Lecture 19/20:

Introduction to Color Science

Computer Graphics and Imaging
UC Berkeley CS184/284A

Primer on Final Project - Spring 2024

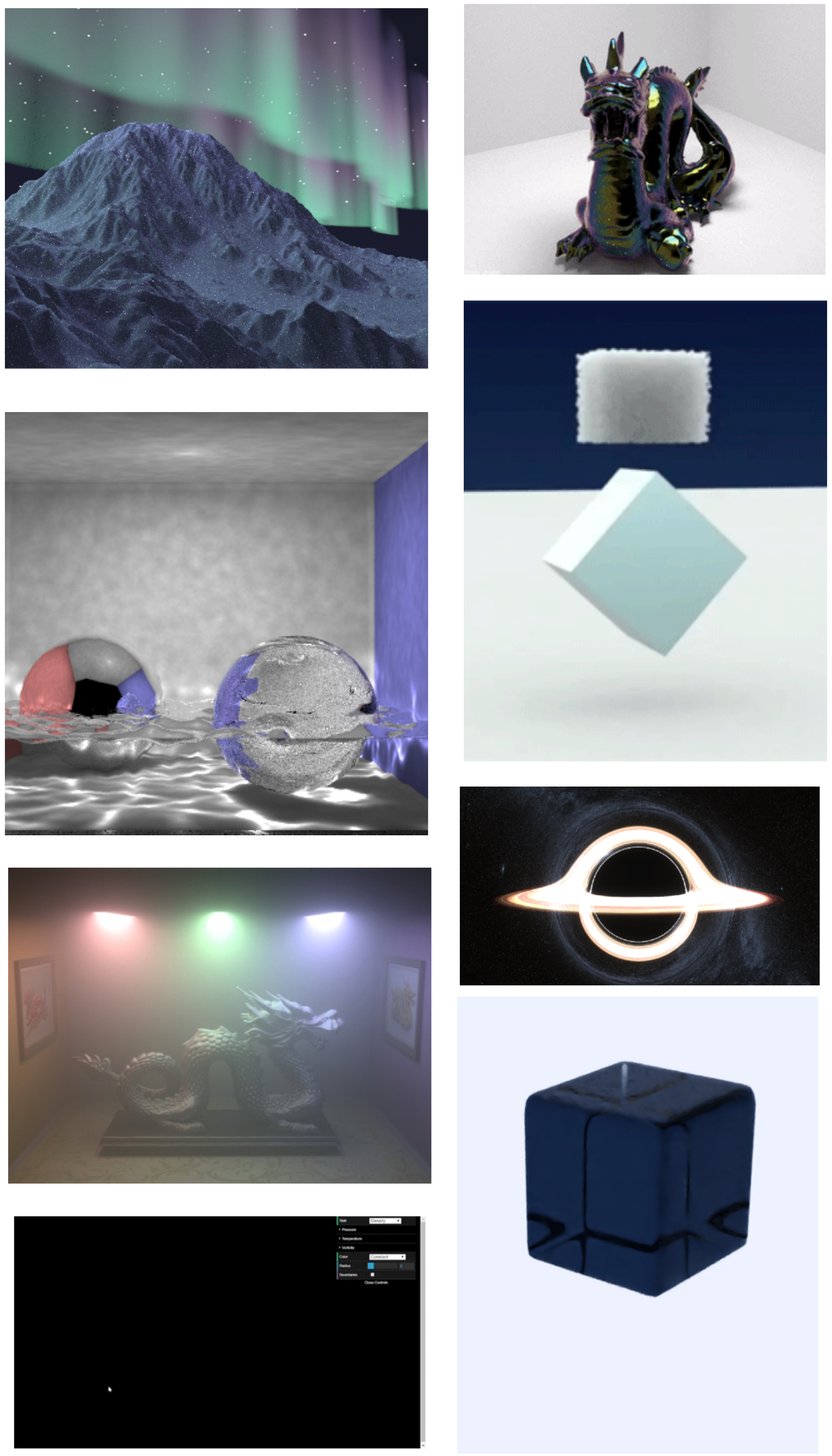
Today is just to get you thinking

Project - reminder

- Build something interesting to you
- Teams of four - choose your team
  - 25% for 184, 40% for 284A

Timeline: 4 weeks (tentative dates)

Feb 22	Teaming Mixer
Today	Teaming Mixer (after lecture)
April 2	Proposals due
April 16	Milestone Due
April 30	Reports/Video Due
May 2-3	Final Presentations, In-Person



Project Inspiration

Showcase winners in recent years:

- 2023: https://cs184.eecs.berkeley.edu/sp23/docs/final_showcase
- 2022: https://cs184.eecs.berkeley.edu/sp22/docs/final_showcase
- 2021: https://cs184.eecs.berkeley.edu/sp21/docs/final_showcase
- 2020: https://cs184.eecs.berkeley.edu/sp20/article/39/final-project-showcase

Ideas:

- Message and links on Ed. This year's project spec will be up soon.

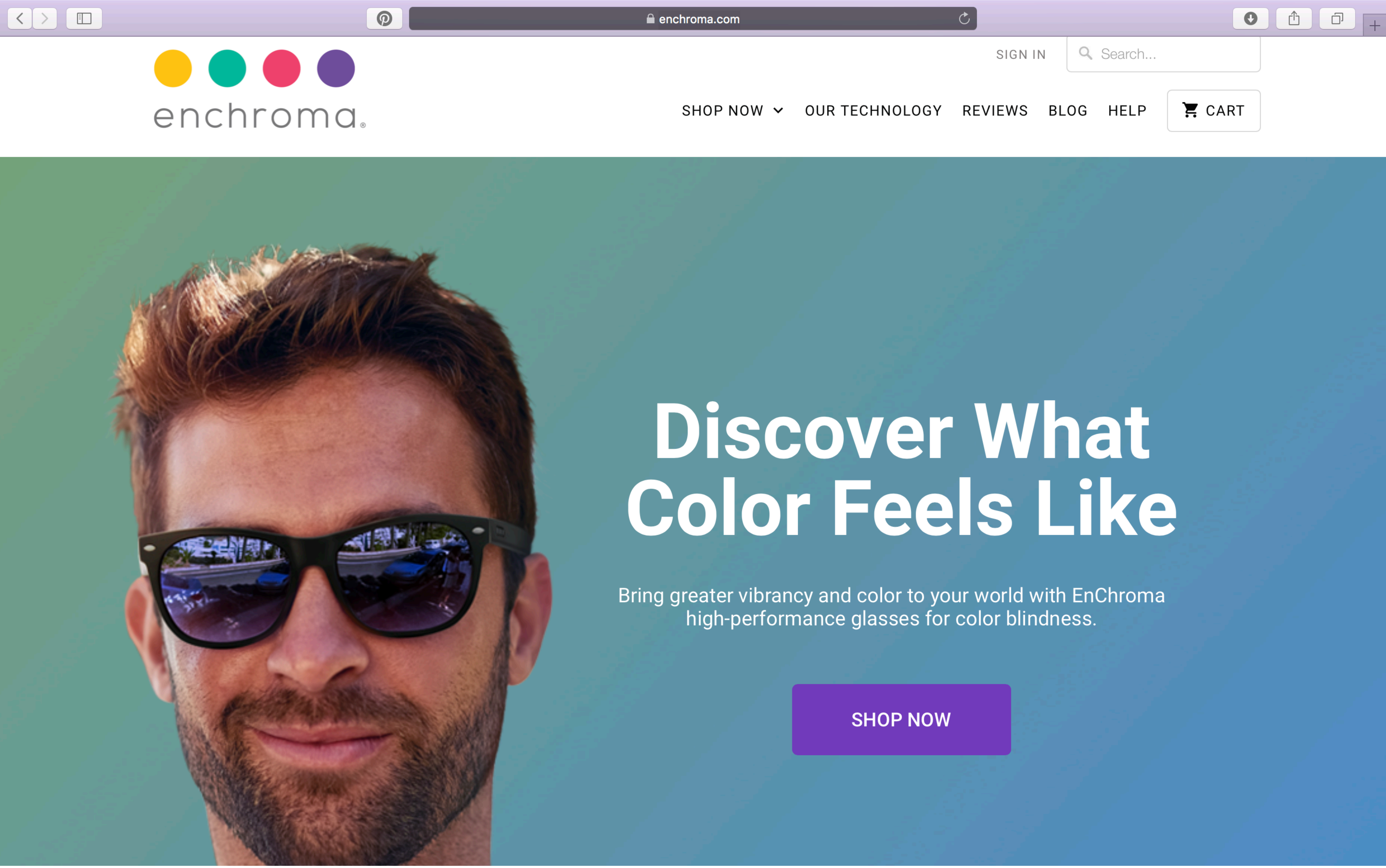
Color is Central to Our Human Experience


UMBRA GEBRANNT
BURNT UMBER
SIENNA
0 94376-89704
8 712079 143084
0 94376-89814
WINSOR & NEWTON
YELLOW
GELB

Wassily Kandinsky, Color Study. Squares with Concentric Circles, 1913
Munich, The Städtische Galerie im Lenbachhaus

Mark Rothko
No. 61. Rust and Blue
1953,
Museum of Contemporary Art, Los Angeles


FIRE ENGINE



Discover What Color Feels Like

Bring greater vibrancy and color to your world with EnChroma high-performance glasses for color blindness.

Color-Blind Reactions to Perceiving New Colors

Enchroma, https://www.youtube.com/watch?v=-rMjUsG--zo

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Enchroma, https://www.youtube.com/watch?v=-rMjUsG--zo

Simulation of Color Blind Perception
(Color Vision Deficiency)

Normal

Protan


Deutan

Tritan

Simulation of Color Blind Perception

Normal

Protan

Deutan

Tritan

Created with Coblis — Color Blindness Simulator

A Person With One Trichromatic Eye and One Deuteranopic Eye

Graham and Hsia, 1959.
“A unilaterally dichromatic subject”.

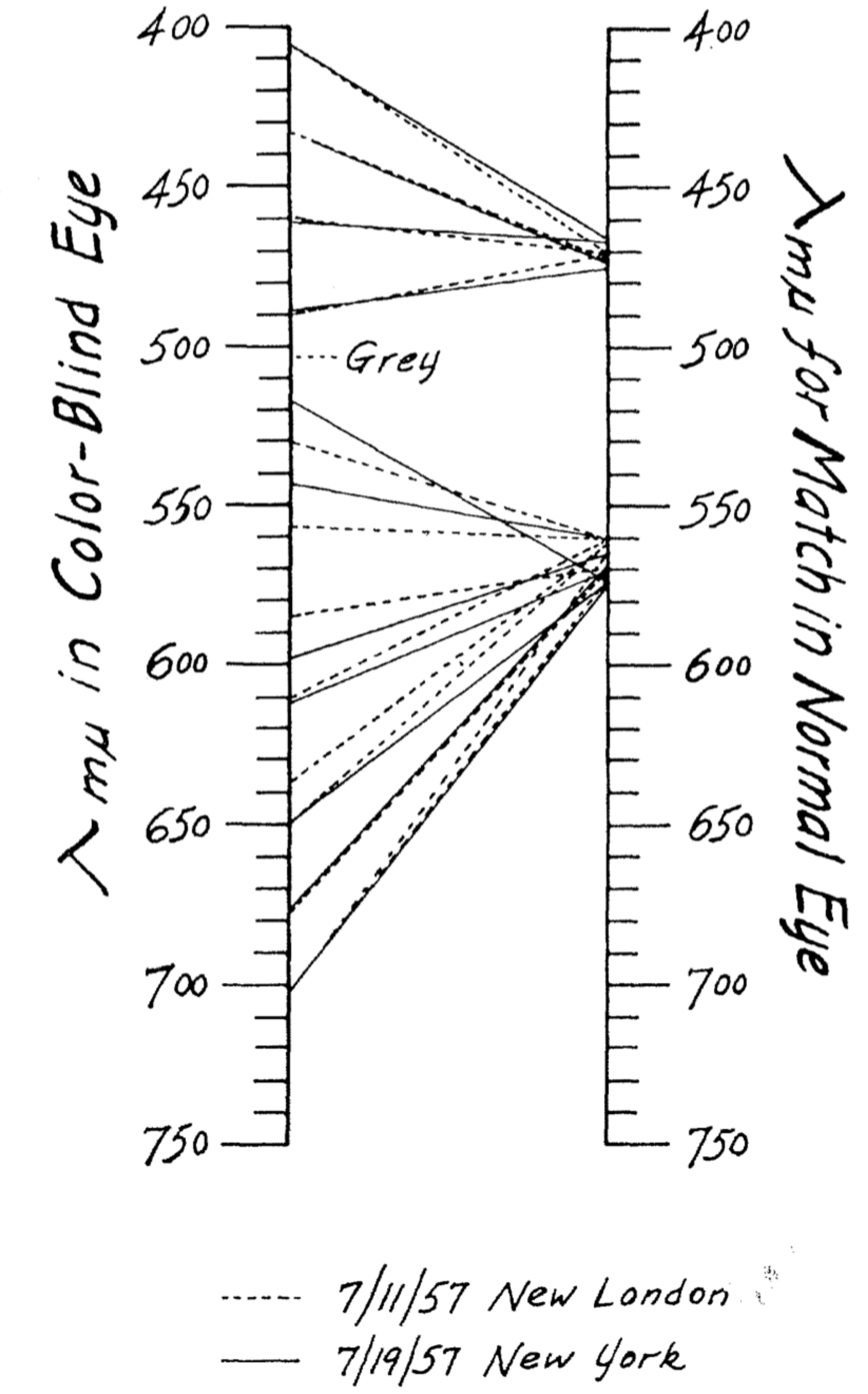
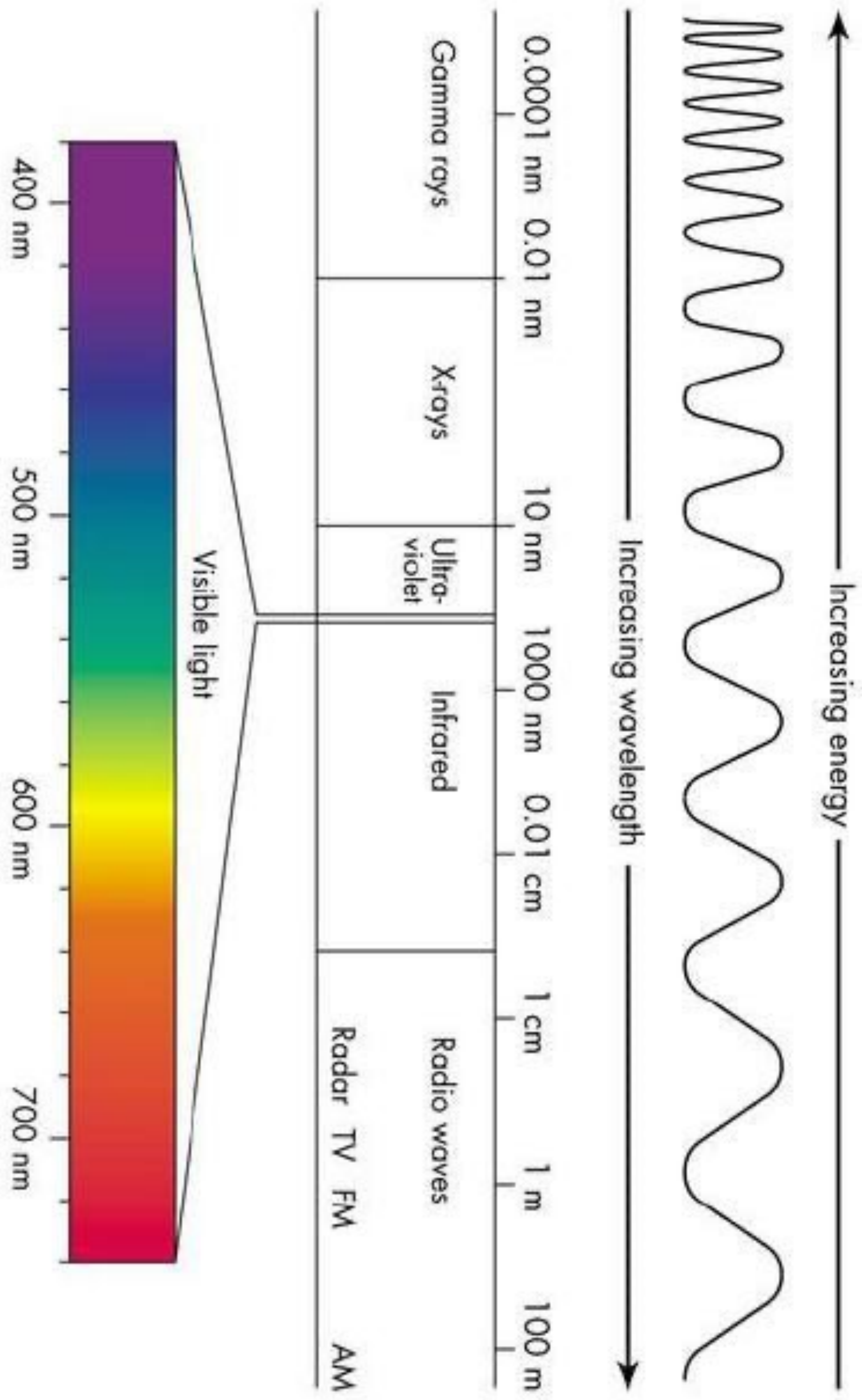


FIG. 2.—Results of the experiment on binocular matching



Source: Munsell

Color is Core to Our Human Visual Sense

Steve McCurry | Reza | Walter Iooss | Steve McCurry
Harold Edgerton | NASA | National Geographic

Normal

Color is Core to Our Human Visual Sense

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Protan

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Deutan

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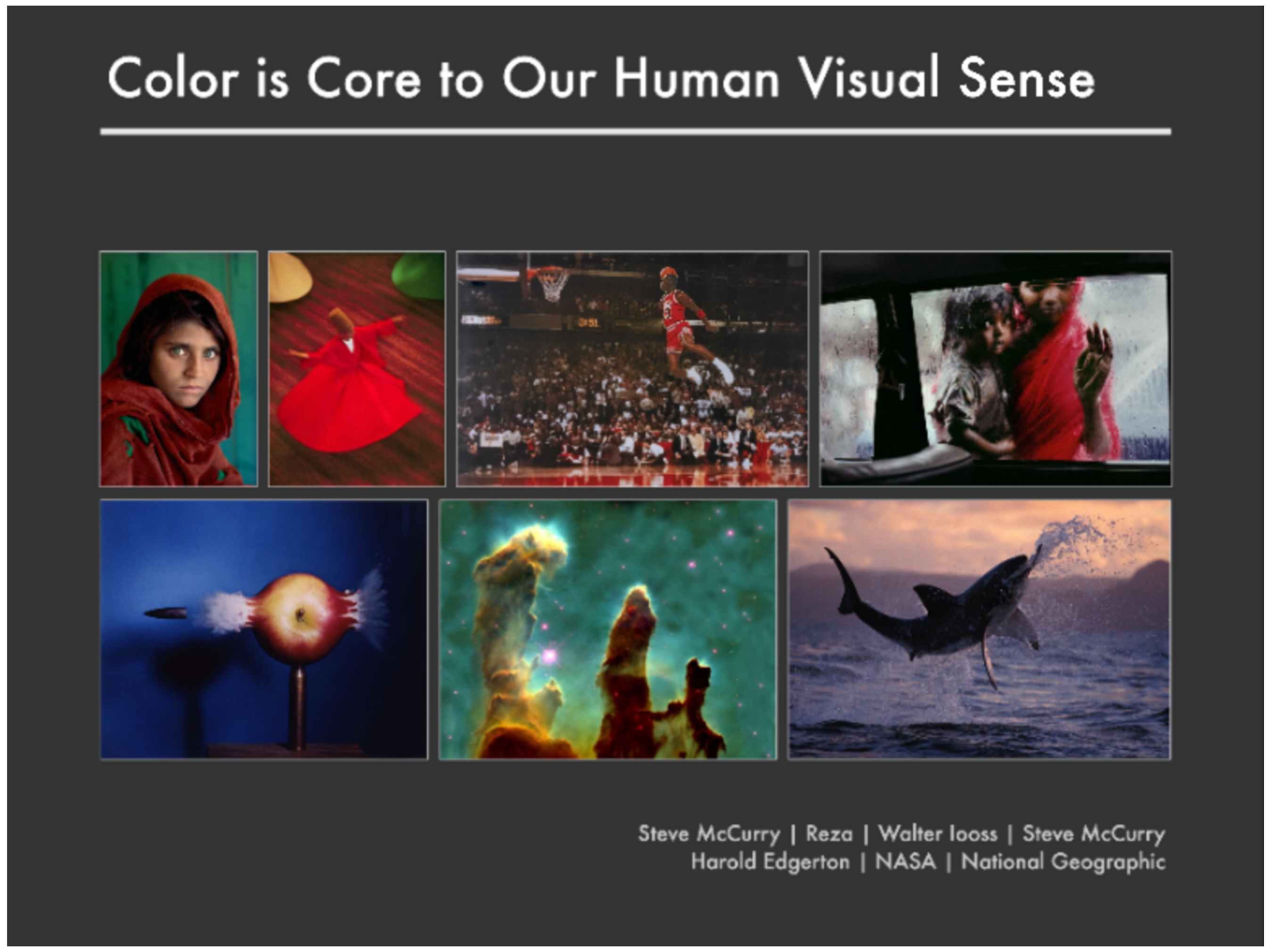
Tritan

Simulation of Color Blind Perception



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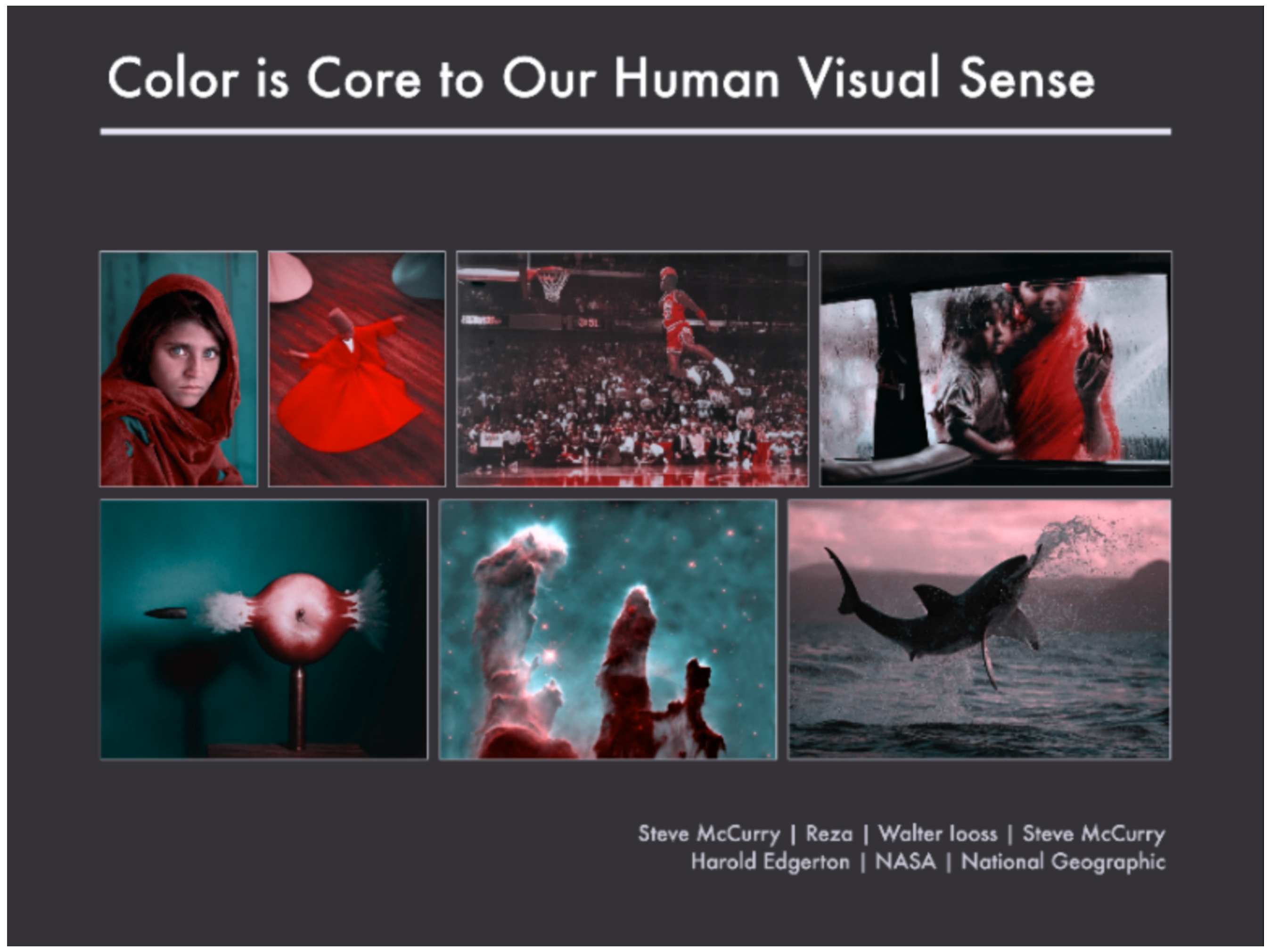


Color is Core to Our Human Visual Sense

Steve McCurry | Reza | Walter Iooss | Steve McCurry
Harold Edgerton | NASA | National Geographic

Tritan

Created with Coblis — Color Blindness Simulator

Chromatic Adaptation

Studying Chromatic Adaptation

Slide credit: Mark Fairchild

Slide credit: Mark Fairchild

A CYAN FILTER

Slide credit: Mark Fairchild

Slide credit: Mark Fairchild

Slide credit: Mark Fairchild

Automatic White Balance - Examples

No white balance (all processed as "daylight")

Automatic White Balance - Examples

Automatic white balance applied (Lightroom implementation)

Automatic White Balance - Examples

AWB + light manual editing

Automatic White Balance

$$\begin{bmatrix} R \\ G \\ B \end{bmatrix} = \begin{bmatrix} \frac{1}{R'_W} & 0 & 0 \\ 0 & \frac{1}{G'_W} & 0 \\ 0 & 0 & \frac{1}{B'_W} \end{bmatrix} \begin{bmatrix} R' \\ G' \\ B' \end{bmatrix}$$

R, G, B **- automatic white balanced output**

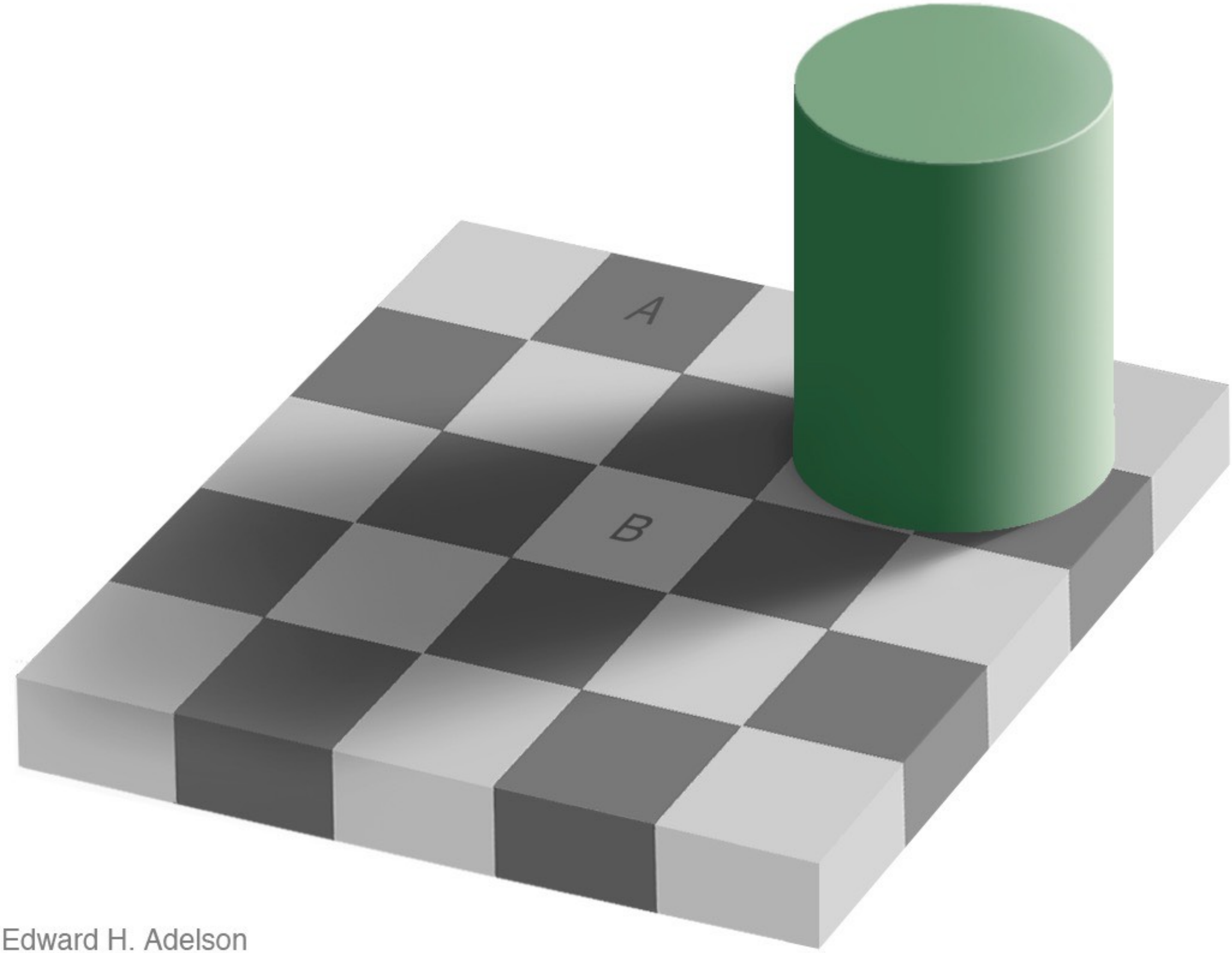
R'_W, G'_W, B'_W **- raw input of white object**

R', G', B' **- raw input**

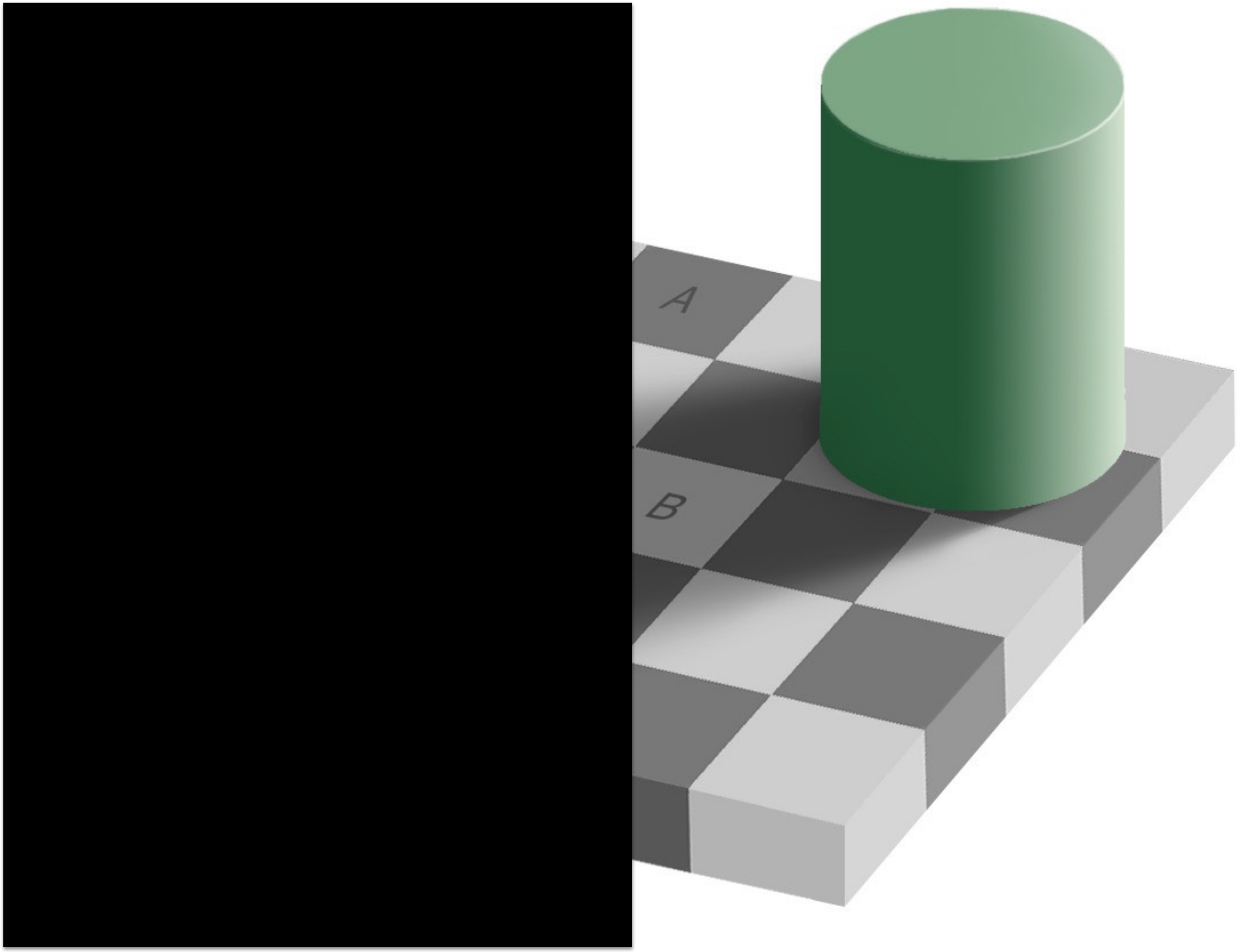
In technical portion of today's lecture, on color reproduction calculations, we will implicitly assume either that:

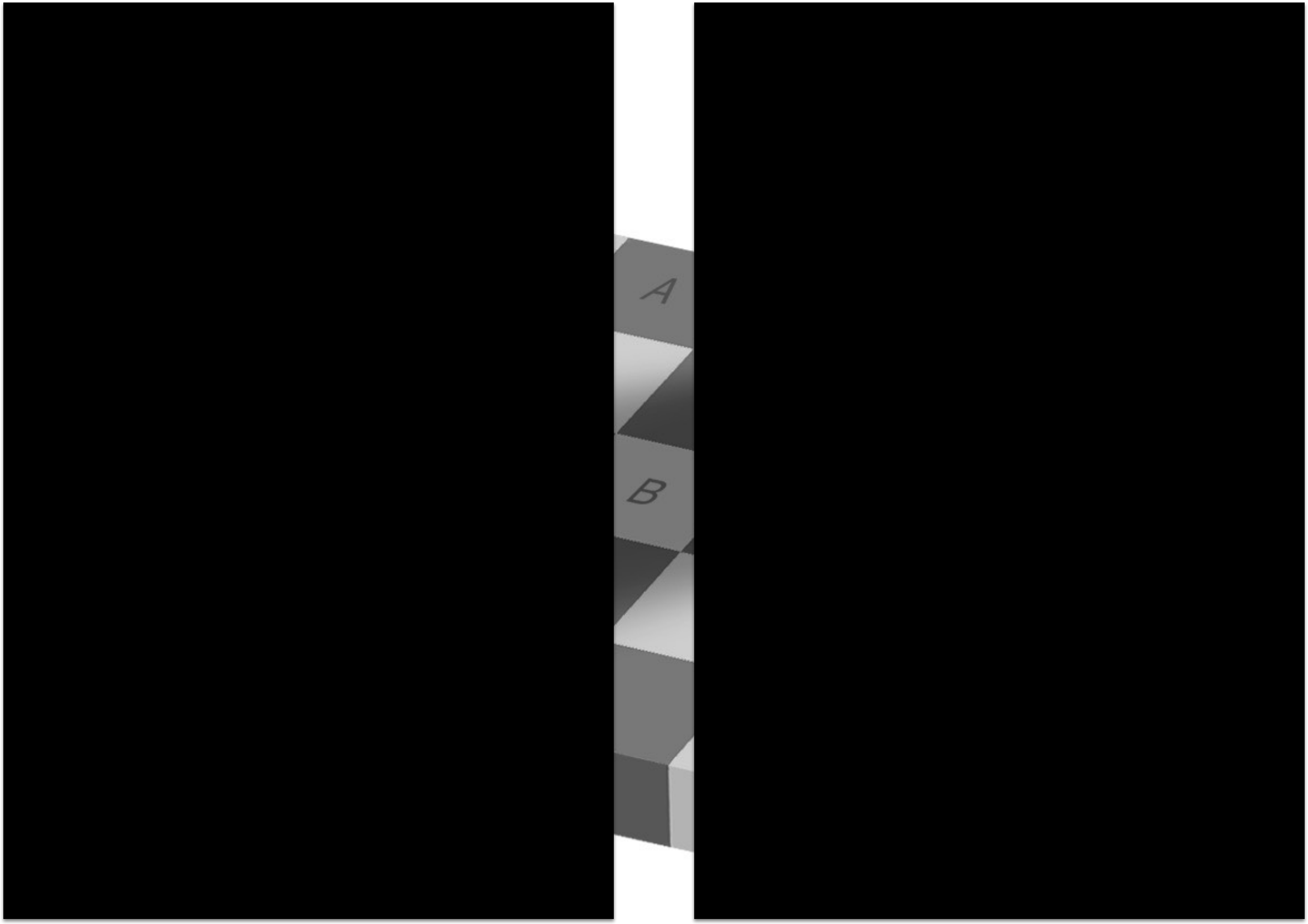
- **Auto white balance has been applied**
- **Or that the viewing conditions of the color reproduction match the original scene**

