Color is Central to Our Human Experience
Wassily Kandinsky, Color Study. Squares with Concentric Circles, 1913
Munich, The Städtische Galerie im Lenbachhaus
Mark Rothko
No. 61. Rust and Blue
1953,
Museum of Contemporary Art, Los Angeles
Discover What Color Feels Like

Bring greater vibrancy and color to your world with EnChroma high-performance glasses for color blindness.

SHOP NOW
Color-Blind Reactions to Perceiving New Colors

Enchroma, https://www.youtube.com/watch?v=-rMjUsG--zo
Color-Blind Reactions to Perceiving New Colors

Enchroma, https://www.youtube.com/watch?v=-rMjUsG--zo
Simulation of Color Blind Perception
(Color Vision Deficiency)
Simulation of Color Blind Perception

Normal

Protan

Deutan

Tritan

Created with Coblis — Color Blindness Simulator
A Person With One Trichromatic Eye and One Deuteranopic Eye

Graham and Hsia, 1959.
“A unilaterally dichromatic subject”.

Source: Munsell

Fig. 2.—Results of the experiment on binocular matching
Color is Core to Our Human Visual Sense

Steve McCurry  |  Reza  |  Walter Iooss  |  Steve McCurry
Harold Edgerton  |  NASA  |  National Geographic
Color is Core to Our Human Visual Sense

Steve McCurry | Reza | Walter Iooss | Steve McCurry
Harold Edgerton | NASA | National Geographic

Protan
Color is Core to Our Human Visual Sense

- Steve McCurry
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Deutan
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Simulation of Color Blind Perception

Color is Core to Our Human Visual Sense

Normal

Protan

Deutan

Tritan

Created with Coblis — Color Blindness Simulator
Chromatic Adaptation
Studying Chromatic Adaptation

Slide credit: Mark Fairchild
A CYAN FILTER

Slide credit: Mark Fairchild
Automatic White Balance - Examples

No white balance (all processed as “daylight”)
Automatic White Balance - Examples

Automatic white balance applied (Lightroom implementation)
Automatic White Balance - Examples

AWB + light manual editing
Automatic White Balance

\[
\begin{bmatrix}
R \\
G \\
B
\end{bmatrix} = \begin{bmatrix}
\frac{1}{R'_W} & 0 & 0 \\
0 & \frac{1}{G'_W} & 0 \\
0 & 0 & \frac{1}{B'_W}
\end{bmatrix} \begin{bmatrix}
R' \\
G' \\
B'
\end{bmatrix}
\]

\(R, G, B\) - automatic white balanced output
\(R'_W, G'_W, B'_W\) - raw input of white object
\(R', G', B'\) - raw input

In technical portion of today’s lecture, on color reproduction calculations, we will implicitly assume either that:

- Auto white balance has been applied
- Or that the viewing conditions of the color reproduction match the original scene
Color Perception is Highly Adaptive
Even simple judgments – such as lightness - depend on brain processing (Anderson and Winawer, Nature, 2005)
Simultaneous Contrast and Surround Effect

Slide credit: Mark Fairchild
Surround Effects

Roberts (1996)
Surround Effects

Roberts (1996)
AfterImages: Perception Operates on "Opponent" Color Axes
Image

Afterimage
keep staring at the black dot.
Color Perception is Complex and Surprising
Watercolor Illusion
Watercolor Illusion
Watercolor Illusion
And Yet, We Understand Color Reproduction As a Quantitative Science
Notice R, G, B sub-pixel geometry.
Effectively three lights at each (x,y) location.
Recall: Real LCD Screen Pixels (Closeup)
Color Reproduction Problem We Will Study

Goal: at each pixel, choose R, G, B values for display so that the output color matches the appearance of the colors in the real world.

Real world damselfly

Display image of damselfly on computer screen
What is Color?
What is Color?

- Color is a phenomenon of human perception; it is not a universal property of light
- Colors are the visual sensations that arise from seeing light of different spectral power distributions

Color Science
Sources of Optical Radiation: PHYSICS
Characterization of Objects: PHYSICS, CHEMISTRY
Perception: ANATOMY, PHYSIOLOGY, PSYCHOLOGY
Physical Basis of Color
Isaac Newton's Experimentum Crucis

Isaac Newton performing his crucial prism experiment – the 'experimentum crucis' – in his Woolsthorpe Manor bedroom.

