward vs Forward Euler Integration

$$\mathbf{x}^{t+1} = \mathbf{x}^t + \Delta t \dot{\mathbf{x}}^t$$

$$\dot{\mathbf{x}}^{t+1} = \dot{\mathbf{x}}^t + \Delta t \ddot{\mathbf{x}}^t = \dot{\mathbf{x}}^t + \Delta t \mathbf{M}^{-1} \mathbf{f}(\mathbf{x}^t, \cancel{\dot{\mathbf{x}}^t})$$

Forward Euler

$$\Delta \mathbf{x}_F = \Delta t \dot{\mathbf{x}}^t$$

$$\Delta \dot{\mathbf{x}}_F = \Delta t \mathbf{M}^{-1} \mathbf{f}^t$$

Backward (semi-implicit) Euler

$$\Delta \mathbf{x}_B = \Delta t \dot{\mathbf{x}}^{t+1}$$

$$\Delta \dot{\mathbf{x}}_B = \Delta t \mathbf{M}^{-1} \mathbf{f}^{t+1} \approx \Delta t \mathbf{M}^{-1} (\mathbf{f}^t + \mathbf{K} \Delta \mathbf{x})$$

Backward vs Forward Euler Integration

$$\Delta \mathbf{x}_B = \Delta t \dot{\mathbf{x}}^{t+1}$$
$$\Delta \dot{\mathbf{x}}_B = \Delta t \mathbf{M}^{-1}(\mathbf{f}^t + \mathbf{K}\Delta \mathbf{x}_B)$$

$$\Delta \mathbf{x}_F = \Delta t \dot{\mathbf{x}}^{t}$$
$$\Delta \dot{\mathbf{x}}_F = \Delta t \mathbf{M}^{-1}\mathbf{f}^t$$

Rewrite with deltas

$$\Delta \mathbf{x}_B = \Delta t \dot{\mathbf{x}}^{t} + \Delta t \Delta \dot{\mathbf{x}}_B$$
$$\Delta \dot{\mathbf{x}}_B = \Delta t \mathbf{M}^{-1}\mathbf{f}^t + \Delta t \mathbf{M}^{-1}\mathbf{K}\Delta \mathbf{x}_B)$$

Combine

$$\Delta \mathbf{x}_B = \Delta t \dot{\mathbf{x}}^{t} + \Delta t^2 \mathbf{M}^{-1}\mathbf{f}^t + \Delta t^2 \mathbf{M}^{-1}\mathbf{K}\Delta \mathbf{x}_B)$$
$$(\mathbf{I} - \Delta t^2 \mathbf{M}^{-1}\mathbf{K})\Delta \mathbf{x}_B = \Delta t \dot{\mathbf{x}}^{t} + \Delta t^2 \mathbf{M}^{-1}\mathbf{f}^t$$

Solve

$$\Delta \mathbf{x}_B = (\mathbf{I} - \Delta t^2 \mathbf{M}^{-1}\mathbf{K})^{-1}(\Delta \mathbf{x}_F + \Delta t^2 \mathbf{M}^{-1}\mathbf{f}^t)$$

Backward vs Forward Euler Integration

$$\Delta \mathbf{x}_B = \Delta t \dot{\mathbf{x}}^{t+1}$$
$$\Delta \dot{\mathbf{x}}_B = \Delta t \mathbf{M}^{-1}(\mathbf{f}^t + \mathbf{K}\Delta \mathbf{x}_B)$$

$$\Delta \mathbf{x}_F = \Delta t \dot{\mathbf{x}}^t$$
$$\Delta \dot{\mathbf{x}}_F = \Delta t \mathbf{M}^{-1}\mathbf{f}^t$$

$$\Delta \mathbf{x}_B = (\mathbf{I} - \Delta t^2 \mathbf{M}^{-1}\mathbf{K})^{-1}(\Delta \mathbf{x}_F + \Delta t^2 \mathbf{M}^{-1}\mathbf{f}^t)$$

Solve

$$\Delta \mathbf{x}_B = \mathbf{V}(\mathbf{I} - \Delta t^2 \mathbf{\Lambda})^{-1}\mathbf{V}^{\mathsf{T}}(\Delta \mathbf{x}_F + \Delta t^2 \mathbf{f}^t)$$
$$\Delta \mathbf{x}_B = \text{filter}(\Delta \mathbf{x}_F + \Delta t^2 \mathbf{f}^t)$$

Note: $\mathbf{K}$, $\mathbf{V}$, $\mathbf{\Lambda}$, and **filter** change over time

Backward vs Forward Euler Integration

$$\Delta \mathbf{x}_F = \Delta t \dot{\mathbf{x}}^t$$
$$\Delta \dot{\mathbf{x}}_F = \Delta t \mathbf{M}^{-1} \mathbf{f}^t$$

$$\Delta \mathbf{x}_B = \mathbf{V}(\mathbf{I} - \Delta t^2 \mathbf{\Lambda})^{-1} \mathbf{V}^\mathsf{T}(\Delta \mathbf{x}_F + \Delta t^2 \mathbf{f}^t)$$

$$\Delta \mathbf{x}_B = \text{filter}(\Delta \mathbf{x}_F + \Delta t^2 \mathbf{f}^t)$$

Note: $\mathbf{K}$, $\mathbf{V}$, $\mathbf{\Lambda}$, and filter change over time

- **PCA vs Eigen System**
  - **Dynamic orthogonality**
  - **Energy leakage**
  - **Limited sample states for PCA**

Acknowledgments

This slide set contain contributions from:

- Kayvon Fatahalian
- David Forsyth
- Pat Hanrahan
- Angjoo Kanazawa
- Steve Marschner
- Ren Ng
- James F. O'Brien
- Mark Pauly