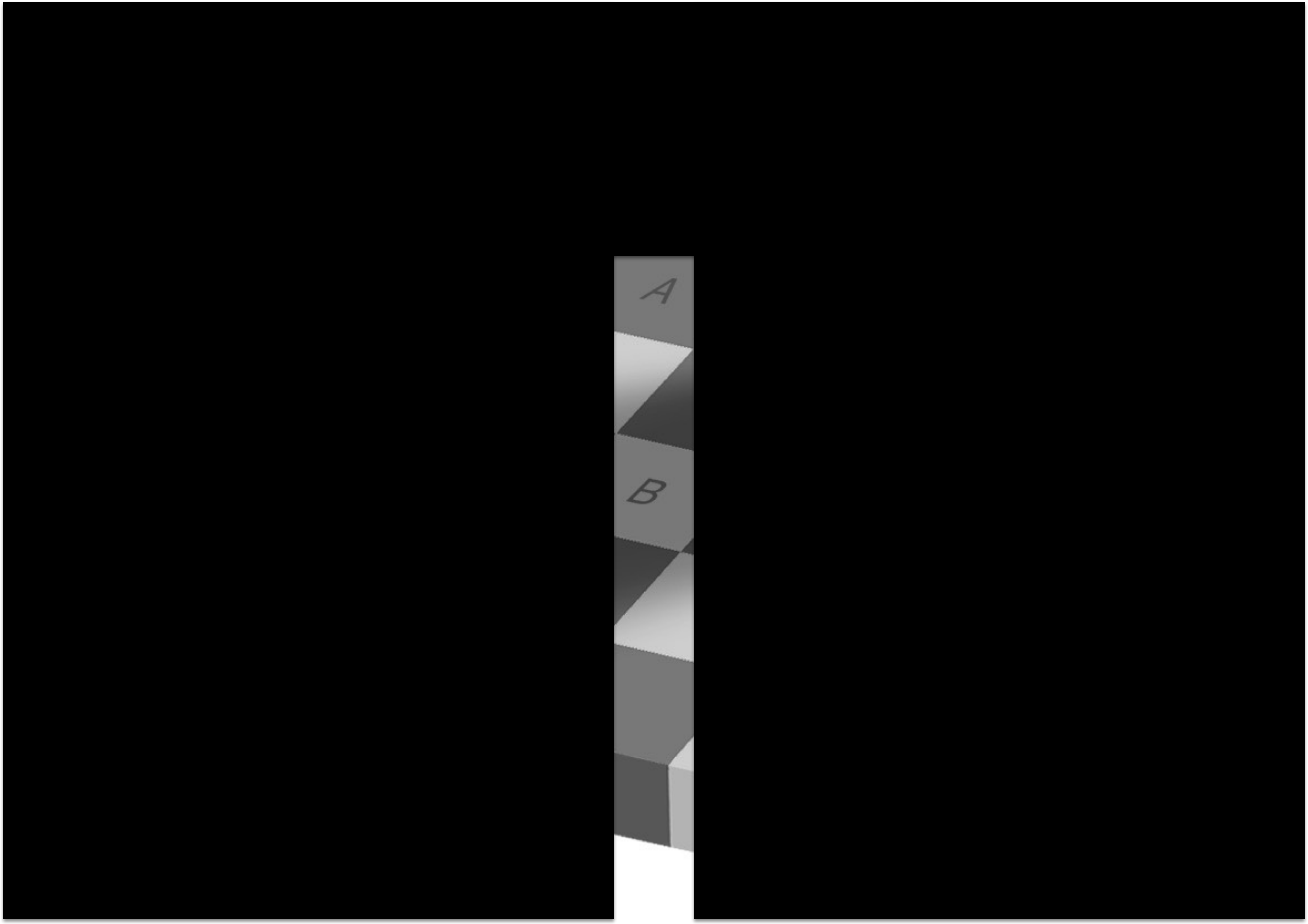
Color Perception is Highly Adaptive

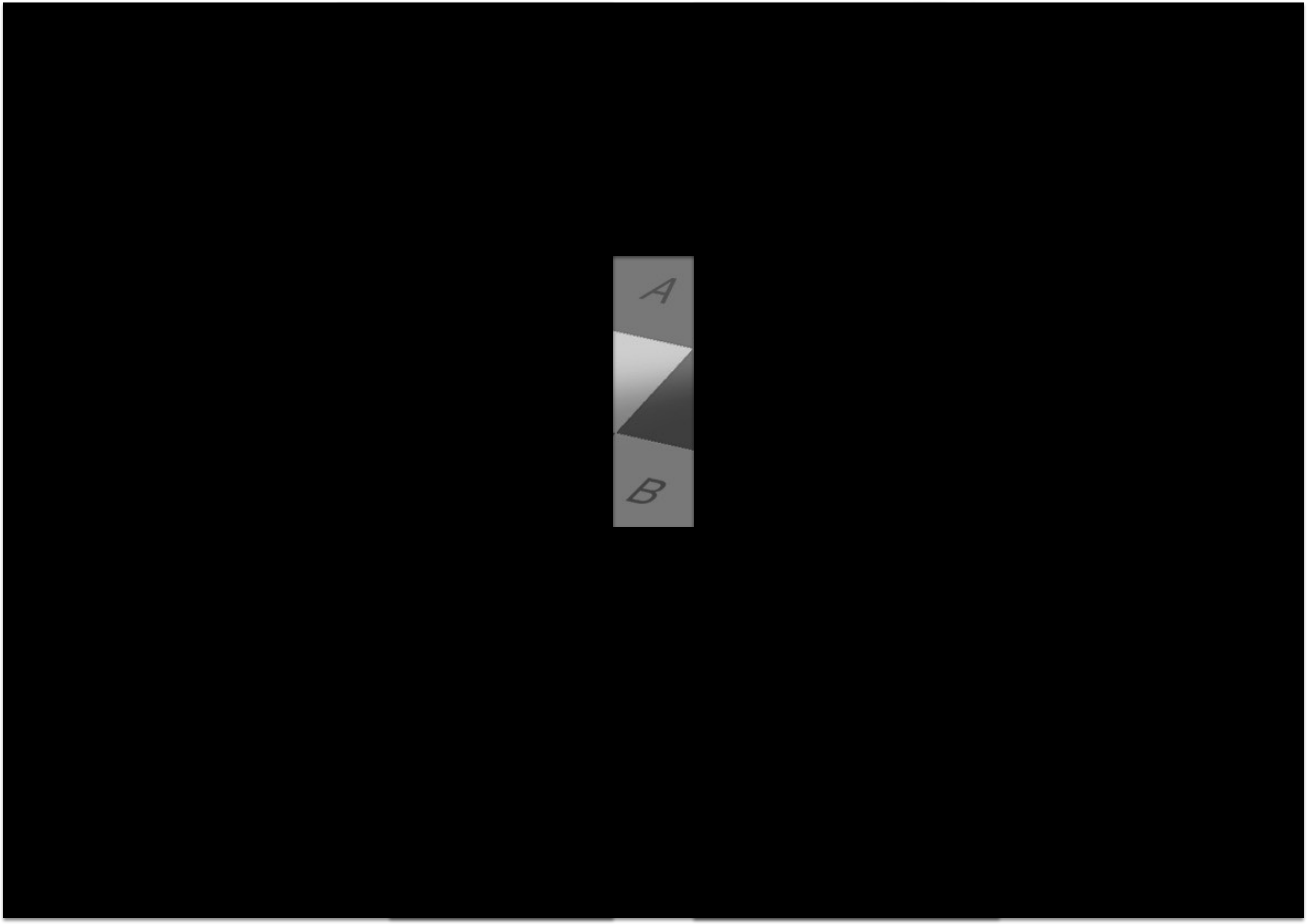


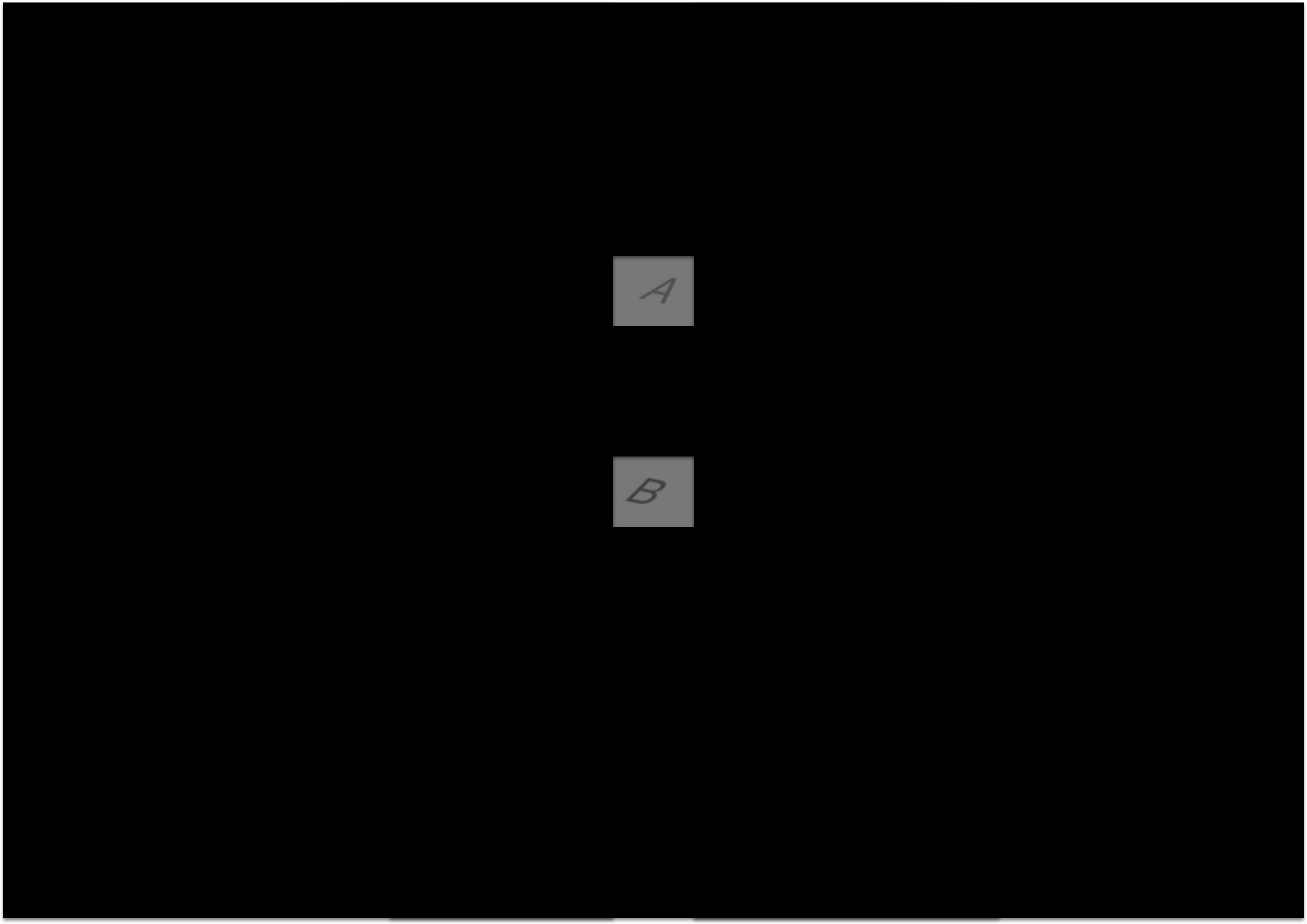
Edward H. Adelson

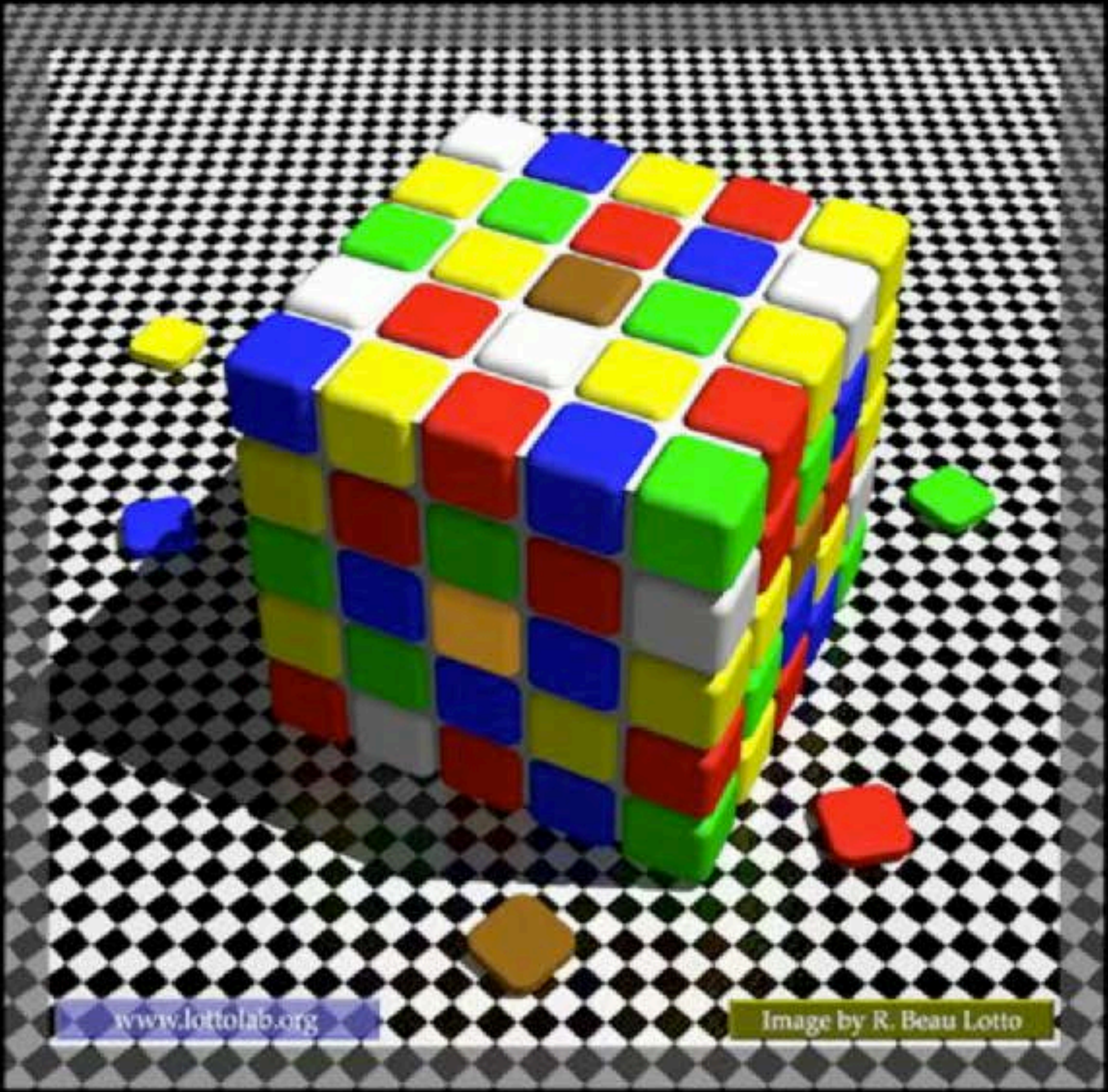

A
B


A
B


A
B

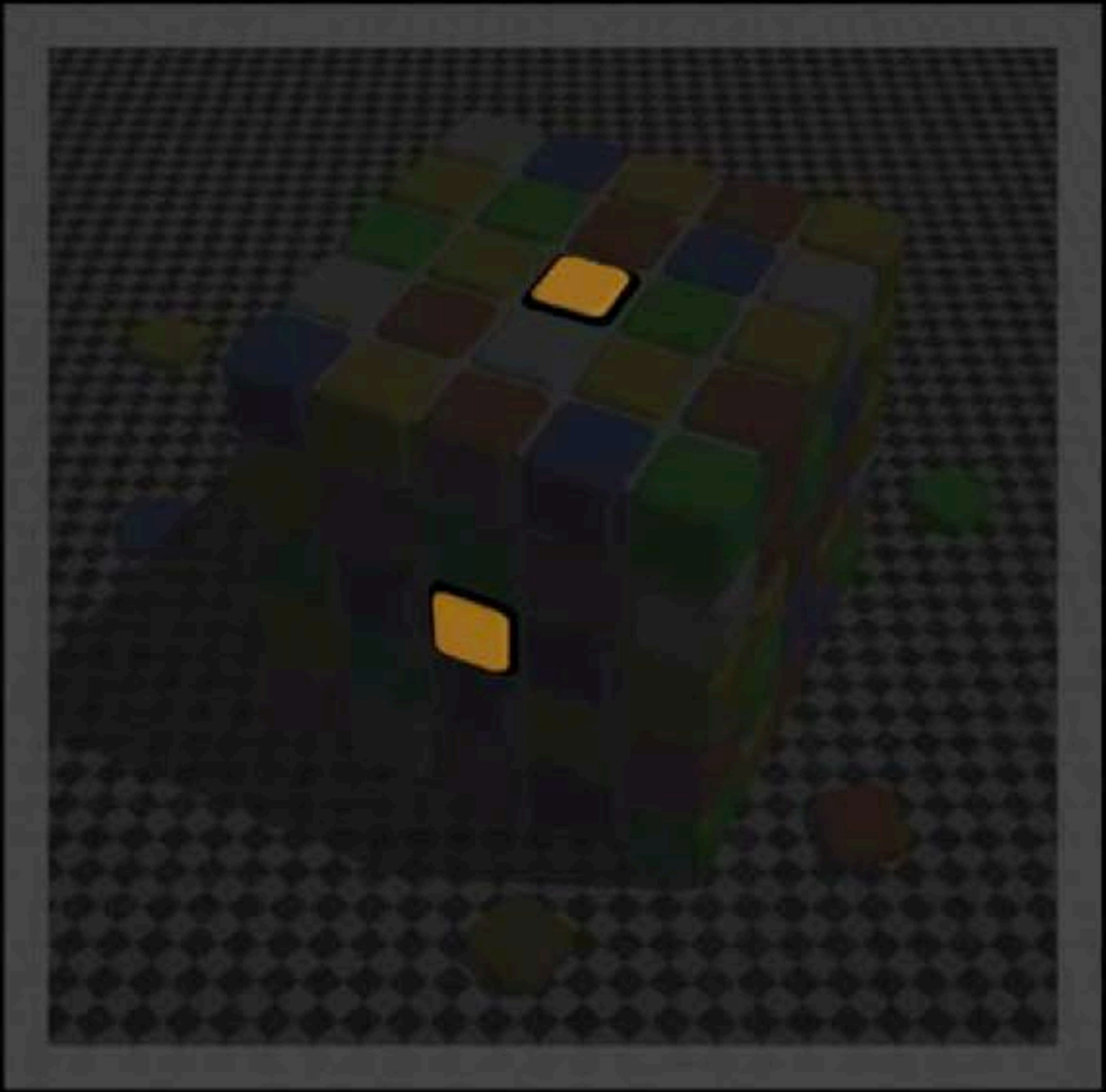

A
B


A
B

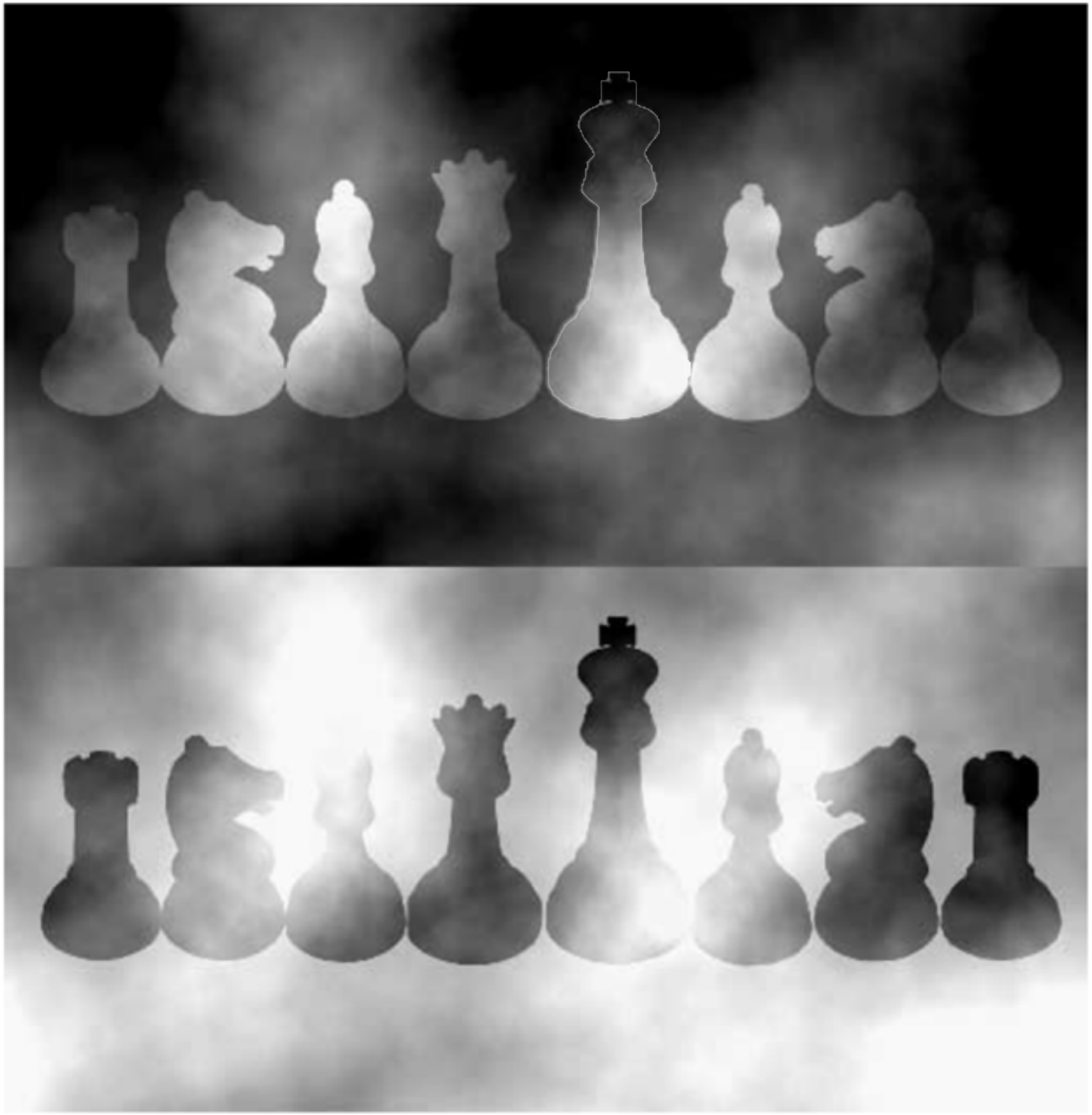


www.lottolab.org

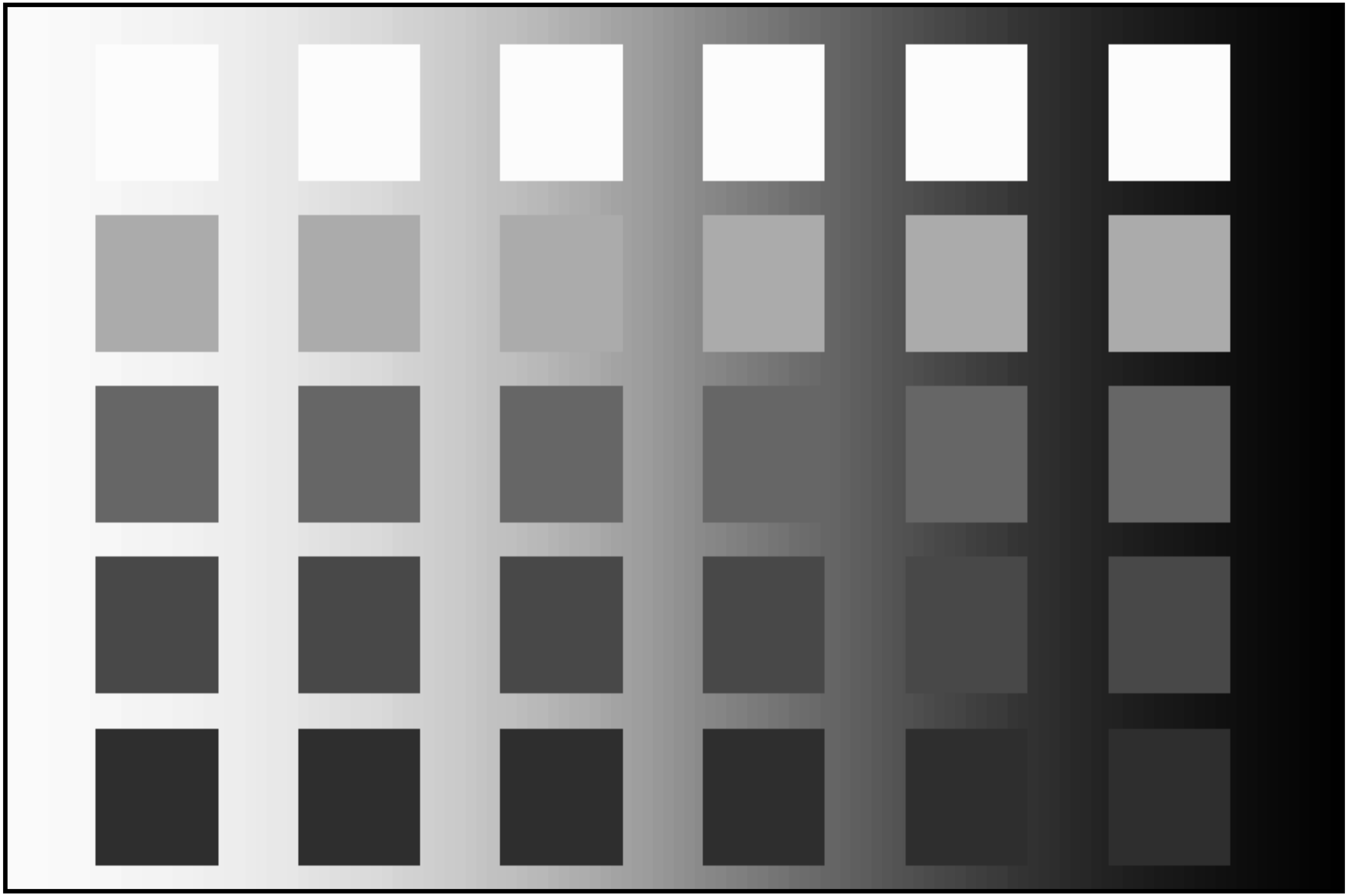
Image by R. Beau Lotto



Even simple judgments – such as lightness - depend on brain processing (Anderson and Winawer, Nature, 2005)

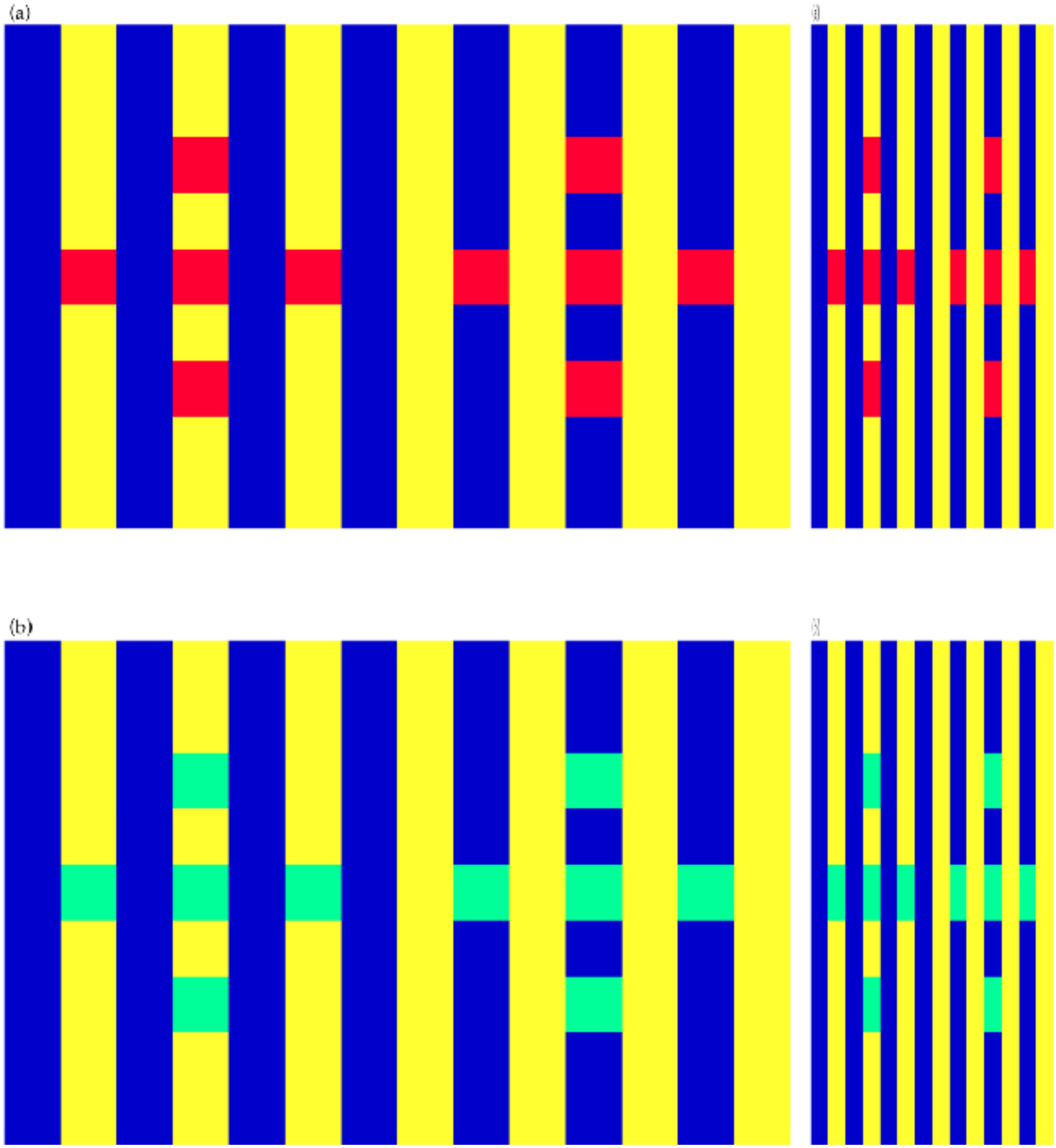


Simultaneous Contrast and Surround Effect



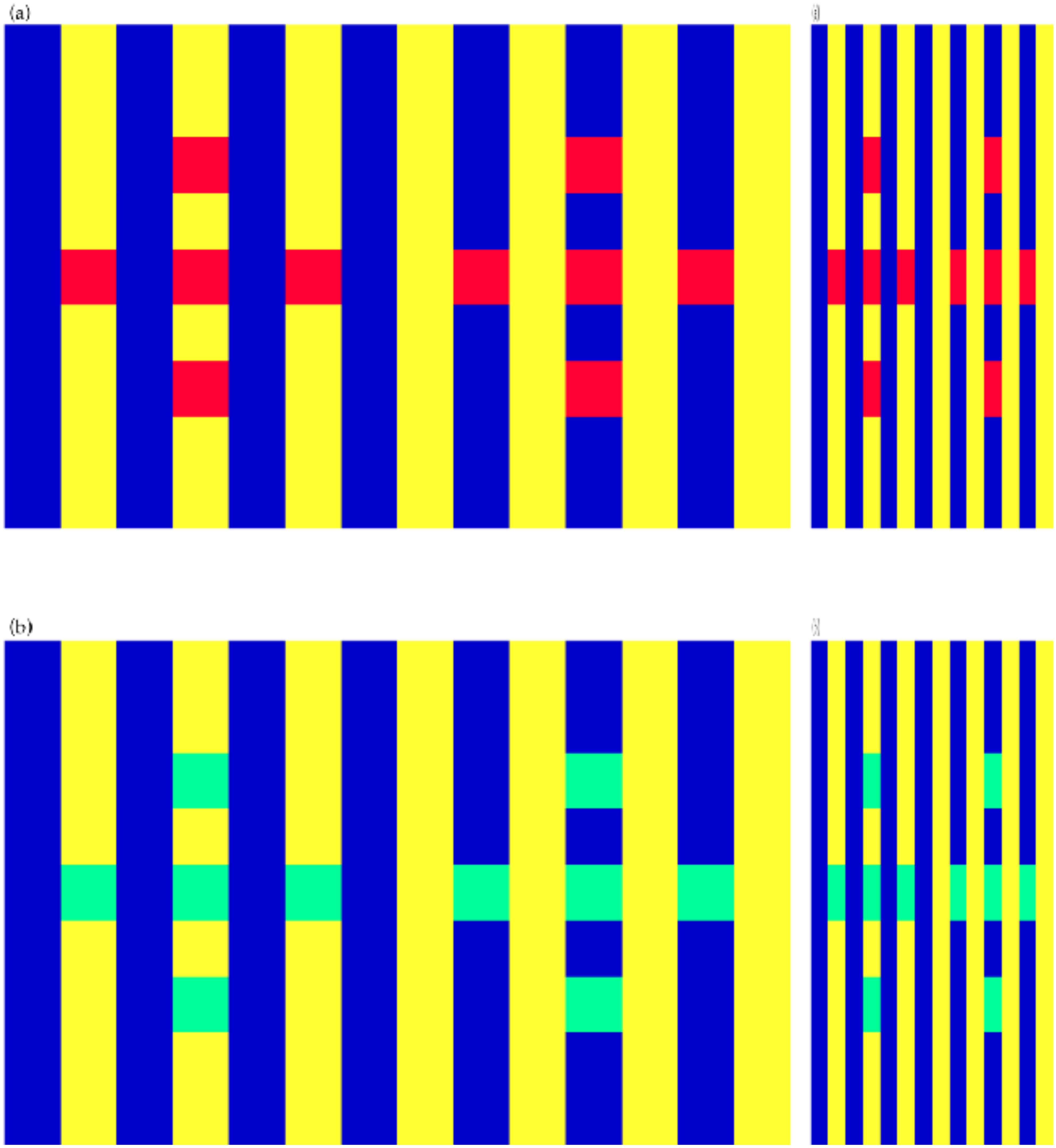
Slide credit: Mark Fairchild

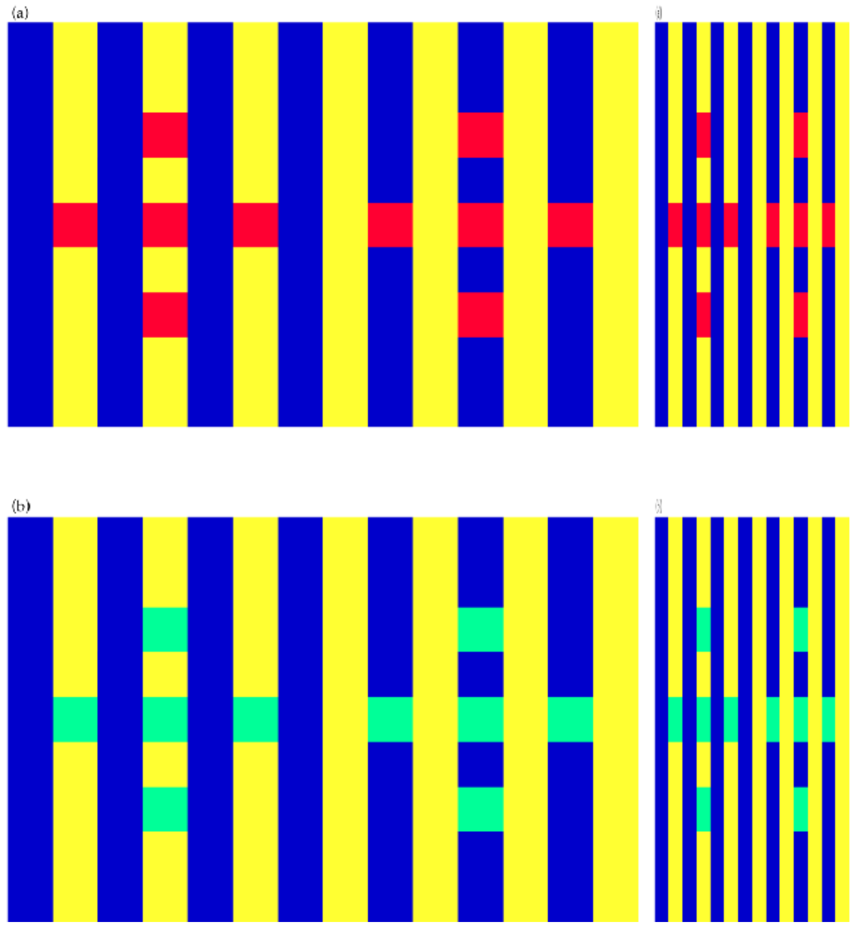
Surround Effects



Roberts (1996)

Surround Effects



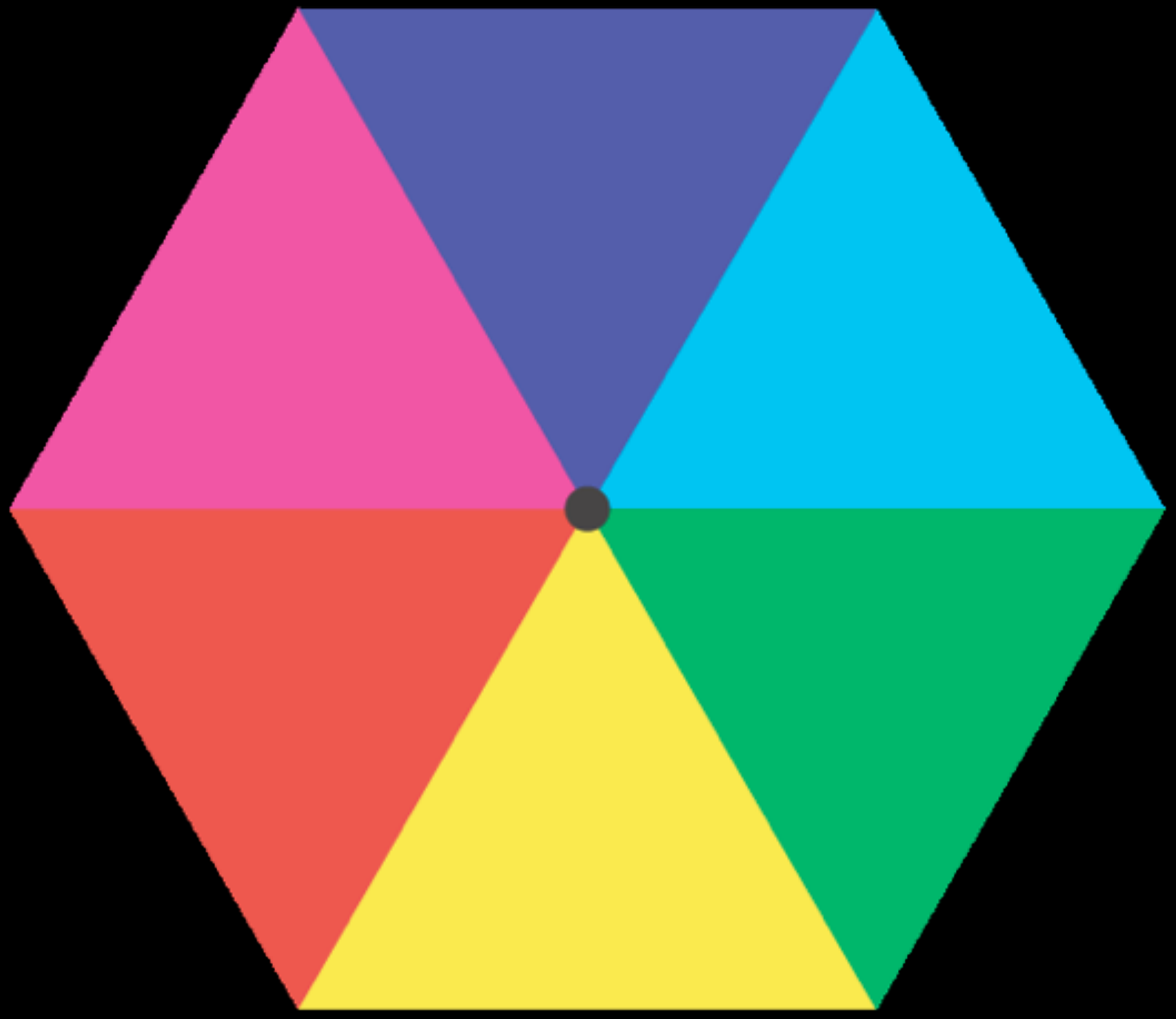


Roberts (1996)

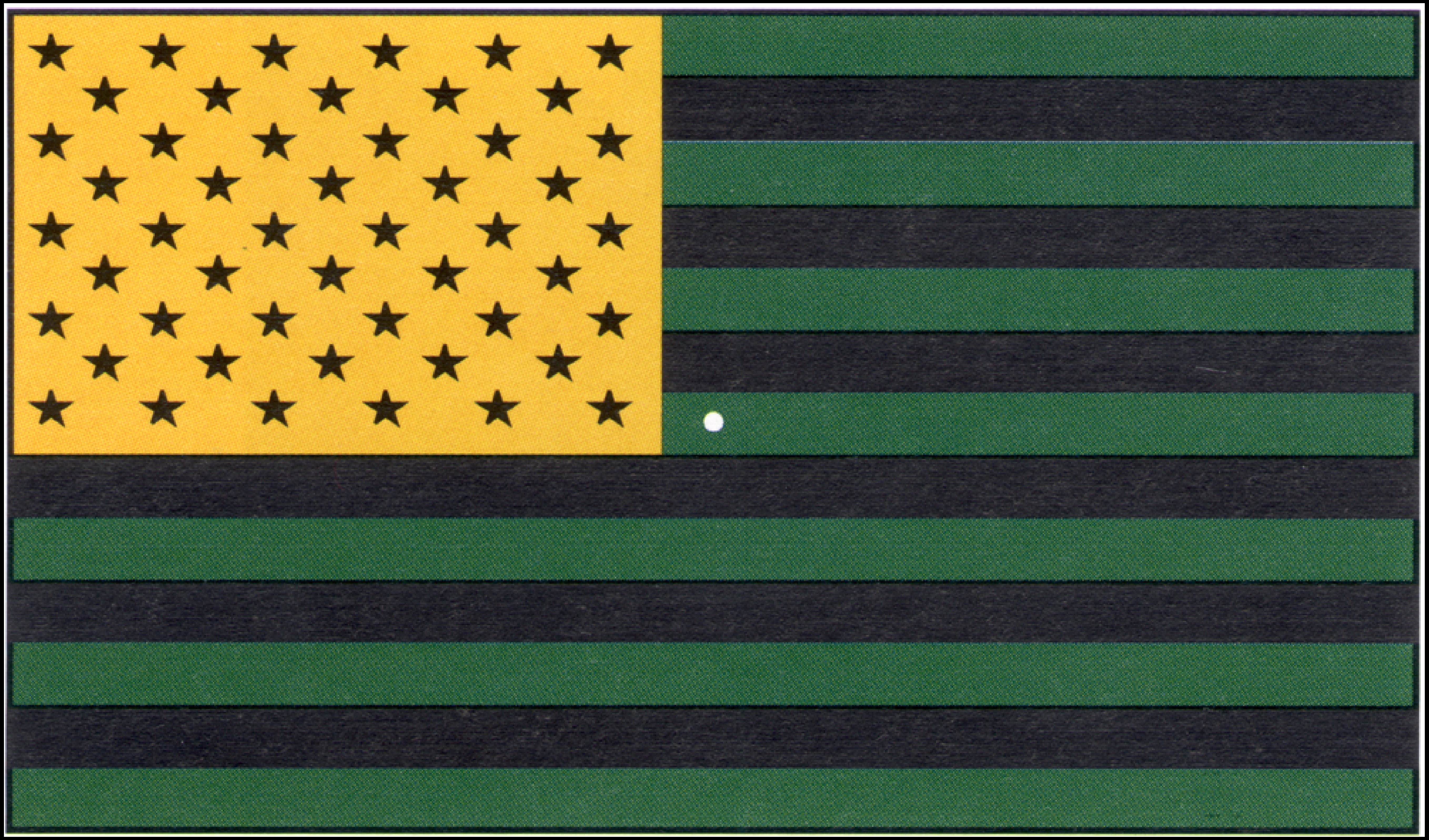
AfterImages: Perception Operates on "Opponent" Color Axes

●

Image



Afterimage



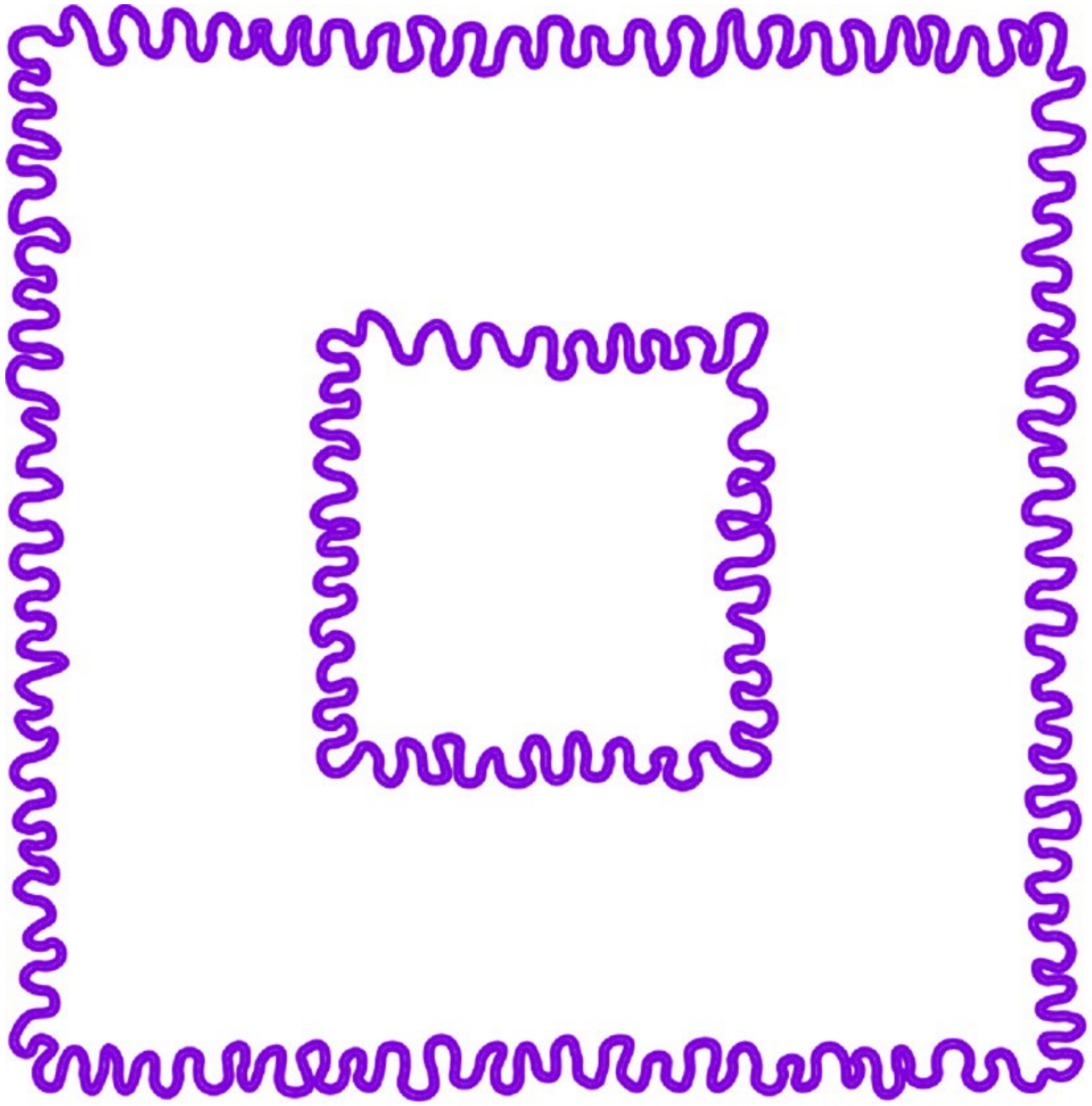

TOTH³

keep staring at the black dot.


johnsadowski.com

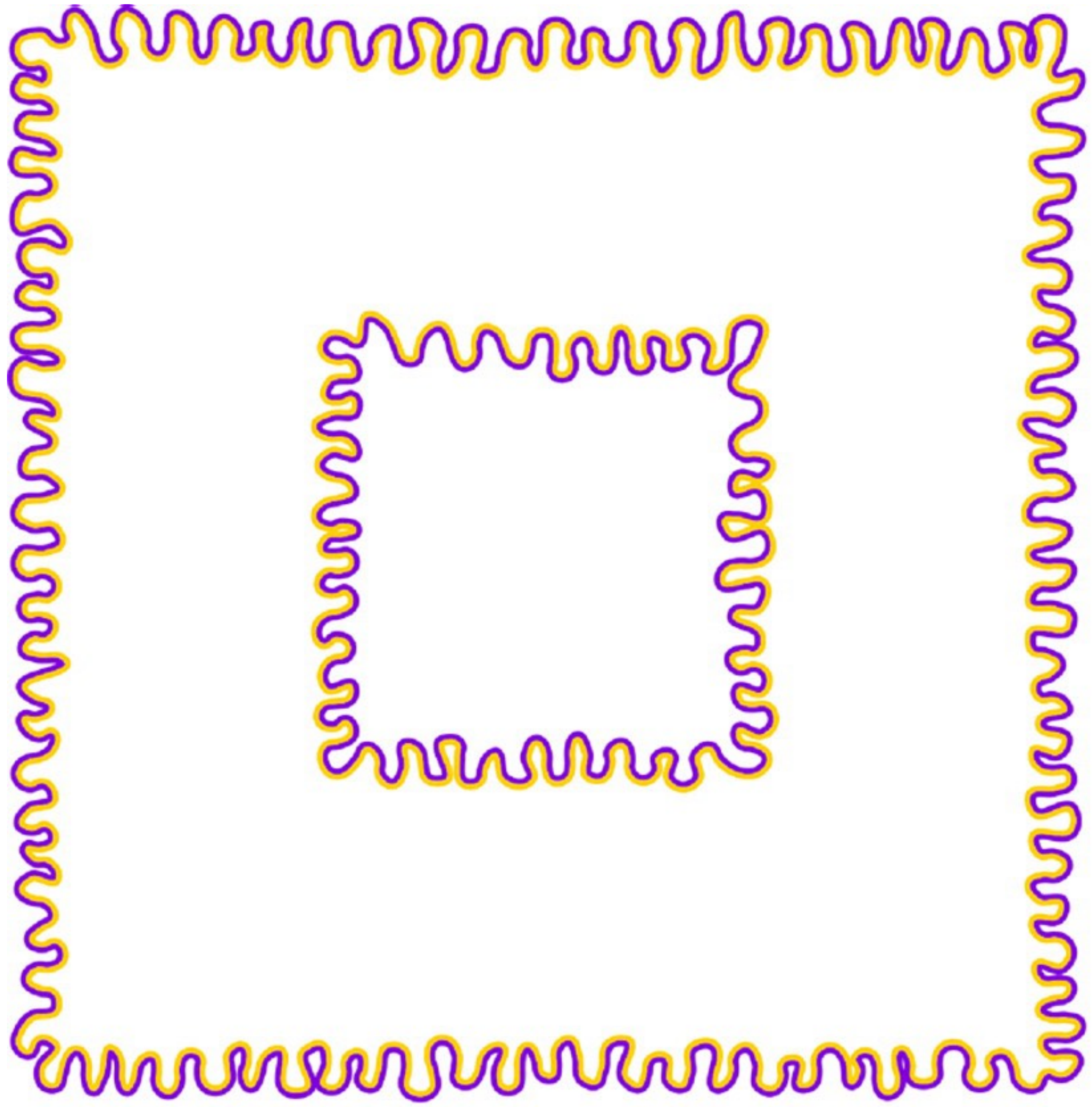
Color Perception is Complex and Surprising

Watercolor Illusion



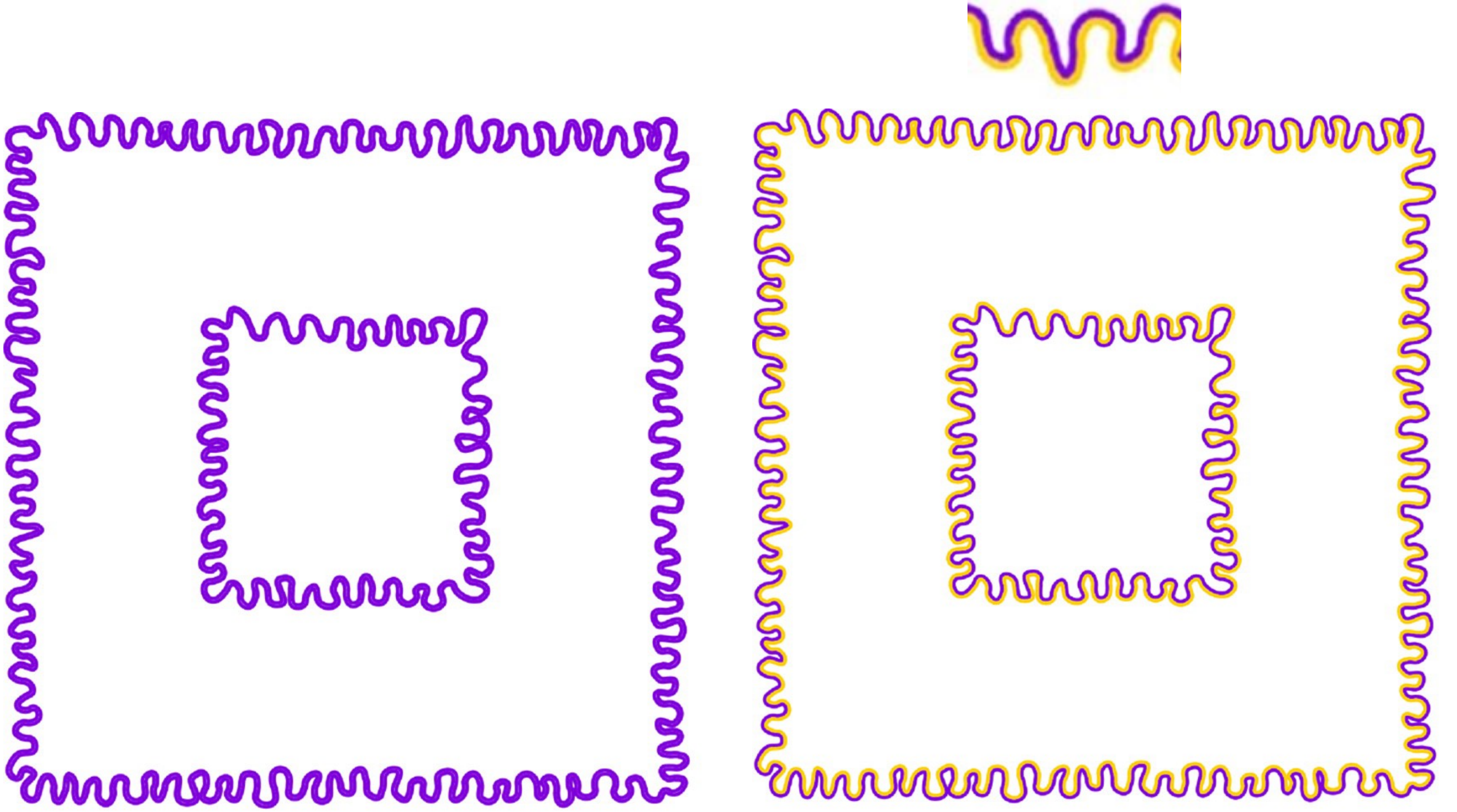
Pinna & Tanca, *JOV*, 2008

Watercolor Illusion



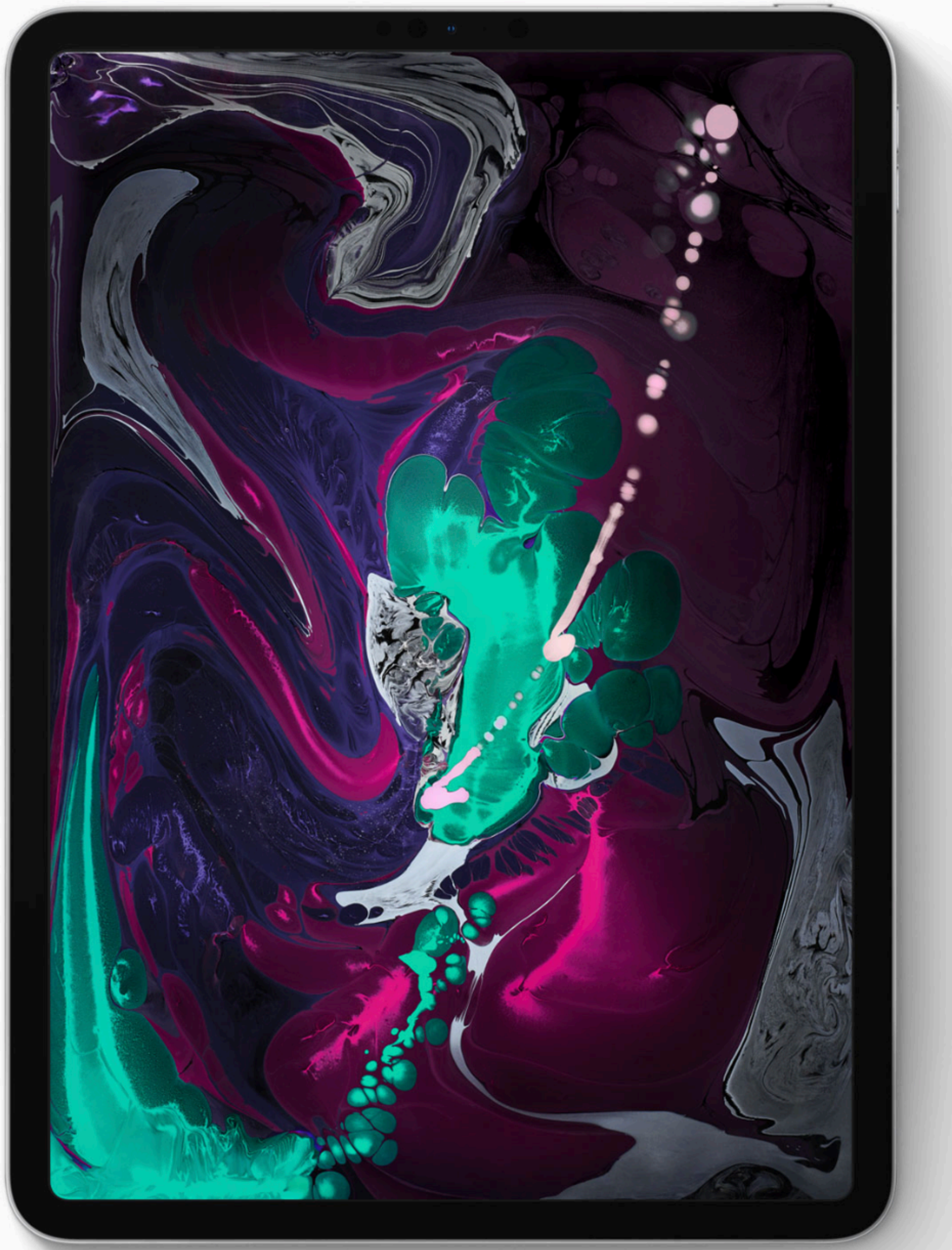
Pinna & Tanca, *JOV*, 2008

Watercolor Illusion



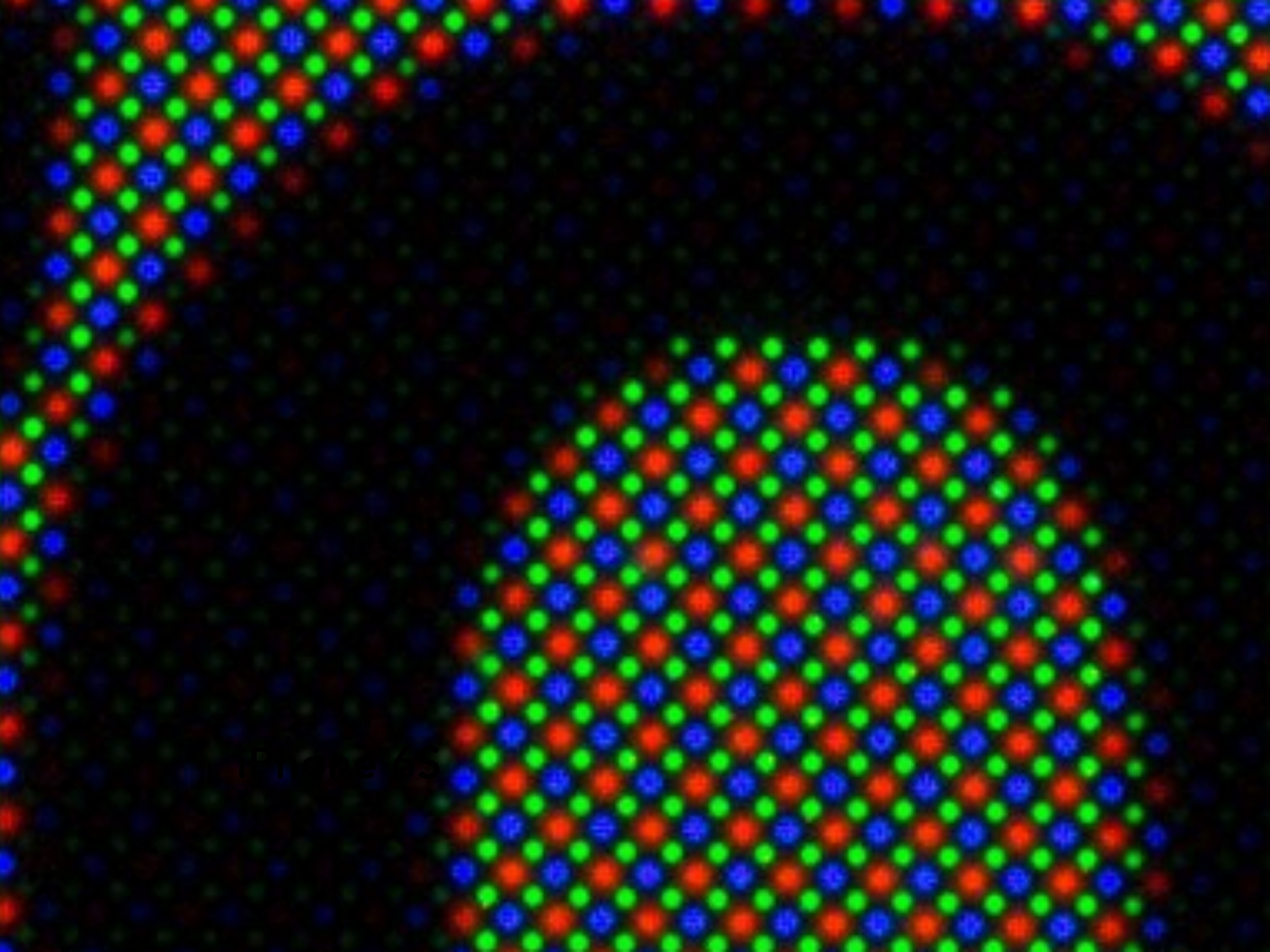
Pinna & Tanca, *JOV*, 2008

And Yet, We Understand Color Reproduction As a Quantitative Science



11″

12.9″

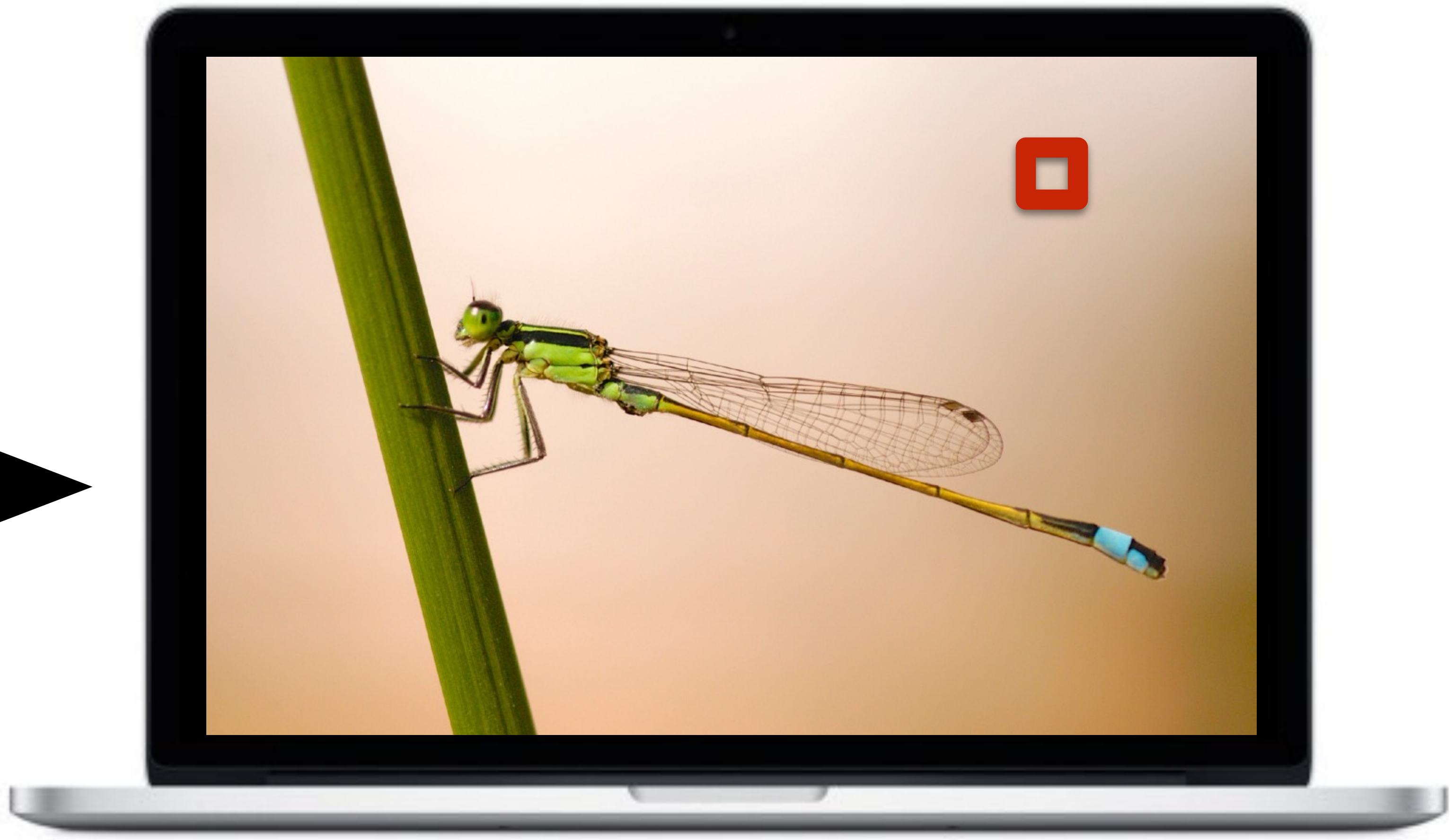


iPhone 6S


Cancel
Send to Lightroom
lavender-73_Original-Edit-Edit
56%
30

Color Reproduction Problem We Will Study

Real world damselfly



Display image of damselfly on computer screen

Goal: at each pixel, choose R, G, B values for display so that the output color matches the appearance of the colors in the real world.