Acrylic painting by Sascha Grusche (17 Dec 2015)

- Newton showed sunlight can be subdivided into a rainbow with a prism
- Resulting light cannot be further subdivided with a second prism
The Visible Spectrum of Light

Electromagnetic radiation

- Oscillations of different frequencies (wavelengths)

[Image of the Visible Spectrum of Light]

Image credit: Licensed under CC BY-SA 3.0 via Commons
A monochromator delivers light of a single wavelength from a light source with broad spectrum. Control which wavelength by angle of prism.
Spectrometer

For unknown light source, use a monochromator to isolate each wavelength of light for measurement
Spectral Power Distribution (SPD)

Salient property in measuring light

- The amount of light present at each wavelength
- Units:
  - radiometric units / nanometer (e.g. watts / nm)
  - Can also be unit-less
- Often use “relative units” scaled to maximum wavelength for comparison across wavelengths when absolute units are not important
Daylight Spectral Power Distributions Vary

![Diagram showing spectral power distributions for blue sky and solar disk.](image)
Spectral Power Distribution of Light Sources

Describes distribution of energy by wavelength

Figure credit: admesy

Ren Ng
Superposition (Linearity) of Spectral Power Distributions
Measuring Light
A Simple Model of a Light Detector

Produces a scalar value (a number) when photons land on it

- Value depends only on the number of photons detected
- Each photon has a probability of being detected that depends on the wavelength
- No way to distinguish between signals caused by light of different wavelengths: there is just a number

This model works for many detectors:

- based on semiconductors (such as in a digital camera)
- based on visual photopigments (such as in human eyes)
A Simple Model of a Light Detector

$$X = \int n(\lambda)p(\lambda) \, d\lambda$$

Credit: Marschner
Mathematics of Light Detection

Same math carries over to spectral power distributions

- Light entering the detector has its spectral power distribution, $s(\lambda)$
- Detector has its spectral sensitivity or spectral response, $r(\lambda)$

$$X = \int s(\lambda) r(\lambda) \, d\lambda$$

- Measured signal
- Detector’s sensitivity
- Input spectrum
If we think of \( s \) and \( r \) as discrete, sampled representations (vectors) rather than continuous functions, this integral operation is a dot product:

\[
X = s \cdot r
\]

We can also write this in matrix form:

\[
X = \begin{bmatrix}
s \end{bmatrix} \begin{bmatrix}
r
\end{bmatrix}
\]
Dimensionality Reduction From $\infty$ to 1

At the detector:

- SPD is a function of wavelength ($\infty$ - dimensional signal)
- Detector result is a scalar value (1 - dimensional signal)
Tristimulus Theory of Color
Searching for a Linear Systems Basis for Colors: The Color Matching Experiment
Maxwell's Crucial Color Matching Experiment

http://designblog.rietveldacademie.nl/?p=68422

Portrait: http://rsta.royalsocietypublishing.org/content/366/1871/1685
Color Matching Experiment

Same idea as spinning top, fancier implementation (Maxwell did this too)
Show test light spectrum on left
Mix “primaries” on right until they match
The primaries need not be RGB
Example Experiment

Test light

Primary lights

Slide credit: Kotani, Durand, Freeman
Example Experiment

Test light

Primary lights

$p_1$ $p_2$ $p_3$

Slide credit: Kotani, Durand, Freeman
Example Experiment

Test light

Primary lights

$p_1$ $p_2$ $p_3$

Slide credit: Kotani, Durand, Freeman
Example Experiment

The primary color amounts needed for a match

Primary lights

Test light

Primary lights

$p_1$ $p_2$ $p_3$

Slide credit: Kotani, Durand, Freeman
Experiment 2: Out of Gamut
Experiment 2: Out of Gamut

Test light

Primary lights

$p_1$ $p_2$ $p_3$
Experiment 2: Out of Gamut

Test light

Primary lights

$p_1 \ p_2 \ p_3$

Slide credit: Kotani, Durand, Freeman
Experiment 2: Out of Gamut

We say a “negative” amount of \( p_2 \) was needed to make the match, because we added it to the test color’s side.

\[
p_1 \quad p_2 \quad p_3
\]

Test light + Primary light

Primary light amounts needed for a match

\[
p_1 \quad p_2 \quad p_3
\]
The Color Matching Experiment is Linear

If matches and matches then matches
What is the Dimensionality of Human Color Perception?

And how do we know?

What is the definition of “dimension” here?

- We can appeal to linear systems theory, where “dimension” equals the rank of a basis for the linear space.

In the color matching experiment, empirically one finds:

- For subjects with “normal” color vision, three primary colors are necessary and sufficient to match any test color. Four primaries work but are unnecessary; two are insufficient.