What is Color?

What is Color?

- Color is a phenomenon of human perception; it is not a universal property of light
- Colors are the visual sensations that arise from seeing light of different spectral power distributions

Color Science

Sources of Optical Radiation: PHYSICS
Characterization of Objects: PHYSICS, CHEMISTRY
Perception: ANATOMY, PHYSIOLOGY, PSYCHOLOGY

Slide credit: Mark Fairchild

Physical Basis of Color

credit: Science Media Group.

Isaac Newton's Experimentum Crucis

Isaac Newton performing his crucial prism experiment – the 'experimentum crucis' – in his Woolsthorpe Manor bedroom.
Acrylic painting by Sascha Grusche (17 Dec 2015)

- Newton showed sunlight can be subdivided into a rainbow with a prism
- Resulting light cannot be further subdivided with a second prism

The Visible Spectrum of Light

Electromagnetic radiation

- Oscillations of different frequencies (wavelengths)

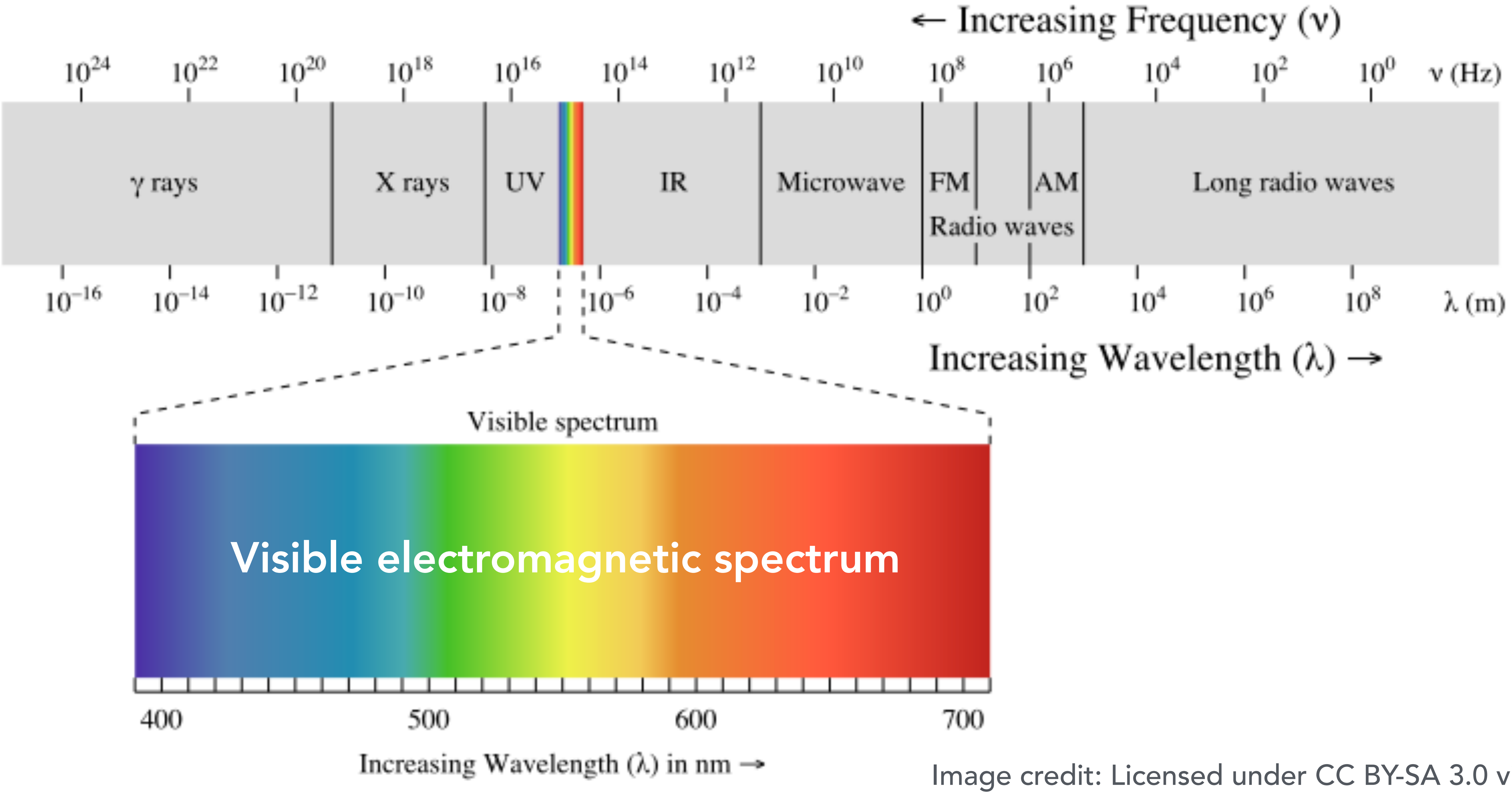
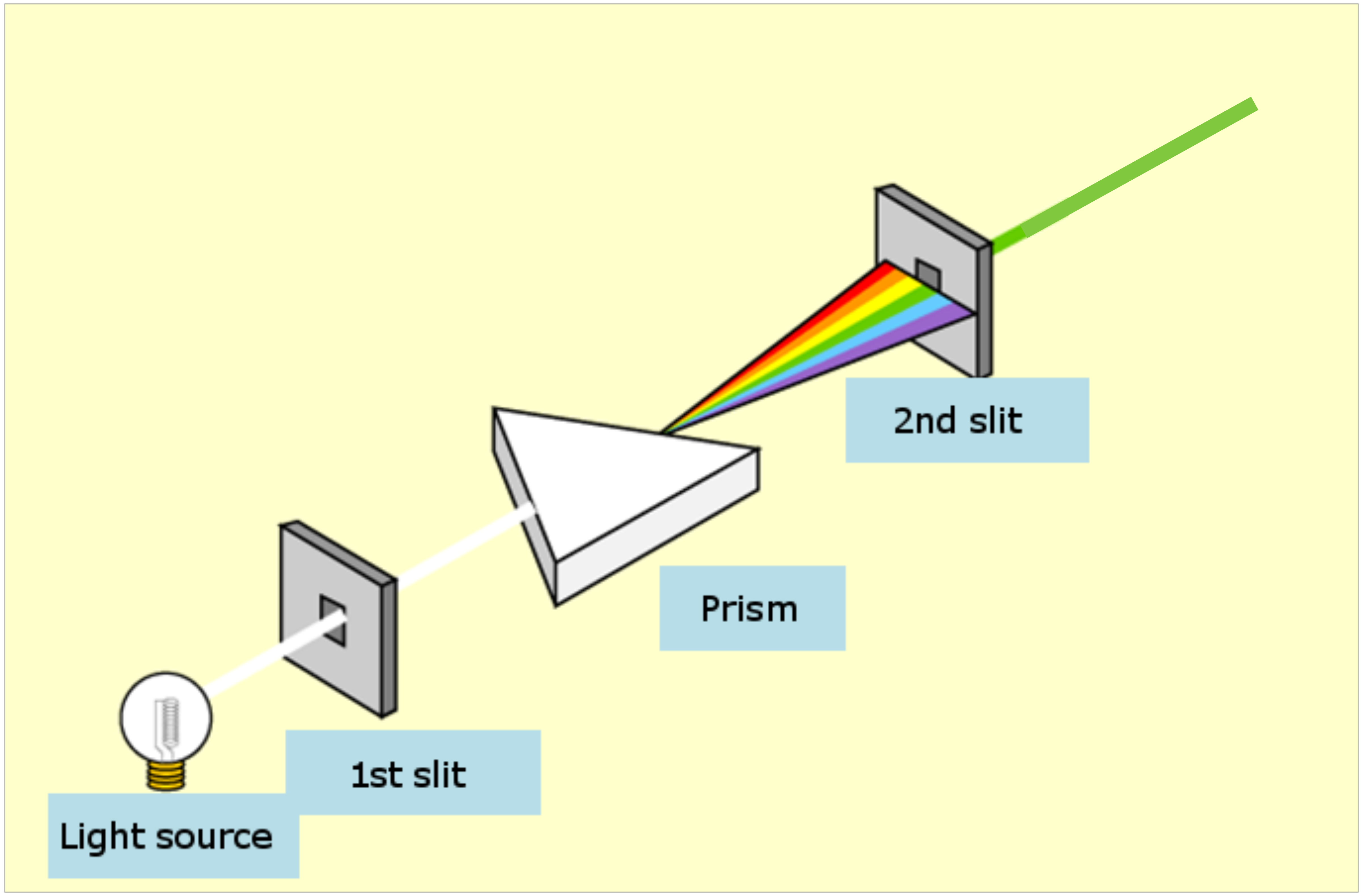


Image credit: Licensed under CC BY-SA 3.0 via Commons
https://commons.wikimedia.org/wiki/File:EM_spectrum.svg#/media/File:EM_spectrum.svg

Monochromator



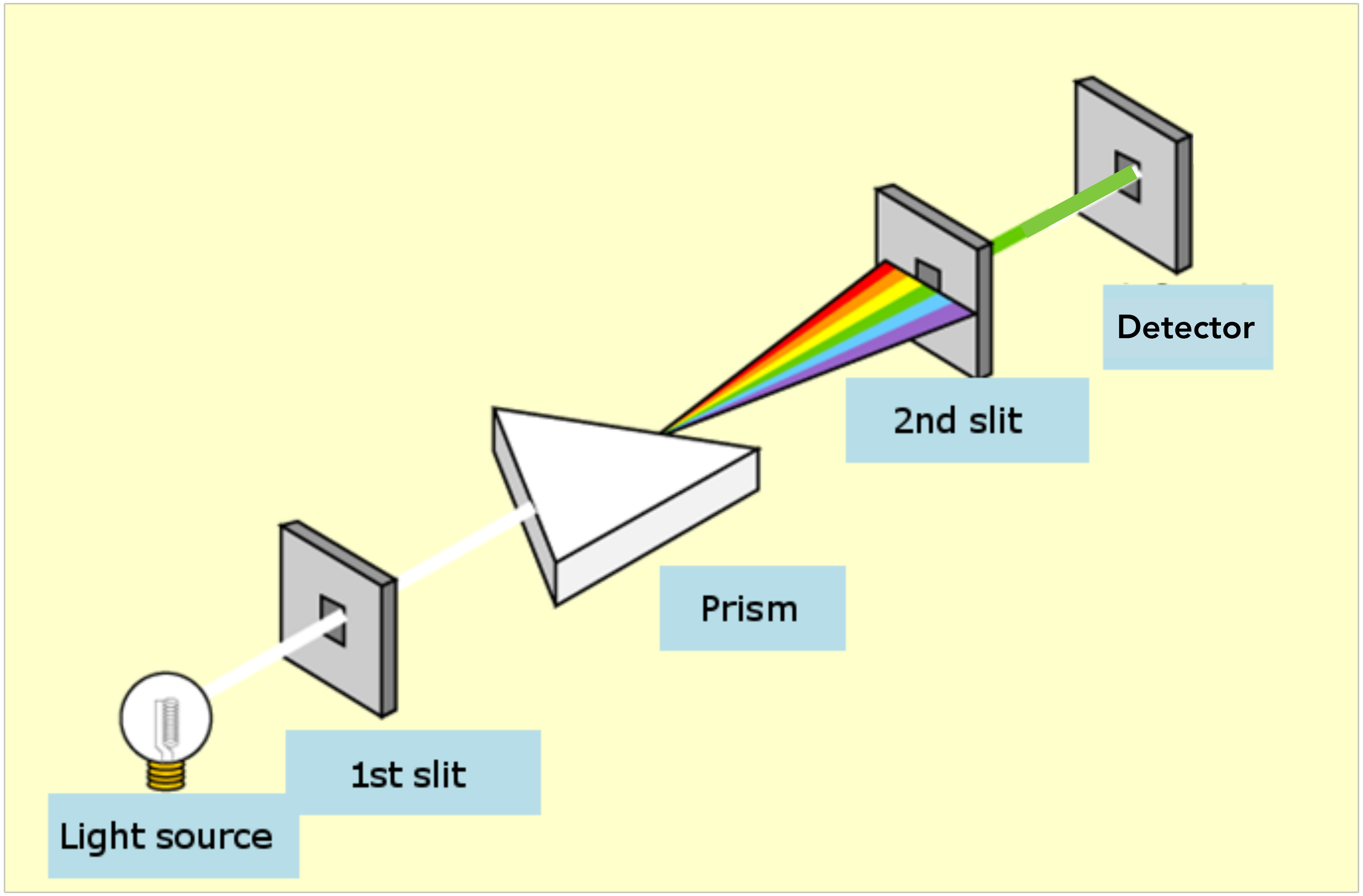
A monochromator delivers light of a single wavelength from a light source with broad spectrum.

Control which wavelength by angle of prism.

Hegyi et al. "Introduction to Practical Biochemistry"

http://elte.prompt.hu/sites/default/files/tananyagok/IntroductionToPracticalBiochemistry/ch04s07.html

Spectrometer



For unknown light source, use a monochromator to isolate each wavelength of light for measurement

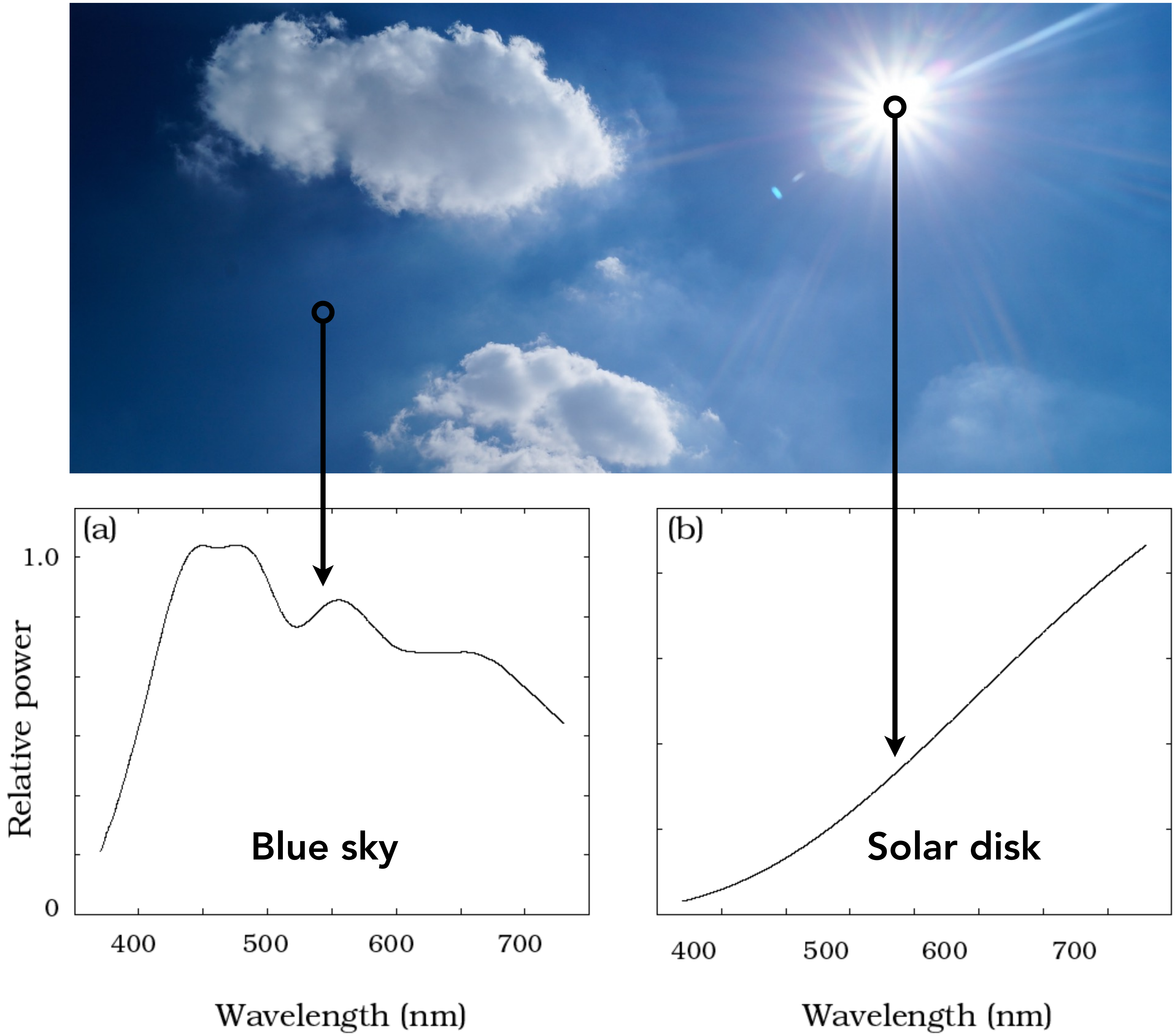
Hegyi et al. "Introduction to Practical Biochemistry"
http://elte.prompt.hu/sites/default/files/tananyagok/IntroductionToPracticalBiochemistry/ch04s07.html

Spectral Power Distribution (SPD)

Salient property in measuring light

- The amount of light present at each wavelength
- Units:
    - radiometric units / nanometer (e.g. watts / nm)
    - Can also be unit-less
- Often use “relative units” scaled to maximum wavelength for comparison across wavelengths when absolute units are not important

Daylight Spectral Power Distributions Vary



[Brian Wandell]

Spectral Power Distribution of Light Sources

Describes distribution of energy by wavelength

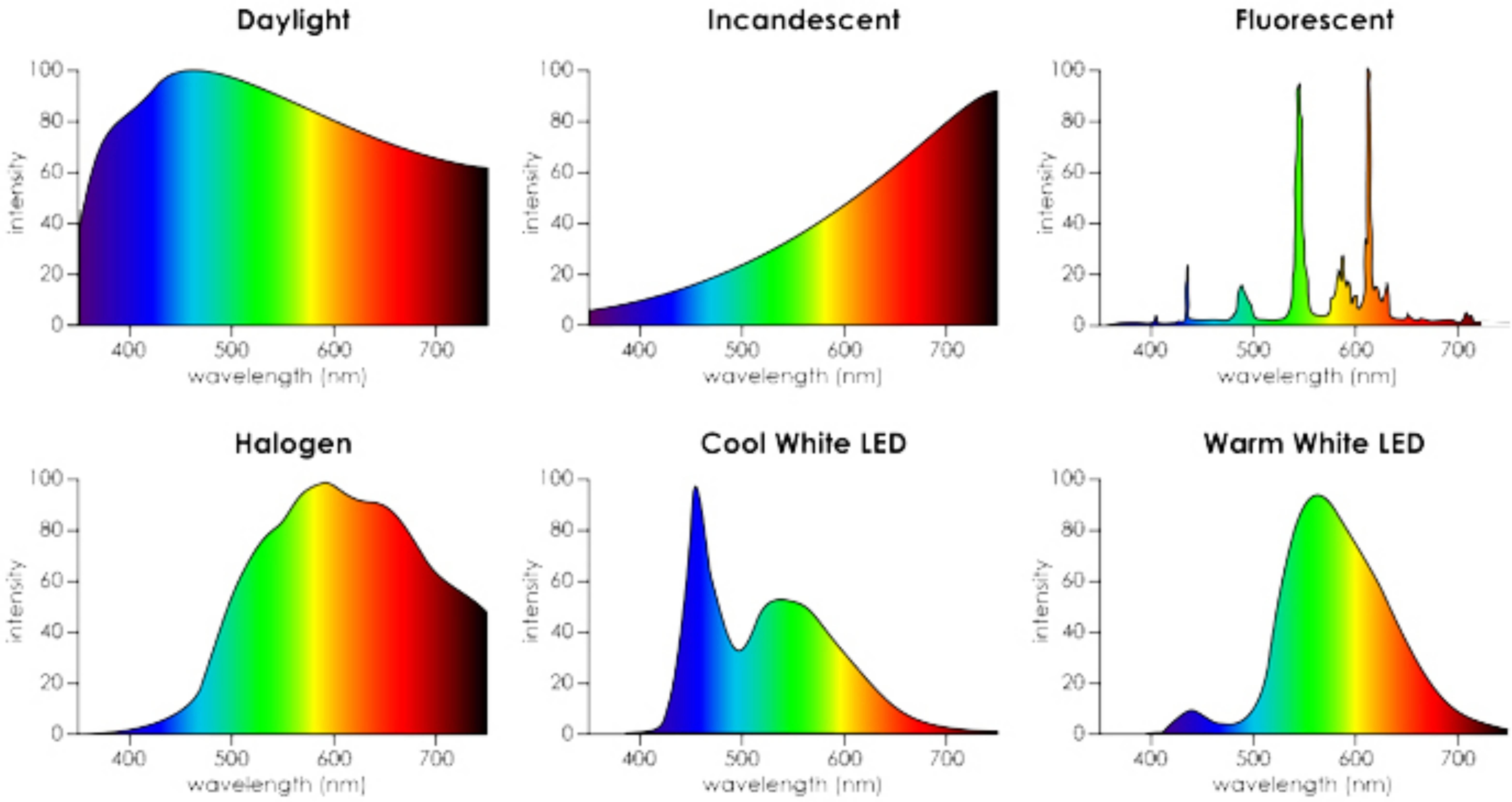
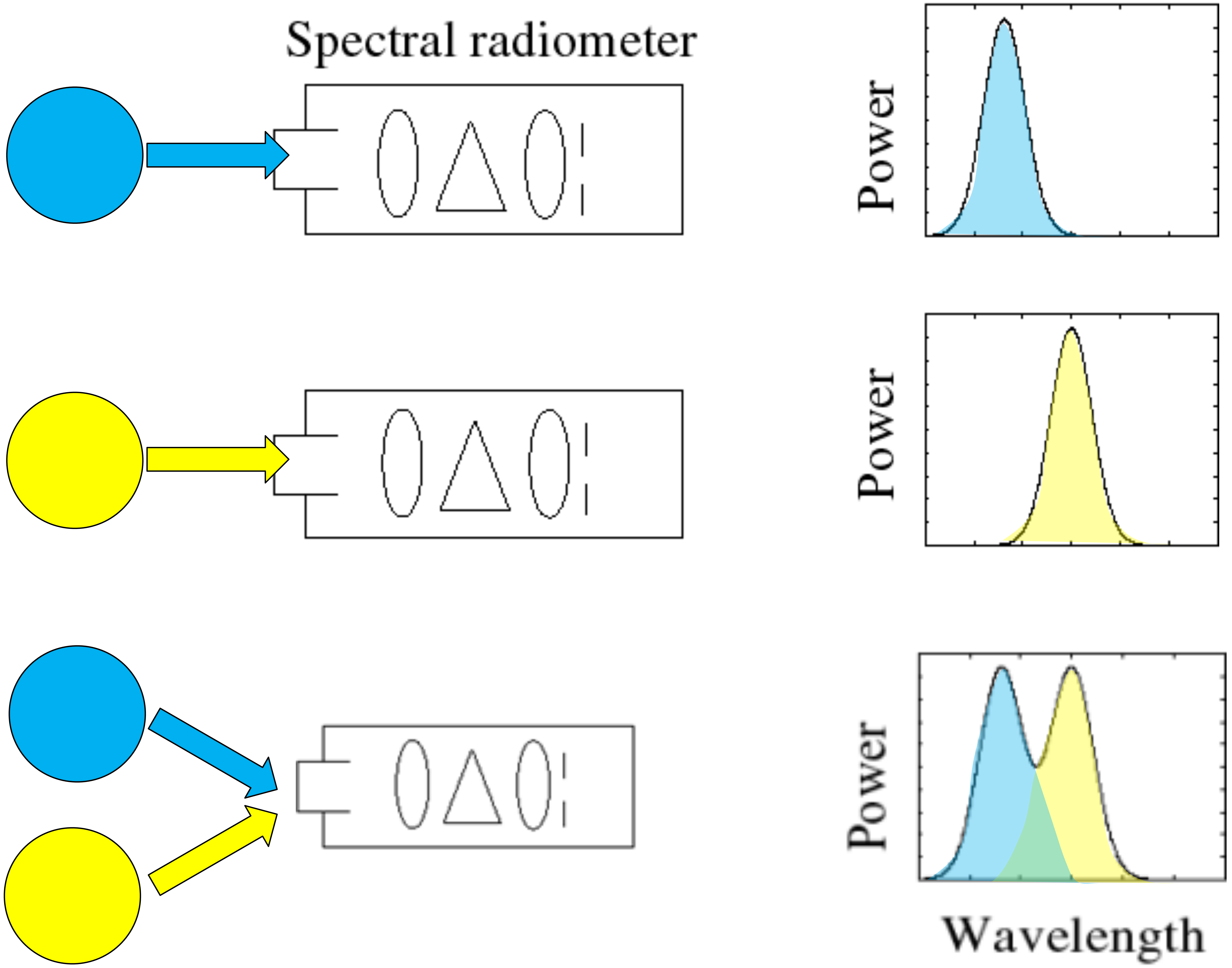


Figure credit: admesy ADVANCED MEASUREMENT SYSTEMS

Superposition (Linearity) of Spectral Power Distributions



[Brian Wandell]

Measuring Light

A Simple Model of a Light Detector

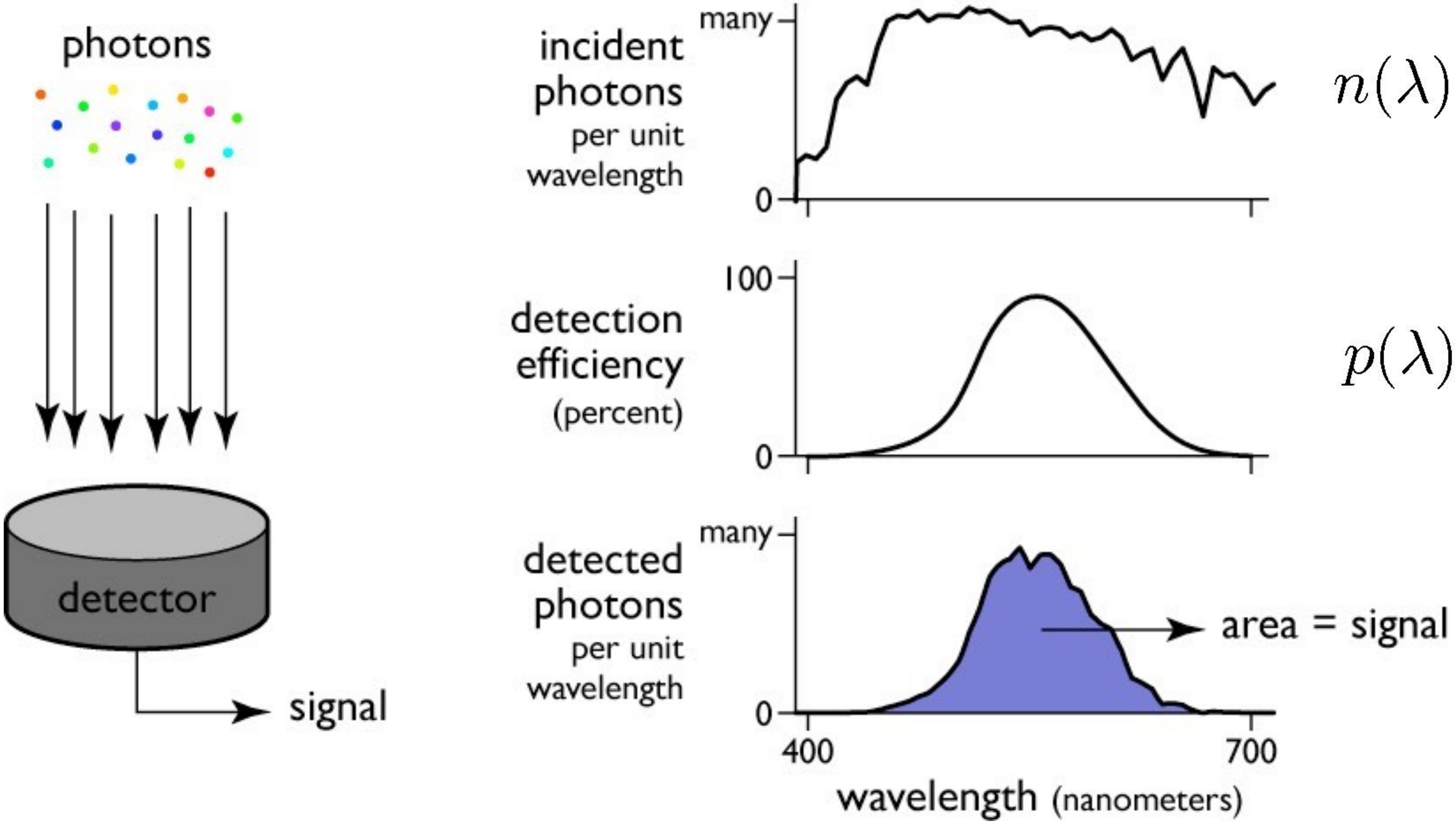
Produces a scalar value (a number) when photons land on it

- Value depends only on the number of photons detected
- Each photon has a probability of being detected that depends on the wavelength
- No way to distinguish between signals caused by light of different wavelengths: there is just a number

This model works for many detectors:

- based on semiconductors (such as in a digital camera)
- based on visual photopigments (such as in human eyes)

A Simple Model of a Light Detector



Credit: Marschner

$$X = \int n(\lambda)p(\lambda)\,d\lambda$$

Mathematics of Light Detection

Same math carries over to spectral power distributions

- Light entering the detector has its spectral power distribution, s(λ)
- Detector has its spectral sensitivity or spectral response, r(λ)

$$\underbrace{X}_{\text{measured signal}} = \int \underbrace{s(\lambda)}_{\text{input spectrum}}\underbrace{r(\lambda)}_{\text{detector's sensitivity}}\, d\lambda$$

Mathematics of Light Detection

If we think of s and r as discrete, sampled representations (vectors) rather than continuous functions, this integral operation is a dot product:

$$X = s \cdot r$$

We can also write this in matrix form:

$$X = \begin{bmatrix} \text{---} & s & \text{---} \end{bmatrix} \begin{bmatrix} | \\ r \\ | \end{bmatrix}$$

Dimensionality Reduction From ∞ to 1

At the detector:

- SPD is a function of wavelength (∞ - dimensional signal)
- Detector result is a scalar value (1 - dimensional signal)

Attendance Time

If you are seated in class, go to this form and sign in:

- https://bit.ly/184attendance

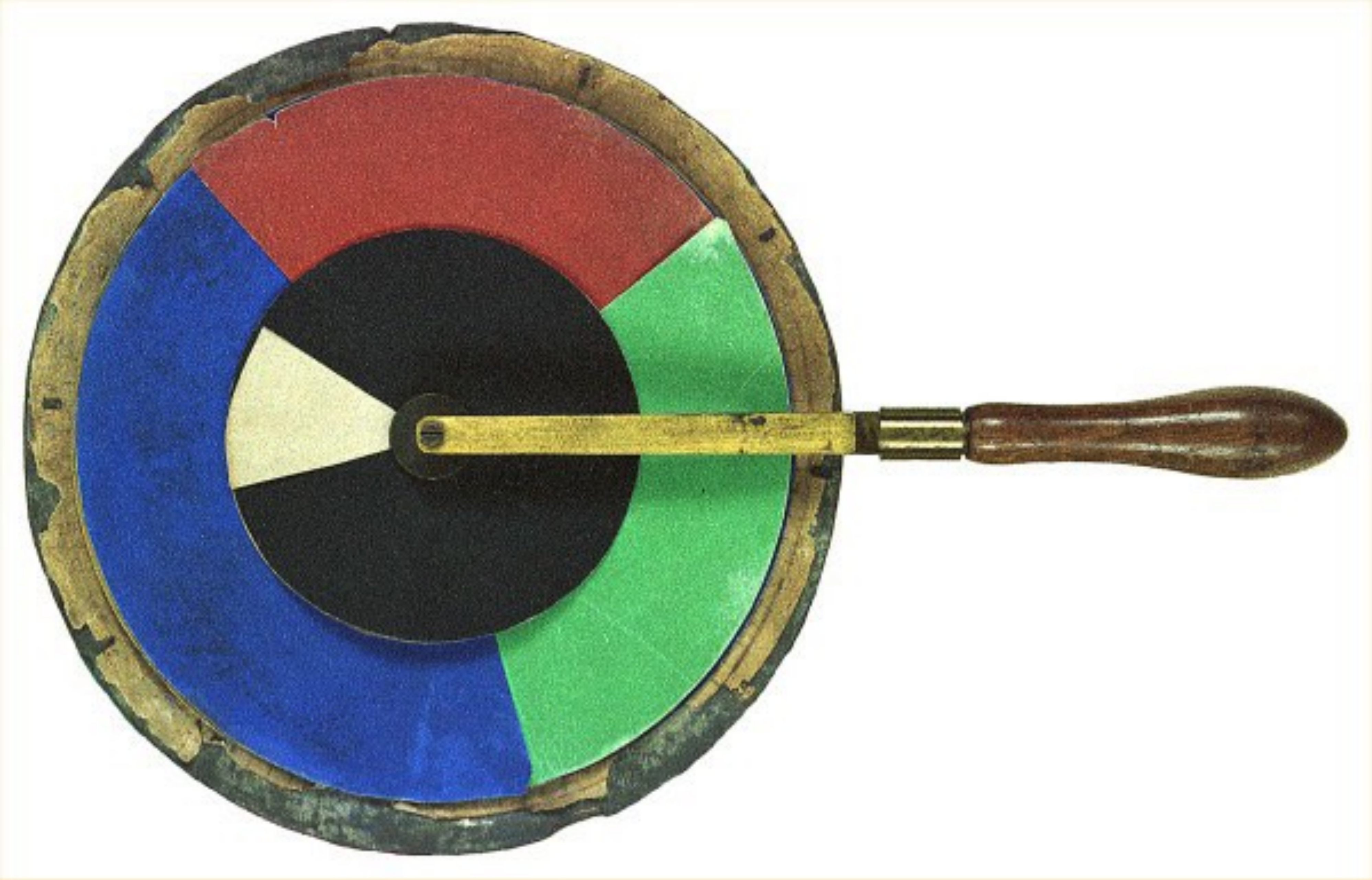
Notes:

- Time-stamp will be taken when you submit form.
  Do it now, won't count later.
- Don't tell friends outside class to fill it out now, because we will audit at some point in semester.
- Failing audit will have large negative consequence. You don't need to, because you have an alternative!

Tristimulus Theory of Color

Searching for a Linear Systems Basis for Colors: The Color Matching Experiment

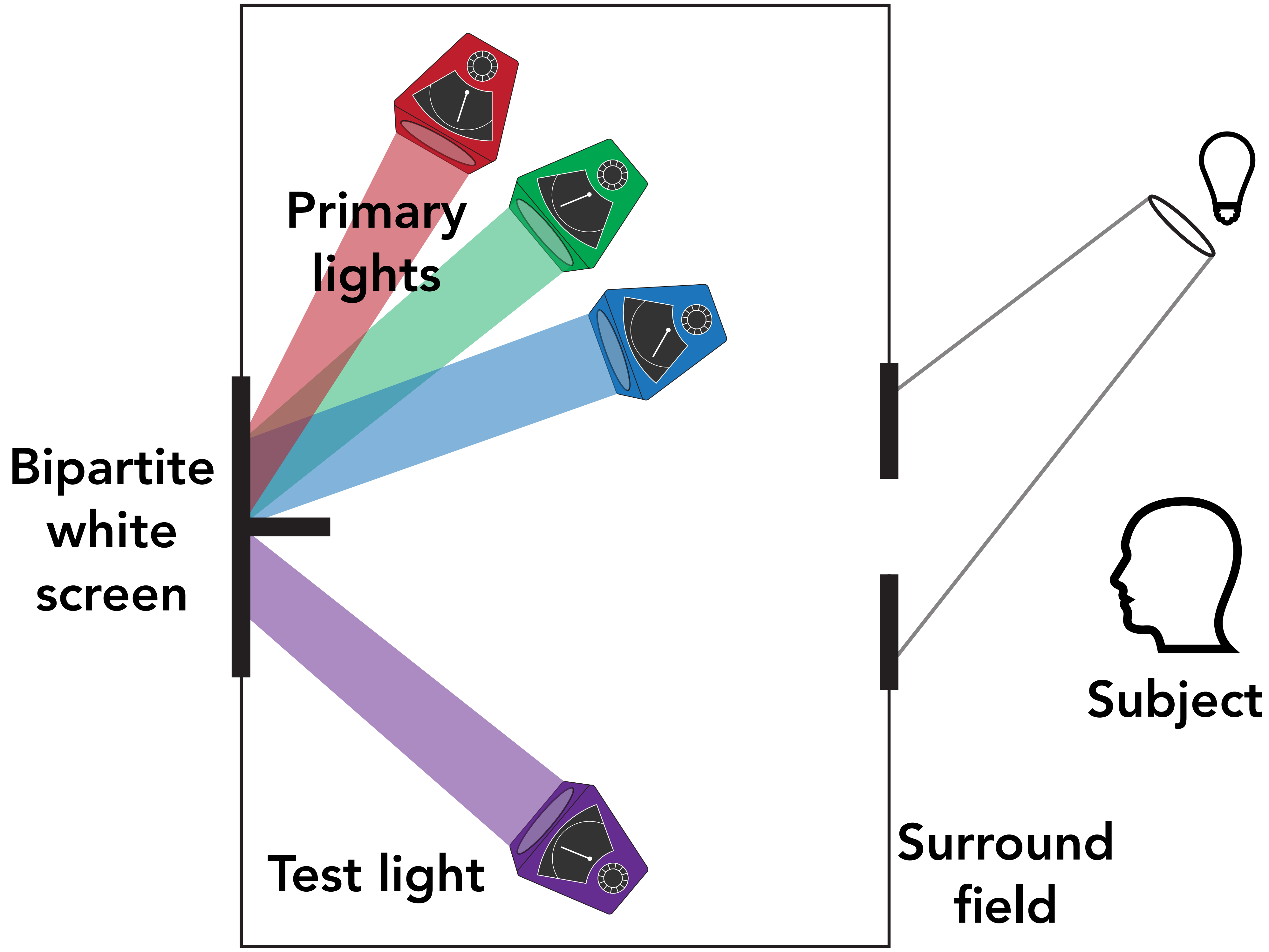
Maxwell's Crucial Color Matching Experiment

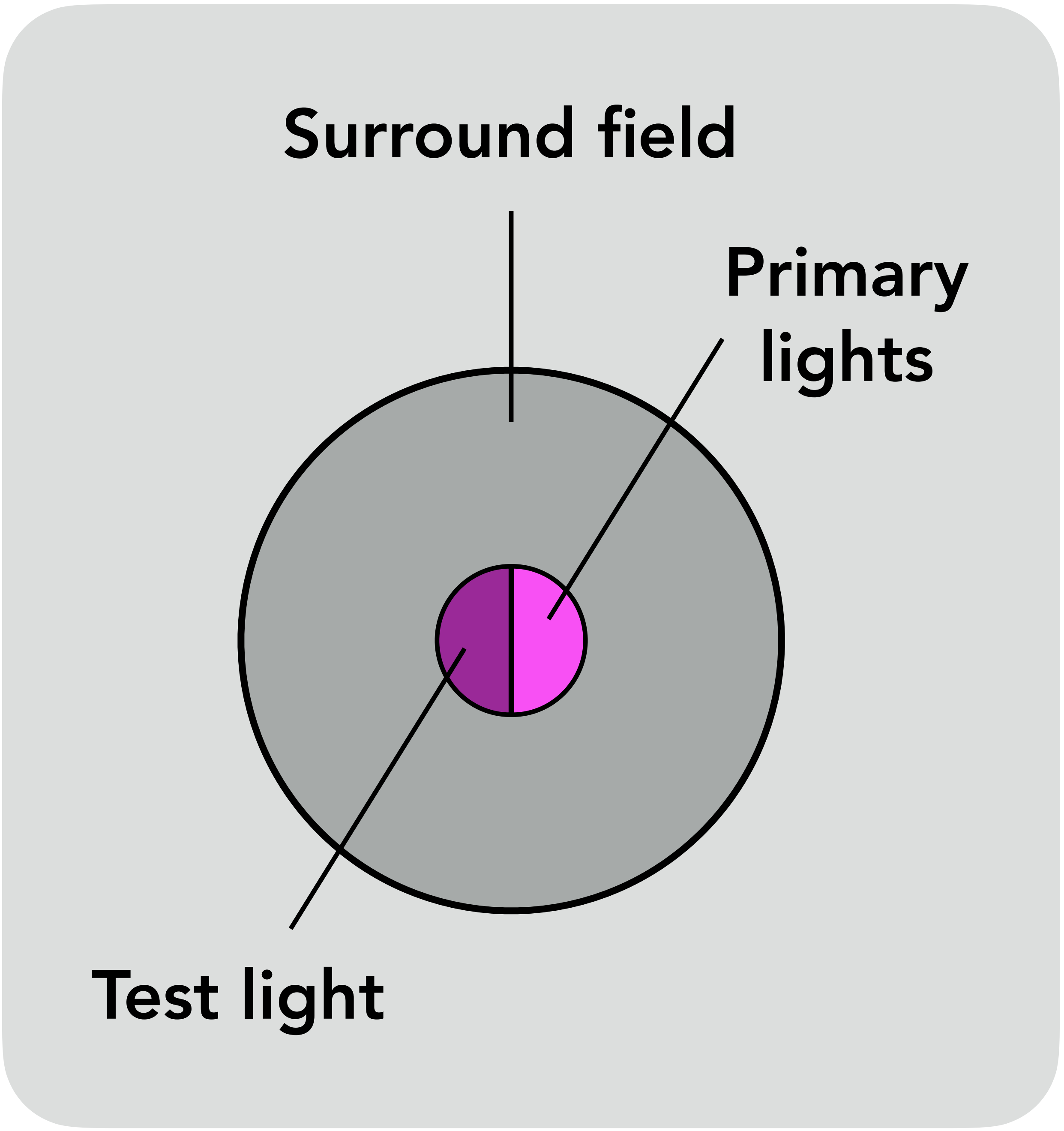


http://designblog.rietveldacademie.nl/?p=68422

Portrait: http://rsta.royalsocietypublishing.org/content/366/1871/1685

Color Matching Experiment





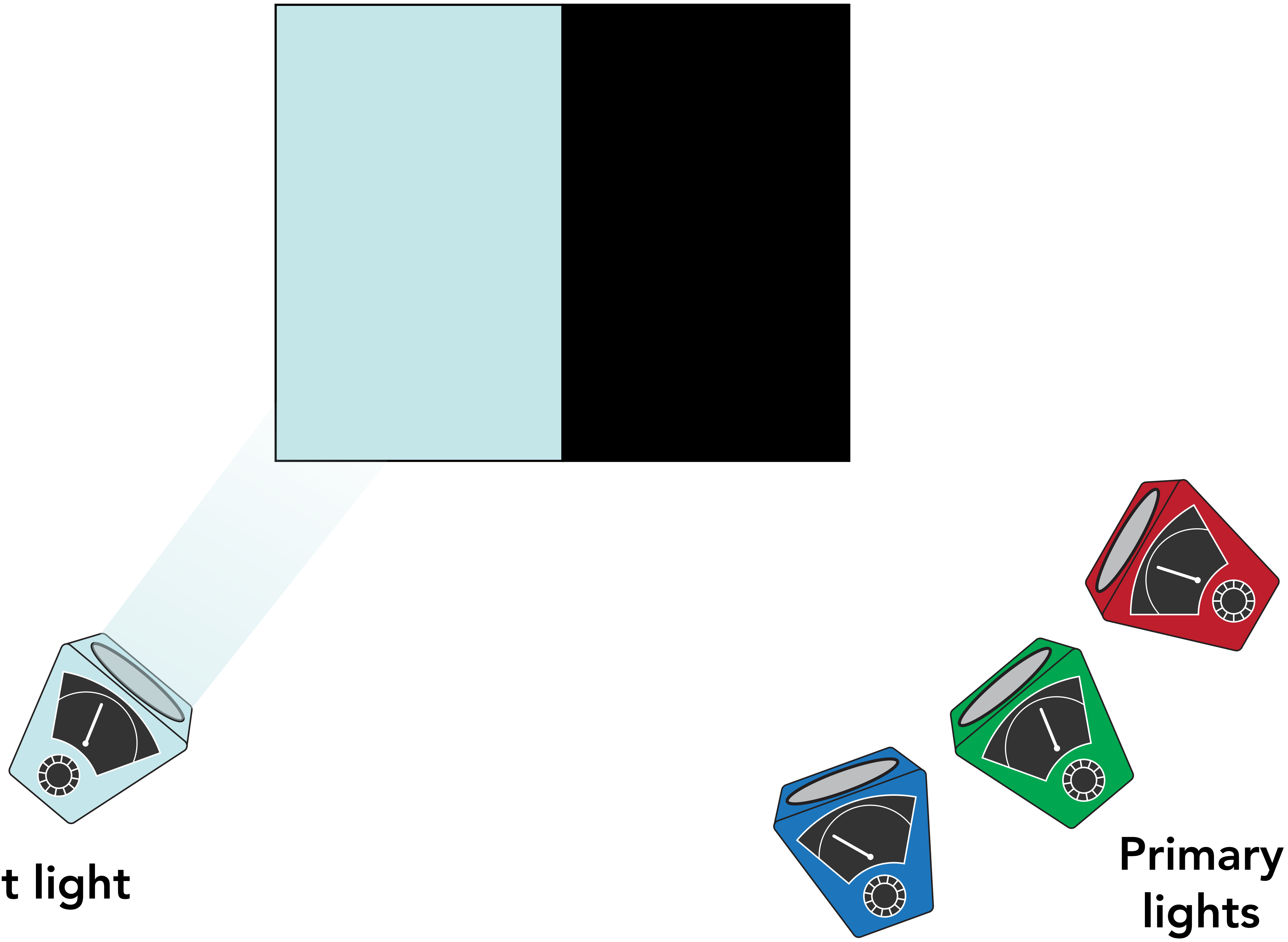
Same idea as spinning top, fancier implementation (Maxwell did this too)

Show test light spectrum on left

Mix “primaries” on right until they match

The primaries need not be RGB

Example Experiment

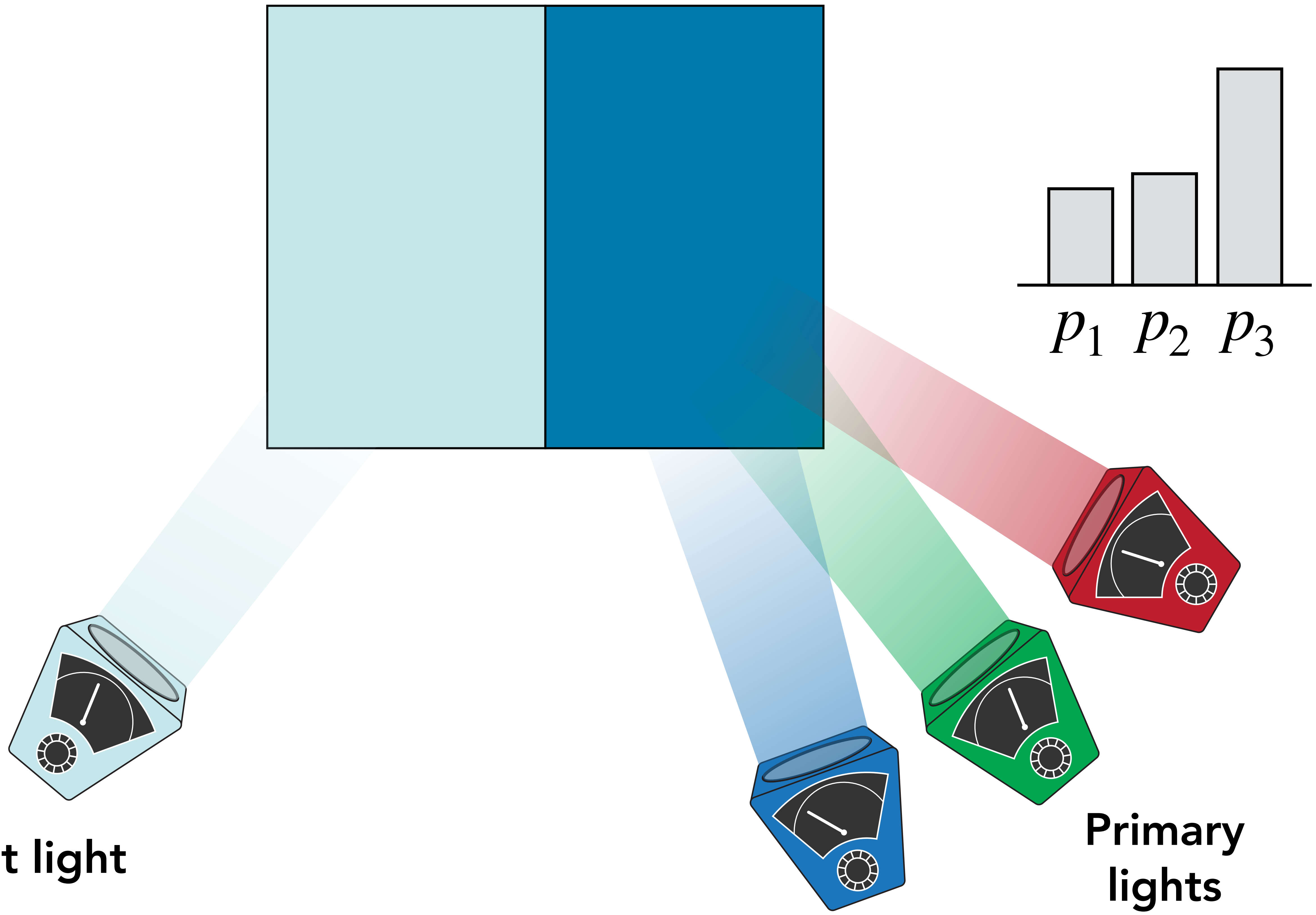


Test light

Primary lights

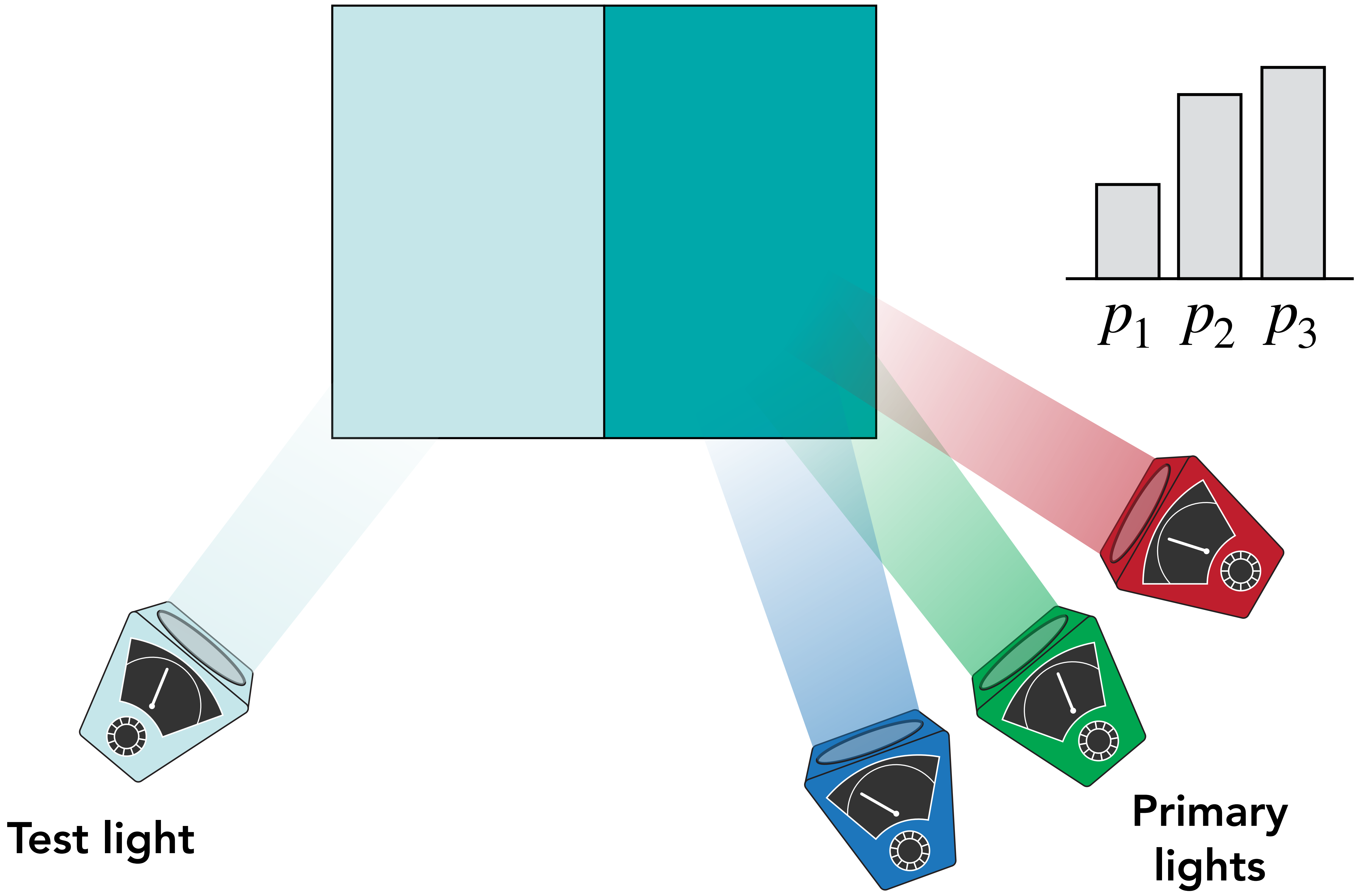
Slide credit: Kotani, Durand, Freeman

Example Experiment



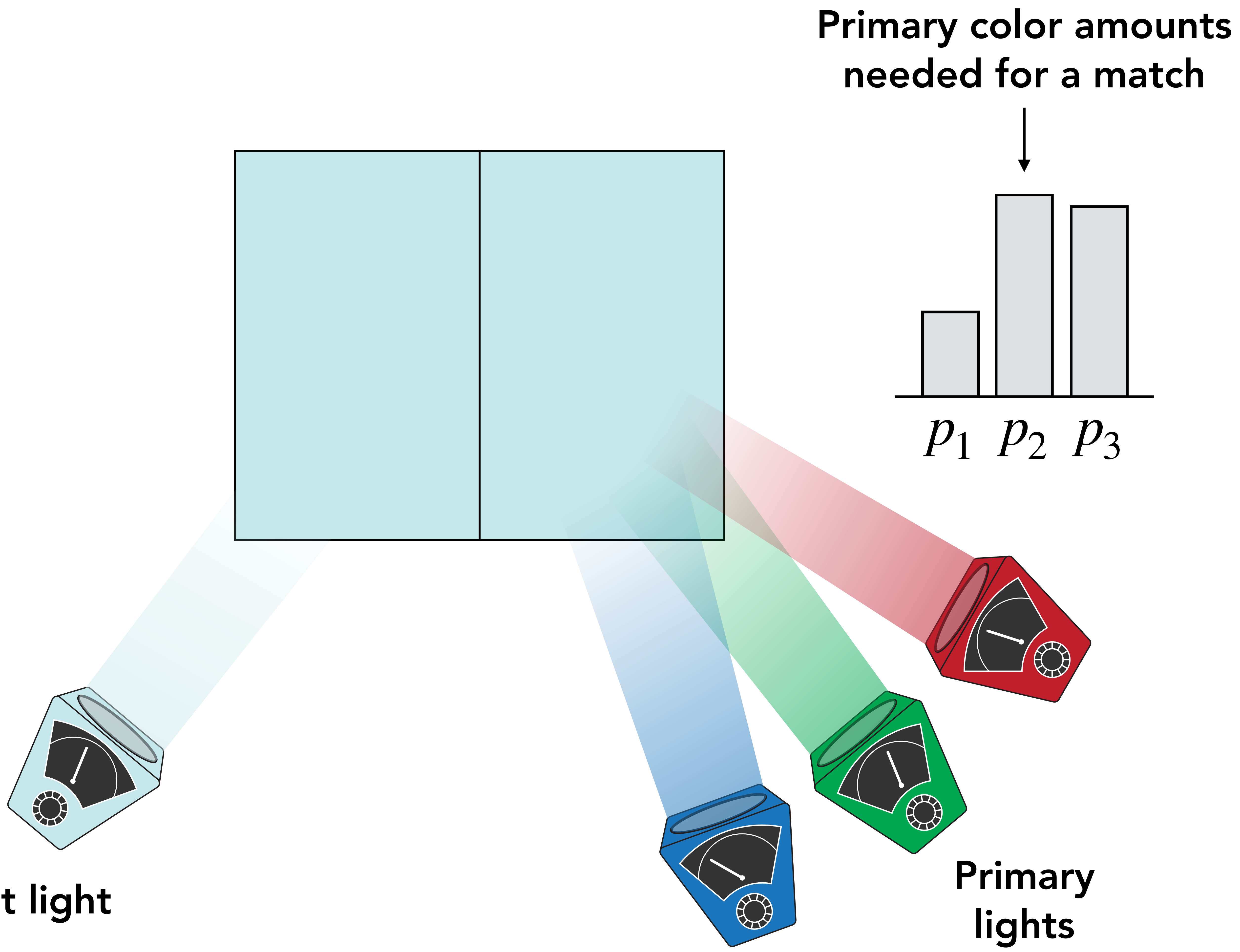
Slide credit: Kotani, Durand, Freeman

Example Experiment



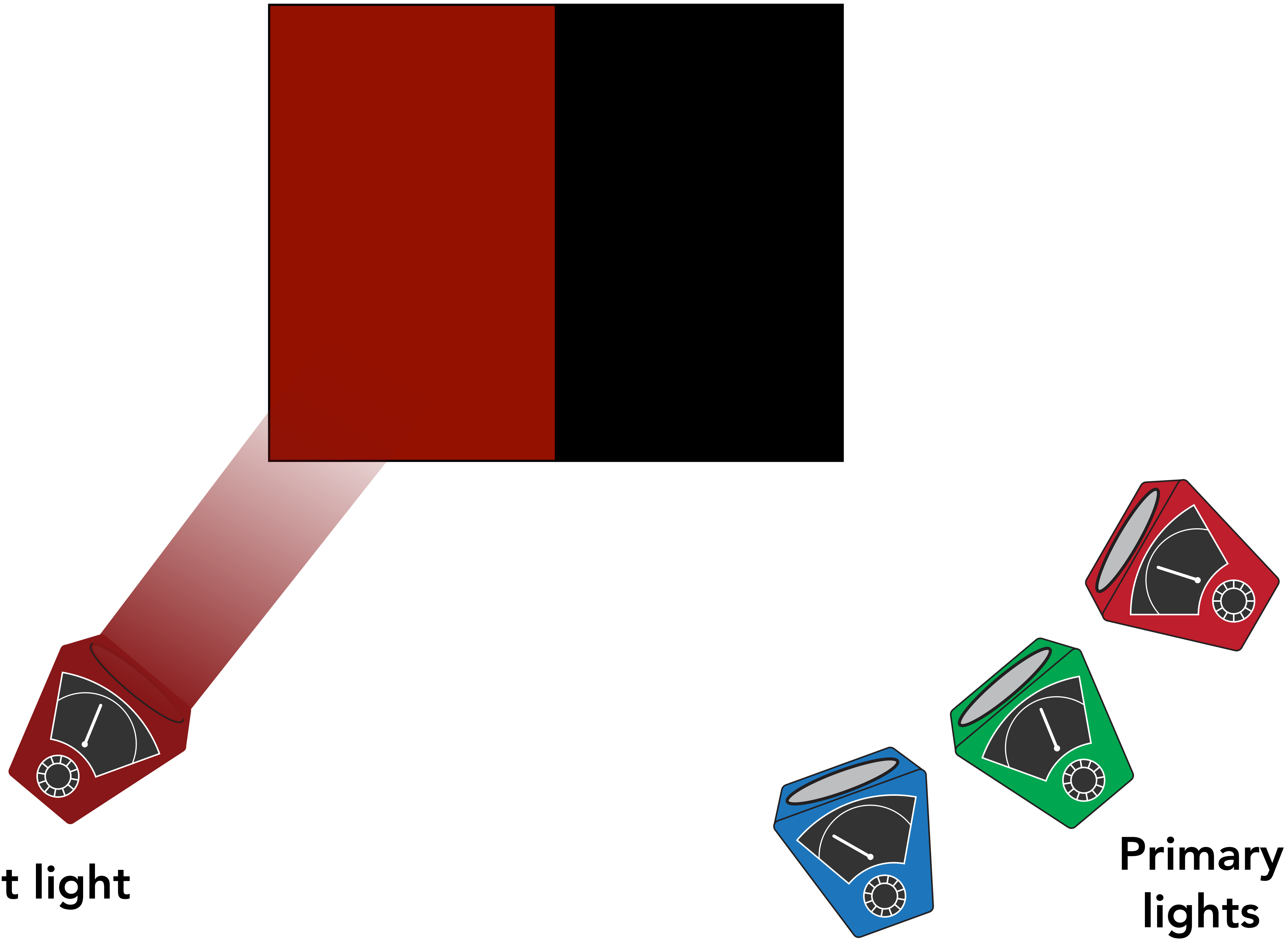
Slide credit: Kotani, Durand, Freeman

Example Experiment



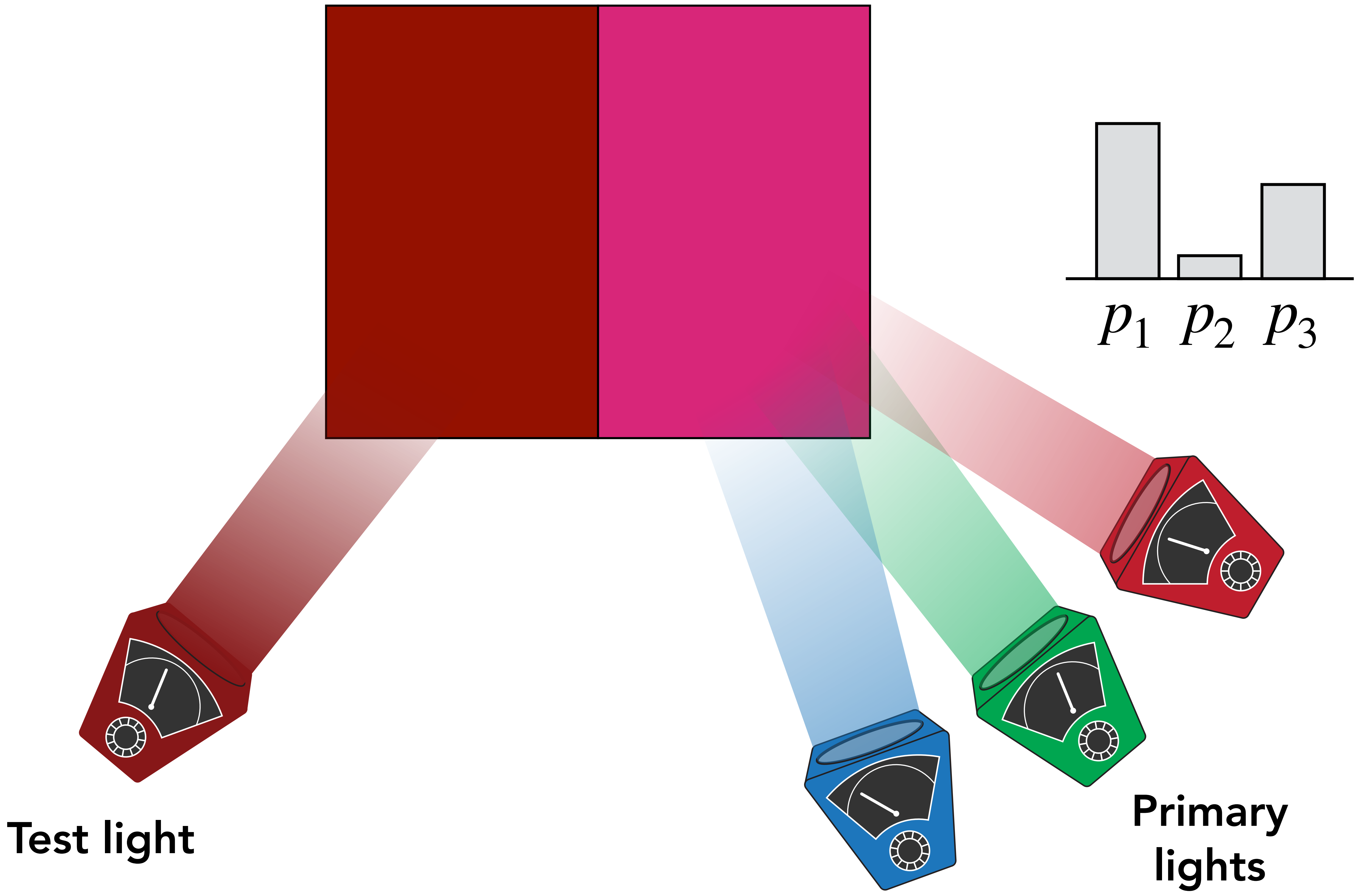
Slide credit: Kotani, Durand, Freeman

Experiment 2: Out of Gamut



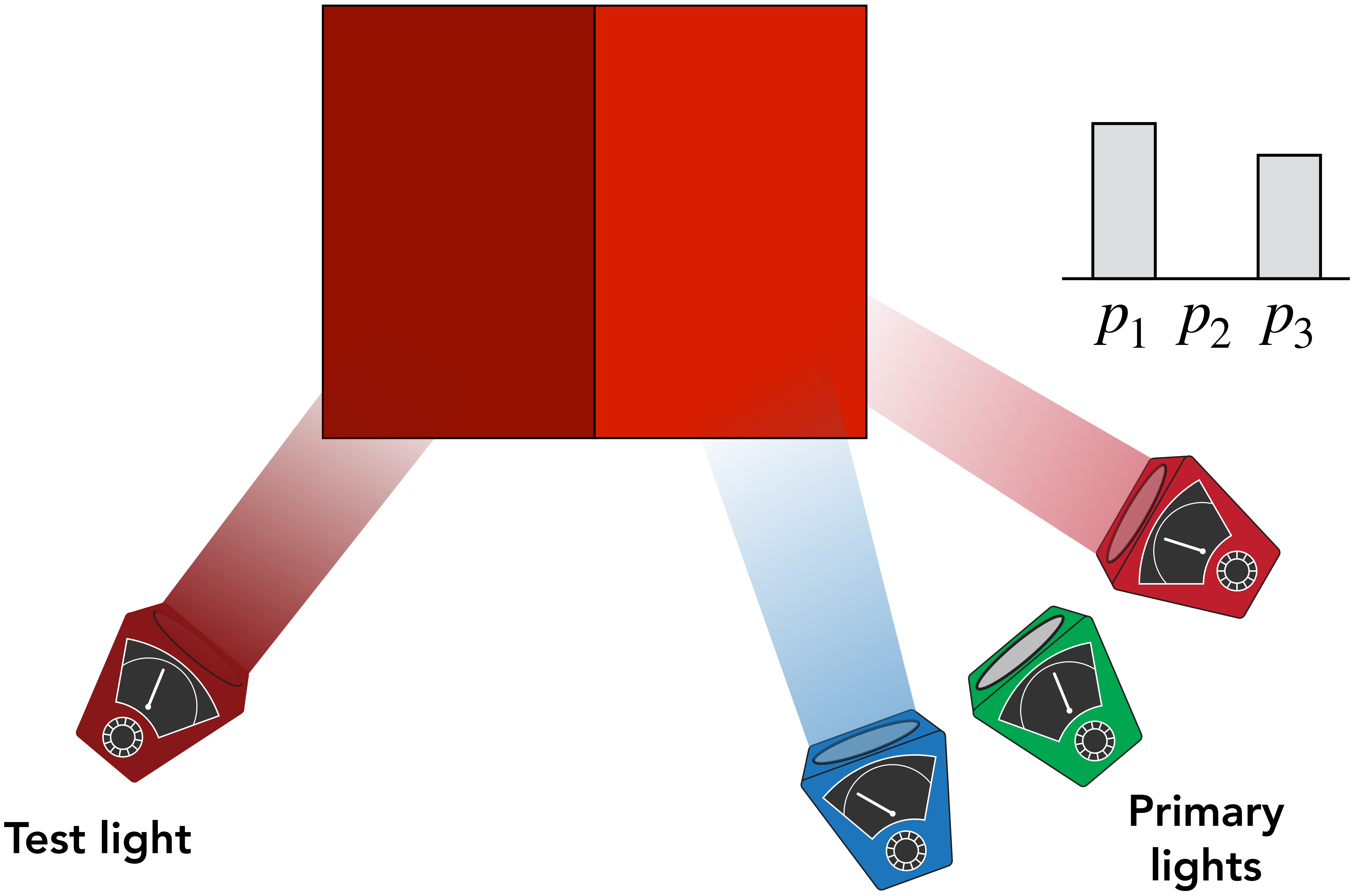
Slide credit: Kotani, Durand, Freeman

Experiment 2: Out of Gamut



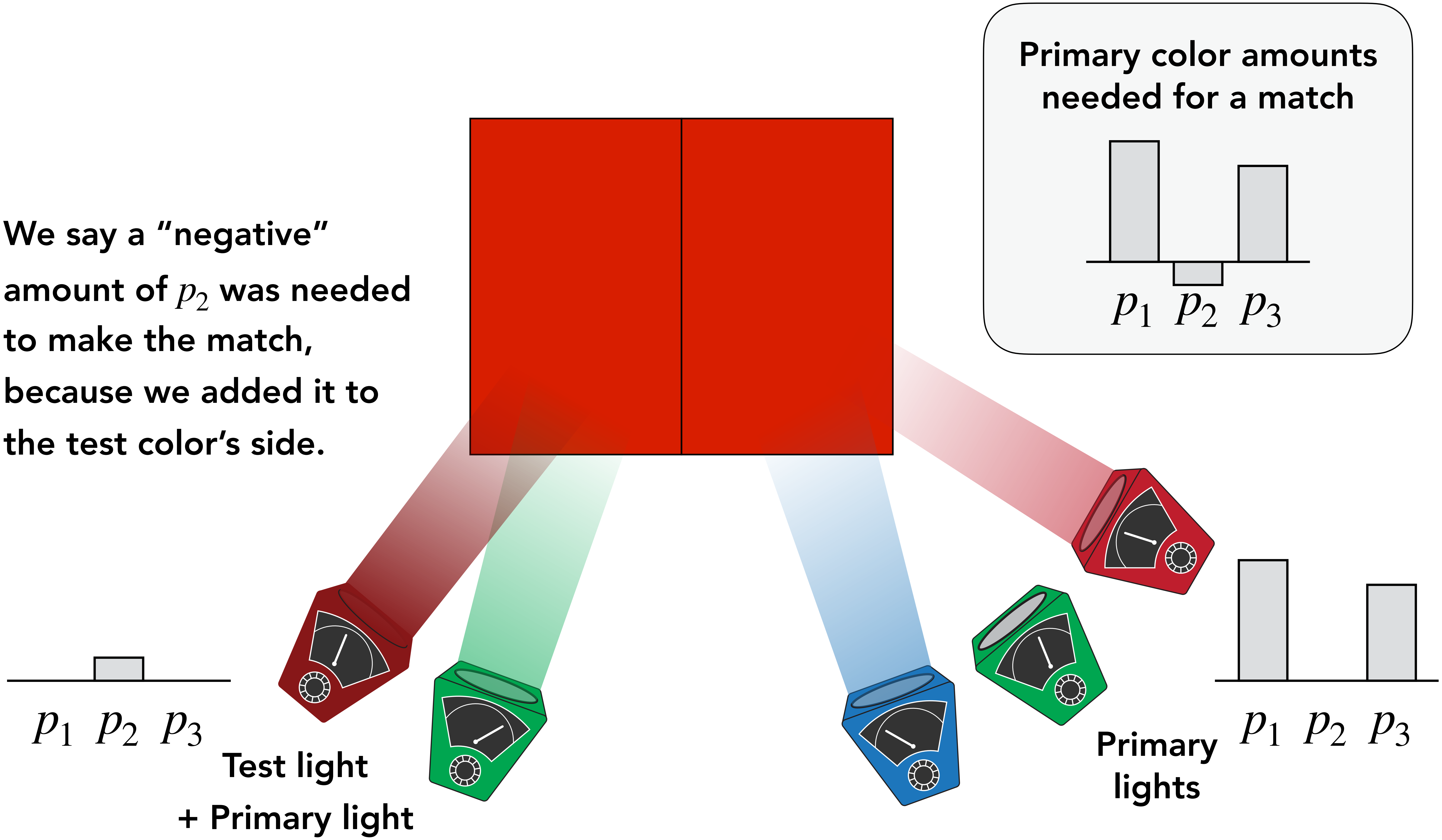
Slide credit: Kotani, Durand, Freeman

Experiment 2: Out of Gamut



Slide credit: Kotani, Durand, Freeman

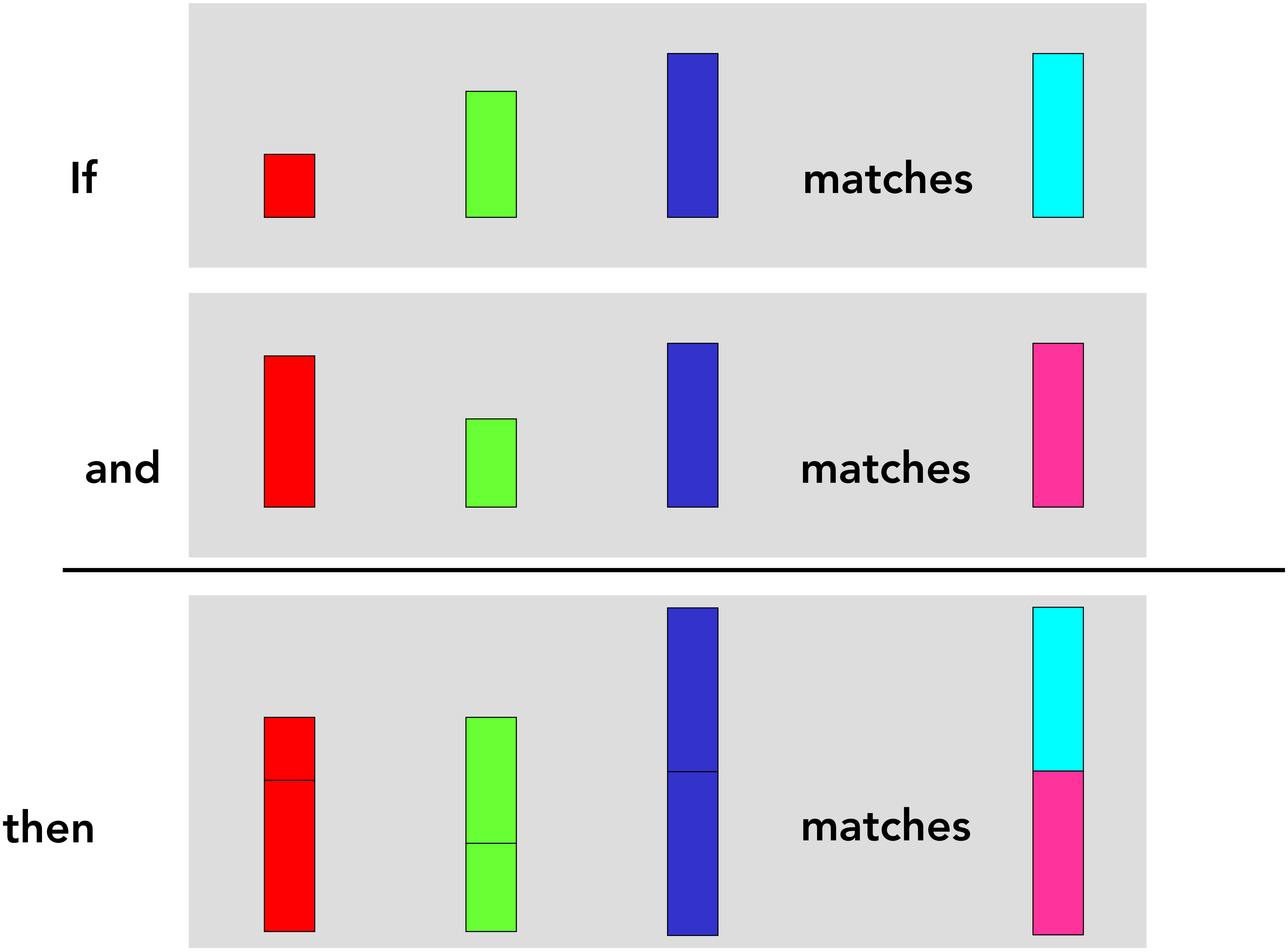
Experiment 2: Out of Gamut



We say a “negative” amount of p_2 was needed to make the match, because we added it to the test color’s side.

Slide credit: Kotani, Durand, Freeman

The Color Matching Experiment is Linear



If

matches

and

matches

then

matches

Brian Wandell

What is the Dimensionality of Human Color Perception?

And how do we know?

What is the definition of “dimension” here?

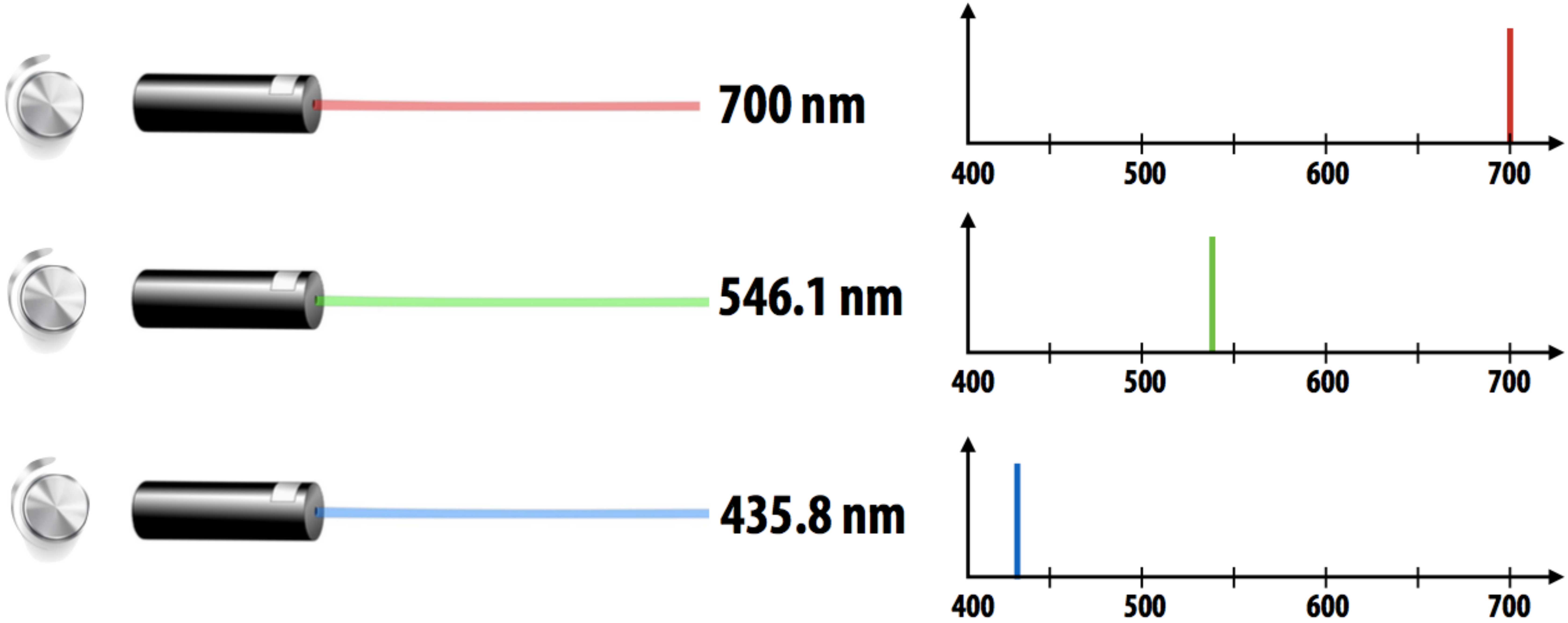
- We can appeal to linear systems theory, where “dimension” equals the rank of a basis for the linear space.

In the color matching experiment, empirically one finds:

- For subjects with “normal” color vision, three primary colors are necessary and sufficient to match any test color. Four primaries work but are unnecessary; two are insufficient.
- For red-green colorblind subjects, only two primary colors are necessary and sufficient to match any test color.

CIE RGB Color Matching Experiment

Same setup as additive color matching before, but primaries are monochromatic light (single wavelength) of the following wavelengths defined by CIE RGB standard

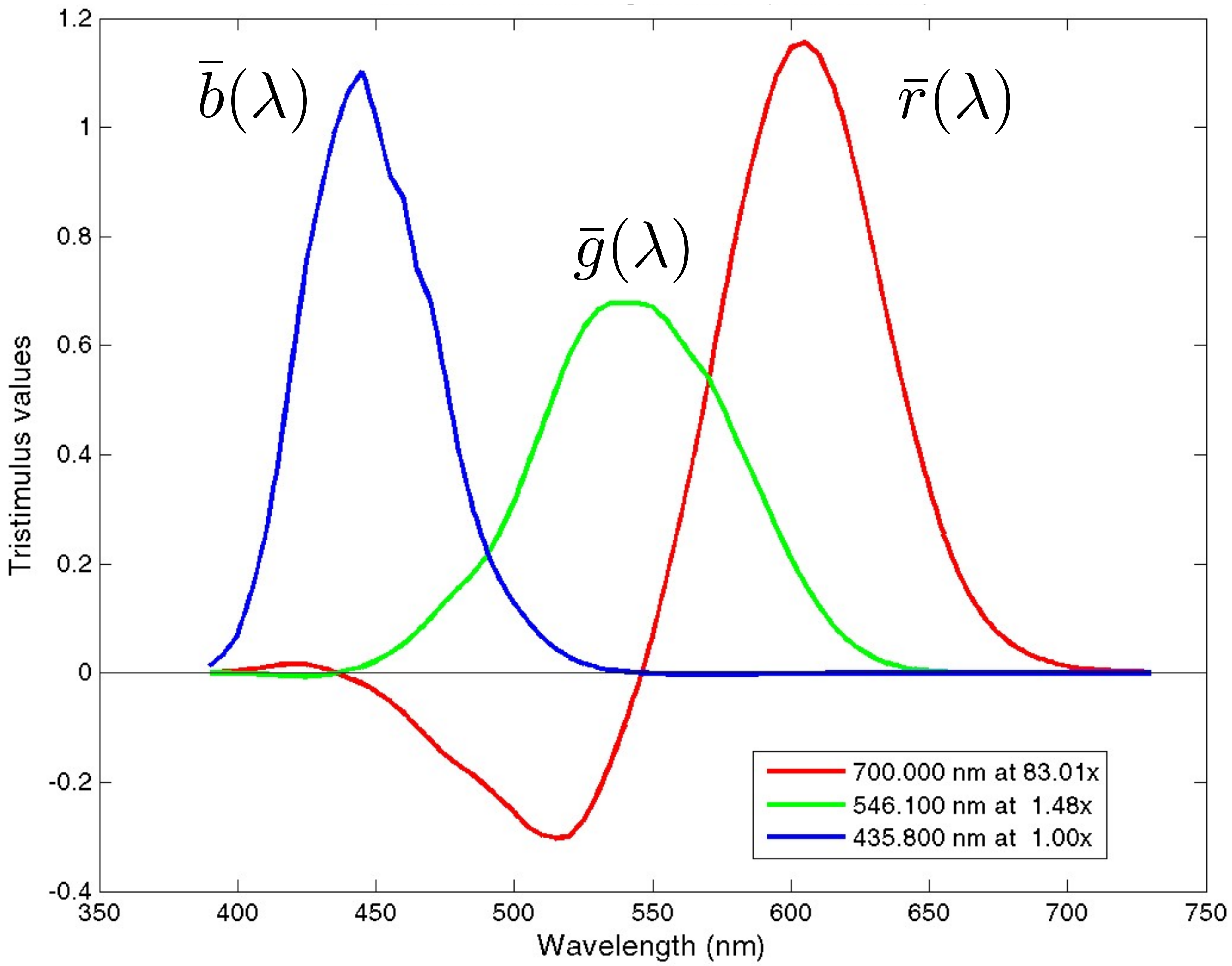


Kayvon Fatahalian

The test light is also a monochromatic light

CIE RGB Color Matching Functions

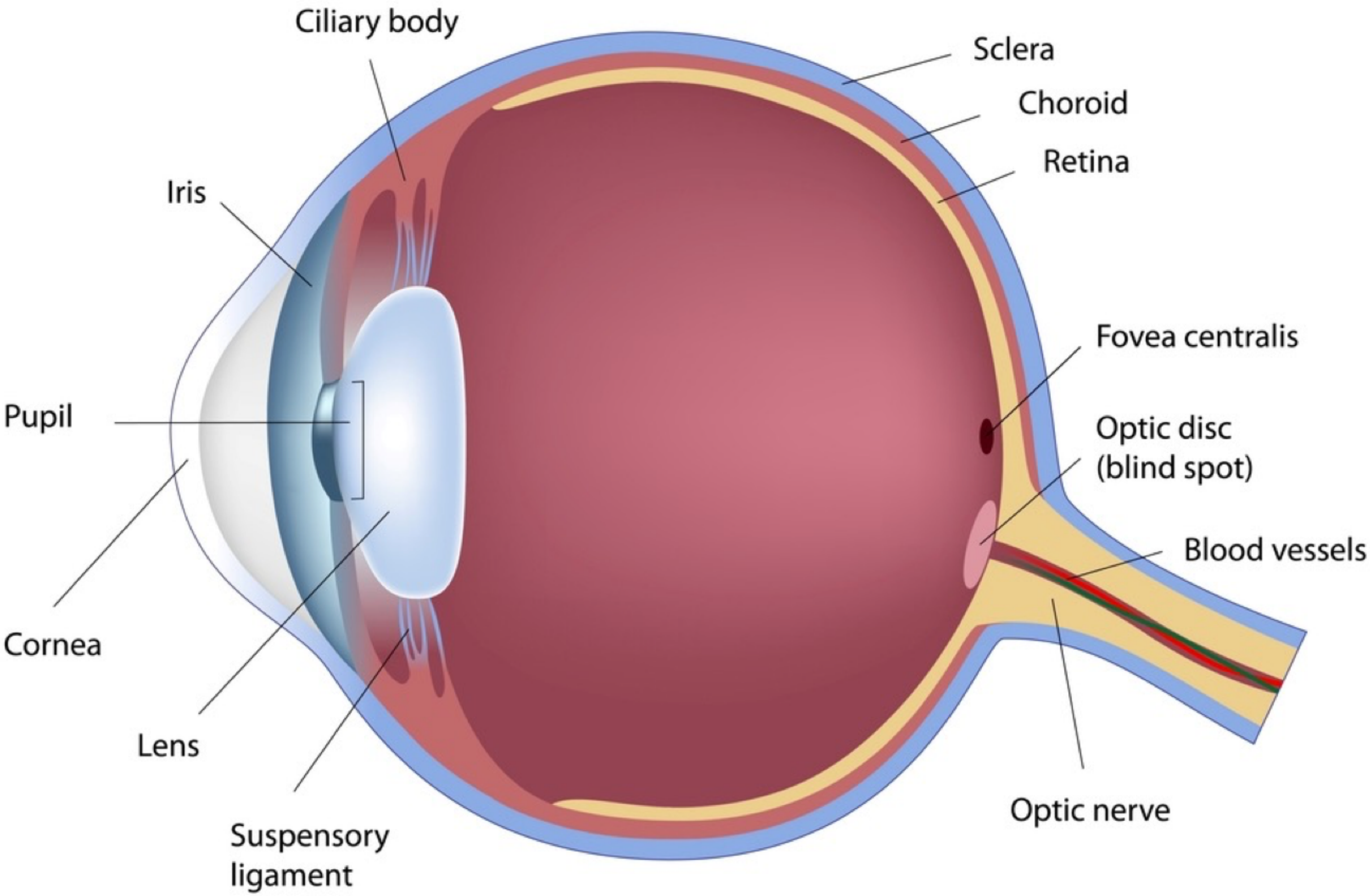
Graph plots how much of each CIE RGB primary light must be combined to match a monochromatic light of wavelength given on x-axis



Careful: these are not response curves or primary spectra!