- For red-green colorblind subjects, only two primary colors are necessary and sufficient to match any test color.
CIE RGB Color Matching Experiment

Same setup as additive color matching before, but primaries are monochromatic light (single wavelength) of the following wavelengths defined by CIE RGB standard:

- 700 nm
- 546.1 nm
- 435.8 nm

The test light is also a monochromatic light
CIE RGB Color Matching Functions

Graph plots how much of each CIE RGB primary light must be combined to match a monochromatic light of wavelength given on x-axis

Careful: these are not response curves or primary spectra!
Biological Basis of Color
Anatomy of The Human Eye
Retinal Photoreceptor Cells: Rods and Cones

Rods are primary receptors in very low light (“scotopic” conditions), e.g. dim moonlight

- ~120 million rods in eye
- Perceive only shades of gray, no color

Cones are primary receptors in typical light levels (“photopic”)

- ~6-7 million cones in eye
- Three types of cones, each with different spectral sensitivity
- Provide sensation of color

http://ebooks.bfwpub.com/life.php Figure 45.18
Photoreceptor Size and Distribution Vary Across Retina

- No rods in fovea
- Cones increase in size and spacing with eccentricity
On the Retina, Three Types of Cone Cells

Three types of cone cells: S, M, and L (corresponding to peak response at short, medium, and long wavelengths)
Spectral Response of Human Cone Cells

Instead of one detector as before, now we have three detectors (S, M, L cone cells), each with a different spectral response curve.

\[
S = \int r_S(\lambda) s(\lambda) \, d\lambda \\
M = \int r_M(\lambda) s(\lambda) \, d\lambda \\
L = \int r_L(\lambda) s(\lambda) \, d\lambda
\]
Example: Spectral Response of Human Cone Cells
Example: Spectral Response of Human Cone Cells

Scene projected onto retina
Example: Spectral Response of Human Cone Cells

Scene projected onto retina
Example: Spectral Response of Human Cone Cells

$$\int s(x, y, \lambda) \cdot r_L(\lambda) \, d\lambda$$
Example: Spectral Response of Human Cone Cells

\[ \int s(x, y, \lambda) \cdot r_M(\lambda) \, d\lambda \]

M cone cell

Credit: Sabesan, http://depts.washington.edu/sabaolab/
Example: Spectral Response of Human Cone Cells

\[ \int s(x, y, \lambda) \cdot r_s(\lambda) \, d\lambda \]

Credit: Sabesan, http://depts.washington.edu/sabaolab/
Spectral Response of Human Cone Cells

Instead of one detector as before, now we have three detectors (S, M, L cone cells), each with a different spectral response curve.

\[
S = \int r_S(\lambda)s(\lambda)\,d\lambda
\]
\[
M = \int r_M(\lambda)s(\lambda)\,d\lambda
\]
\[
L = \int r_L(\lambda)s(\lambda)\,d\lambda
\]

Brainard, Color and the Cone Mosaic, 2015.
Spectral Response of Human Cone Cells

Instead of one detector as before, now we have three detectors (S, M, L cone cells), each with a different spectral response curve.

Written as vector dot products:

\[ S = r_S \cdot s \]
\[ M = r_M \cdot s \]
\[ L = r_L \cdot s \]

Matrix formulation:

\[
\begin{bmatrix}
  S \\
  M \\
  L
\end{bmatrix} =
\begin{bmatrix}
  r_S & & \\
  r_M & & \\
  r_L & & \\
\end{bmatrix}
\begin{bmatrix}
  s
\end{bmatrix}
\]
Dimensionality Reduction From $\infty$ to 3

At each position on the human retina:

- SPD is a function of wavelength ($\infty$ - dimensional signal)
- 3 types of cones near that position produce three scalar values (3 - dimensional signal)

What about 2D images?

- The dimensionality reduction described above is happening at every 2D position in our visual field
The Human Visual System

- Human eye does not measure and brain does not receive information about each wavelength of light
- Rather, the eye measures three response values only (S, M, L) at each position in visual field, and this is only spectral info available to brain
- This is the result of integrating the incoming spectrum against response functions of S, M, L cones
Metamerism
Metamers

Metameters are two different spectra ($\infty$-dim) that project to the same (S,M,L) (3-dim) response.