Biological Basis of Color

Anatomy of The Human Eye



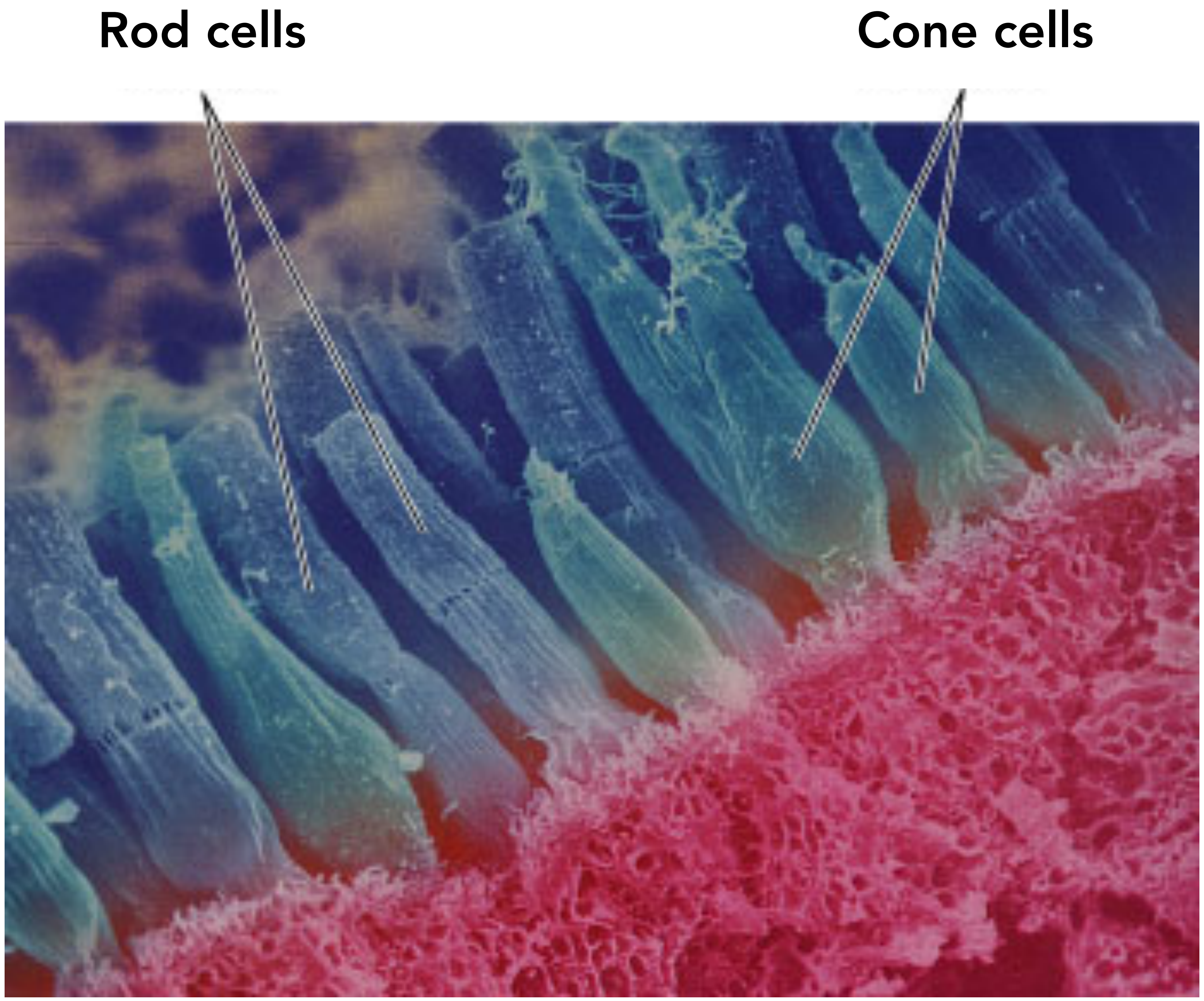
Retinal Photoreceptor Cells: Rods and Cones

Rods are primary receptors in very low light ("scotopic" conditions), e.g. dim moonlight

- ~120 million rods in eye
- Perceive only shades of gray, no color

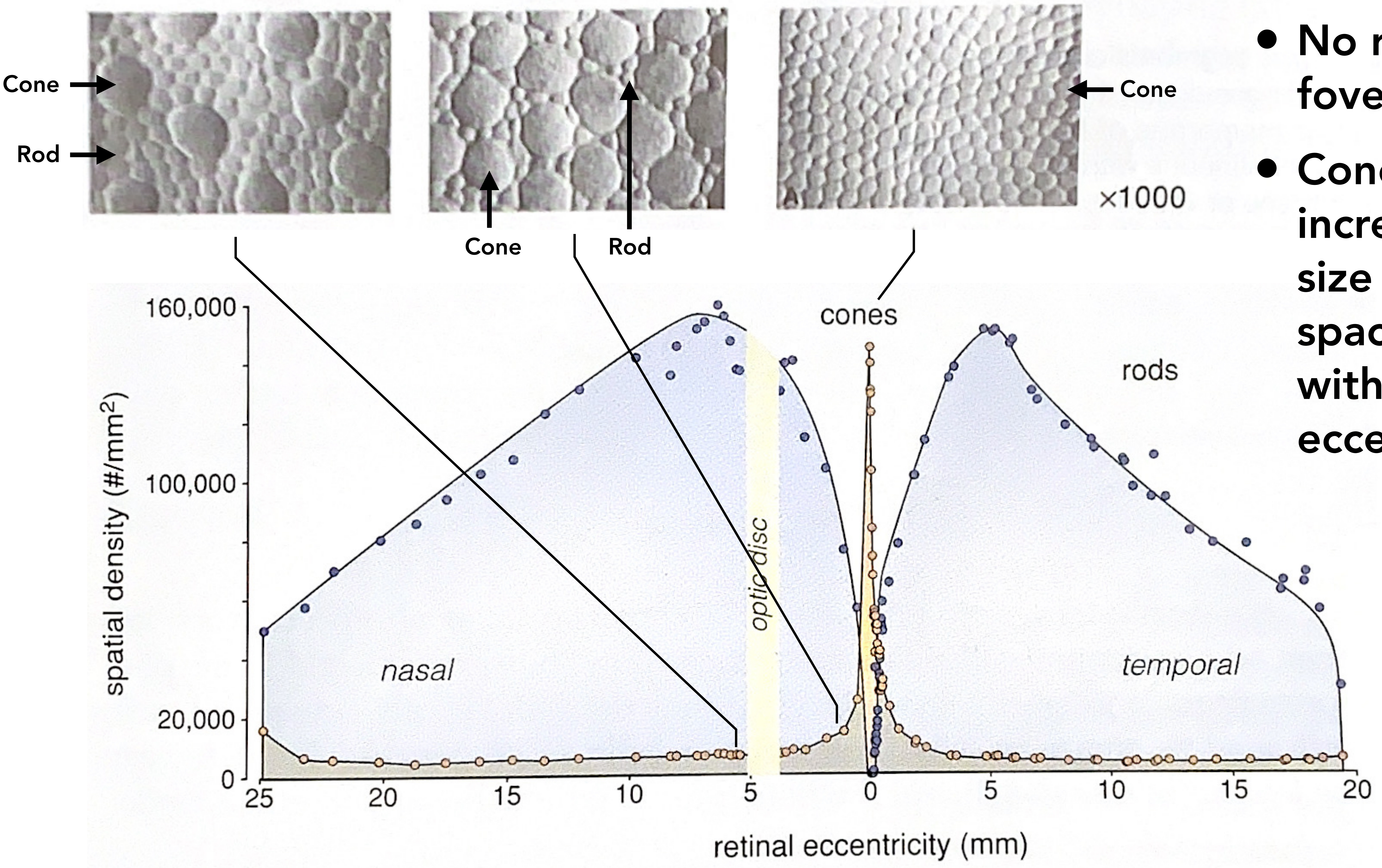
Cones are primary receptors in typical light levels ("photopic")

- ~6-7 million cones in eye
- Three types of cones, each with different spectral sensitivity
- Provide sensation of color



http://ebooks.bfwpub.com/life.php Figure 45.18

Photoreceptor Size and Distribution Vary Across Retina

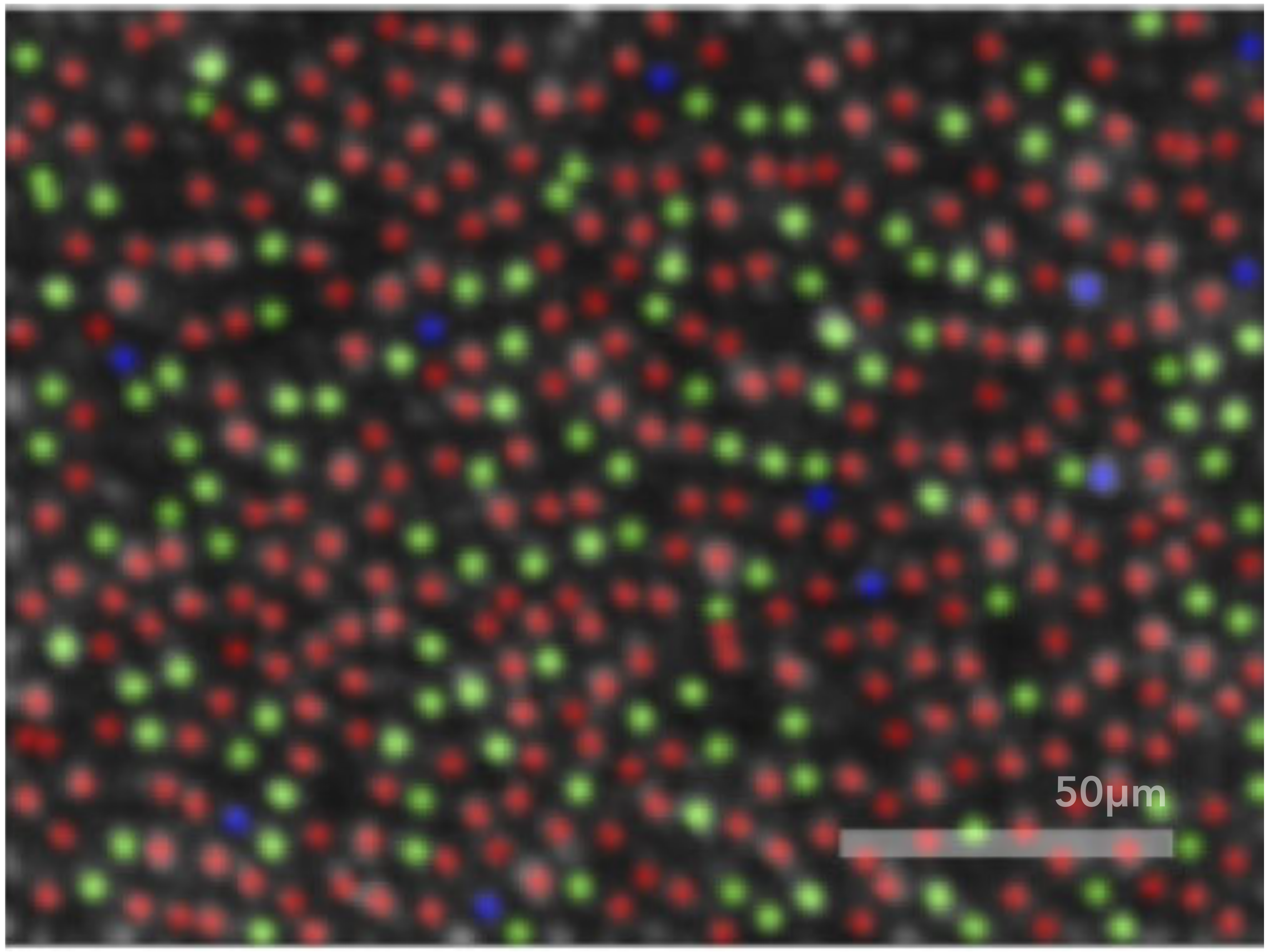


after Østerberg, 1935; as modified by Rodieck, 1988

- No rods in fovea
- Cones increase in size and spacing with eccentricity

Rodieck, p. 42

On the Retina, Three Types of Cone Cells



Sabesan Lab, UW. Pandiyan et al. 2020.

Three types of cone cells: S, M, and L (corresponding to peak response at short, medium, and long wavelengths)

Credit: http://depts.washington.edu/sabaolab/

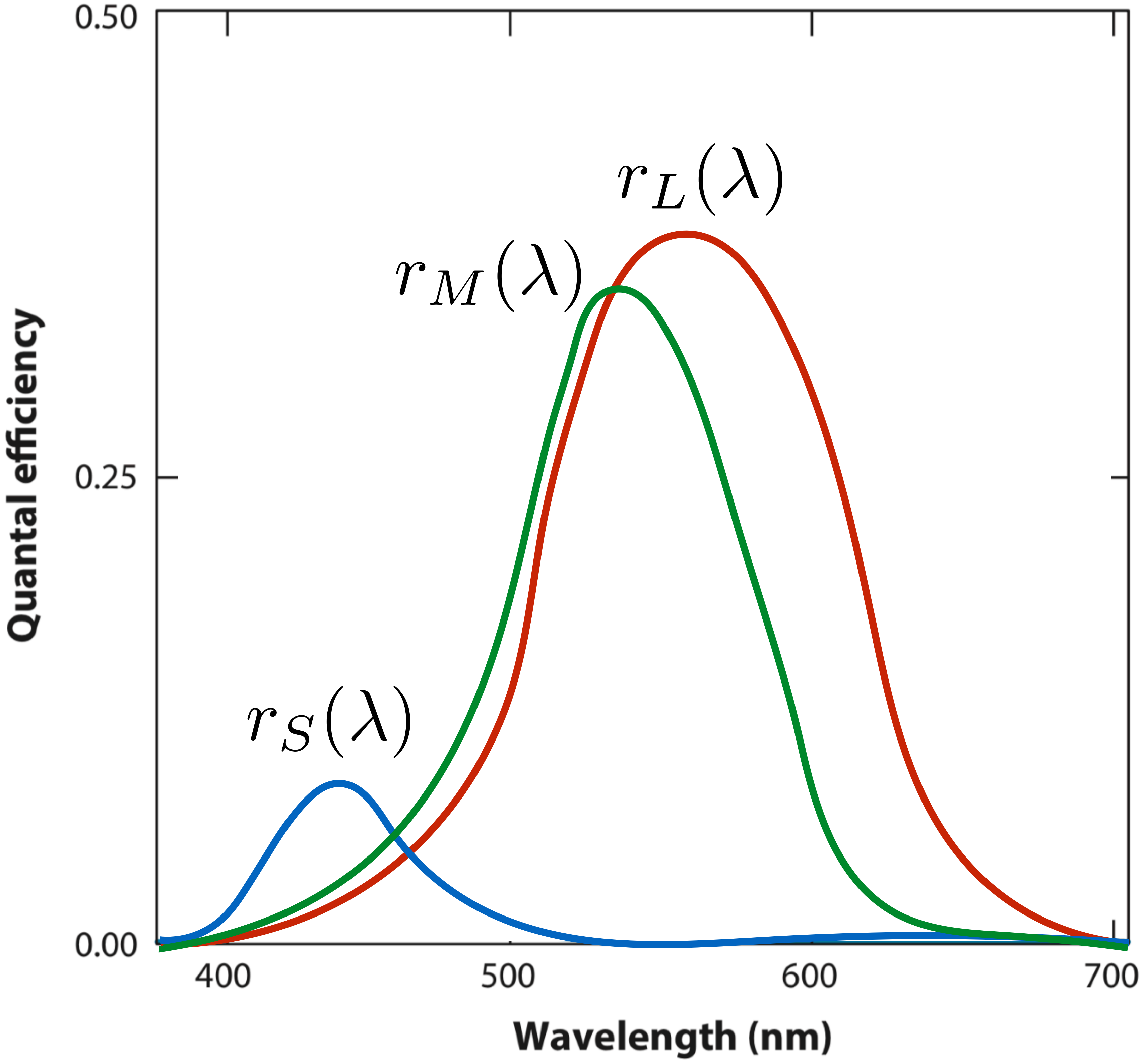
Spectral Response of Human Cone Cells

Instead of one detector as before, now we have three detectors (S, M, L cone cells), each with a different spectral response curve

$$S = \int r_S(\lambda) s(\lambda)\, d\lambda$$

$$M = \int r_M(\lambda) s(\lambda)\, d\lambda$$

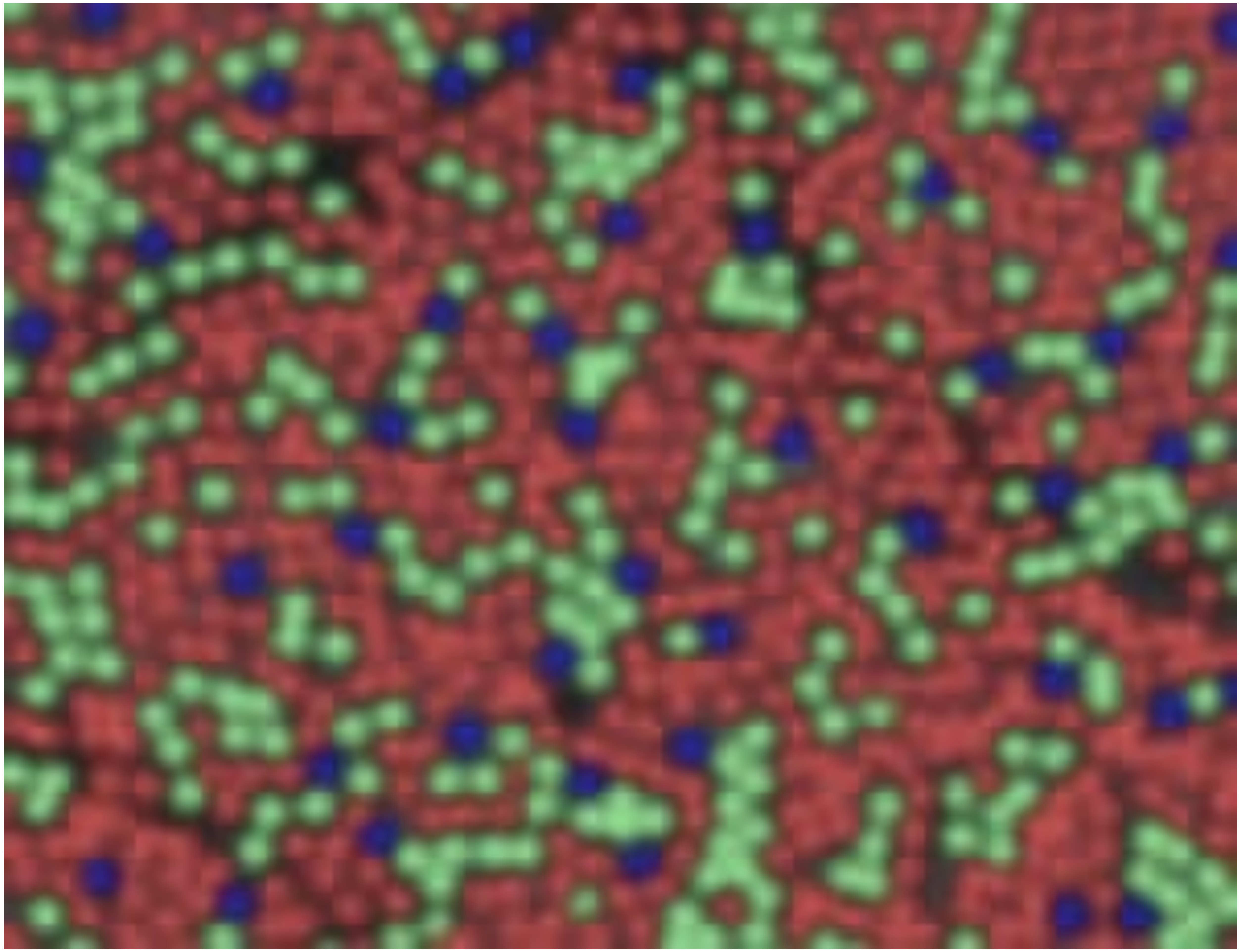
$$L = \int r_L(\lambda) s(\lambda)\, d\lambda$$



Brainard, Color and the Cone Mosaic, 2015.

Example: Spectral Response of Human Cone Cells

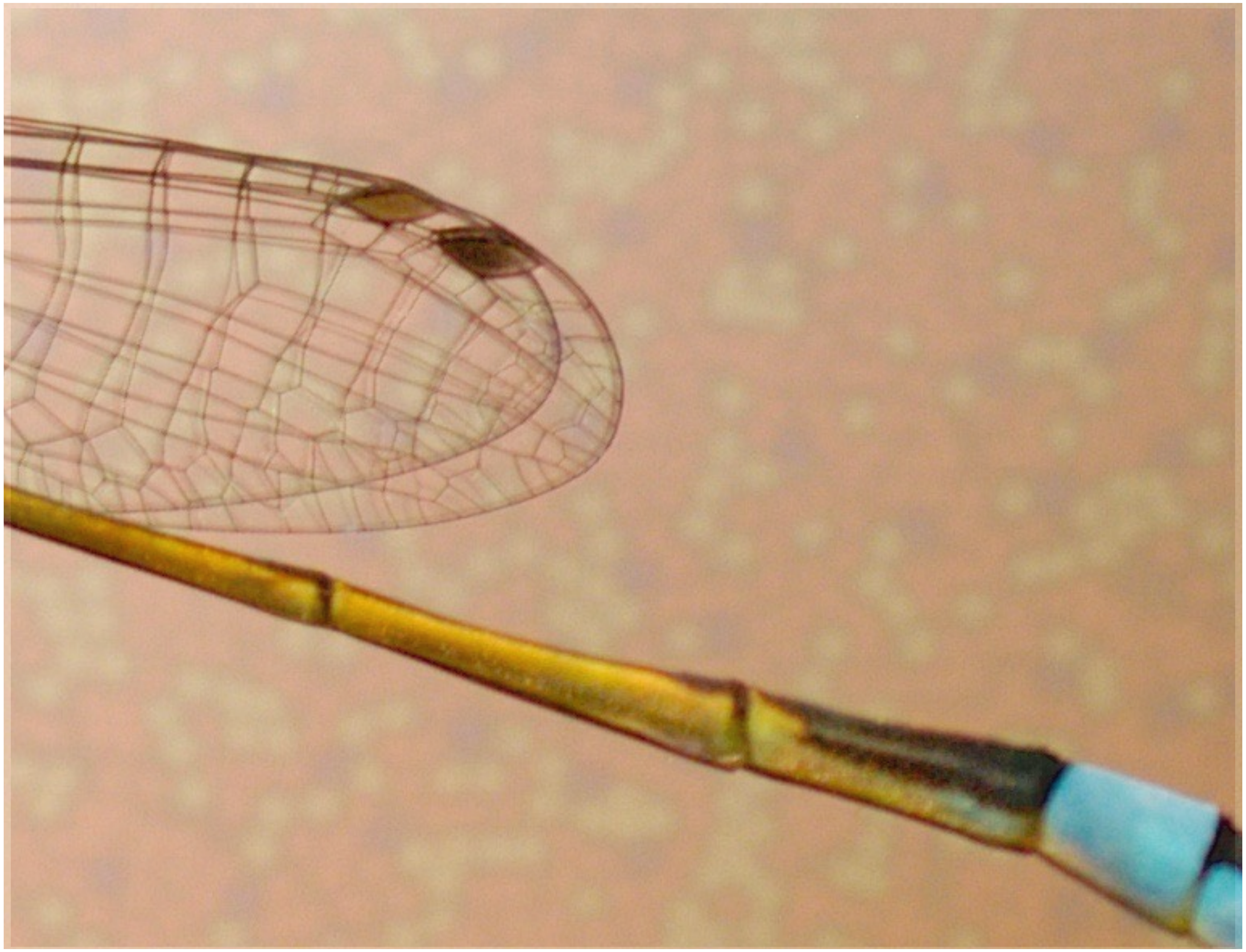
Example: Spectral Response of Human Cone Cells



Scene projected onto retina

Credit: Sabesan, http://depts.washington.edu/sabaolab/

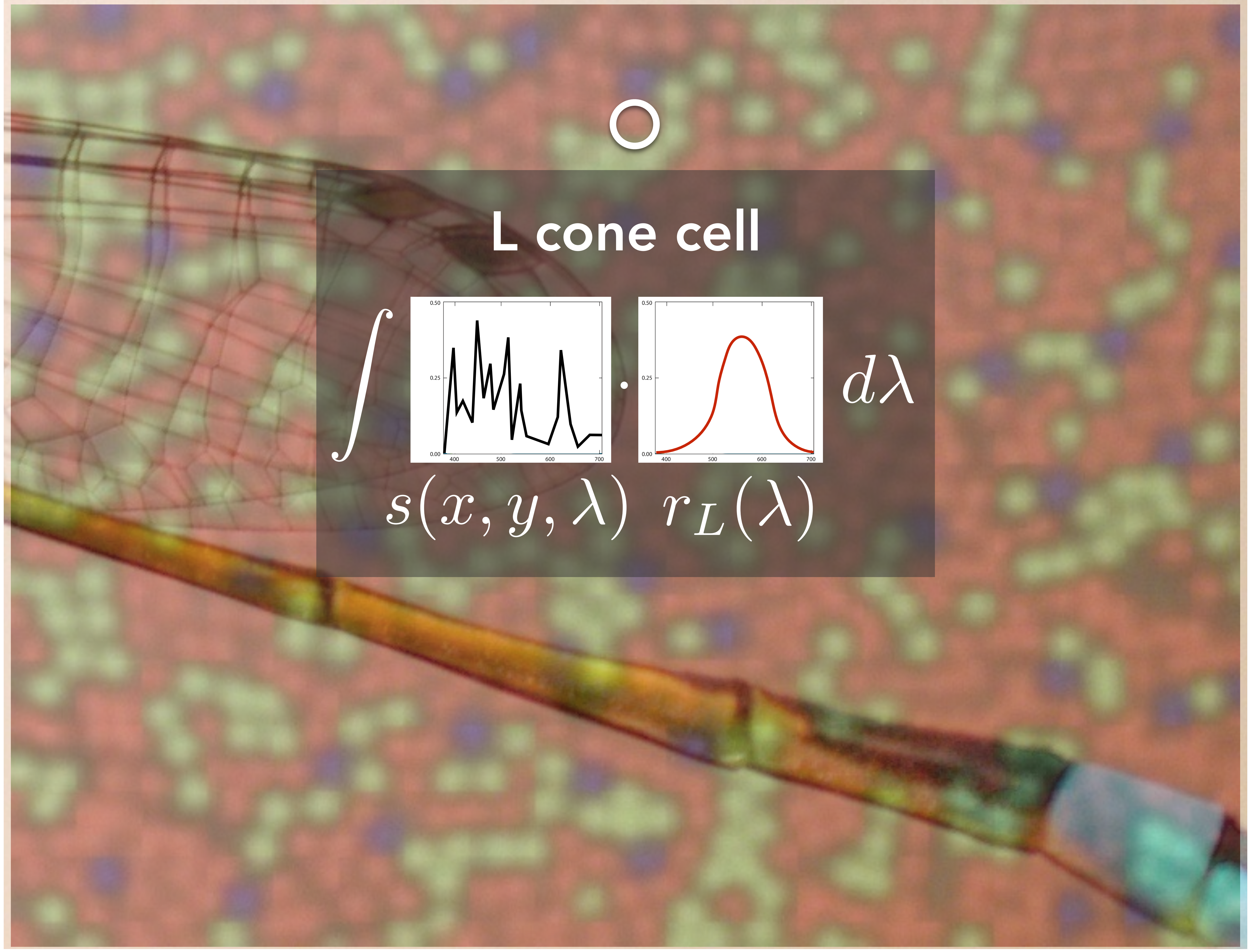
Example: Spectral Response of Human Cone Cells



Scene projected onto retina

Credit: Sabesan, http://depts.washington.edu/sabaolab/

Example: Spectral Response of Human Cone Cells

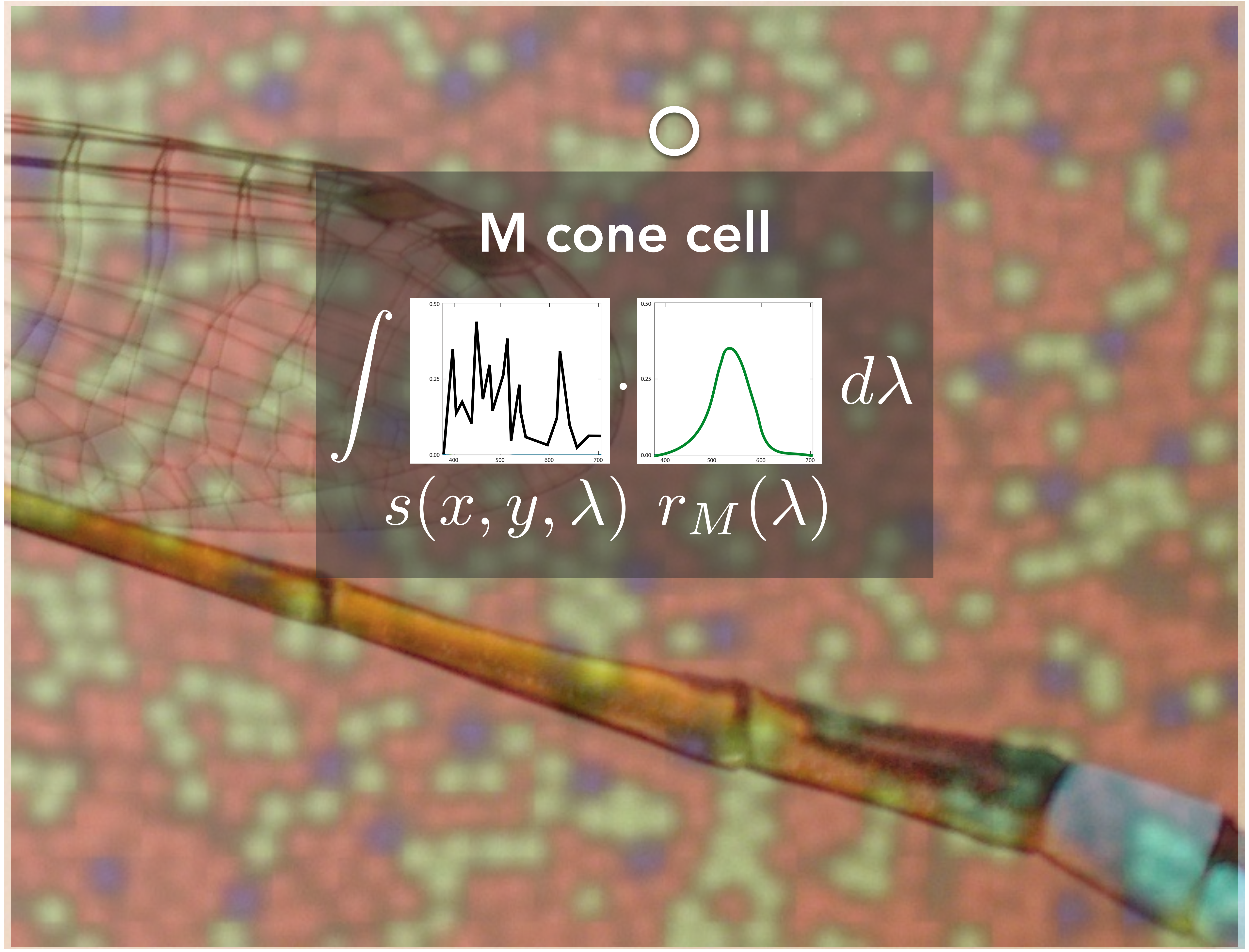


L cone cell

$$\int s(x,y,\lambda) \cdot r_L(\lambda) \, d\lambda$$

Credit: Sabesan, http://depts.washington.edu/sabaolab/

Example: Spectral Response of Human Cone Cells

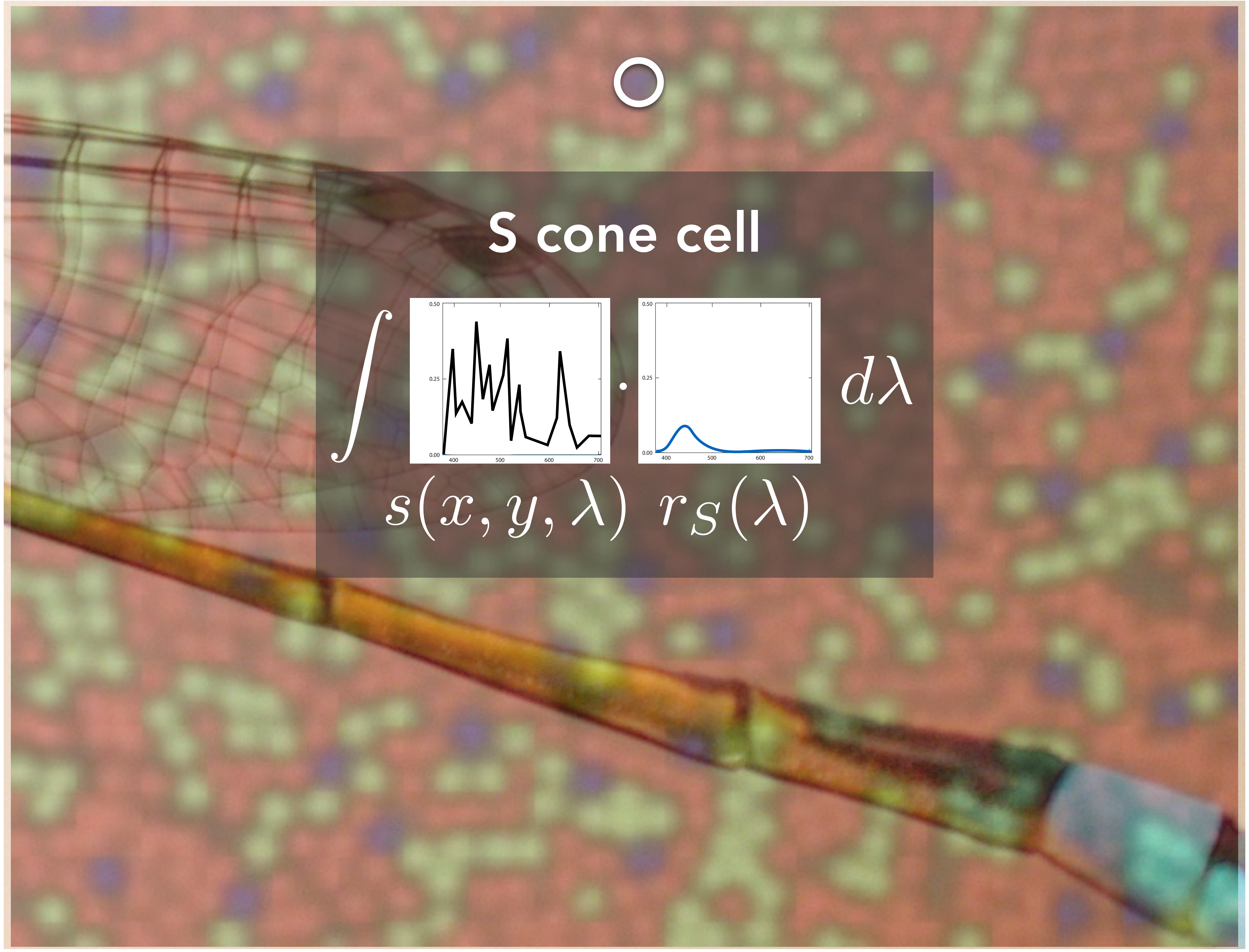


M cone cell

$$\int s(x, y, \lambda) \cdot r_M(\lambda) \, d\lambda$$

Credit: Sabesan, http://depts.washington.edu/sabaolab/

Example: Spectral Response of Human Cone Cells



S cone cell

$$\int s(x,y,\lambda)\cdot r_S(\lambda)\,d\lambda$$

Credit: Sabesan, http://depts.washington.edu/sabaolab/

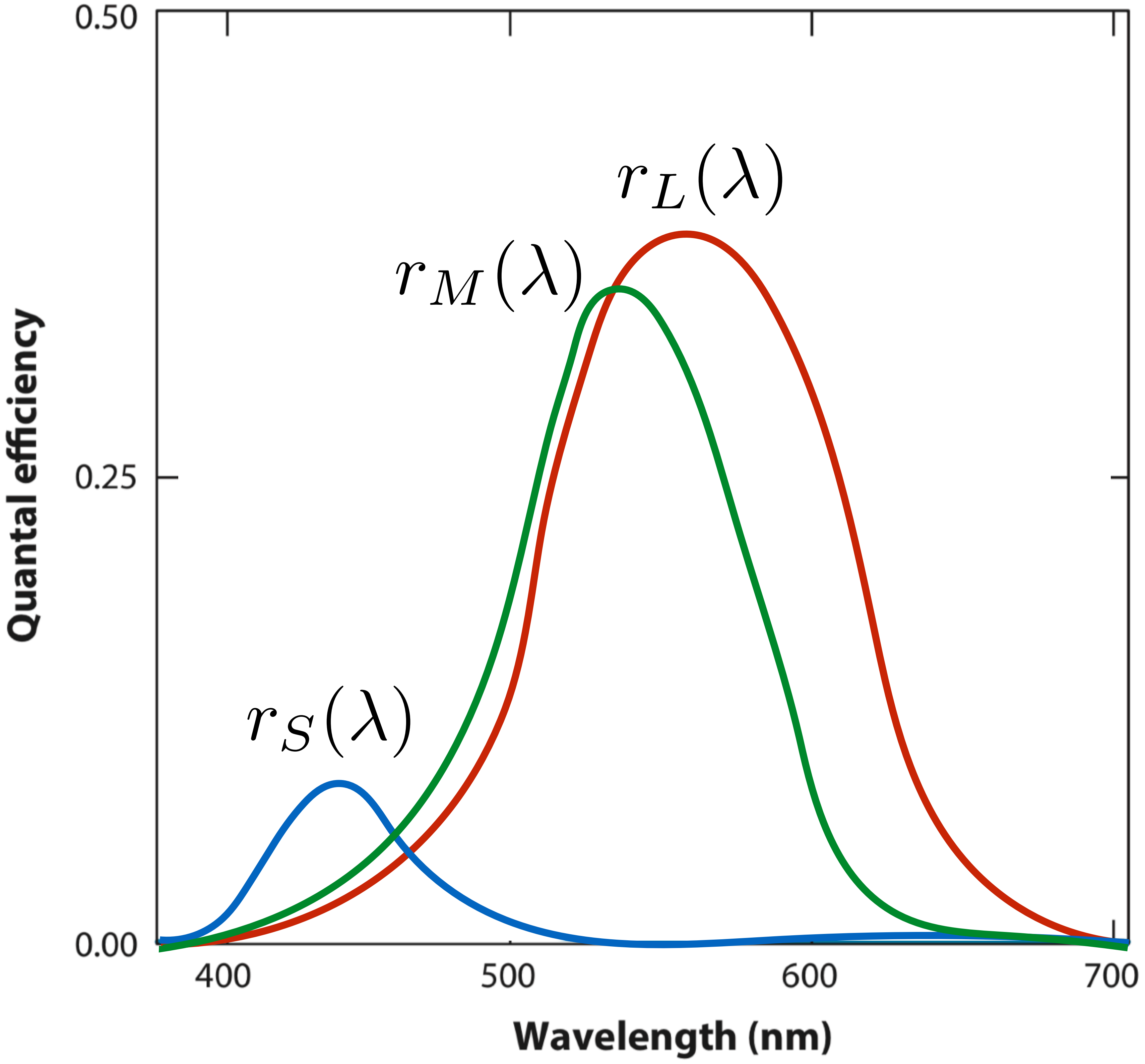
Spectral Response of Human Cone Cells

Instead of one detector as before, now we have three detectors (S, M, L cone cells), each with a different spectral response curve

$$S = \int r_S(\lambda) s(\lambda)\, d\lambda$$

$$M = \int r_M(\lambda) s(\lambda)\, d\lambda$$

$$L = \int r_L(\lambda) s(\lambda)\, d\lambda$$



Brainard, Color and the Cone Mosaic, 2015.

Spectral Response of Human Cone Cells

Instead of one detector as before, now we have three detectors (S, M, L cone cells), each with a different spectral response curve

Written as vector dot products:

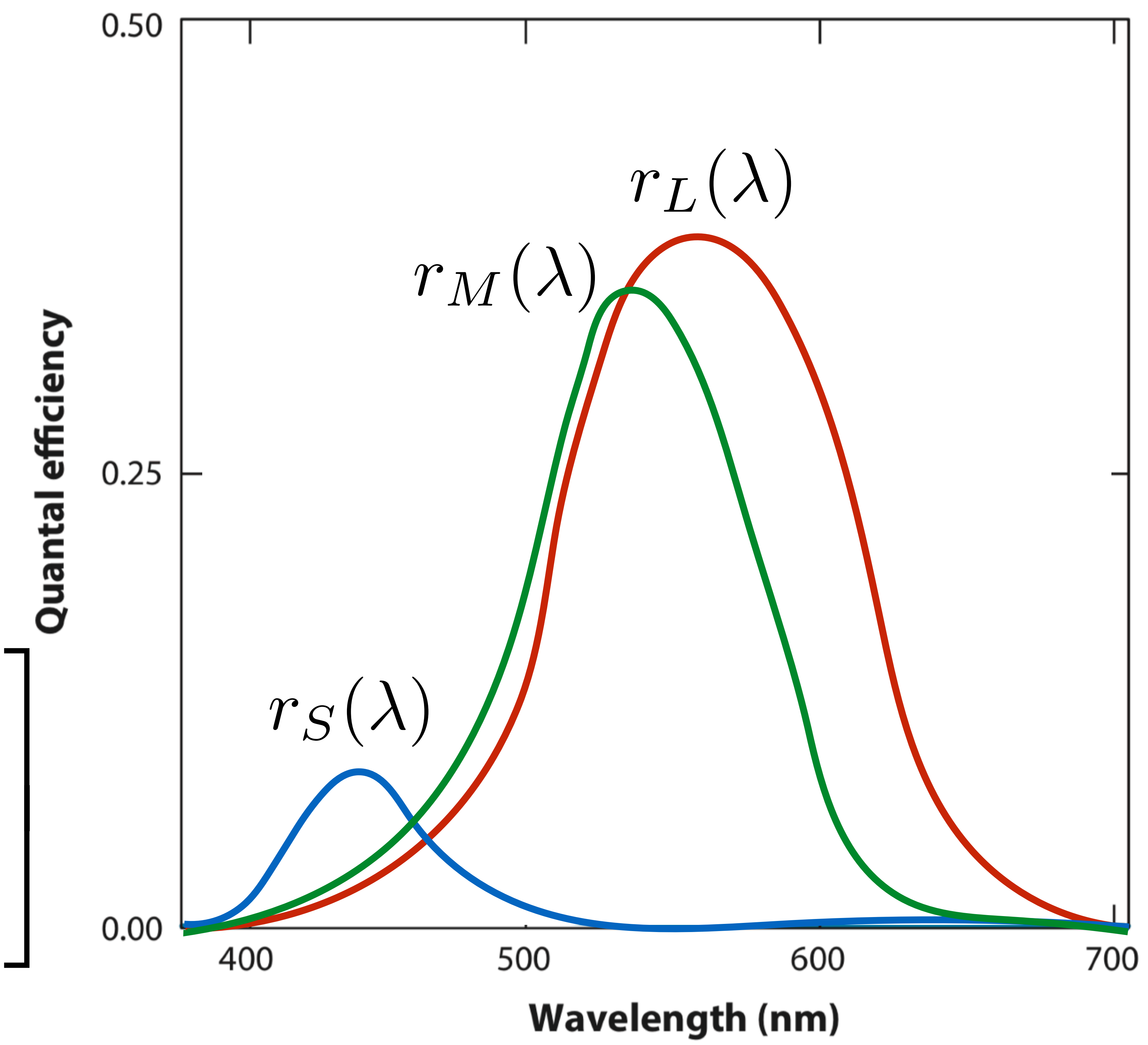
$$S = r_S \cdot s$$

$$M = r_M \cdot s$$

$$L = r_L \cdot s$$

Matrix formulation:

$$\begin{bmatrix} S \\ M \\ L \end{bmatrix} = \begin{bmatrix} \text{—} & r_S & \text{—} \\ \text{—} & r_M & \text{—} \\ \text{—} & r_L & \text{—} \end{bmatrix} \begin{bmatrix} | \\ s \\ | \end{bmatrix}$$



Brainard, Color and the Cone Mosaic, 2015.

Dimensionality Reduction From ∞ to 3

At each position on the human retina:

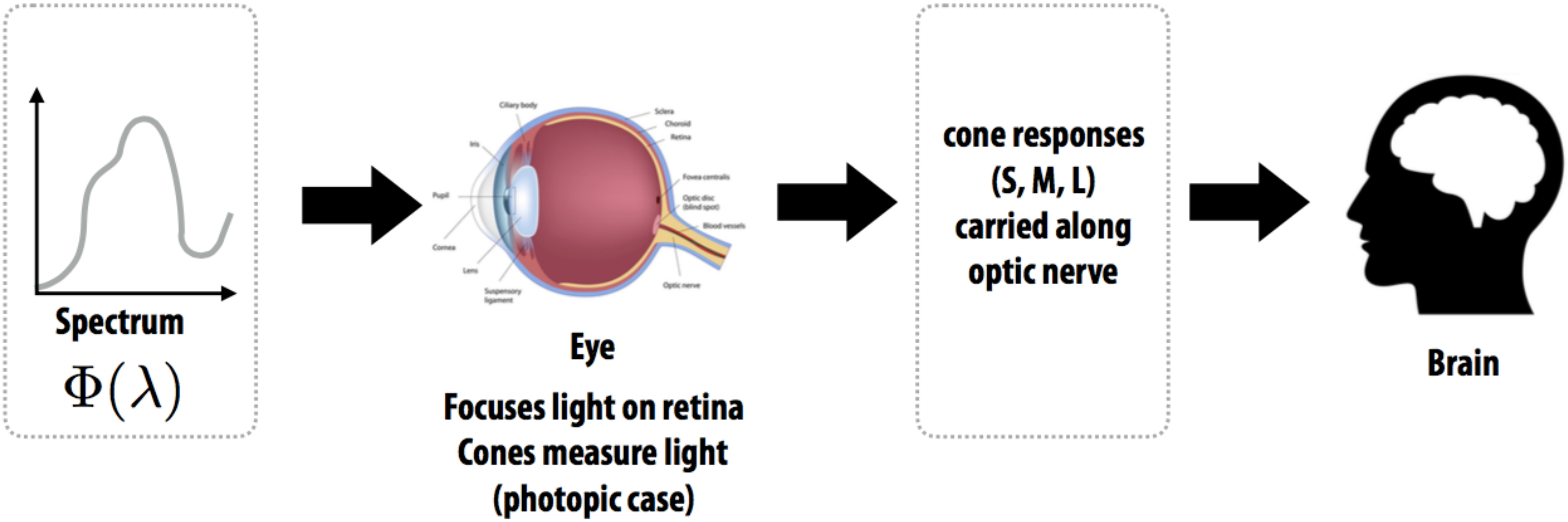
- SPD is a function of wavelength (∞ - dimensional signal)
- 3 types of cones near that position produce three scalar values (3 - dimensional signal)

What about 2D images?

- The dimensionality reduction described above is happening at every 2D position in our visual field

The Human Visual System

- Human eye does not measure and brain does not receive information about each wavelength of light
- Rather, the eye measures three response values only (S, M, L) at each position in visual field, and this is only spectral info available to brain
  - This is the result of integrating the incoming spectrum against response functions of S, M, L cones



Spectrum

$\Phi(\lambda)$

Eye

Focuses light on retina
Cones measure light
(photopic case)

cone responses (S, M, L) carried along optic nerve

Brain

Kayvon Fatahalian

Metamerism

Metamers

Metameters are two different spectra (∞-dim) that project to the same (S,M,L) (3-dim) response.

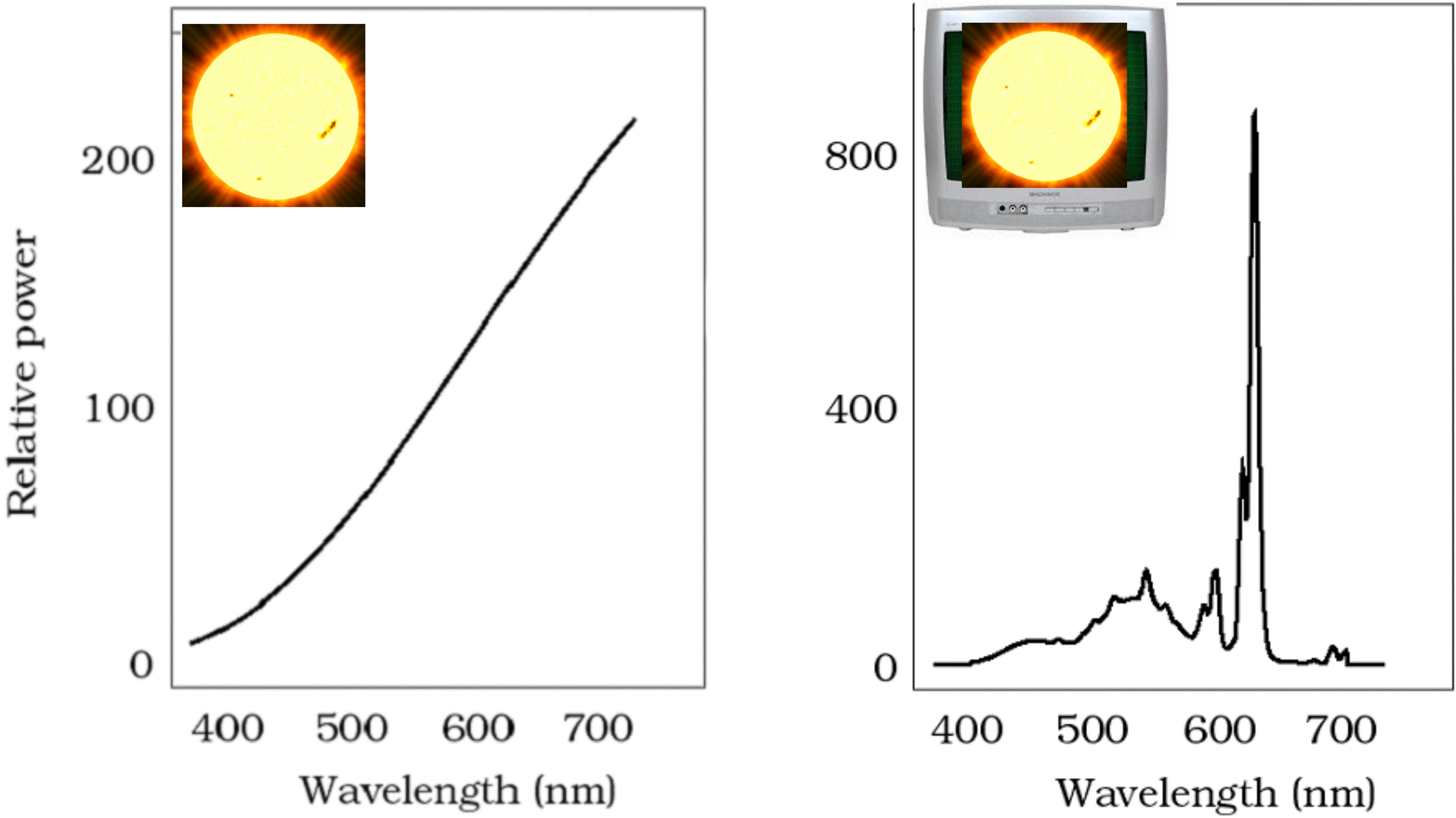
- These will appear to have the same color to a human

The existence of metamers is critical to color reproduction

- Don’t have to reproduce the full spectrum of a real world scene
- Example: A metamer can reproduce the perceived color of a real-world scene on a display with pixels of only three colors

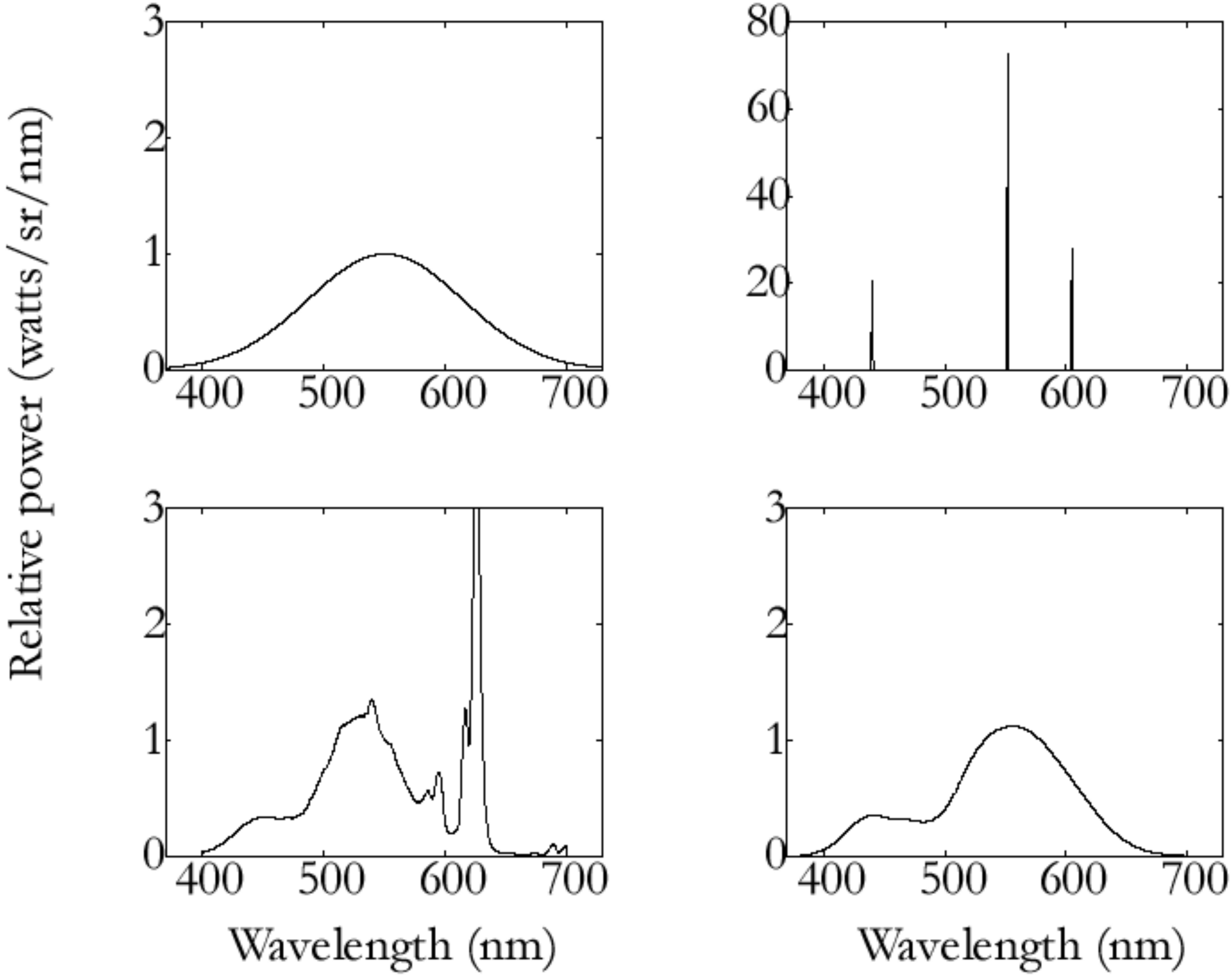
Metamerism

Color matching is an important illusion that is understood quantitatively



Brian Wandell

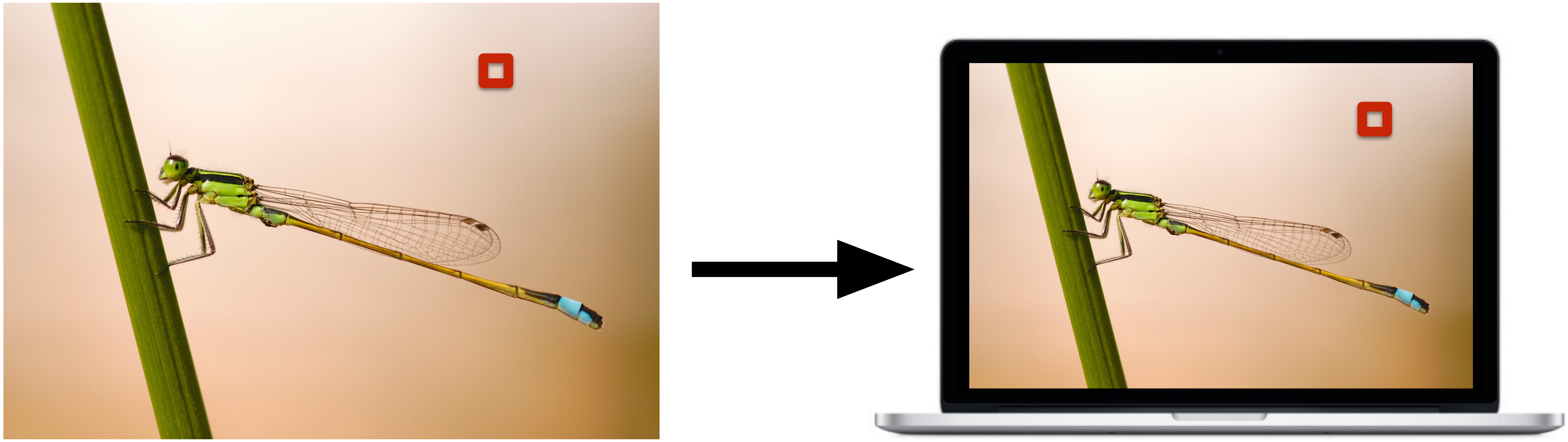
Metamerism is a Big Effect



Brian Wandell

Color Reproduction

Color Reproduction Problem



Target real spectrum $s(\lambda)$

Display outputs spectrum
$R\,s_R(\lambda) + G\,s_G(\lambda) + B\,s_B(\lambda)$

Goal: at each pixel, choose R, G, B values for display so that the output color matches the appearance of the target color in the real world.

Additive Color

- Given a set of primary lights, each with its own spectral distribution (e.g. R,G,B display pixels):

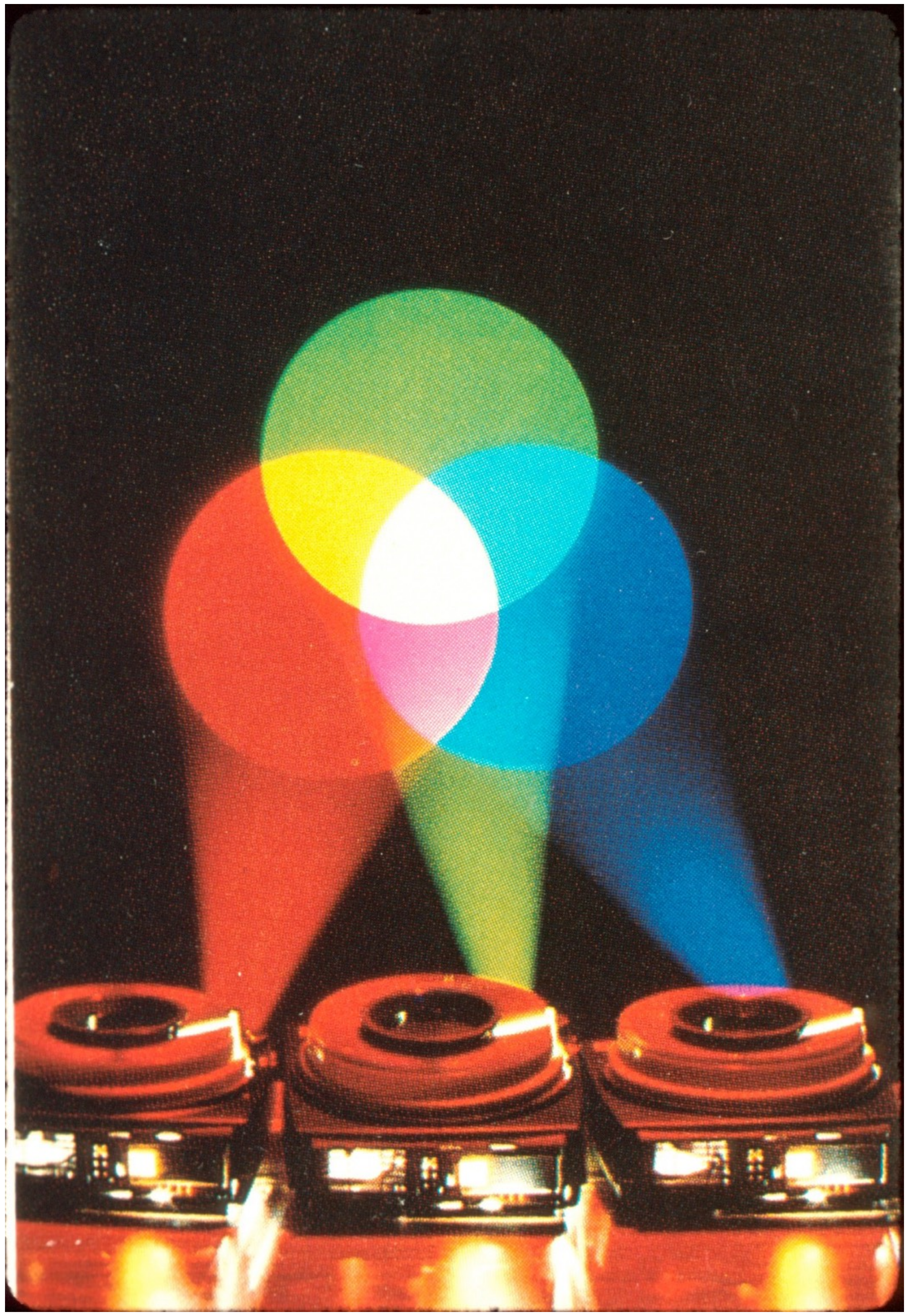
$$s_R(\lambda),\ s_G(\lambda),\ s_B(\lambda)$$

- We can adjust the brightness of these lights and add them together to produce a linear subspace of spectral distribution:

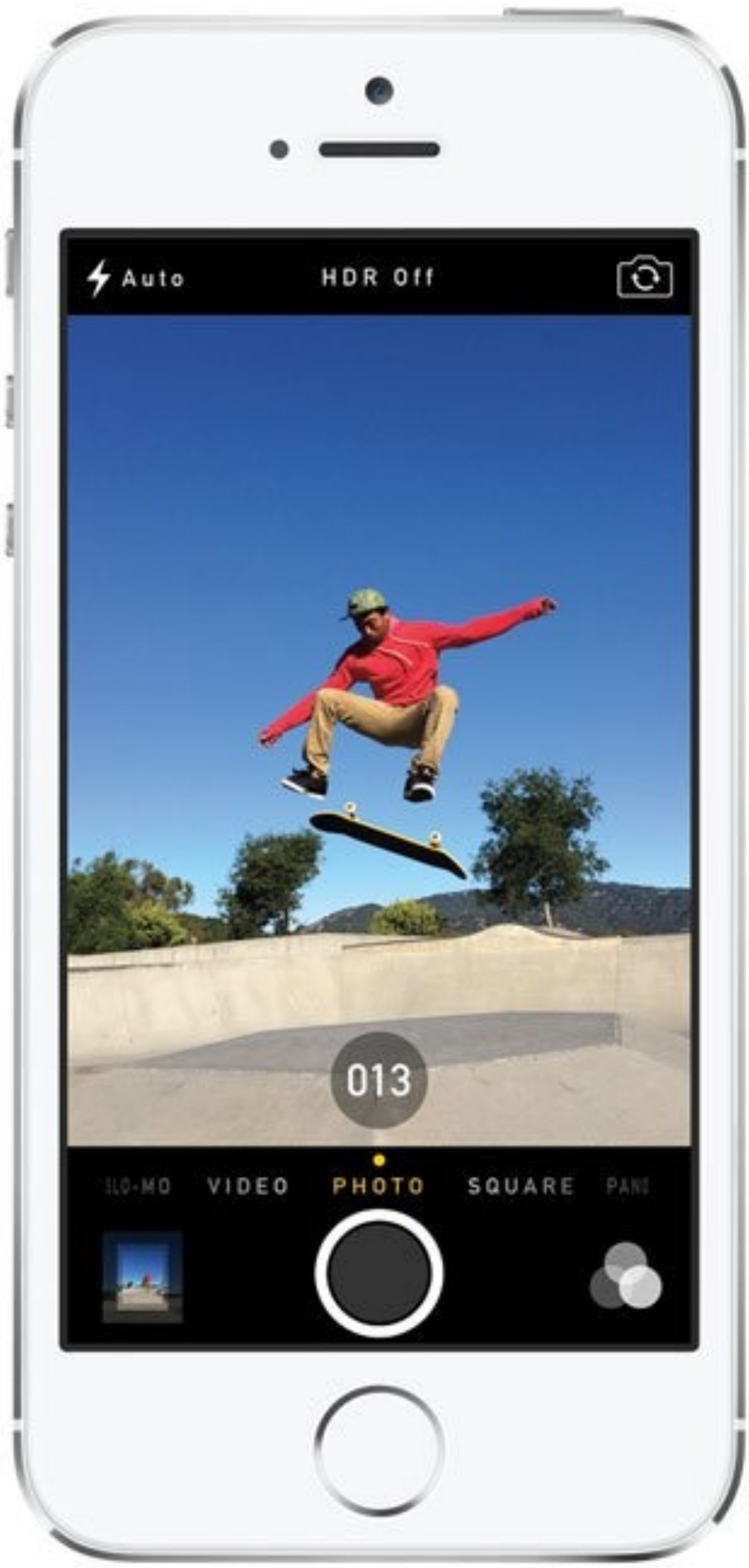
$$R\,s_R(\lambda) + G\,s_G(\lambda) + B\,s_B(\lambda)$$

- The color is now described by the scalar values:

$$R,\ G,\ B$$



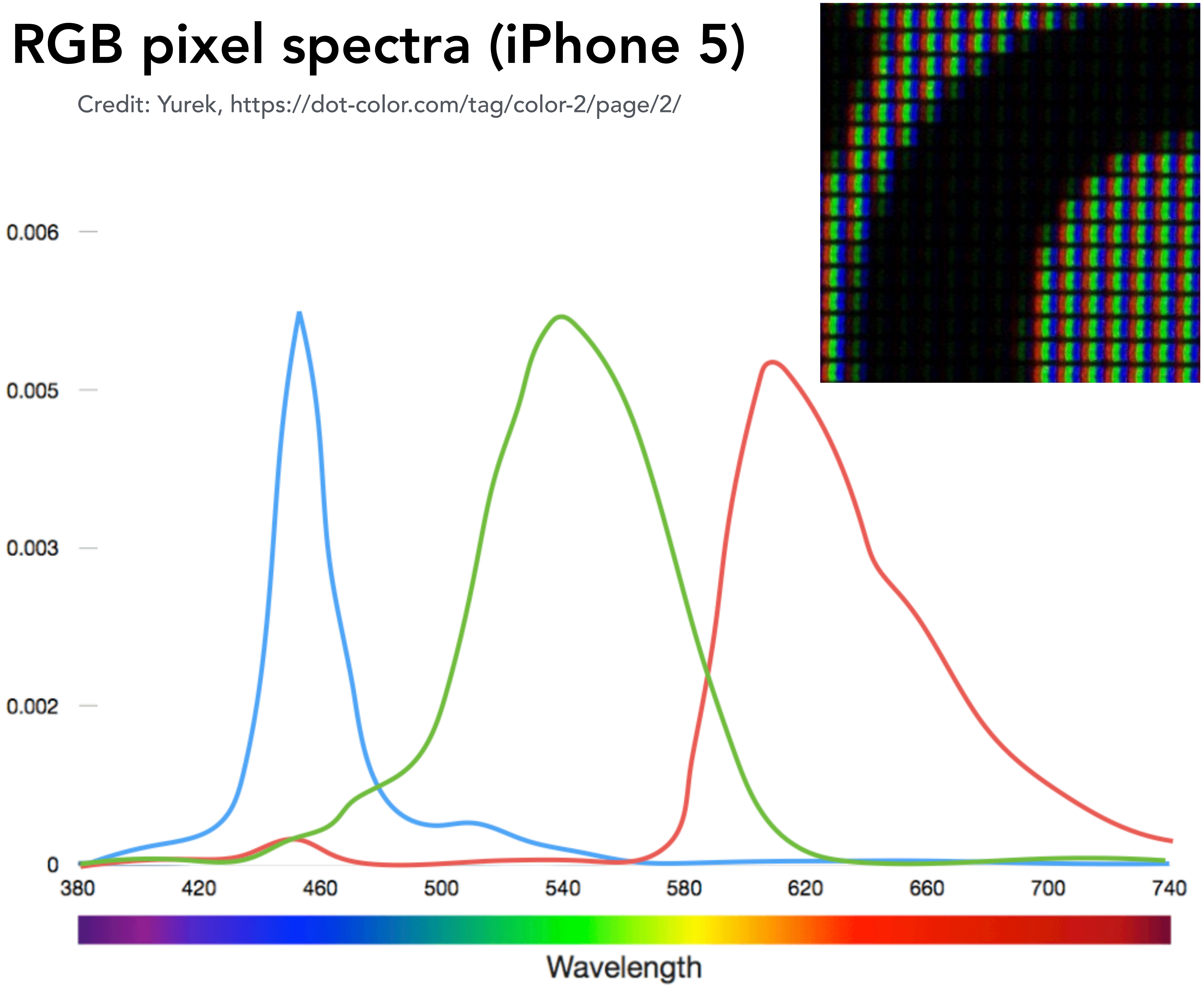
Example RGB Emission Spectra ("Color Primaries") for Phone Display



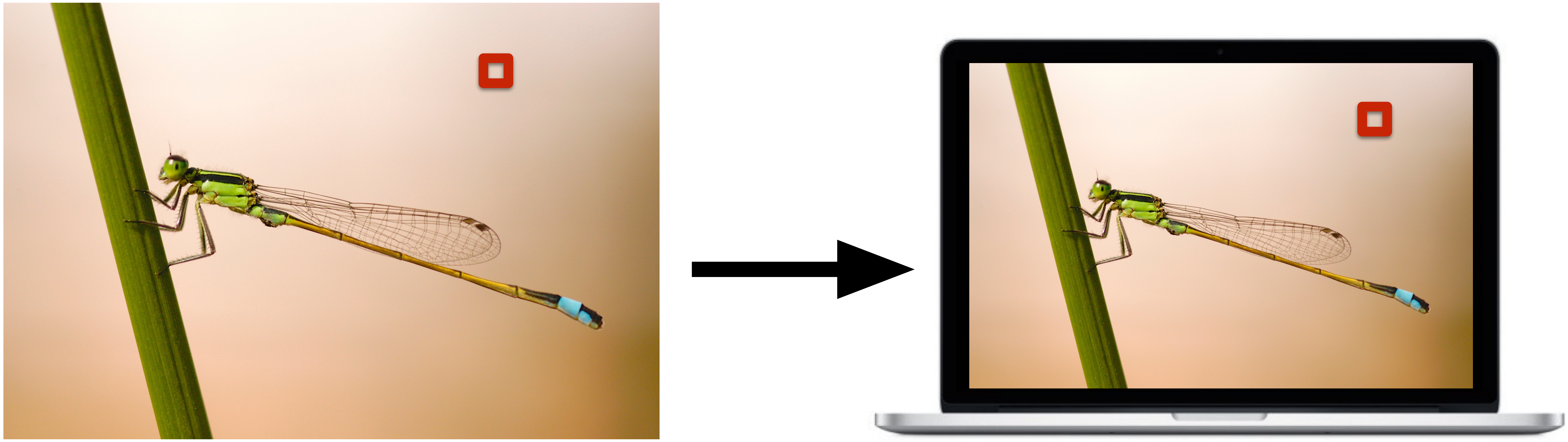
https://www.macrumors.com/roundup/iphone-5s/

RGB pixel spectra (iPhone 5)

Credit: Yurek, https://dot-color.com/tag/color-2/page/2/



Color Reproduction Problem



Target real spectrum $s(\lambda)$

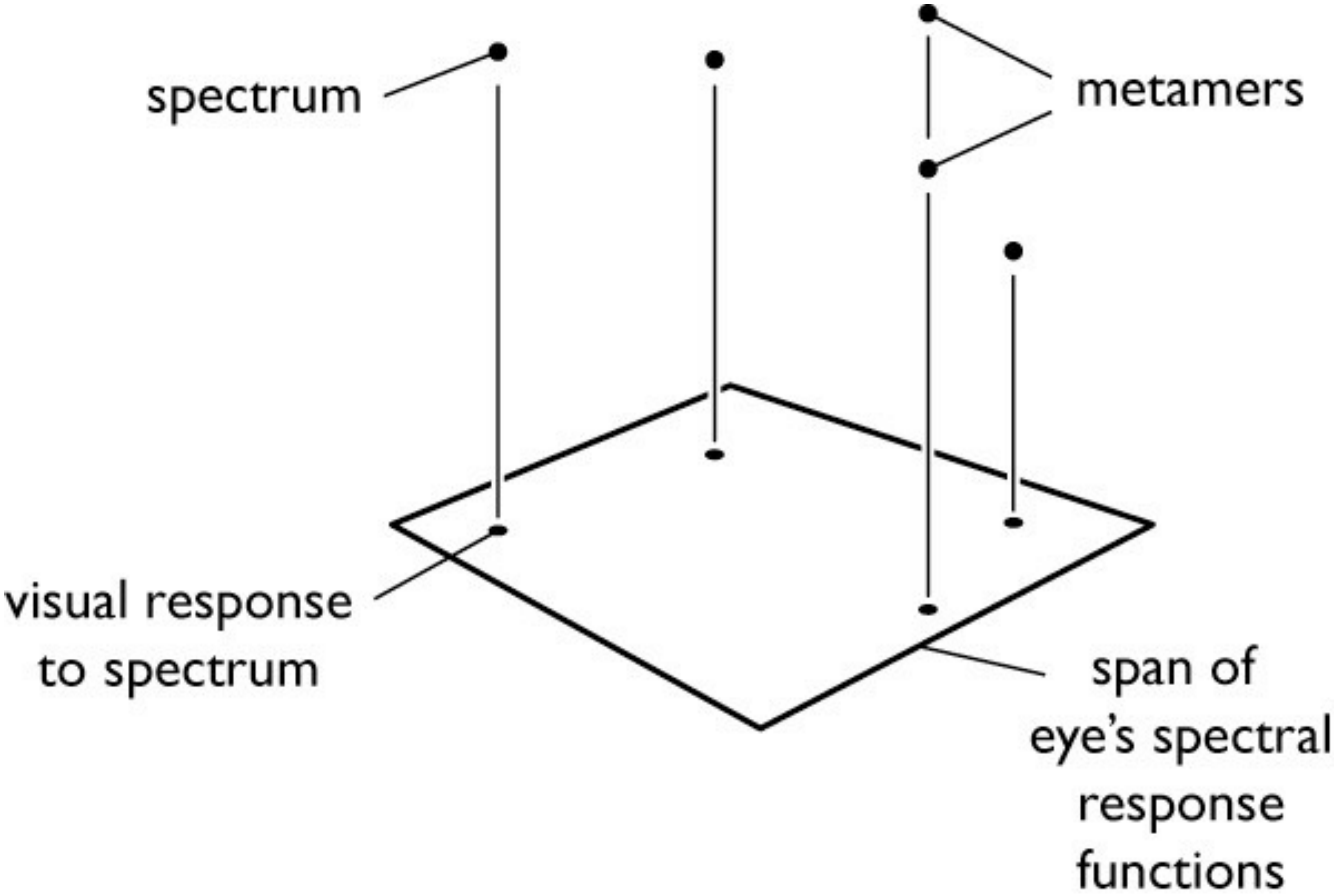
Display outputs spectrum
$$R\,s_R(\lambda) + G\,s_G(\lambda) + B\,s_B(\lambda)$$

Goal: at each pixel, choose R, G, B values for display so that the output color matches the appearance of the target color in the real world.

Pseudo-Geometric Interpretation

We are projecting a high dimensional vector (wavelength spectrum function) onto a low-dimensional subspace (SML visual response)

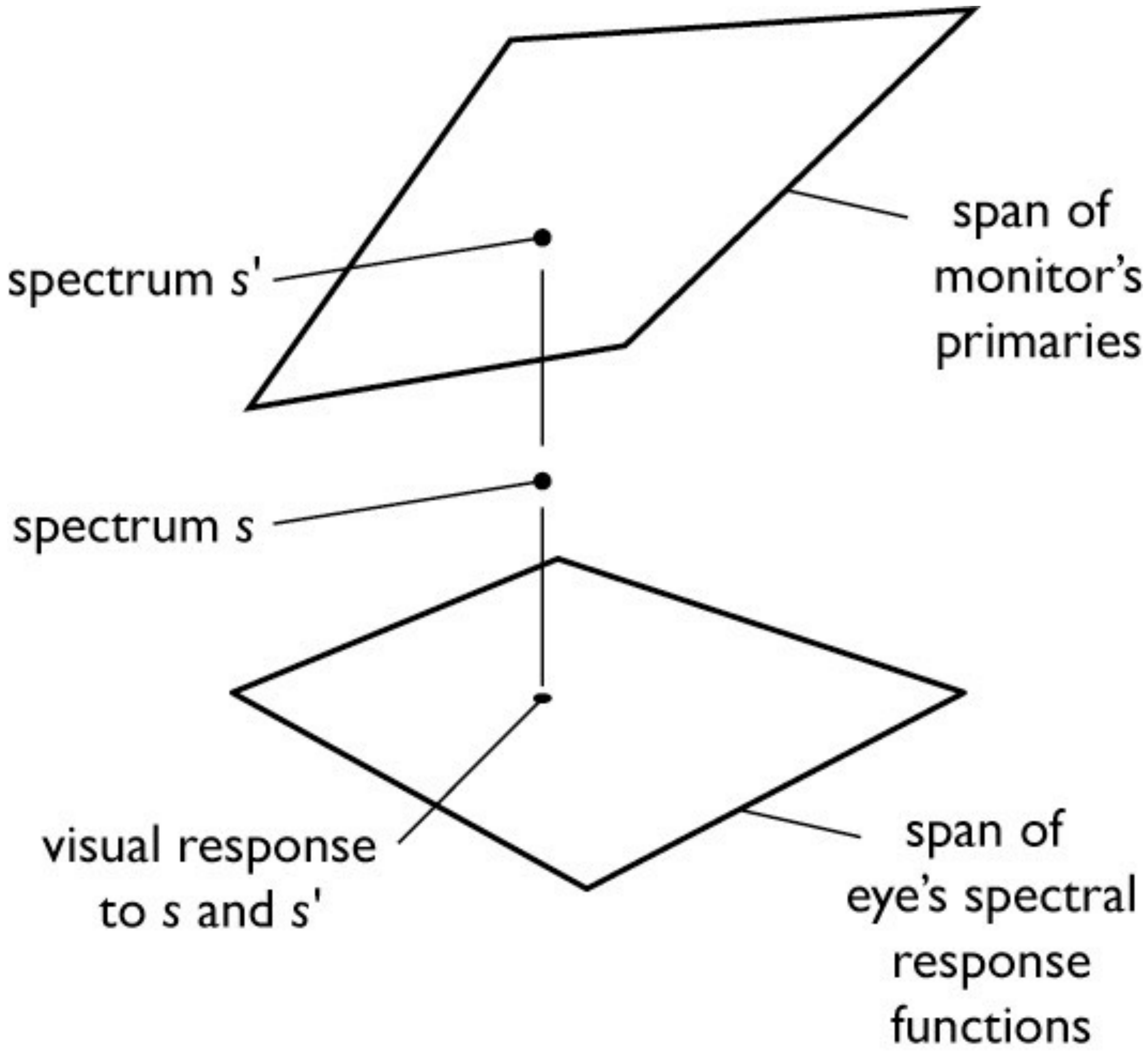
- Differences that are perpendicular to the basis vectors of the low-dimensional space are not detectable



Slide credit: Steve Marschner

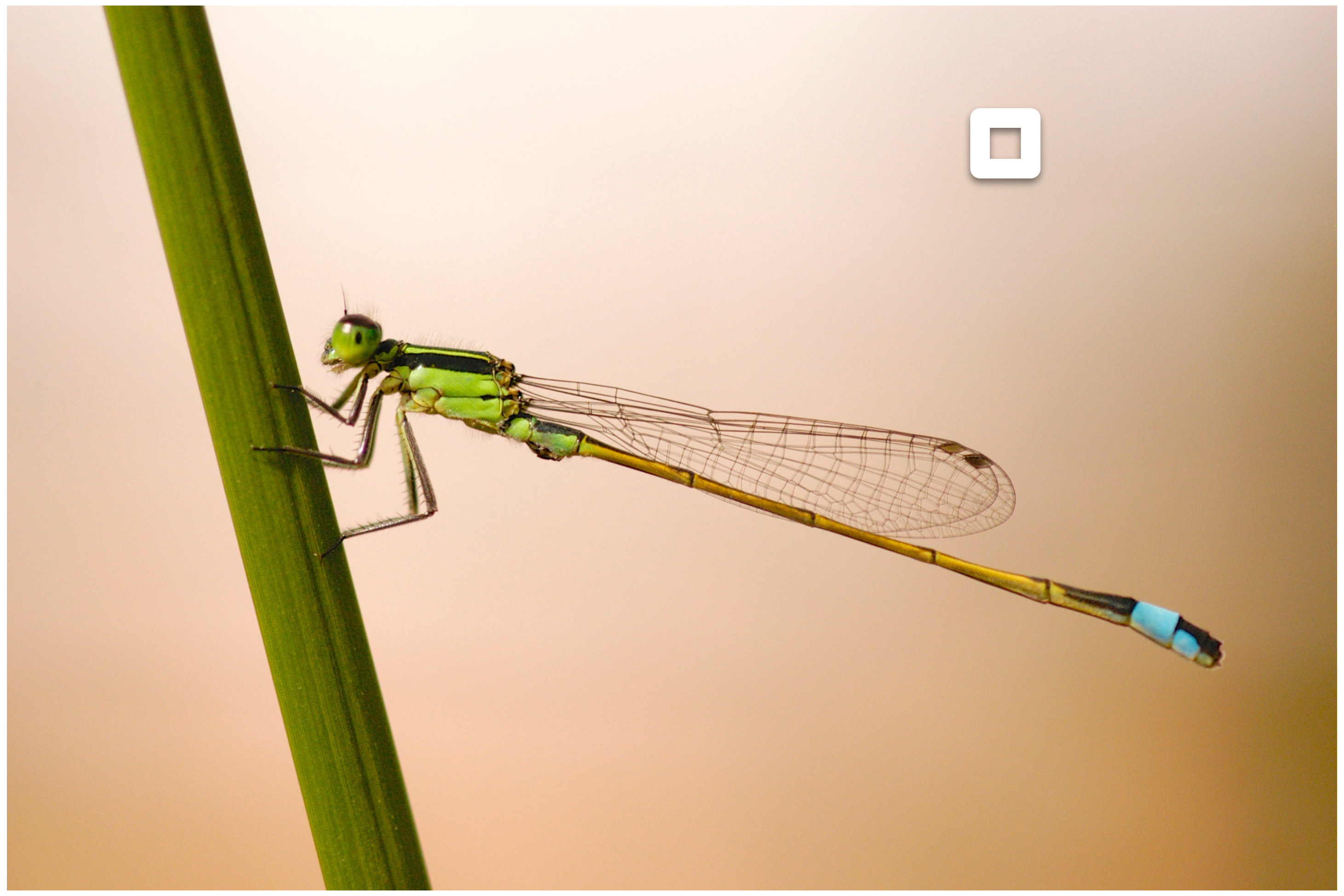
Pseudo-Geometric Interpretation of Color Reproduction

- The display can only produce a low-dimensional subspace of all possible spectra (linear combinations of display primaries)
- In color reproduction, for a given spectrum s (high dimensional), we want to choose a spectrum s′ in the display's low-dimensional subspace, such that s′ and s project to the same response in the low-dimensional subspace of the eye's SML response

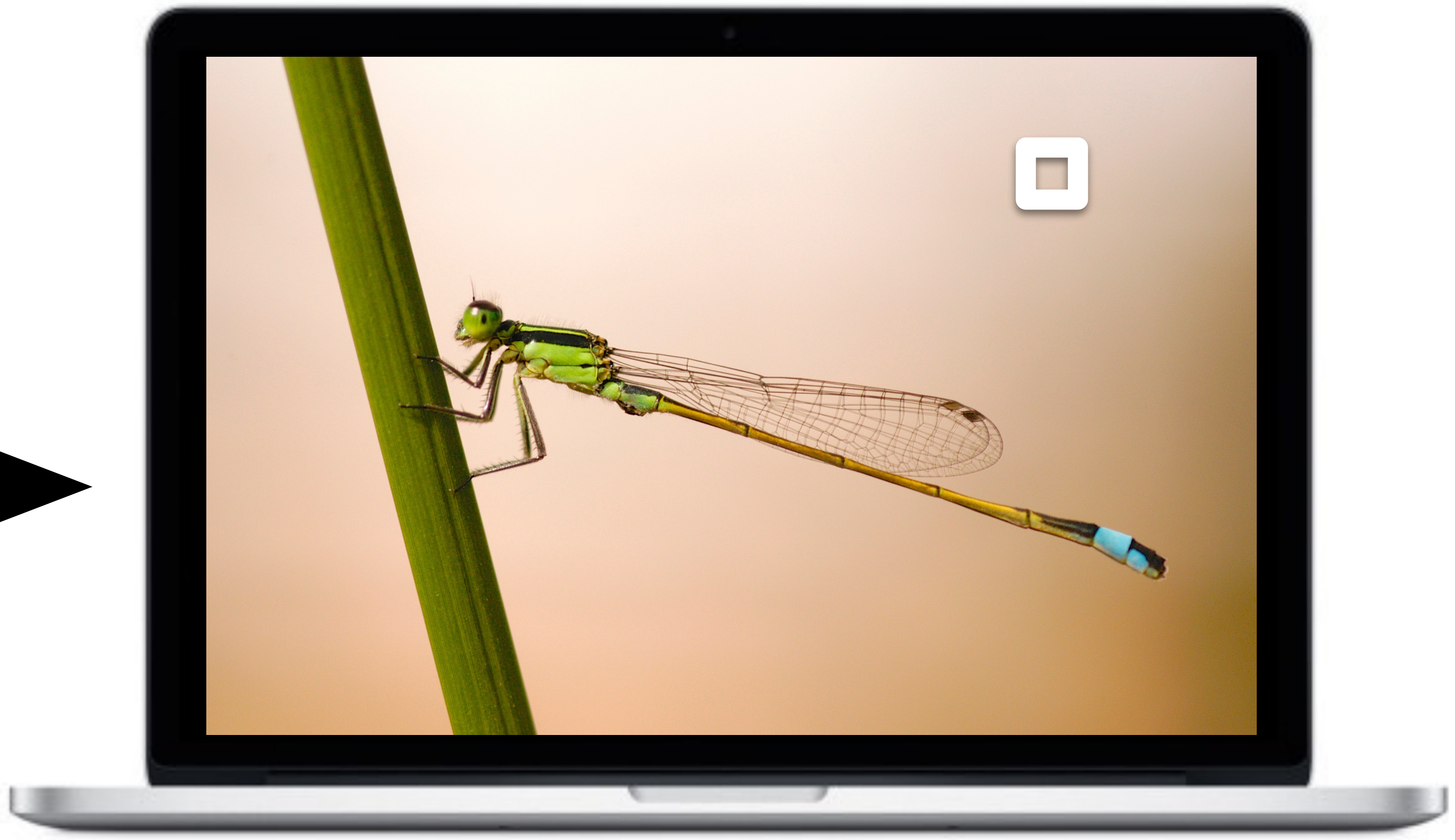


Slide credit: Steve Marschner

Color Reproduction as Linear Algebra



Input spectrum *s*



What R, G, B values?

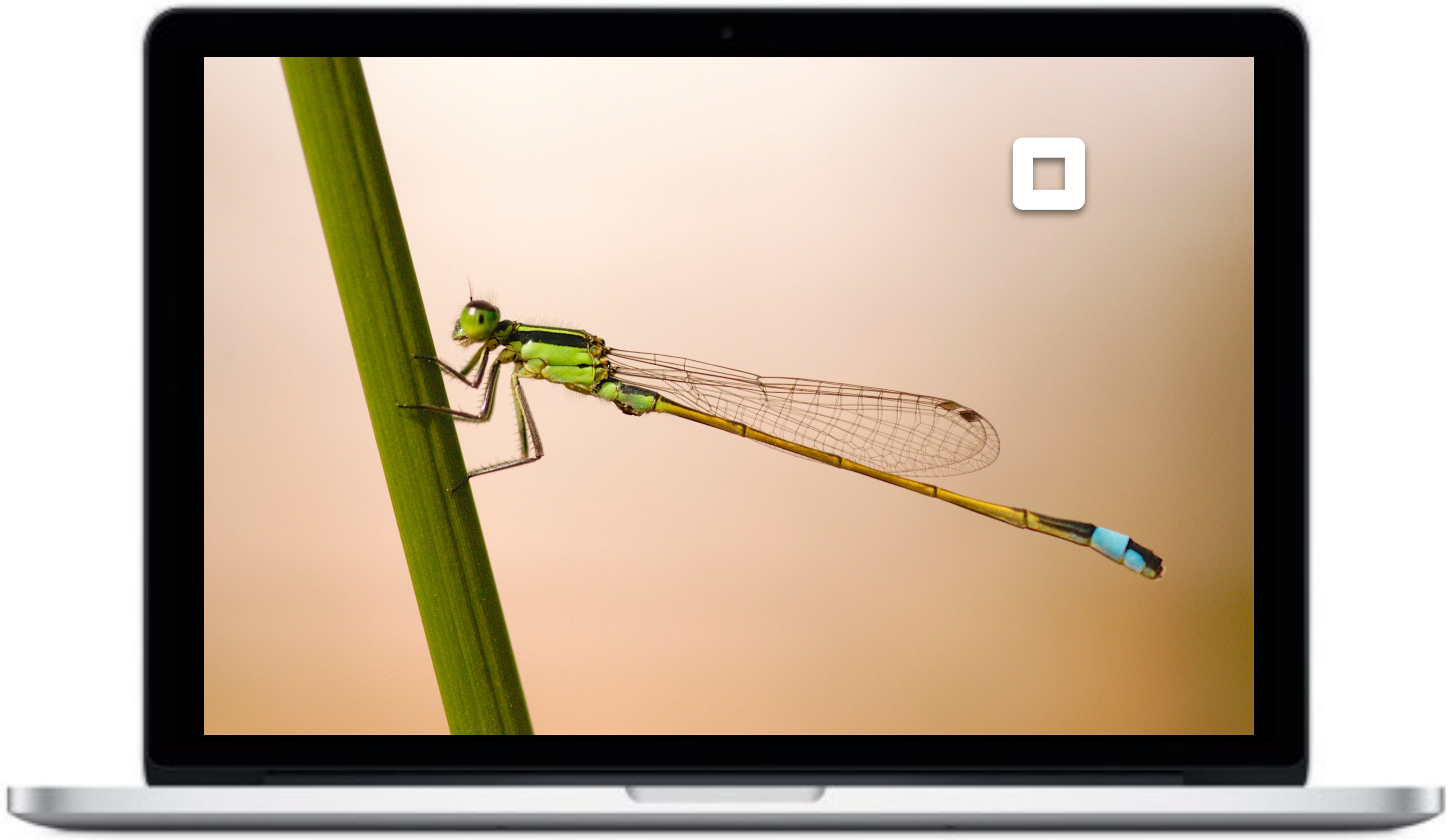
$$\begin{bmatrix} R \\ G \\ B \end{bmatrix} = \begin{bmatrix} \text{—} & ? & \text{—} \\ \text{—} & ? & \text{—} \\ \text{—} & ? & \text{—} \end{bmatrix} \begin{bmatrix} | \\ s \\ | \end{bmatrix}$$

Color Reproduction as Linear Algebra

Spectrum produced by display given values R,G,B:

$$s_{\text{disp}}(\lambda) = R\, s_R(\lambda) + G\, s_G(\lambda) + B\, s_B(\lambda)$$

$$\Longrightarrow \begin{bmatrix} | \\ s_{\text{disp}} \\ | \end{bmatrix} = \begin{bmatrix} | & | & | \\ s_R & s_G & s_B \\ | & | & | \end{bmatrix} \begin{bmatrix} R \\ G \\ B \end{bmatrix}$$



Color Reproduction as Linear Algebra

What color do we perceive when we look at the display?

$$\begin{bmatrix} S \\ M \\ L \end{bmatrix}_{\text{disp}} = \begin{bmatrix} — & r_S & — \\ — & r_M & — \\ — & r_L & — \end{bmatrix} \begin{bmatrix} | \\ s_{\text{disp}} \\ | \end{bmatrix}$$

$$= \begin{bmatrix} — & r_S & — \\ — & r_M & — \\ — & r_L & — \end{bmatrix} \begin{bmatrix} | & | & | \\ s_R & s_G & s_B \\ | & | & | \end{bmatrix} \begin{bmatrix} R \\ G \\ B \end{bmatrix}$$

We want this displayed spectrum to be a metamer for the real-world target spectrum.

Color Reproduction as Linear Algebra

Color perceived for display spectra with values R,G,B

$$\begin{bmatrix} S \\ M \\ L \end{bmatrix}_{\text{disp}} = \begin{bmatrix} — & r_S & — \\ — & r_M & — \\ — & r_L & — \end{bmatrix} \begin{bmatrix} | & | & | \\ s_R & s_G & s_B \\ | & | & | \end{bmatrix} \begin{bmatrix} R \\ G \\ B \end{bmatrix}$$

Color perceived for real scene spectra, s

$$\begin{bmatrix} S \\ M \\ L \end{bmatrix}_{\text{real}} = \begin{bmatrix} — & r_S & — \\ — & r_M & — \\ — & r_L & — \end{bmatrix} \begin{bmatrix} | \\ s \\ | \end{bmatrix}$$

How do we reproduce the color of s? Set these lines equal and solve for R,G,B as a function of s!

Color Reproduction as Linear Algebra

Solution:

$$\begin{bmatrix} \text{---} & r_S & \text{---} \\ \text{---} & r_M & \text{---} \\ \text{---} & r_L & \text{---} \end{bmatrix} \begin{bmatrix} | & | & | \\ s_R & s_G & s_B \\ | & | & | \end{bmatrix} \begin{bmatrix} R \\ G \\ B \end{bmatrix} = \begin{bmatrix} \text{---} & r_S & \text{---} \\ \text{---} & r_M & \text{---} \\ \text{---} & r_L & \text{---} \end{bmatrix} \begin{bmatrix} | \\ s \\ | \end{bmatrix}$$

$$\Longrightarrow \begin{bmatrix} R \\ G \\ B \end{bmatrix} = \left(\begin{bmatrix} \text{---} & r_S & \text{---} \\ \text{---} & r_M & \text{---} \\ \text{---} & r_L & \text{---} \end{bmatrix} \begin{bmatrix} | & | & | \\ s_R & s_G & s_B \\ | & | & | \end{bmatrix} \right)^{-1} \begin{bmatrix} \text{---} & r_S & \text{---} \\ \text{---} & r_M & \text{---} \\ \text{---} & r_L & \text{---} \end{bmatrix} \begin{bmatrix} | \\ s \\ | \end{bmatrix}$$

Color Reproduction as Linear Algebra

Solution (form #1):

$$\underset{\text{3x1}}{\begin{bmatrix} R \\ G \\ B \end{bmatrix}} = \left(\underbrace{\underset{\text{3xN}}{\begin{bmatrix} — & r_S & — \\ — & r_M & — \\ — & r_L & — \end{bmatrix}} \underset{\text{Nx3}}{\begin{bmatrix} | & | & | \\ s_R & s_G & s_B \\ | & | & | \end{bmatrix}}}_{\text{3x3}} \right)^{-1} \underbrace{\underset{\text{3xN}}{\begin{bmatrix} — & r_S & — \\ — & r_M & — \\ — & r_L & — \end{bmatrix}} \underset{\text{Nx1}}{\begin{bmatrix} | \\ s \\ | \end{bmatrix}}}_{\text{3x1}}$$

Solution (form #2):

$$\underset{\text{3x1}}{RGB} = (\underbrace{\underset{\text{3xN}}{\mathbf{M}_{SML}}\,\underset{\text{Nx3}}{\mathbf{M}_{RGB}})^{-1}\underset{\text{3xN}}{\mathbf{M}_{SML}}}_{\text{3xN}}\,\underset{\text{Nx1}}{s}$$

Color Reproduction as Linear Algebra

Solution (form #3):

$$\begin{bmatrix} R \\ G \\ B \end{bmatrix} = \underbrace{\begin{bmatrix} r_S \cdot s_R & r_S \cdot s_G & r_S \cdot s_B \\ r_M \cdot s_R & r_M \cdot s_G & r_M \cdot s_B \\ r_L \cdot s_R & r_L \cdot s_G & r_L \cdot s_B \end{bmatrix}^{-1} \begin{bmatrix} — & r_S & — \\ — & r_M & — \\ — & r_L & — \end{bmatrix}}_{\text{3xN}} \begin{bmatrix} | \\ s \\ | \end{bmatrix}$$

Color Matching Functions

Recall the color matching functions from the matching experiment

$$\begin{bmatrix} R \\ G \\ B \end{bmatrix} = \left(\begin{bmatrix} — & r_S & — \\ — & r_M & — \\ — & r_L & — \end{bmatrix} \begin{bmatrix} | & | & | \\ s_R & s_G & s_B \\ | & | & | \end{bmatrix} \right)^{-1} \begin{bmatrix} — & r_S & — \\ — & r_M & — \\ — & r_L & — \end{bmatrix} \begin{bmatrix} | \\ s \\ | \end{bmatrix}$$

$$= \underbrace{\begin{bmatrix} r_S \cdot s_R & r_S \cdot s_G & r_S \cdot s_B \\ r_M \cdot s_R & r_M \cdot s_G & r_M \cdot s_B \\ r_L \cdot s_R & r_L \cdot s_G & r_L \cdot s_B \end{bmatrix}^{-1} \begin{bmatrix} — & r_S & — \\ — & r_M & — \\ — & r_L & — \end{bmatrix}}_{\text{3xN}} \begin{bmatrix} | \\ s \\ | \end{bmatrix}$$

This 3xN matrix contains, as row vectors,
"color matching functions"
associated with the primary lights s_R, s_G, s_B.

Color Reproduction Issue: No Negative Light

R,G,B values must be positive

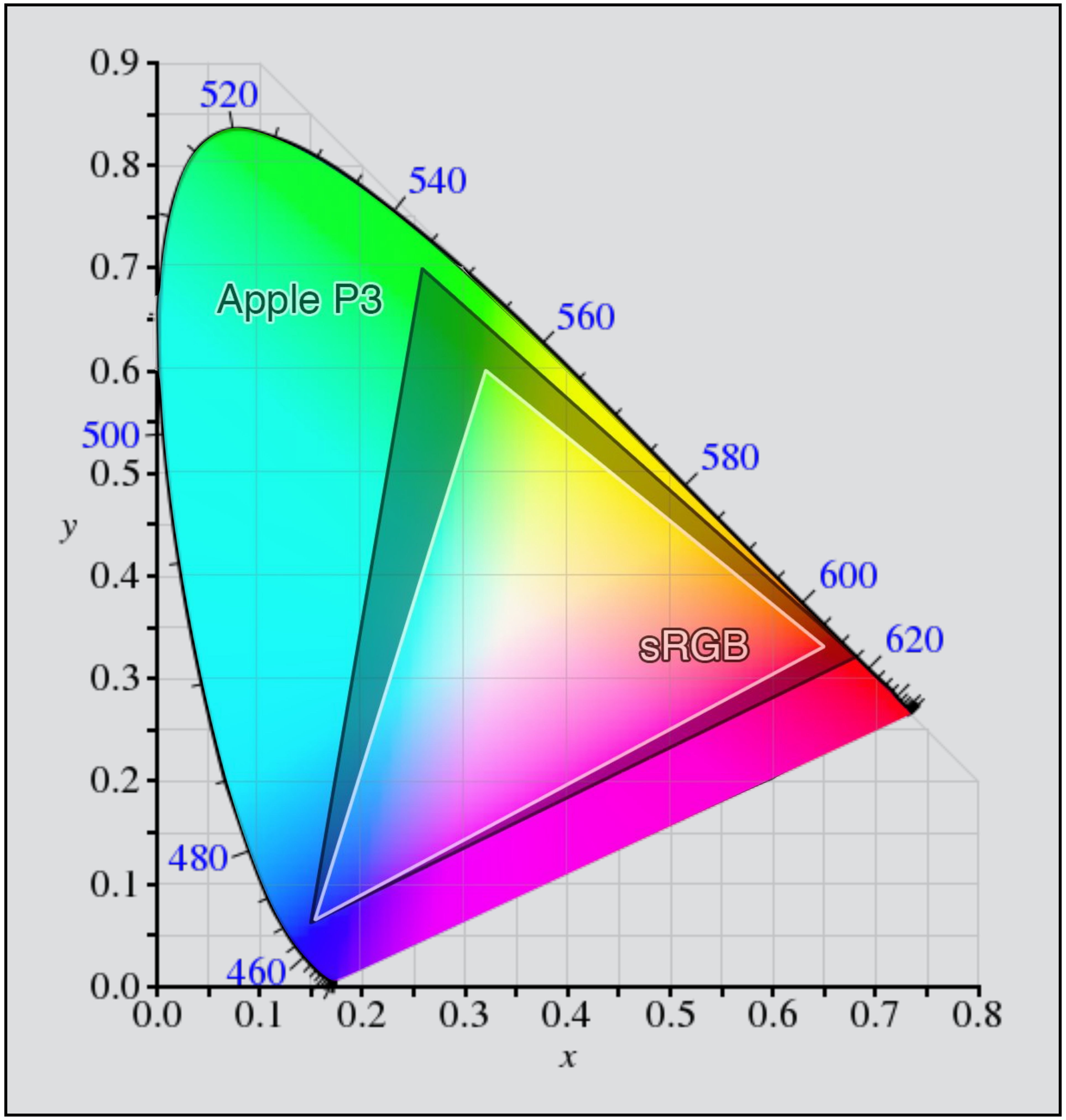
- Display primaries can’t emit negative light
- But solution formulas can certainly produce negative R,G,B values

What do negative R,G,B values mean?