- These will appear to have the same color to a human

The existence of metamers is critical to color reproduction

- Don’t have to reproduce the full spectrum of a real world scene

- Example: A metamer can reproduce the perceived color of a real-world scene on a display with pixels of only three colors
Metamerism

Color matching is an important illusion that is understood quantitatively
Metamerism is a Big Effect

Relative power (watts/str/nm)

Wavelength (nm)
Color Reproduction
Color Reproduction Problem

Target real spectrum $s(\lambda)$

Display outputs spectrum $R s_R(\lambda) + G s_G(\lambda) + B s_B(\lambda)$

Goal: at each pixel, choose R, G, B values for display so that the output color matches the appearance of the target color in the real world.
Additive Color

• Given a set of primary lights, each with its own spectral distribution (e.g. R, G, B display pixels):
  \[ s_R(\lambda), s_G(\lambda), s_B(\lambda) \]

• We can adjust the brightness of these lights and add them together to produce a linear subspace of spectral distribution:
  \[ R s_R(\lambda) + G s_G(\lambda) + B s_B(\lambda) \]

• The color is now described by the scalar values:
  \[ R, G, B \]
Example RGB Emission Spectra ("Color Primaries") for Phone Display

RGB pixel spectra (iPhone 5)


https://www.macrumors.com/roundup/iphone-5s/
Color Reproduction Problem

Target real spectrum \( s(\lambda) \)

Display outputs spectrum
\[
R \ s_R(\lambda) + G \ s_G(\lambda) + B \ s_B(\lambda)
\]

Goal: at each pixel, choose R, G, B values for display so that the output color matches the appearance of the target color in the real world.
Pseudo-Geometric Interpretation

We are projecting a high dimensional vector (wavelength spectrum function) onto a low-dimensional subspace (SML visual response)

- Differences that are perpendicular to the basis vectors of the low-dimensional space are not detectable
Pseudo-Geometric Interpretation of Color Reproduction

- The display can only produce a low-dimensional subspace of all possible spectra (linear combinations of display primaries).
- In color reproduction, for a given spectrum \( s \) (high dimensional), we want to choose a spectrum \( s' \) in the display’s low-dimensional subspace, such that \( s' \) and \( s \) project to the same response in the low-dimensional subspace of the eye’s SML response.
Color Reproduction as Linear Algebra

Input spectrum \( s \)  

\[
\begin{bmatrix}
R \\
G \\
B
\end{bmatrix} = \begin{bmatrix}
\_ & ? & \_ \\
\_ & ? & \_ \\
\_ & ? & \_
\end{bmatrix} \begin{bmatrix}
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Color Reproduction as Linear Algebra

Spectrum produced by display given values R, G, B:

\[ s_{\text{disp}}(\lambda) = R \ s_R(\lambda) + G \ s_G(\lambda) + B \ s_B(\lambda) \]

\[ \implies [s_{\text{disp}}] = [s_R \ s_G \ s_B] \begin{bmatrix} R \\ G \\ B \end{bmatrix} \]
Color Reproduction as Linear Algebra

What color do we perceive when we look at the display?

\[
\begin{bmatrix}
S \\
M \\
L
\end{bmatrix}_{\text{disp}} =
\begin{bmatrix}
\quad r_S \\
\quad r_M \\
\quad r_L
\end{bmatrix}
\begin{bmatrix}
S_{\text{disp}} \\
R \\
G \\
B
\end{bmatrix}
\]

We want this displayed spectrum to be a metamer for the real-world target spectrum.
Color Reproduction as Linear Algebra

Color perceived for display spectra with values R,G,B

\[
\begin{bmatrix}
S \\
M \\
L
\end{bmatrix}_{\text{disp}} =
\begin{bmatrix}
r_S & \_ & \_ \\
r_M & \_ & \_ \\
r_L & \_ & \_
\end{bmatrix}
\begin{bmatrix}
s_R \\
s_G \\
s_B
\end{bmatrix}
\begin{bmatrix}
R \\
G \\
B
\end{bmatrix}
\]

Color perceived for real scene spectra, s

\[
\begin{bmatrix}
S \\
M \\
L
\end{bmatrix}_{\text{real}} =
\begin{bmatrix}
r_S & \_ & \_ \\
r_M & \_ & \_ \\
r_L & \_ & \_
\end{bmatrix}
\begin{bmatrix}
s_R \\
s_G \\
s_B
\end{bmatrix}
\]

How do we reproduce the color of s? Set these lines equal and solve for R,G,B as a function of s!
Color Reproduction as Linear Algebra