- Display can’t physically reproduce the desired color
- Desired color is outside the display’s color gamut

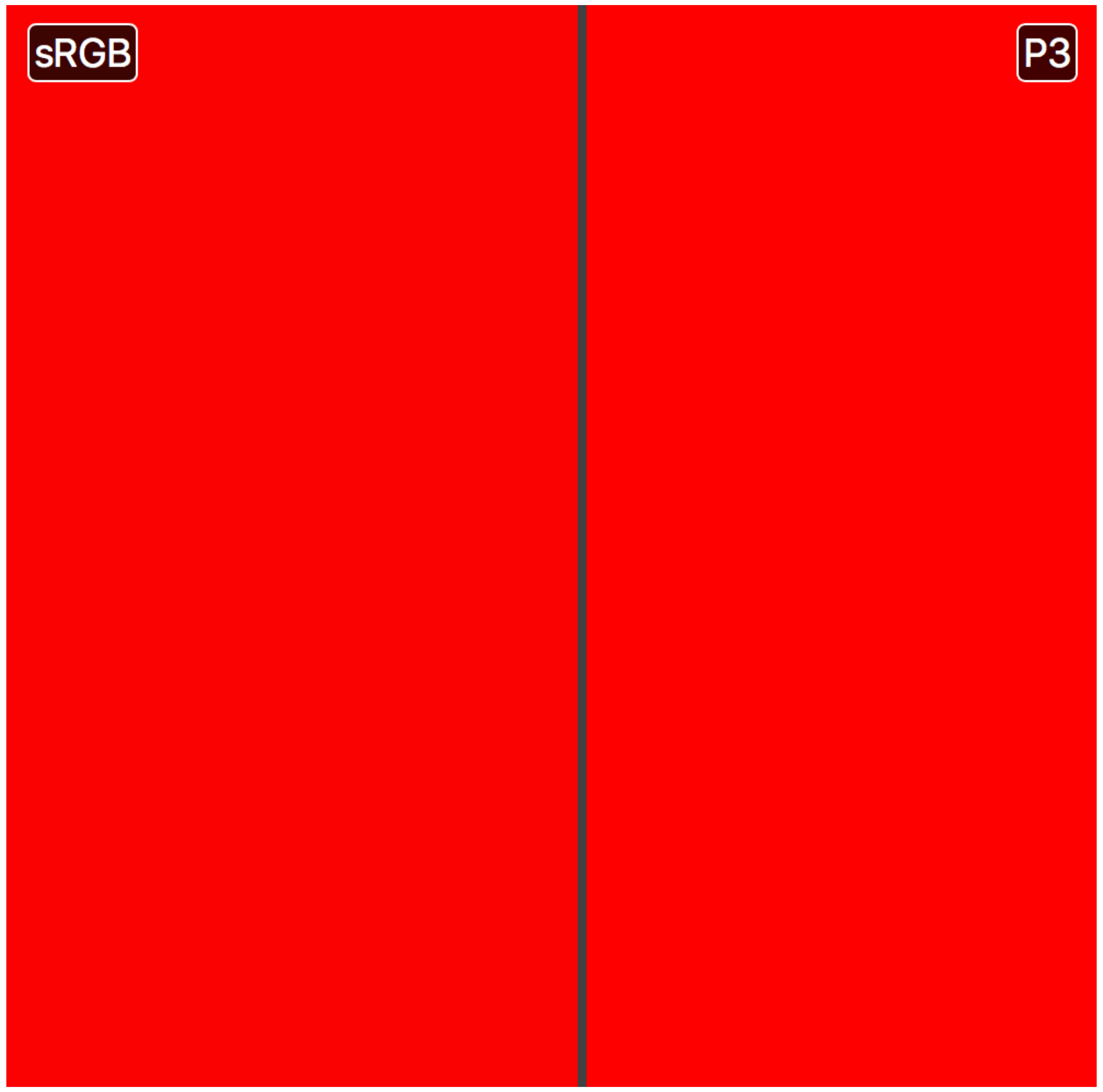
Gamut

Example: Color Gamut for sRGB and Apple P3



Wikipedia

Comparing sRGB and Wide Gamut P3 Color Spaces

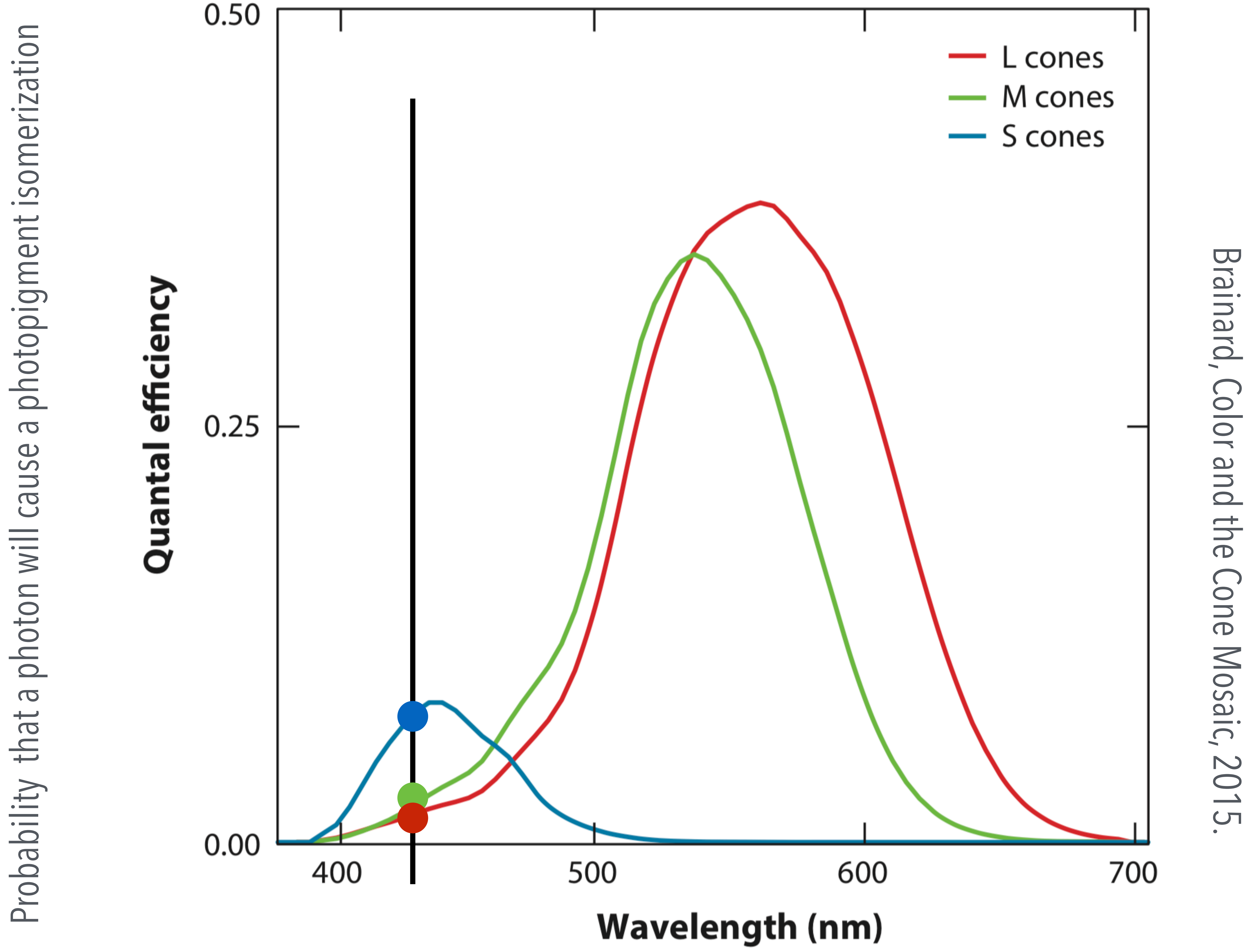


Interactive Color Space Comparison:

https://webkit.org/blog-files/color-gamut/comparison.html

- Needs a wide-gamut physical display
- I can see differences clearly on my MacBook Pro, less so on LG display

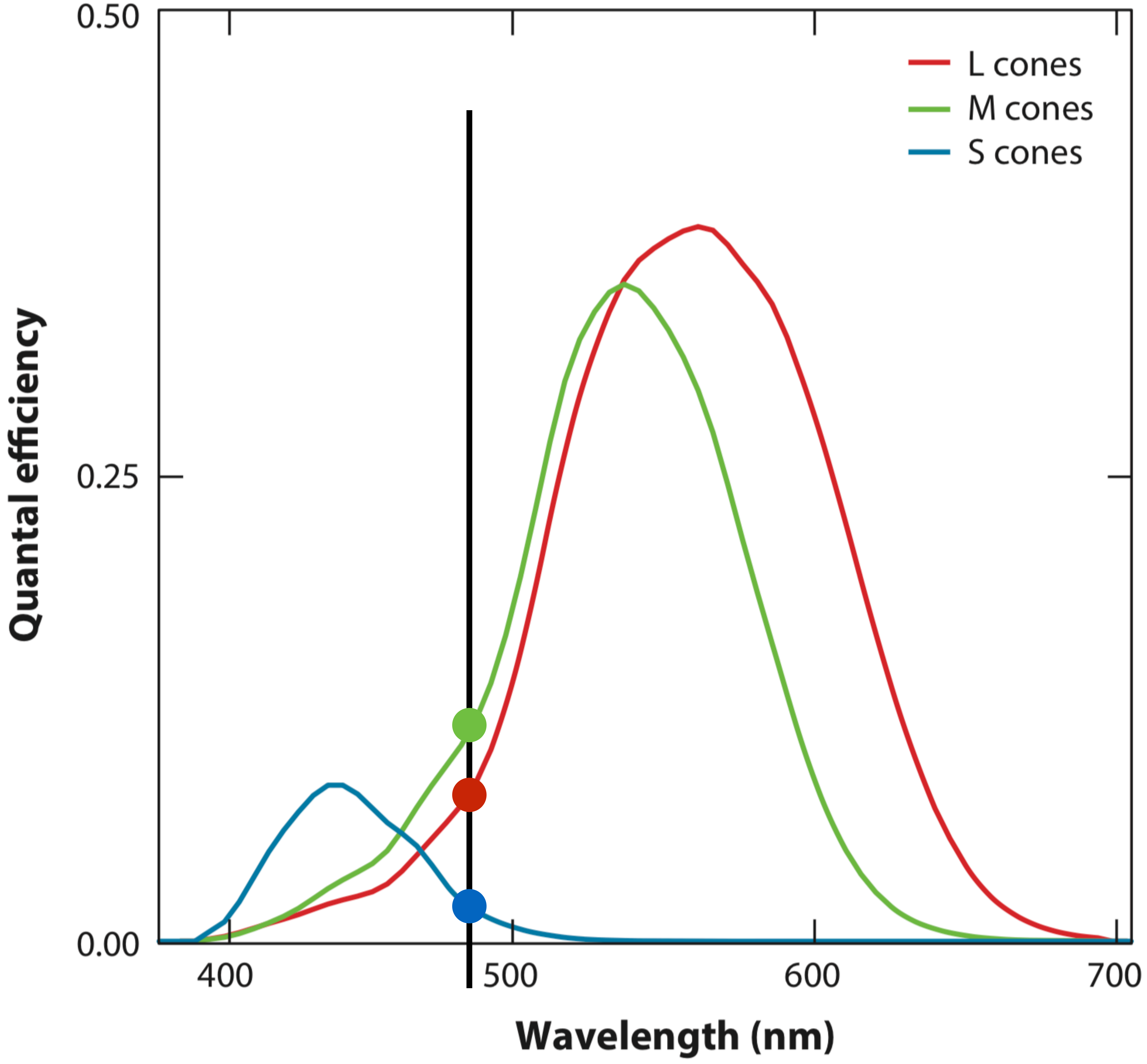
LMS Response Values for Each Wavelength



Brainard, Color and the Cone Mosaic, 2015.

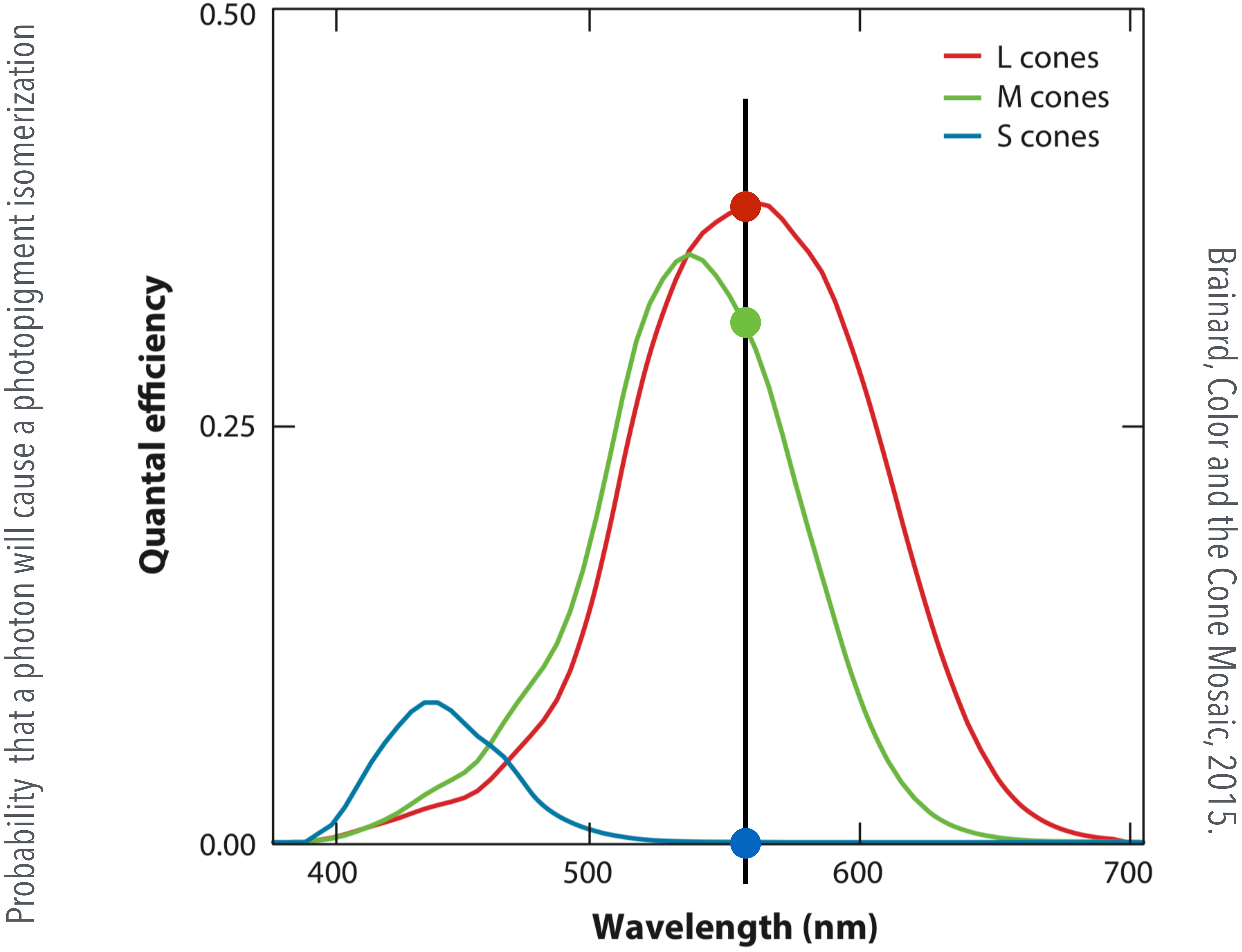
LMS Response Values for Each Wavelength

Probability that a photon will cause a photopigment isomerization



Brainard, Color and the Cone Mosaic, 2015.

LMS Response Values for Each Wavelength



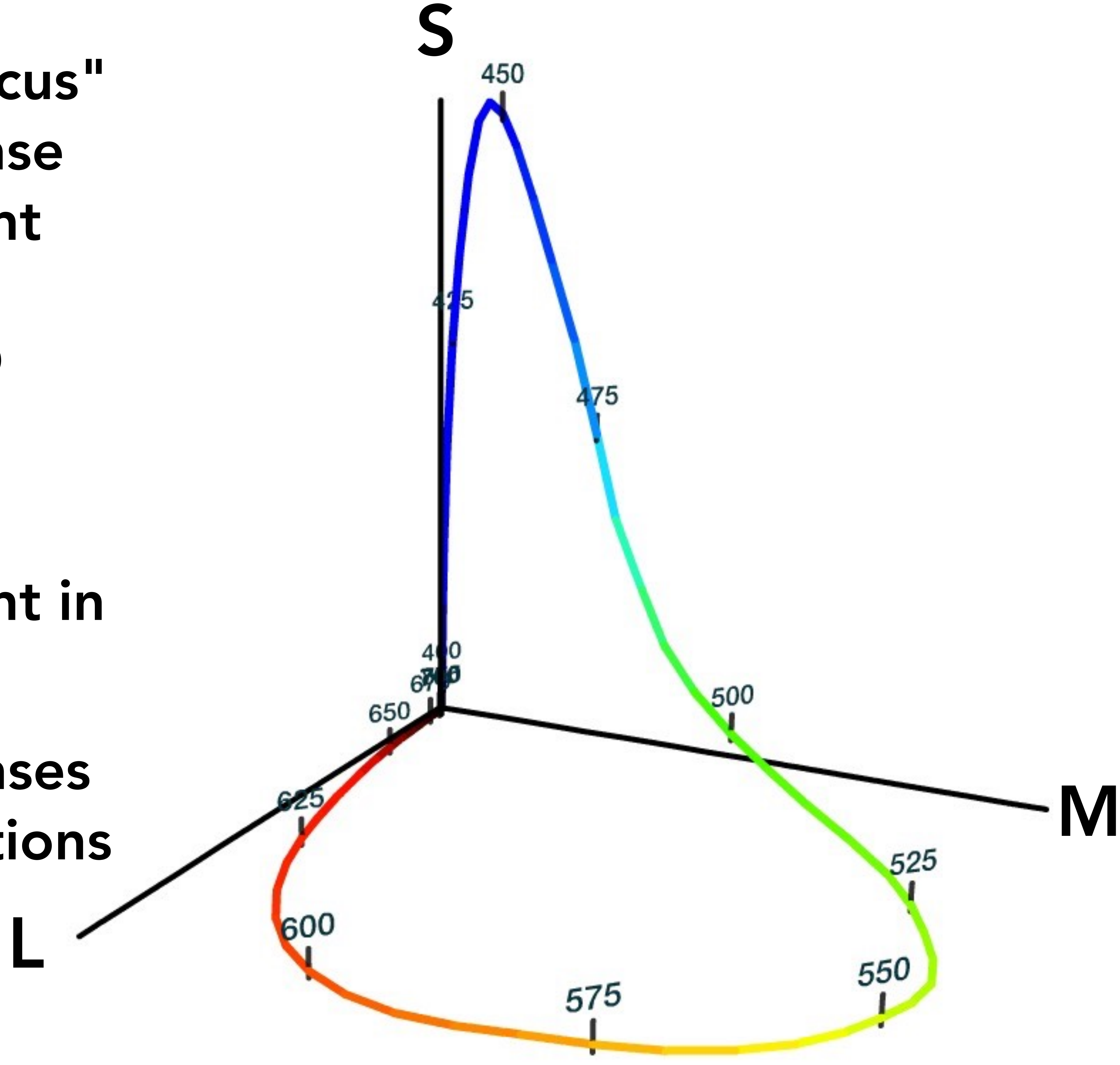
Brainard, Color and the Cone Mosaic, 2015.

LMS Responses Plotted as 3D Color Space

Visualization of "spectral locus" of human cone cells’ response to monochromatic light (light with energy in a single wavelength) as points in 3D space.

This is a plot of the S, M, L response functions as a point in 3D space.

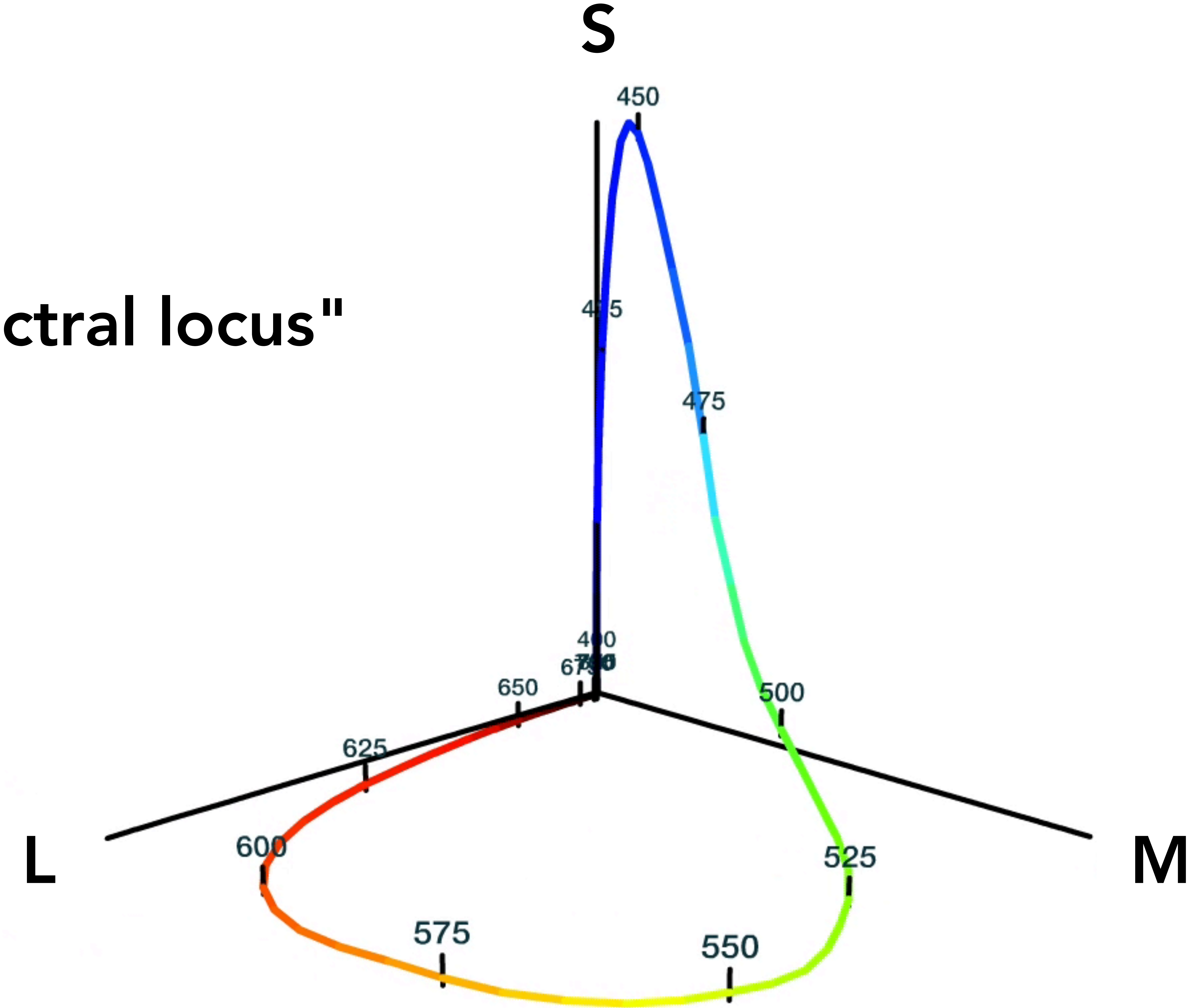
Space of all possible responses are positive linear combinations of points on this curve.



Marc Levoy

LMS Responses Plotted as 3D Color Space

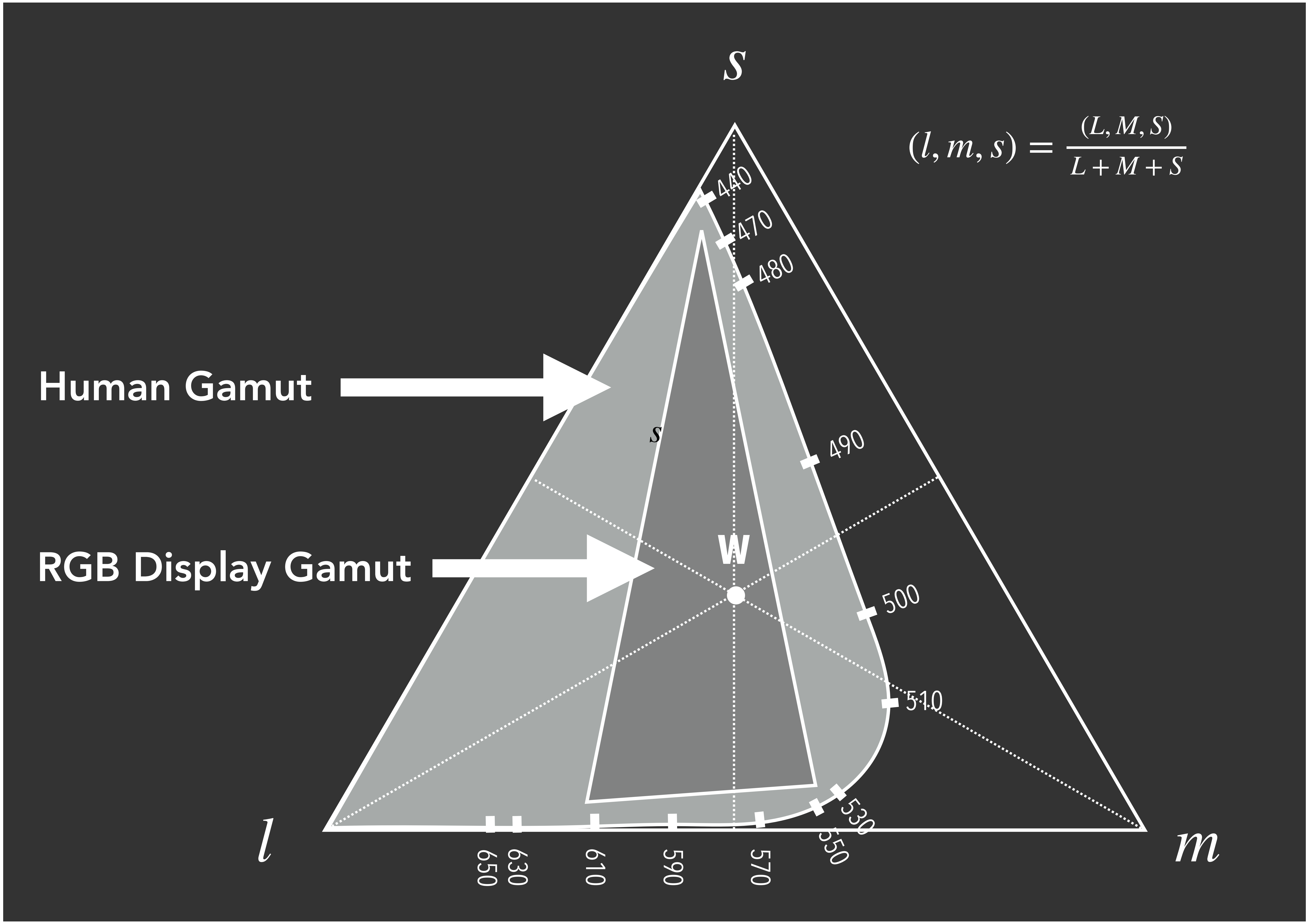
"Spectral locus"



https://graphics.stanford.edu/courses/cs178-10/applets/locus.html

Dektar, Adams, Levoy

Chromaticity Diagram (Maxwellian)



$$(l, m, s) = \frac{(L, M, S)}{L + M + S}$$

Human Gamut

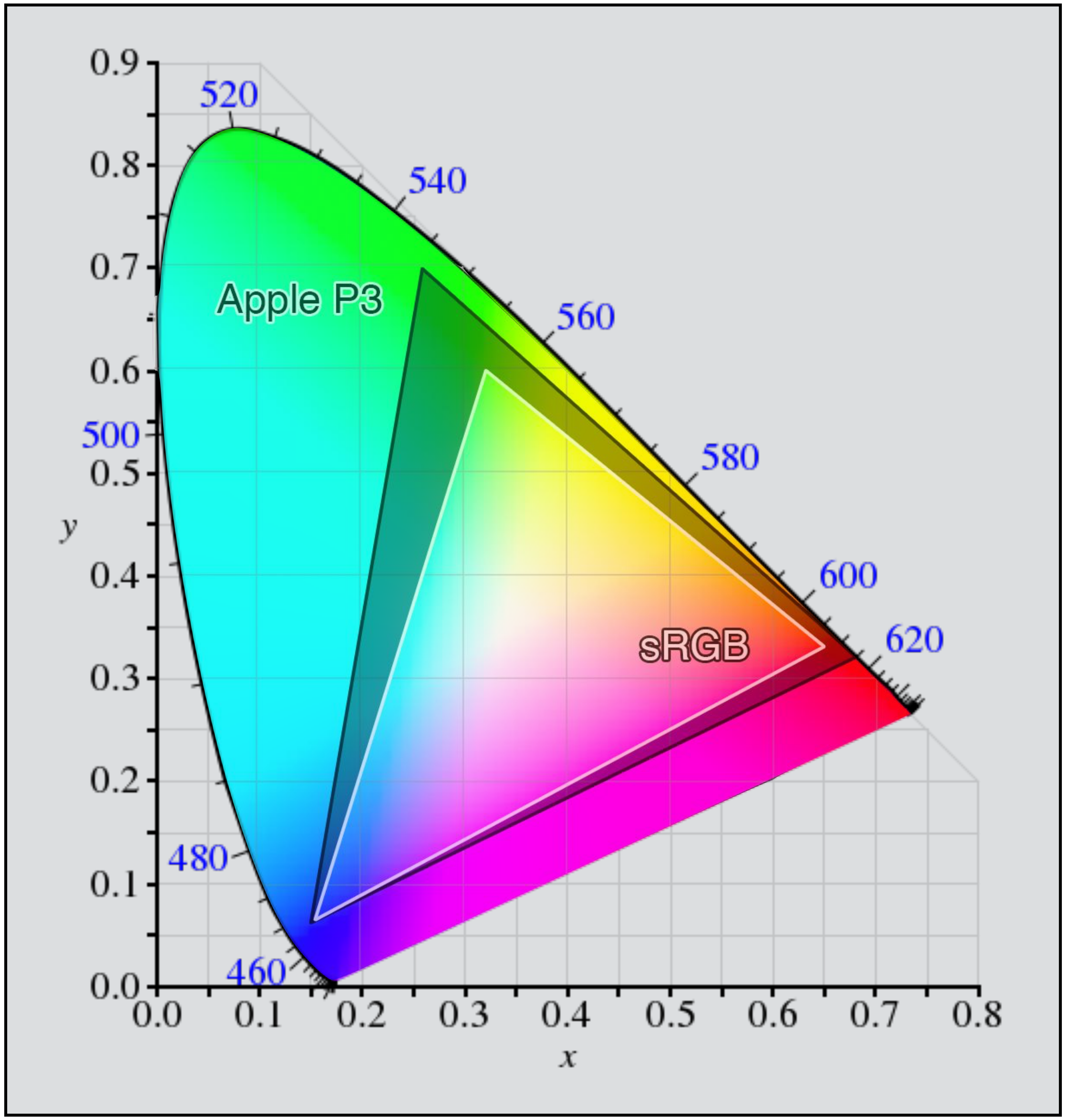
RGB Display Gamut

Perspective projection of spectral locus
looking diagonally down at origin from (1,1,1)

Chromaticity Diagram (CIE 1931 xy)

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} 1.9121 & -1.1121 & 0.2019 \\ 0.3709 & 0.6291 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} L \\ M \\ S \end{bmatrix}$$

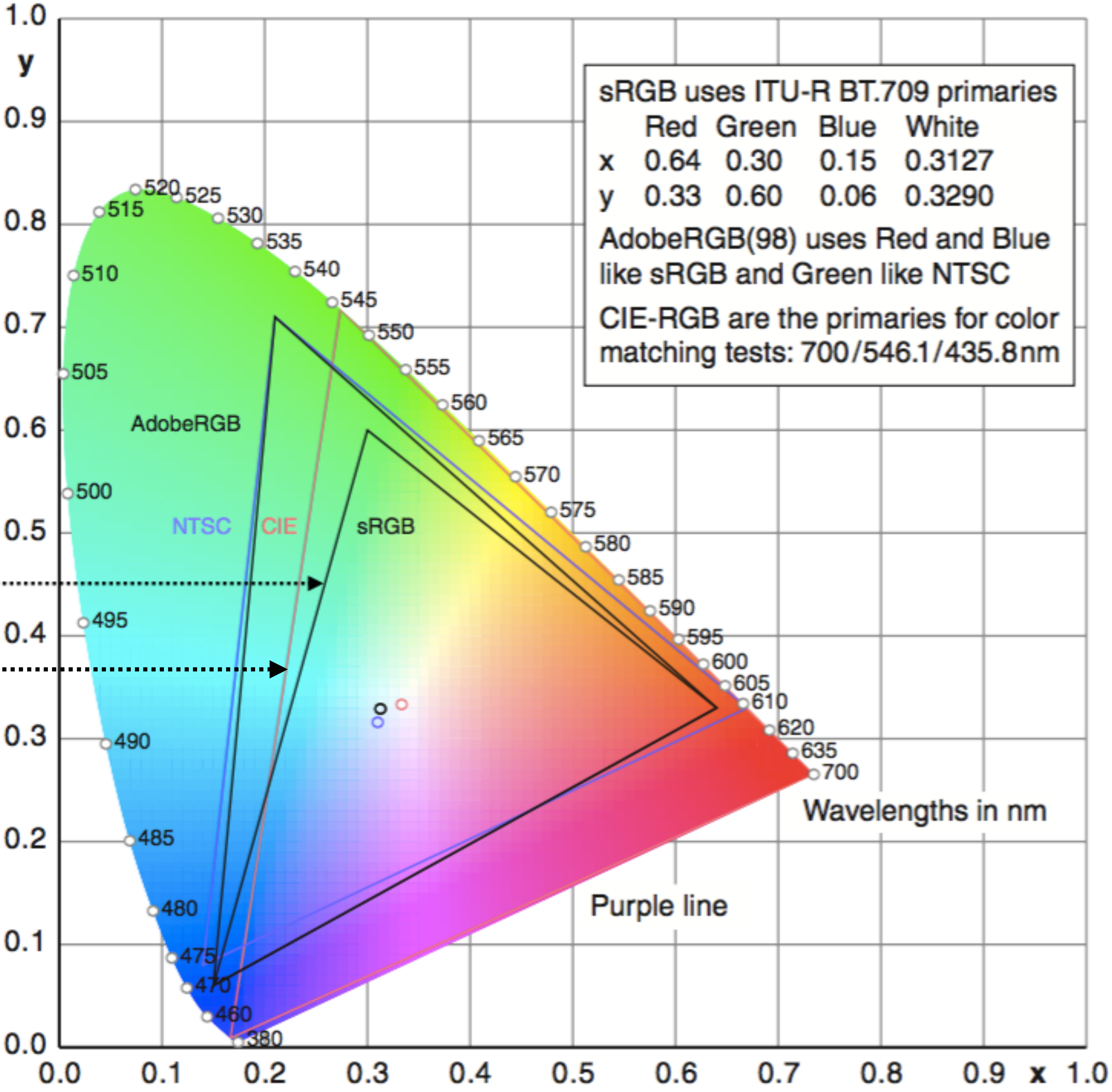
$$(x, y) = \frac{(X, Y)}{X + Y + Z}$$



Wikipedia

Color Gamut

sRGB is a common color space used throughout the internet



sRGB uses ITU-R BT.709 primaries

	Red	Green	Blue	White
x	0.64	0.30	0.15	0.3127
y	0.33	0.60	0.06	0.3290

AdobeRGB(98) uses Red and Blue like sRGB and Green like NTSC

CIE-RGB are the primaries for color matching tests: 700/546.1/435.8nm

CIE RGB are the monochromatic primaries used for color matching tests described earlier

Color Representation

Color Spaces

Need three numbers to specify a color

- But what three numbers?
- A color space is an answer to this question
- Same color has different coordinates in different color spaces. E.g. RGB, XYZ, Lab, HSV, …

Common example: display color space

- Define a color by what R, G, B scalar values will produce it on your display
  - As before, $s(\lambda) = r(\lambda)R + g(\lambda)G + b(\lambda)B$ for some spectra r, g, b
- Device dependent (depends on primary spectra, gamma, …)
  - Therefore if I choose R,G,B by looking at my display and send it to you, you may not see the same color
- Also leaves out some colors (limited gamut), e.g. vivid yellow
  - Because in file formats R, G, B usually constrained to be non-negative

Standard Color Spaces

Standardized RGB (sRGB)

- makes a particular monitor RGB standard
- other color devices simulate that monitor by calibration
- sRGB is usable as an interchange space; still widely used today, though other standards common now
- gamut is still limited

The Historical "Standard" Color Space: CIE XYZ

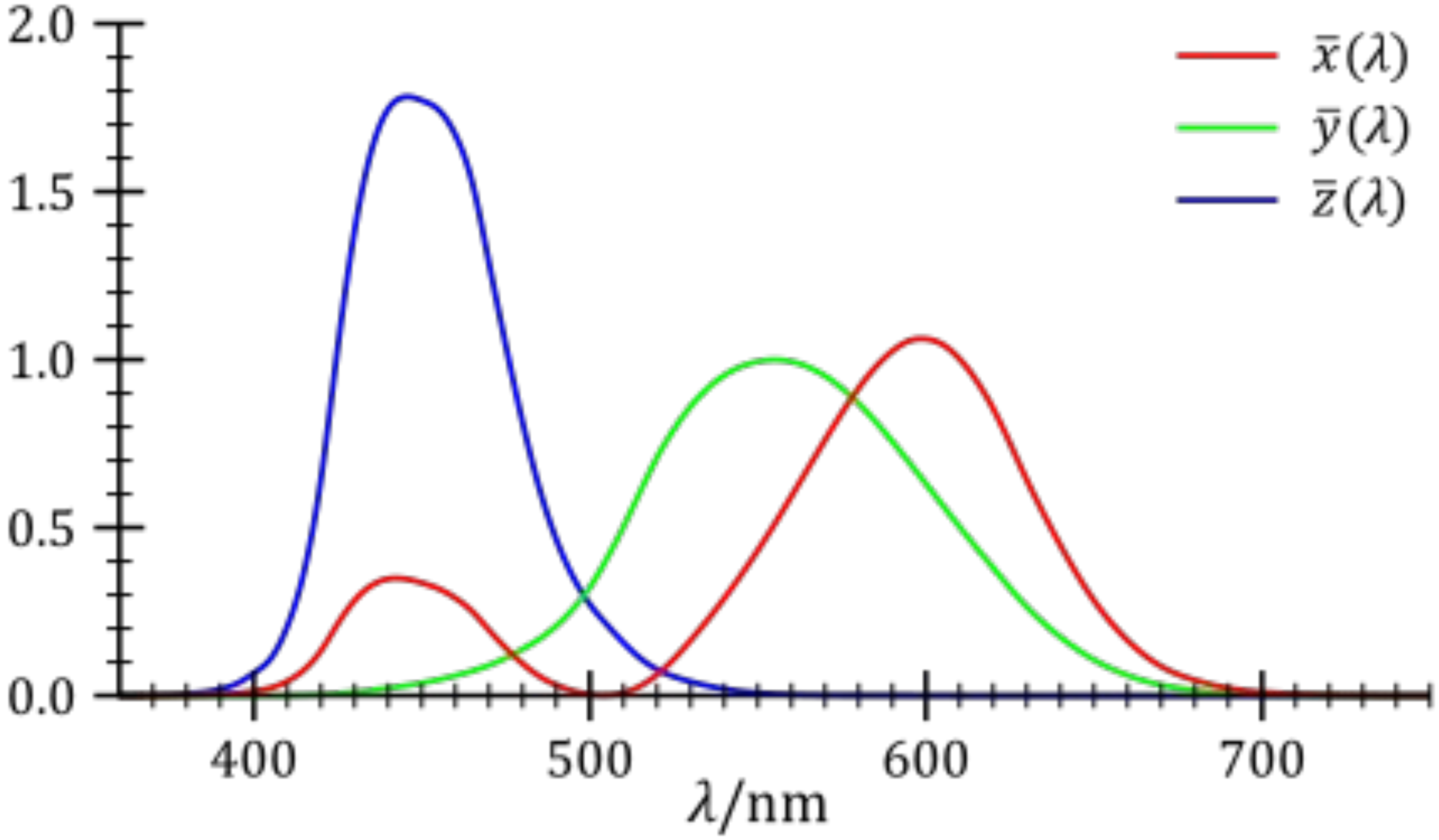
Imaginary set of standard color primaries X, Y, Z

Designed such that

- X, Y, Z span all observable colors
- Matching functions are strictly positive
- Y is luminance (brightness absent color)

Imaginary because can only be realized with primaries that are negative at some wavelengths

CIE XYZ color matching functions

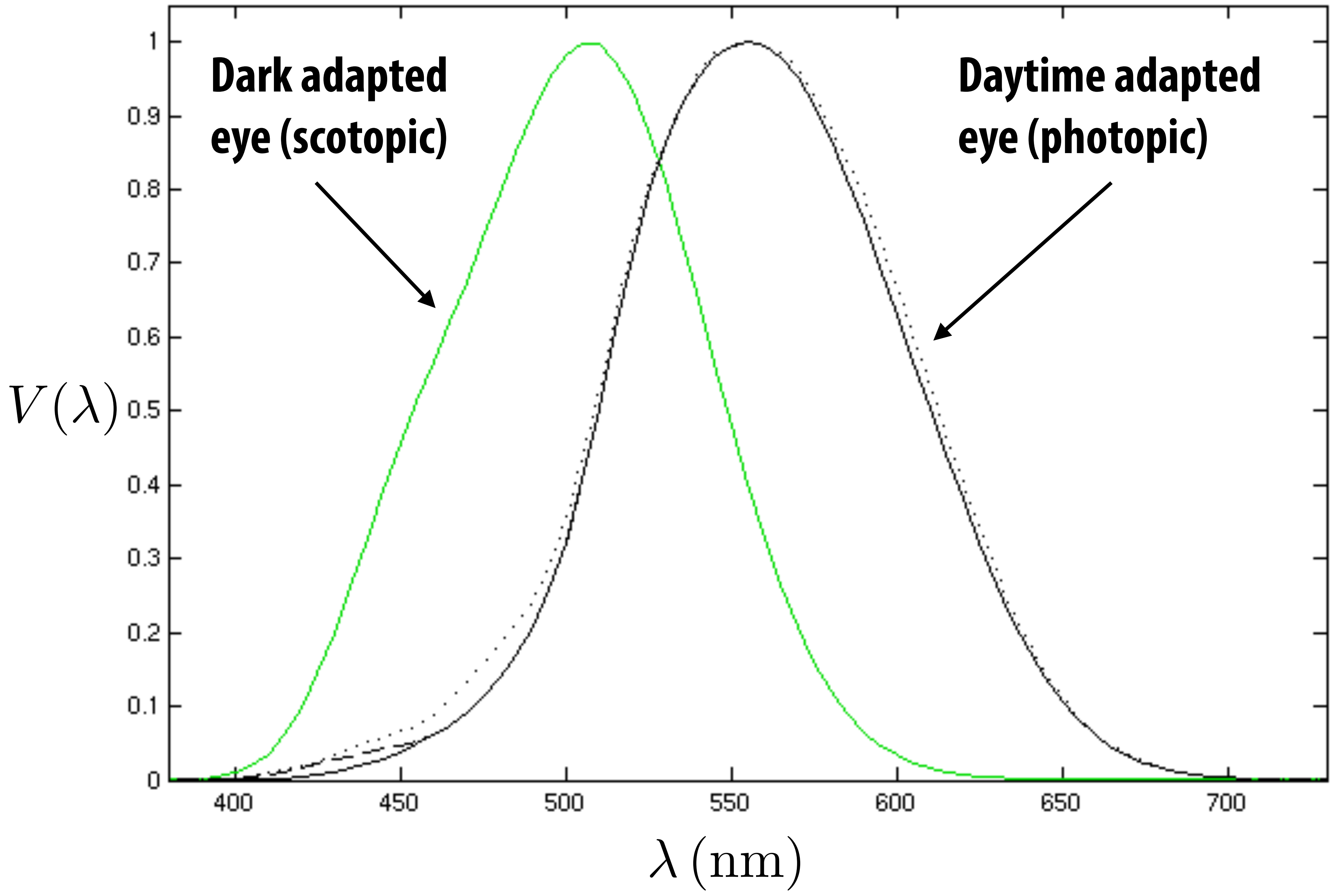


Luminance (Lightness)

Integral of radiance scaled by the visual luminous efficiency

$$Y = \int \Phi(\lambda)\, V(\lambda)\, \mathrm{d}\lambda$$

Luminous efficiency $V(\lambda)$ is a measure of how bright a light at a given wavelength is perceived by a human



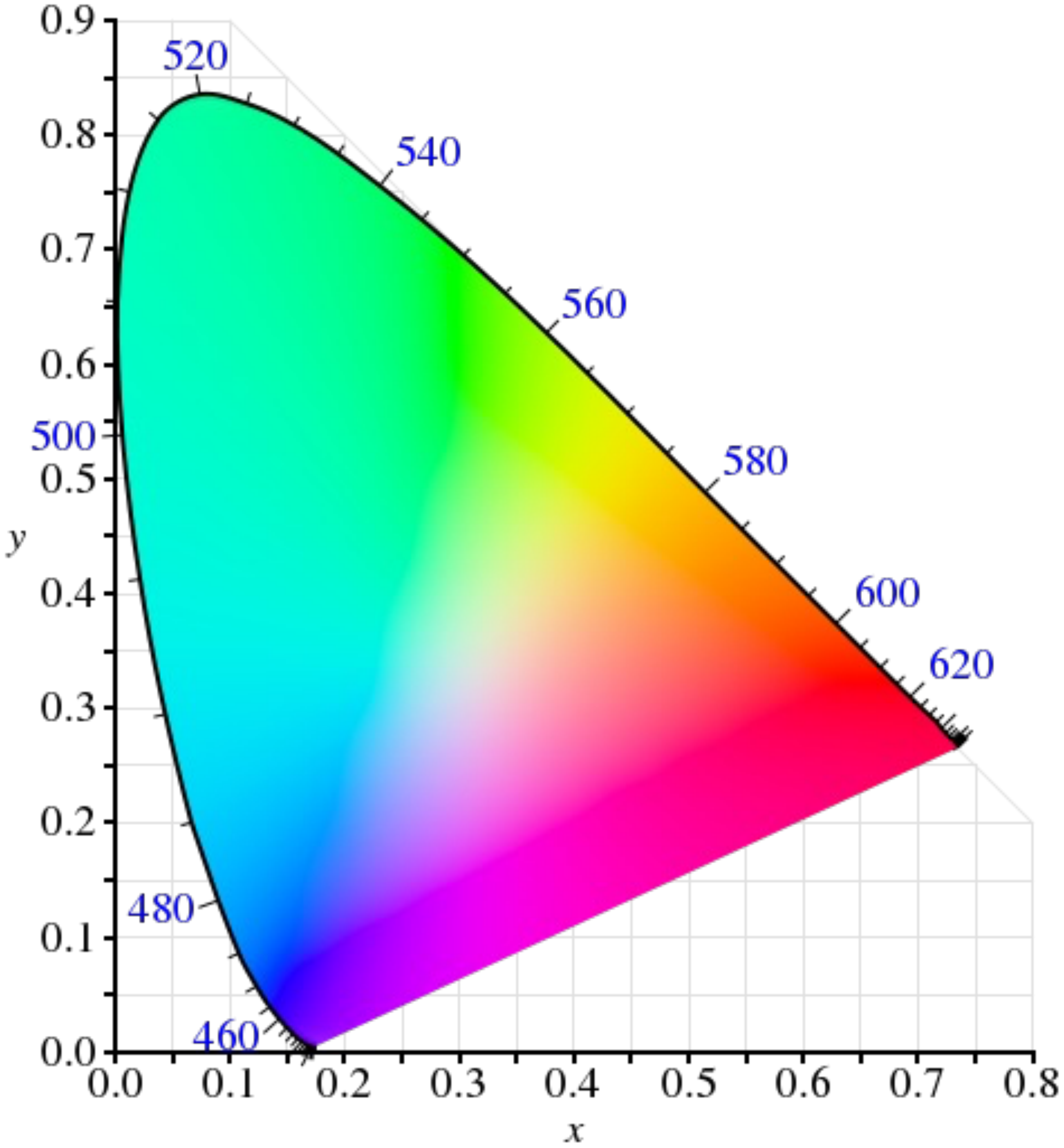
https://upload.wikimedia.org/wikipedia/commons/a/a0/Luminosity.png