Solution:

\[
\begin{bmatrix}
S & r_S \\
M & r_M \\
L & r_L \\
\end{bmatrix}
\begin{bmatrix}
s_R \\
s_G \\
s_B \\
\end{bmatrix}
\begin{bmatrix}
R \\
G \\
B \\
\end{bmatrix}
= \begin{bmatrix}
r_S \\
r_M \\
r_L \\
\end{bmatrix}
\begin{bmatrix}
s \\
\end{bmatrix}
\Rightarrow
\begin{bmatrix}
R \\
G \\
B \\
\end{bmatrix}
= \left(\begin{bmatrix}
S & r_S \\
M & r_M \\
L & r_L \\
\end{bmatrix}
\begin{bmatrix}
s_R \\
s_G \\
s_B \\
\end{bmatrix}\right)^{-1}
\begin{bmatrix}
r_S \\
r_M \\
r_L \\
\end{bmatrix}
\begin{bmatrix}
s \\
\end{bmatrix}
\]
Color Reproduction as Linear Algebra

Solution (form #1):

\[
\begin{bmatrix}
R \\
G \\
B
\end{bmatrix} = \left( \begin{bmatrix}
\phantom{r_s} & r_S & \phantom{s_R} \\
\phantom{r_M} & r_M & \phantom{s_G} \\
\phantom{r_L} & r_L & \phantom{s_B}
\end{bmatrix}
\begin{bmatrix}
s_R & s_G & s_B
\end{bmatrix} \right)^{-1}
\begin{bmatrix}
r_S & \phantom{s_R} & \phantom{s_B} \\
r_M & \phantom{s_G} & \phantom{s_B} \\
r_L & \phantom{s_G} & \phantom{s_B}
\end{bmatrix}
\begin{bmatrix}
s
\end{bmatrix}
\]

Solution (form #2):

\[
RGB = (M_{SML} M_{RGB})^{-1} M_{SML} s
\]
Color Reproduction as Linear Algebra

Solution (form #3):

\[
\begin{bmatrix}
R \\
G \\
B
\end{bmatrix} = \begin{bmatrix}
rs \cdot sr & rs \cdot sg & rs \cdot sb \\
rm \cdot sr & rm \cdot sg & rm \cdot sb \\
rl \cdot sr & rl \cdot sg & rl \cdot sb
\end{bmatrix}^{-1}
\begin{bmatrix}
\_ & rs & \_ \\
\_ & rm & \_ \\
\_ & rl & \_ \\
\end{bmatrix}
\begin{bmatrix}
s
\end{bmatrix}
\] 

3xN
Color Matching Functions

Recall the color matching functions from the matching experiment

\[
\begin{bmatrix}
R \\
G \\
B
\end{bmatrix}
= 
\left( \begin{bmatrix}
\_ & r_S \\
\_ & r_M \\
\_ & r_L
\end{bmatrix}
\begin{bmatrix}
s_R \\
s_G \\
s_B
\end{bmatrix} \right)^{-1}
\begin{bmatrix}
r_S \\
r_M \\
r_L
\end{bmatrix}
\begin{bmatrix}
\_ \\
\_ \\
\_ \\
\end{bmatrix}
\begin{bmatrix}
s \\
\end{bmatrix}
\]

\[
= \begin{bmatrix}
r_S \cdot s_R & r_S \cdot s_G & r_S \cdot s_B \\
r_M \cdot s_R & r_M \cdot s_G & r_M \cdot s_B \\
r_L \cdot s_R & r_L \cdot s_G & r_L \cdot s_B
\end{bmatrix}^{-1}
\begin{bmatrix}
r_S \\
r_M \\
r_L
\end{bmatrix}
\begin{bmatrix}
s \\
\end{bmatrix}
\]

This 3xN matrix contains, as row vectors, “color matching functions” associated with the primary lights \( s_R, s_G, s_B \).
Color Reproduction Issue: No Negative Light

R,G,B values must be positive

- Display primaries can’t emit negative light
- But solution formulas can certainly produce negative R,G,B values

What do negative R,G,B values mean?

- Display can’t physically reproduce the desired color
- Desired color is outside the display’s color gamut
Gamut
Example: Color Gamut for sRGB and Apple P3
Comparing sRGB and Wide Gamut P3 Color Spaces

Interactive Color Space Comparison:
https://webkit.org/blog-files/color-gamut/comparison.html

• Needs a wide-gamut physical display
• I can see differences clearly on my MacBook Pro, less so on LG display
LMS Response Values for Each Wavelength

Probability that a photon will cause a photopigment isomerization

Wavelength (nm)

Quantal efficiency

L cones
M cones
S cones

Brainard, Color and the Cone Mosaic, 2015.

CS184/284A
Ren Ng
LMS Response Values for Each Wavelength

Brainard, Color and the Cone Mosaic, 2015.
LMS Response Values for Each Wavelength

Probability that a photon will cause a photopigment isomerization

Brainard, Color and the Cone Mosaic, 2015.
LMS Responses Plotted as 3D Color Space

Visualization of "spectral locus" of human cone cells’ response to monochromatic light (light with energy in a single wavelength) as points in 3D space.

This is a plot of the S, M, L response functions as a point in 3D space.

Space of all possible responses are positive linear combinations of points on this curve.
LMS Responses Plotted as 3D Color Space

"Spectral locus"

https://graphics.stanford.edu/courses/cs178-10/applets/locus.html
Chromaticity Diagram (Maxwellian)

Human Gamut

RGB Display Gamut

Perspective projection of spectral locus looking diagonally down at origin from (1,1,1)
Chromaticity Diagram (CIE 1931 xy)

\[
\begin{bmatrix}
X \\
Y \\
Z
\end{bmatrix} = \begin{bmatrix}
1.9121 & -1.1121 & 0.2019 \\
0.3709 & 0.6291 & 0 \\
0 & 0 & 1
\end{bmatrix} \begin{bmatrix}
L \\
M \\
S
\end{bmatrix}
\]

\[
x = \frac{X}{|X| + |Y| + |Z|}
\]

\[
y = \frac{Y}{|X| + |Y| + |Z|}
\]
Color Gamut

sRGB is a common color space used throughout the internet.

CIE RGB are the monochromatic primaries used for color matching tests described earlier.

sRGB uses ITU-R BT.709 primaries
Red  Green  Blue  White
x  0.64  0.30  0.15  0.3127
y  0.33  0.60  0.06  0.3290

AdobeRGB(98) uses Red and Blue like sRGB and Green like NTSC

CIE-RGB are the primaries for color matching tests: 700/546.1/435.8nm
Color Representation
Color Spaces

Need three numbers to specify a color

- But what three numbers?
- A color space is an answer to this question
- Same color has different coordinates in different color spaces. E.g. RGB, XYZ, Lab, HSV, ...

Common example: display color space

- Define a color by what R, G, B scalar values will produce it on your display
  - As before, \( s(\lambda) = r(\lambda)R + g(\lambda)G + b(\lambda)B \) for some spectra \( r, g, b \)
- Device dependent (depends on primary spectra, gamma, …)
  - Therefore if I choose R,G,B by looking at my display and send it to you, you may not see the same color
- Also leaves out some colors (limited gamut), e.g. vivid yellow
  - Because in file formats R, G, B usually constrained to be non-negative
Standard Color Spaces

Standardized RGB (sRGB)

• makes a particular monitor RGB standard
• other color devices simulate that monitor by calibration
• sRGB is usable as an interchange space; still widely used today, though other standards common now
• gamut is still limited
The Historical “Standard” Color Space: CIE XYZ

Imaginary set of standard color primaries $X$, $Y$, $Z$

Designed such that

- $X$, $Y$, $Z$ span all observable colors
- Matching functions are strictly positive
- $Y$ is luminance (brightness absent color)

Imaginary because can only be realized with primaries that are negative at some wavelengths
Luminance (Lightness)

Integral of radiance scaled by the visual luminous efficiency

\[
Y = \int \Phi(\lambda) \, V(\lambda) \, d\lambda
\]

Luminous efficiency \( V(\lambda) \) is a measure of how bright a light at a given wavelength is perceived by a human.

https://upload.wikimedia.org/wikipedia/commons/a/a0/Luminosity.png
Separating Luminance, Chromaticity

Luminance: $Y$

Chromaticity: $x, y, z$, defined as

$$x = \frac{X}{X + Y + Z}$$
$$y = \frac{Y}{X + Y + Z}$$
$$z = \frac{Z}{X + Y + Z}$$

- since $x + y + z = 1$, we only need to record two of the three
- usually choose $x$ and $y$, leading to $(x, y, Y)$ coords
CIE 1931 xy Chromaticity Diagram

Pure (saturated) spectral colors around the edge of the plot

Less pure (desaturated) colors in the interior of the plot

White at the centroid of the plot (1/3, 1/3)
Perceptually Organized Color Spaces
HSV Color Space (Hue-Saturation-Value)

Axes correspond to artistic characteristics of color
HSV Color Space (Hue-Saturation-Value)

Perceptual dimensions of color

Hue
- the “kind” of color, regardless of attributes
- colorimetric correlate: dominant wavelength
- artist’s correlate: the chosen pigment color

Saturation
- the “colorfulness”
- colorimetric correlate: purity
- artist’s correlate: fraction of paint from the colored tube

Lightness (or value)
- the overall amount of light
- colorimetric correlate: luminance
- artist’s correlate: tints are lighter, shades are darker
CIELAB (AKA L\*a\*b\*)

A perceptually-organized color space that acts as a simple and useful color appearance model

Features

• Chromatic adaptation (white balance)
• Predicts color appearance
  • Opponent color encoding
  • Formulas for hue, chroma, lightness
• Perceptual uniformity (non-linear warping)
**CIELAB Definition**

### CIE XYZ --> CIE LAB

\[
L^* = 116 \cdot f\left(\frac{Y}{Y_n}\right) - 16 \\
\alpha^* = 500 \left( f\left(\frac{X}{X_n}\right) - f\left(\frac{Y}{Y_n}\right) \right) \\
b^* = 200 \left( f\left(\frac{Y}{Y_n}\right) - f\left(\frac{Z}{Z_n}\right) \right)
\]

where

\[
f(t) = \begin{cases} 
\frac{3\sqrt{t}}{t} + \frac{4}{29} & \text{if } t > \delta^3 \\
\frac{6}{29} & \text{otherwise}
\end{cases}
\]

\[
\delta = \frac{6}{29}
\]

### CIE LAB --> CIE XYZ

\[
X = X_n f^{-1}\left(\frac{L^* + 16}{116} + \frac{\alpha^*}{500}\right) \\
Y = Y_n f^{-1}\left(\frac{L^* + 16}{116}\right) \\
Z = Z_n f^{-1}\left(\frac{L^* + 16}{116} - \frac{b^*}{200}\right)
\]

where

\[
f^{-1}(t) = \begin{cases} 
\frac{t^3}{3\delta^2} & \text{if } t > \delta \\
3\delta^2 \left( t - \frac{4}{29} \right) & \text{otherwise}
\end{cases}
\]

and where \( \delta = 6/29 \).

\( X_n, Y_n \) and \( Z_n \) are the CIE XYZ coordinates of the reference white point.
CIELAB Has Chromatic Adaptation (Reference White)

CIELAB Color Spaces

CIELAB has Chromatic Adaptation (Reference White)

CIELAB

CIEXYZ --> CIELAB

\[ L^* = 116 \left( \frac{Y}{Y_n} \right)^{1/3} - 16 \]
\[ a^* = 500 \left( f \left( \frac{X}{X_n} \right) - f \left( \frac{Y}{Y_n} \right) \right) \]
\[ b^* = 200 \left( f \left( \frac{Y}{Y_n} \right) - f \left( \frac{Z}{Z_n} \right) \right) \]

where

\[ f(t) = \begin{cases} 
\frac{3\sqrt{t}}{t^{3/2}} + \frac{4}{29} & \text{if } t > \delta^3 \\
0 & \text{otherwise}
\end{cases} \]
\[ \delta = \frac{6}{29} \]

\( X_n, Y_n \) and \( Z_n \) are the CIEXYZ coordinates of the reference white point

\( Y_n = \text{Reference White} (100, 0, 0) \)

Yn = Reference White (100, 0, 0)

Lightness

Munsell Atlas

Courtesy Gretag - Macbeth

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Tristimulus space: range of whites

Perceptual space: unique white

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Luminance & \( L^* \)

\( L^* \) is a function of normalized luminance

Range 0 to 100

\[ L^* = 116 \left( \frac{Y}{Y_n} \right)^{1/3} - 16 \]

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The \( L^* \) Function

Combination power + linear functions

- Not 1/3 power
- Best fit is 1/2.43

\[ L^* \sim 100 \left( \frac{Y}{Y_n} \right)^{1/2.43} \]

Similar issue in other non-linear lightness specifications
CIELAB As a Color Appearance Model

Hue, chroma, lightness

Not L*, a*, b*

\[ C^* = \sqrt{a^*^2 + b^*^2} \]

\[ h^\circ = \arctan \left( \frac{b^*}{a^*} \right) \]
CIE XYZ is Not Perceptually Uniform

- In the xy chromaticity diagram at left, MacAdam ellipses show regions of perceptually equivalent color (ellipses enlarged 10x)
CIELAB Aims for Perceptual Uniformity

From Henrich et al. 2011

https://iovs.arvojournals.org/article.aspx?articleid=2187751
Perceptual Normalization Function Applies to L*, a*, b*

CIEXYZ --> CIELAB

\[
L^* = 116 \cdot f \left( \frac{Y}{Y_n} \right) - 16
\]
\[
a^* = 500 \left( f \left( \frac{X}{X_n} \right) - f \left( \frac{Y}{Y_n} \right) \right)
\]
\[
b^* = 200 \left( f \left( \frac{Y}{Y_n} \right) - f \left( \frac{Z}{Z_n} \right) \right)
\]

where

\[
f(t) = \begin{cases} 
\frac{3}{2} \sqrt[3]{t} & \text{if } t > \delta^3 \\
\frac{t}{3\delta^2} + \frac{4}{29} & \text{otherwise}
\end{cases}
\]

\[
\delta = \frac{6}{29}
\]

\(X_n, Y_n, Z_n\) are the CIEXYZ coordinates of the reference white point

CS184/284A  
Slide credit: Maureen Stone  
Ren Ng
CIELAB Gives a Recommended Color Difference Metric

Other color spaces we looked at (RGB, CIEXYZ, HSV) are not perceptually uniform and are not recommended for color difference calculations

- E.g. a pair of colors that look similar to a human observer may have R,G,B coordinates further apart than another pair of colors that look quite different

- Try converting colors to CIELAB coordinates for applications that need to quantify differences in color appearance

Distance between \((L^*, a^*, b^*)\) coordinates for two colors is a recommended color difference metric that is approximately perceptually uniform

\[
\Delta E_{ab}^* = \sqrt{\left(L_2^* - L_1^*\right)^2 + \left(a_2^* - a_1^*\right)^2 + \left(b_2^* - b_1^*\right)^2} \quad \text{("Delta E")}
\]

- Caveat: \(\Delta E_{ab}^*\) is not perfect (e.g. large differences, and differences between highly saturated colors are inaccurate). CIEDE2000 is a more complex/accurate metric based on color appearance models
Additive vs Subtractive Color
or
Beam Colors vs Object Colors
Additive Color

- red
- green
- blue
- yellow
- cyan
- magenta

Slide courtesy K. Breeden
Subtractive (Actually Multiplicative) Color

Shining white light on various colored pigments

Slide courtesy K. Breeden
Beam Colors and Additive Color
Object Colors - Multiplicative Color

The “Seven Sisters”, Sussex. Such white chalk cliffs are the primordial objects.

Sulphur crystals (the element, bright yellow) and cinnabar (a deep red mercury(II) sulphide) on Dolomite.

Reflection (range 0–100%) spectra of sulphur and cinnabar. The wavelength range involves the infrared, the visual range is about 0.4–0.75µm. Notice that these spectra are roughly of an all-or-none type. There are no signs of anything special at some “yellow or red wavelength” as many naive persons are wont to think.

CS184/284A  Koenderink, Colour in the Wild. 2018  Ren Ng
Things to Remember

Physics of Light

• Spectral power distribution (SPD)
• Superposition (linearity)

Tristimulus theory of color

• Spectral response of human cone cells (S, M, L)
• Metamers - different SPDs with the same perceived color
• Color reproduction mathematics
• Color matching experiment, per-wavelength matching functions

Color spaces

• CIE RGB, XYZ, xy chromaticity, LAB (and DeltaE), HSV
• Gamut
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Credit to


Calvin and Hobbes

Panel 1:

Honey, you're missing a beautiful sunset out here!

Panel 2:

I'll count to 10, and then... POK!

Panel 3:

Dad, how come old photographs are always black and white? Didn't they have color film back then?

Panel 4:

Sure they did. In fact, those old photographs are in color. It's just the world was black and white then.

Panel 5:

Yep. The world didn't turn color until sometime in the 1930s, and it was pretty grainy color for a while, too.

Panel 6:

Well, truth is stranger than fiction.

Panel 7:

That's really weird.

Panel 8:

But then why are old paintings in color? If the world was black and white, wouldn't artists have painted it that way?

Panel 9:

Not necessarily. A lot of great artists were insane.

Panel 10:

But how could they have painted in color anyway? Wouldn't their paints have been shades of gray back then?

Panel 11:

Of course. But they turned colors like everything else did in the '30s.

Panel 12:

So why didn't old black and white photos turn color too?

Panel 13:

Because they were color pictures of black and white. Remember?

Panel 14:

The world is a complicated place, Hobbes.

Panel 15:

Whenever it seems that way, I take a nap in a tree and wait for dinner.