Separating Luminance, Chromaticity

Luminance: ***Y***

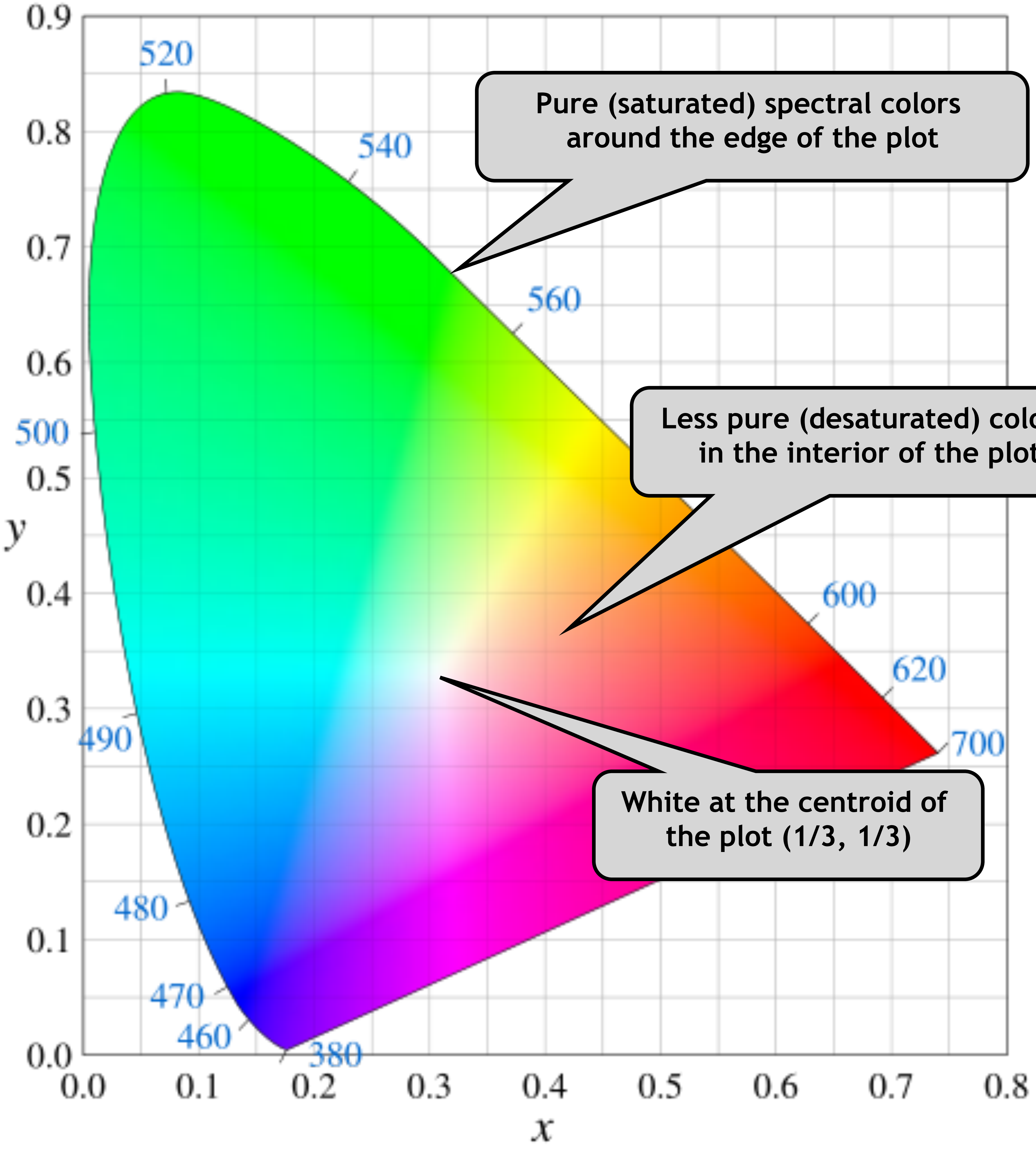
Chromaticity: ***x, y, z,*** **defined as**

$$x = \frac{X}{X+Y+Z}$$
$$y = \frac{Y}{X+Y+Z}$$
$$z = \frac{Z}{X+Y+Z}$$



- since $x + y + z = 1$, we only need to record two of the three
  - usually choose x and y, leading to (x, y, Y) coords

CIE 1931 xy Chromaticity Diagram



Pure (saturated) spectral colors around the edge of the plot

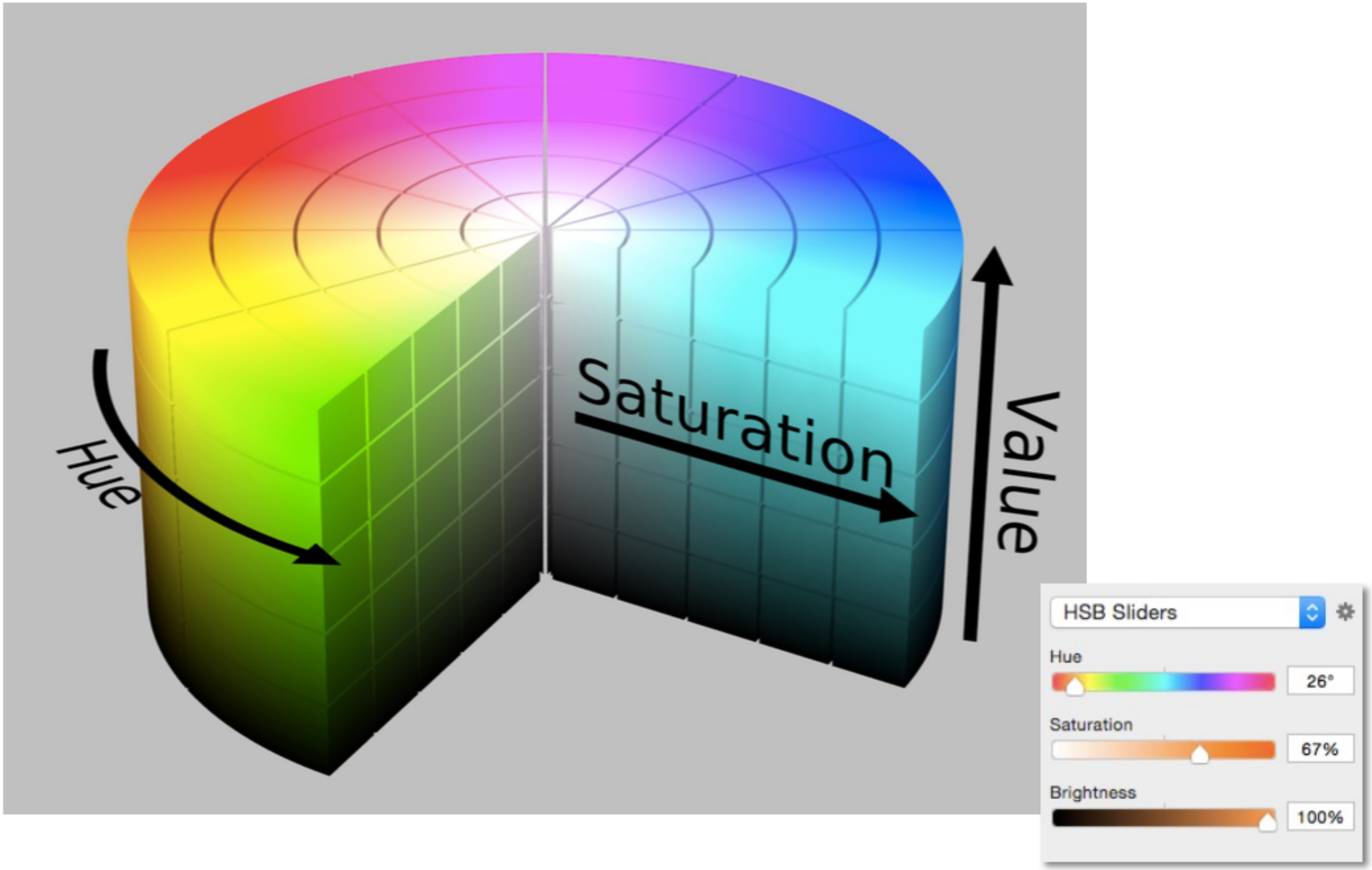
Less pure (desaturated) colors in the interior of the plot

White at the centroid of the plot (1/3, 1/3)

Perceptually Organized Color Spaces

HSV Color Space (Hue-Saturation-Value)

Axes correspond to artistic characteristics of color



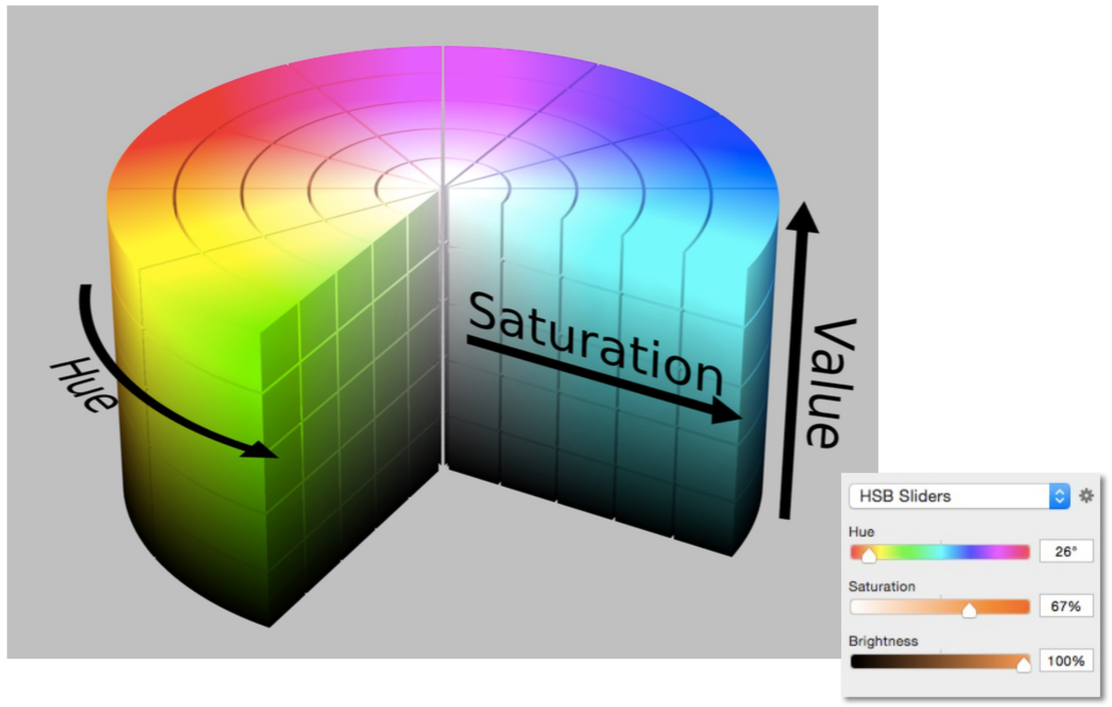
HSB Sliders

Hue: 26°

Saturation: 67%

Brightness: 100%

HSV Color Space (Hue-Saturation-Value)



Hue 26°

Saturation 67%

Brightness 100%

Perceptual dimensions of color

Hue

- the "kind" of color, regardless of attributes
- colorimetric correlate: dominant wavelength
- artist's correlate: the chosen pigment color

Saturation

- the "colorfulness"
- colorimetric correlate: purity
- artist's correlate: fraction of paint from the colored tube

Lightness (or value)

- the overall amount of light
- colorimetric correlate: luminance
- artist's correlate: tints are lighter, shades are darker

CIELAB (AKA L*a*b*)

A perceptually-organized color space that acts as a simple and useful color appearance model

Features

- Chromatic adaptation (white balance)
- Predicts color appearance
  - Opponent color encoding
  - Formulas for hue, chroma, lightness
- Perceptual uniformity (non-linear warping)

CIELAB Definition

CIEXYZ --> CIELAB

$$L^\star = 116\, f\!\left(\frac{Y}{Y_\mathrm{n}}\right) - 16$$

$$a^\star = 500\left(f\!\left(\frac{X}{X_\mathrm{n}}\right) - f\!\left(\frac{Y}{Y_\mathrm{n}}\right)\right)$$

$$b^\star = 200\left(f\!\left(\frac{Y}{Y_\mathrm{n}}\right) - f\!\left(\frac{Z}{Z_\mathrm{n}}\right)\right)$$

where

$$f(t) = \begin{cases} \sqrt[3]{t} & \text{if } t > \delta^3 \\ \frac{t}{3\delta^2} + \frac{4}{29} & \text{otherwise} \end{cases}$$

$$\delta = \frac{6}{29}$$

CIELAB --> CIEXYZ

$$X = X_\mathrm{n} f^{-1}\left(\frac{L^\star + 16}{116} + \frac{a^\star}{500}\right)$$

$$Y = Y_\mathrm{n} f^{-1}\left(\frac{L^\star + 16}{116}\right)$$

$$Z = Z_\mathrm{n} f^{-1}\left(\frac{L^\star + 16}{116} - \frac{b^\star}{200}\right)$$

where

$$f^{-1}(t) = \begin{cases} t^3 & \text{if } t > \delta \\ 3\delta^2\left(t - \frac{4}{29}\right) & \text{otherwise} \end{cases}$$

and where $\delta = 6/29$.

X_n, Y_n and Z_n are the CIEXYZ coordinates of the reference white point

Wikipedia

CIELAB Has Chromatic Adaptation (Reference White)

CIEXYZ --> CIELAB

$$L^\star = 116\, f\left(\frac{Y}{Y_\mathrm{n}}\right) - 16$$

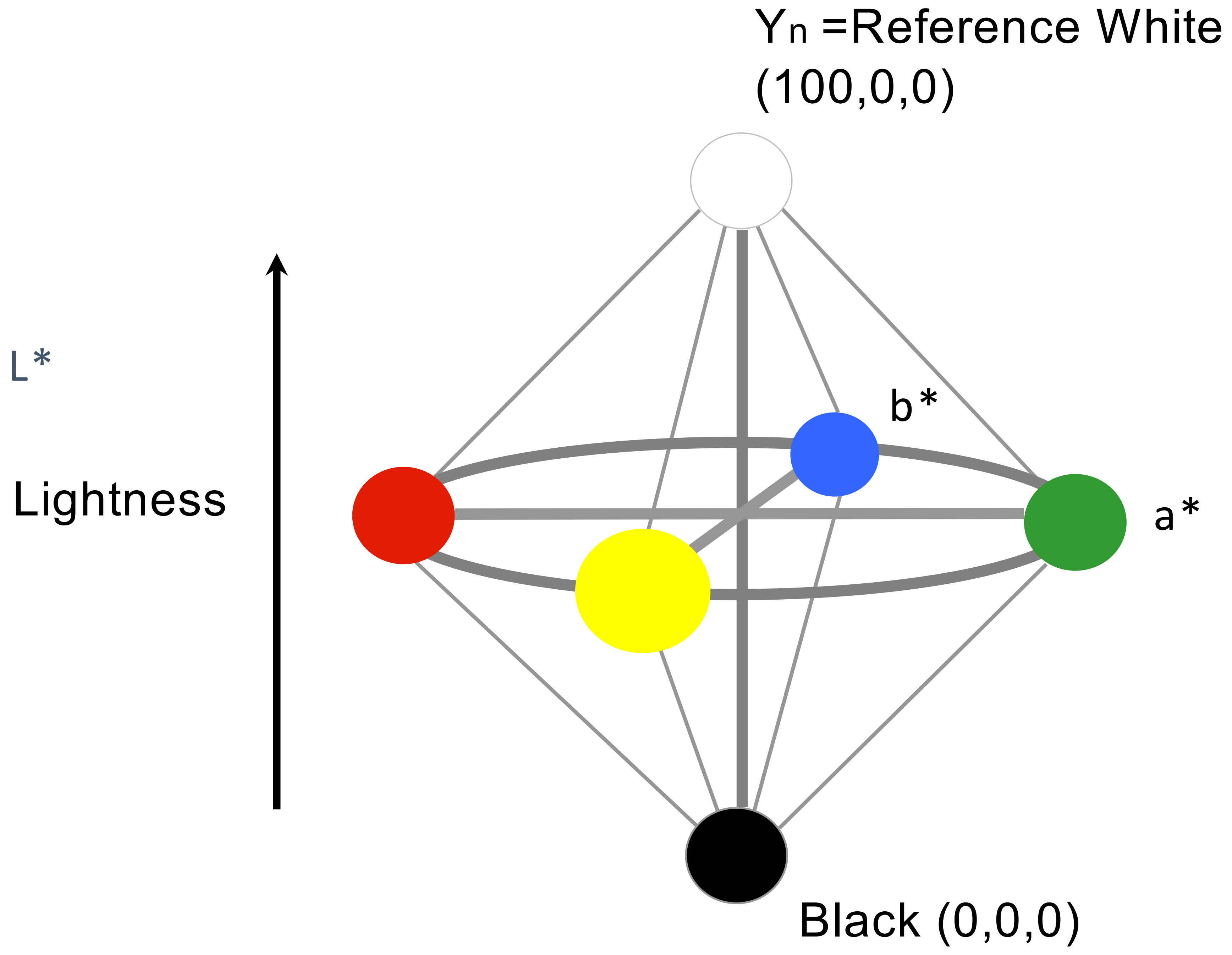
$$a^\star = 500\left(f\left(\frac{X}{X_\mathrm{n}}\right) - f\left(\frac{Y}{Y_\mathrm{n}}\right)\right)$$

$$b^\star = 200\left(f\left(\frac{Y}{Y_\mathrm{n}}\right) - f\left(\frac{Z}{Z_\mathrm{n}}\right)\right)$$

where

$$f(t) = \begin{cases} \sqrt[3]{t} & \text{if } t > \delta^3 \\ \frac{t}{3\delta^2} + \frac{4}{29} & \text{otherwise} \end{cases}$$

$$\delta = \frac{6}{29}$$



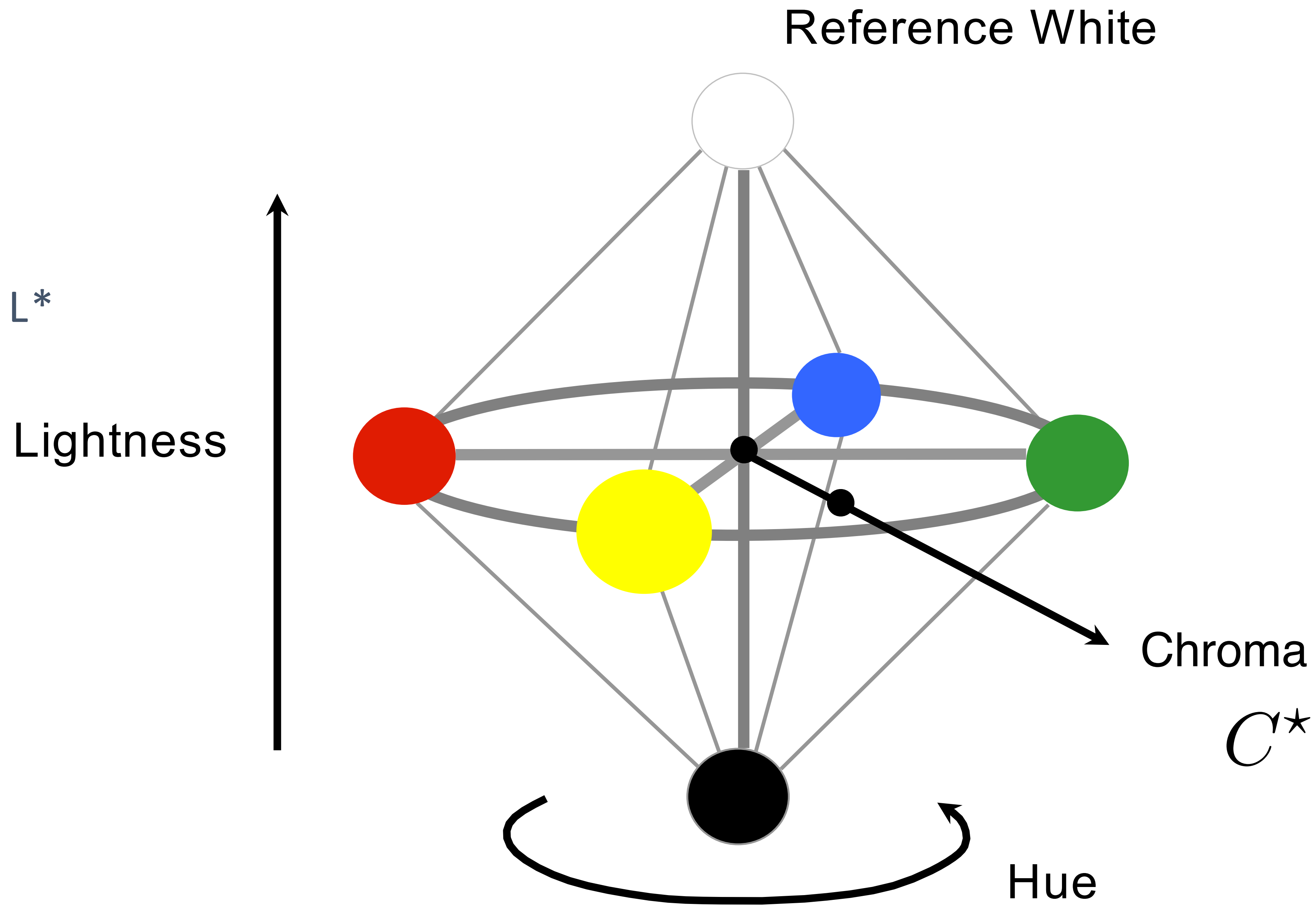
X_n, Y_n and Z_n are the CIEXYZ coordinates of the reference white point

Slide credit: Maureen Stone

CIELAB As a Color Appearance Model

Hue, chroma, lightness

Not L*, a*, b*



Reference White

L*

Lightness

Chroma

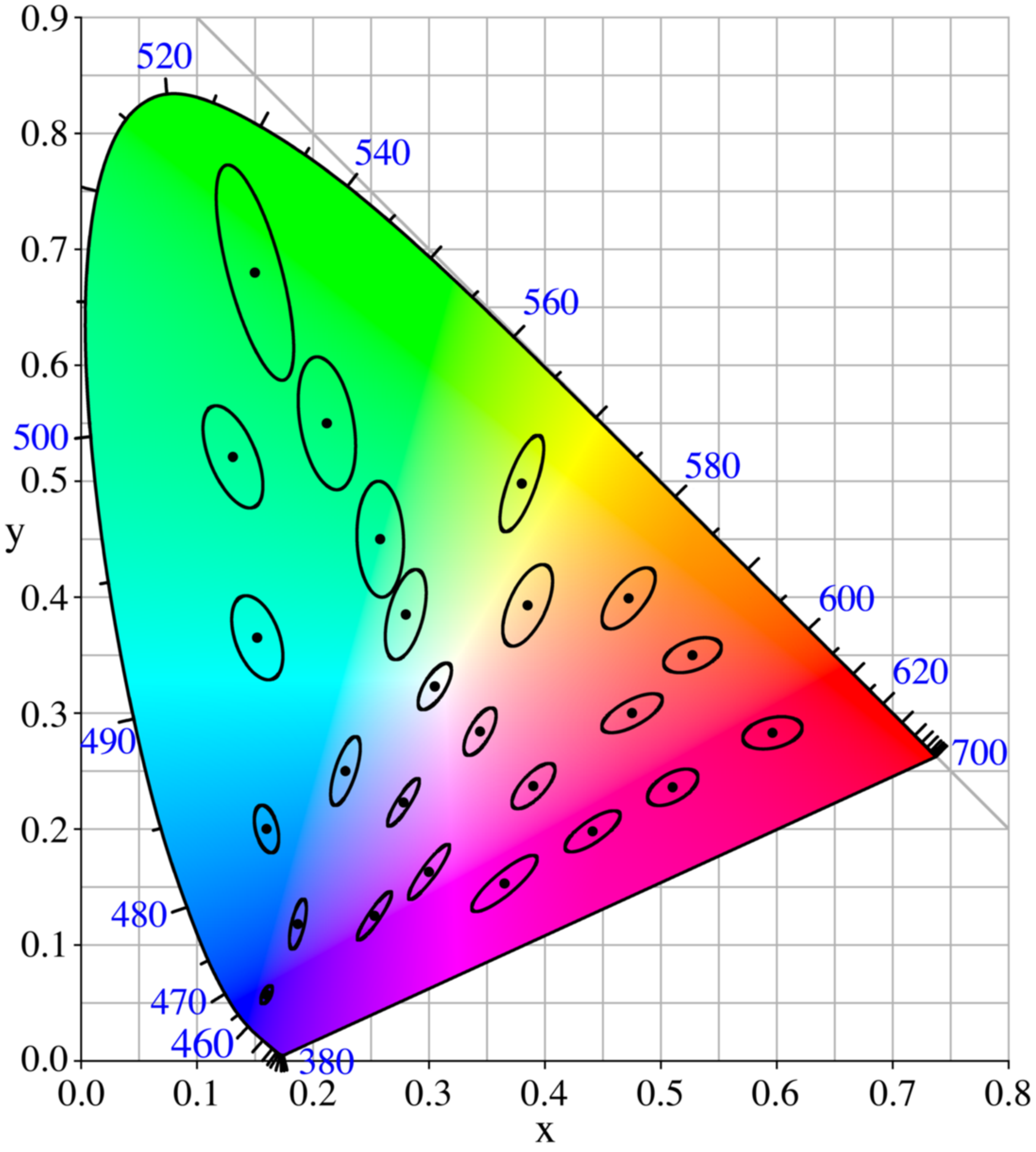
$$C^{\star} = \sqrt{{a^*}^2 + {b^*}^2}$$

Hue

$$h^{\circ} = \arctan\left(\frac{b^*}{a^*}\right)$$

Adapted from slide by M. Stone

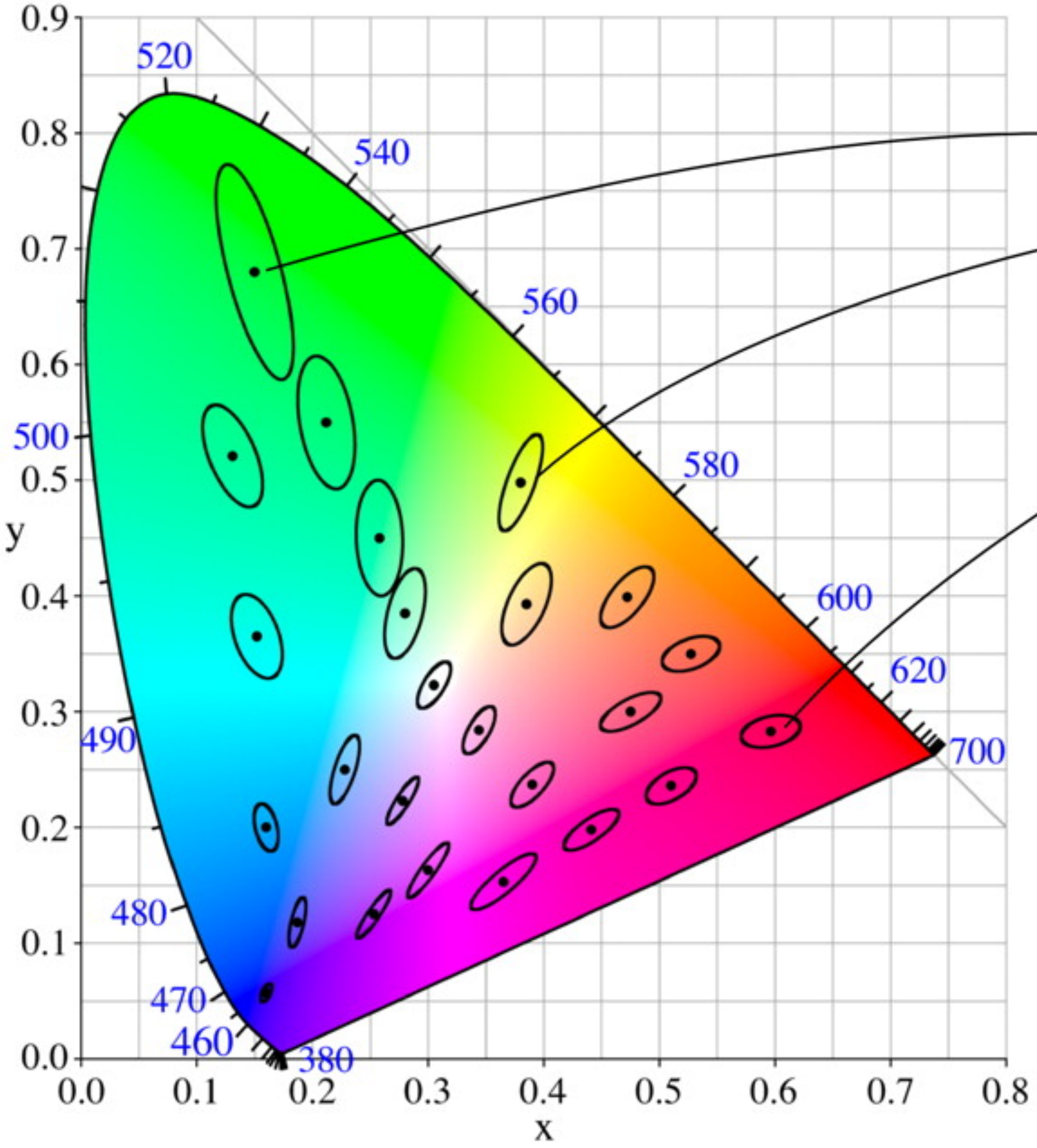
CIEXYZ is Not Perceptually Uniform



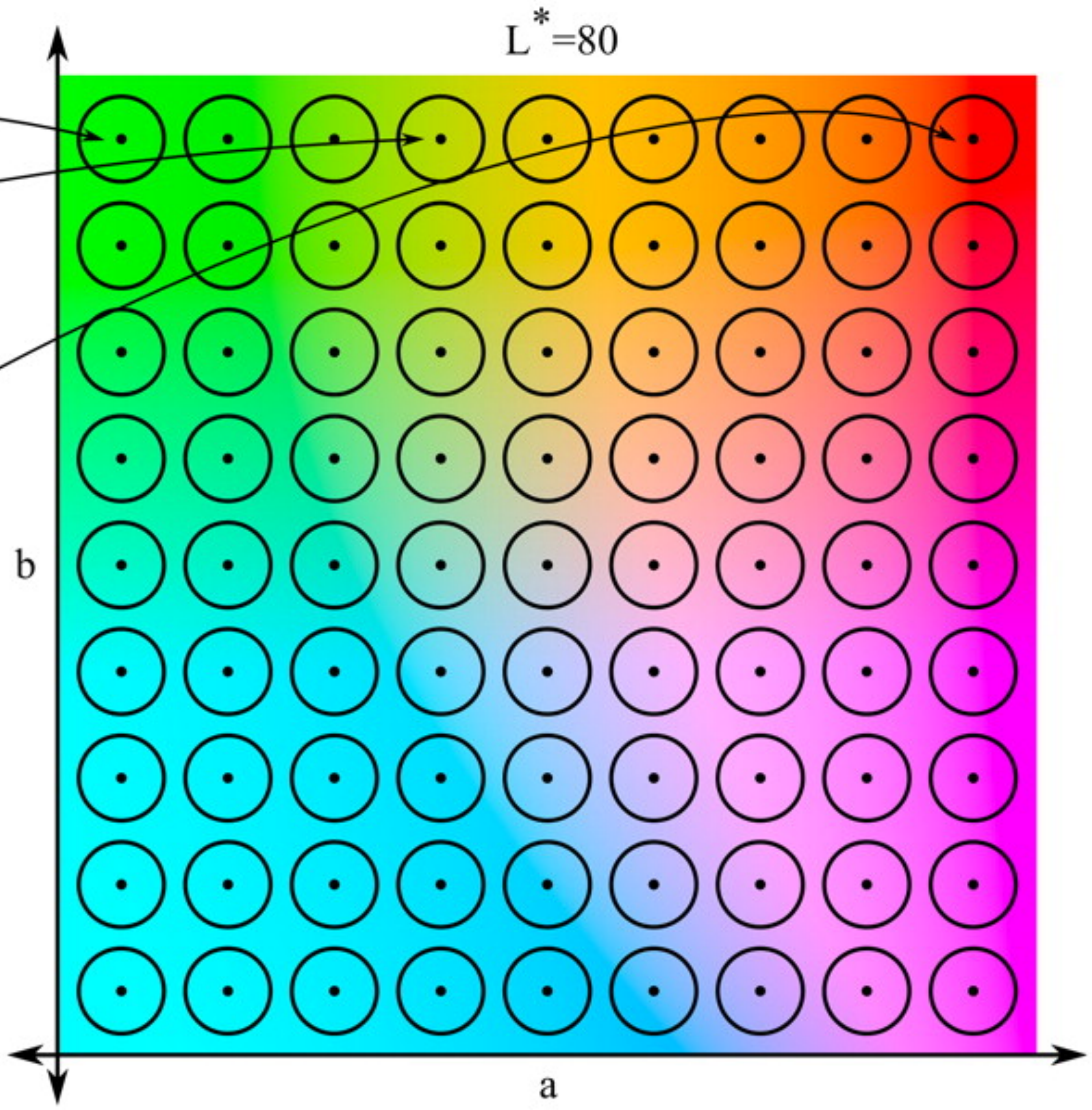
Wikipedia

- In the xy chromaticity diagram at left, MacAdam ellipses show regions of perceptually equivalent color (ellipses enlarged 10x)

CIELAB Aims for Perceptual Uniformity



CIE 1931



CIELAB (CIE 1976 L*,a*,b*)

From Henrich et al. 2011
https://iovs.arvojournals.org/article.aspx?articleid=2187751

Perceptual Normalization Function Applies to L*, a*, b*

CIEXYZ --> CIELAB

$$L^\star = 116\, f\left(\frac{Y}{Y_\mathrm{n}}\right) - 16$$

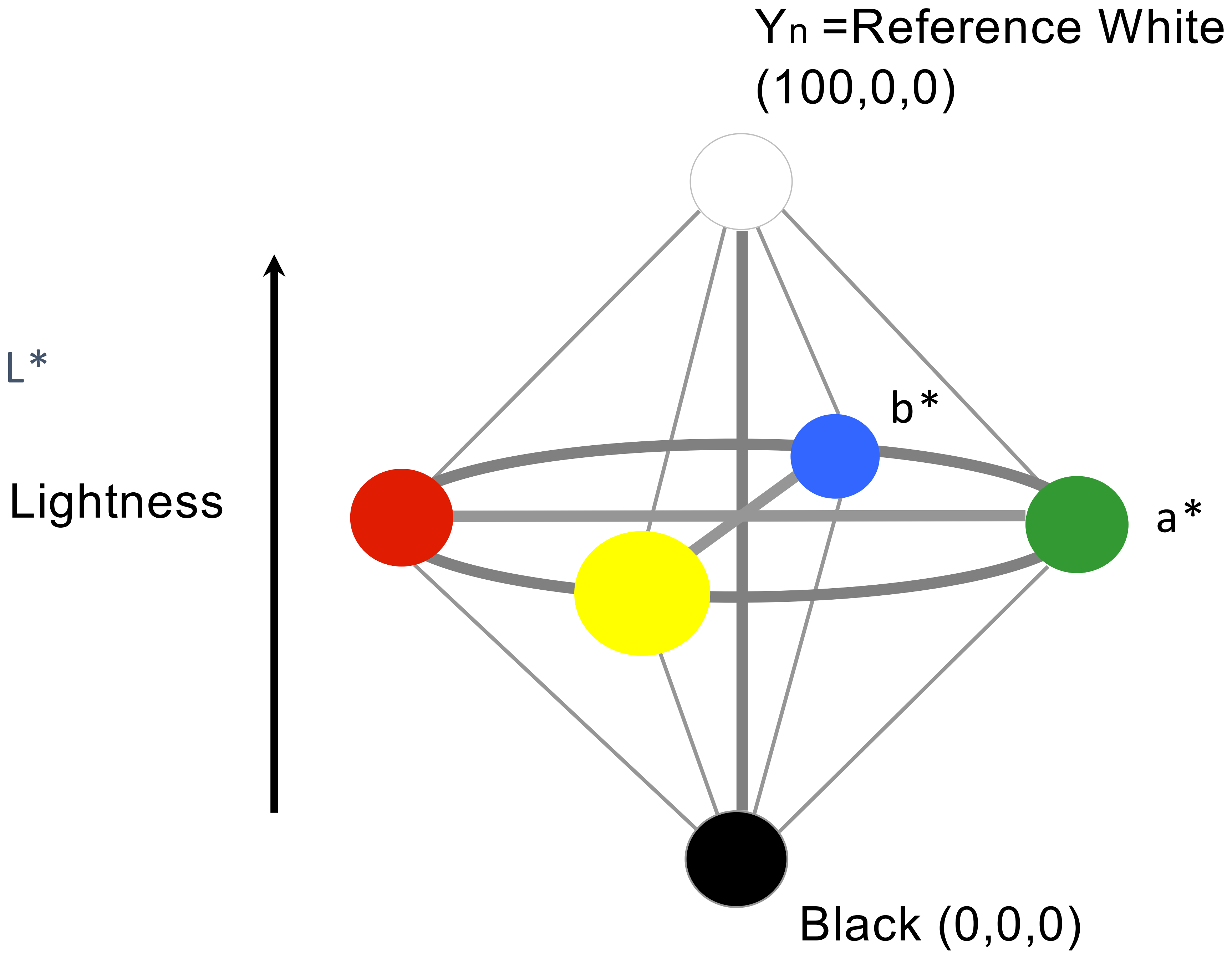
$$a^\star = 500\left(f\left(\frac{X}{X_\mathrm{n}}\right) - f\left(\frac{Y}{Y_\mathrm{n}}\right)\right)$$

$$b^\star = 200\left(f\left(\frac{Y}{Y_\mathrm{n}}\right) - f\left(\frac{Z}{Z_\mathrm{n}}\right)\right)$$

where

$$f(t) = \begin{cases} \sqrt[3]{t} & \text{if } t > \delta^3 \\ \frac{t}{3\delta^2} + \frac{4}{29} & \text{otherwise} \end{cases}$$

$$\delta = \frac{6}{29}$$



X_n, Y_n and Z_n are the CIEXYZ coordinates of the reference white point

Slide credit: Maureen Stone

CIELAB Gives a Recommended Color Difference Metric

Other color spaces we looked at (RGB, CIEXYZ, HSV) are not perceptually uniform and are not recommended for color difference calculations

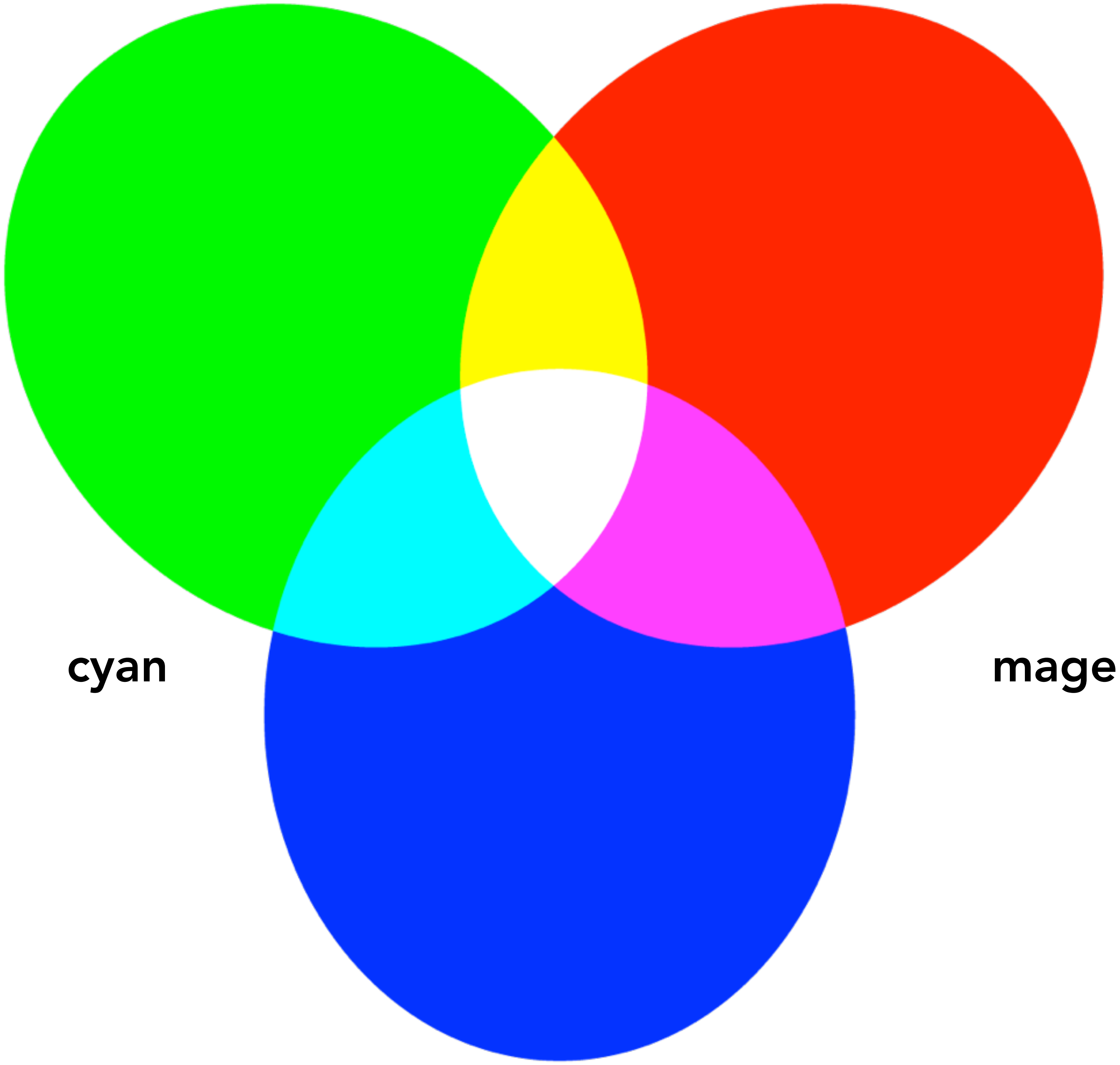
- E.g. a pair of colors that look similar to a human observer may have R,G,B coordinates further apart than another pair of colors that look quite different
- Try converting colors to CIELAB coordinates for applications that need to quantify differences in color appearance

Distance between (L*, a*, b*) coordinates for two colors is a recommended color difference metric that is approximately perceptually uniform

- $\Delta E^*_{ab} = \sqrt{\left(L^*_2 - L^*_1\right)^2 + \left(a^*_2 - a^*_1\right)^2 + \left(b^*_2 - b^*_1\right)^2}$ ("Delta E")
- Caveat: ΔE^*_{ab} is not perfect (e.g. large differences, and differences between highly saturated colors are inaccurate). CIEDE2000 is a more complex/accurate metric based on color appearance models

Additive vs Subtractive Color
or
Beam Colors vs Object Colors

Additive Color



yellow

green

red

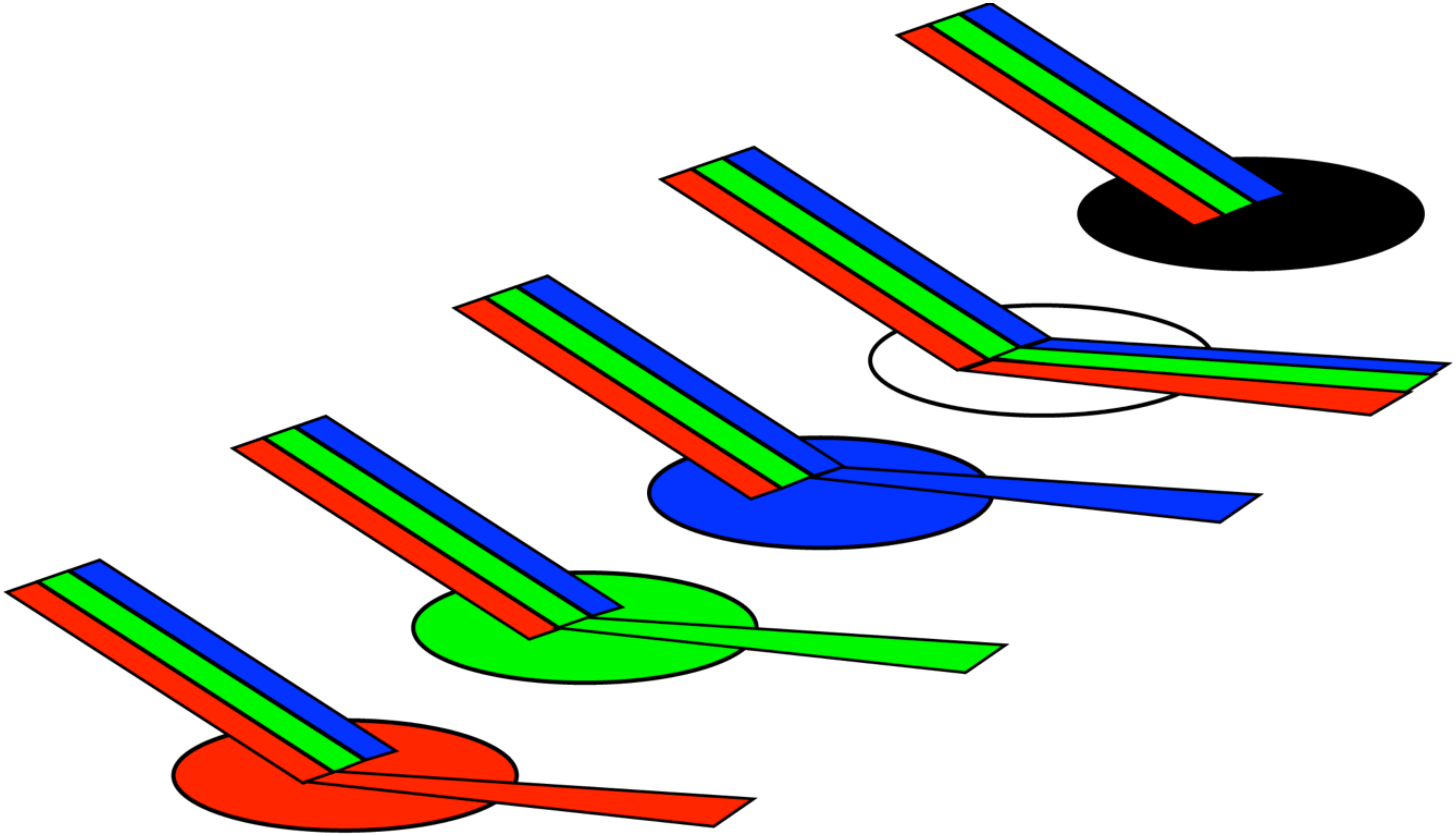
cyan

magenta

blue

Slide courtesy K. Breeden

Subtractive (Actually Multiplicative) Color



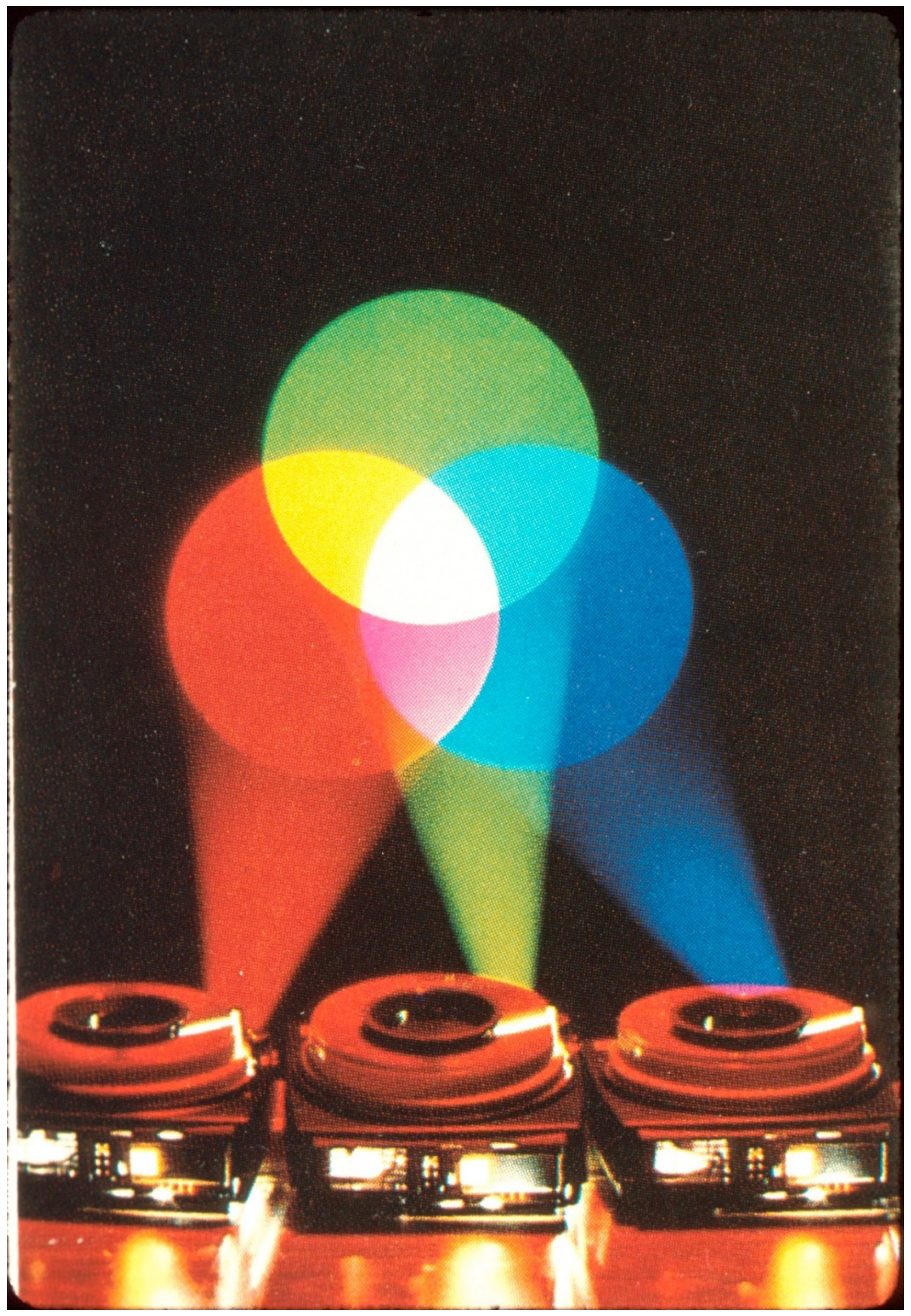
Shining white light on various colored pigments

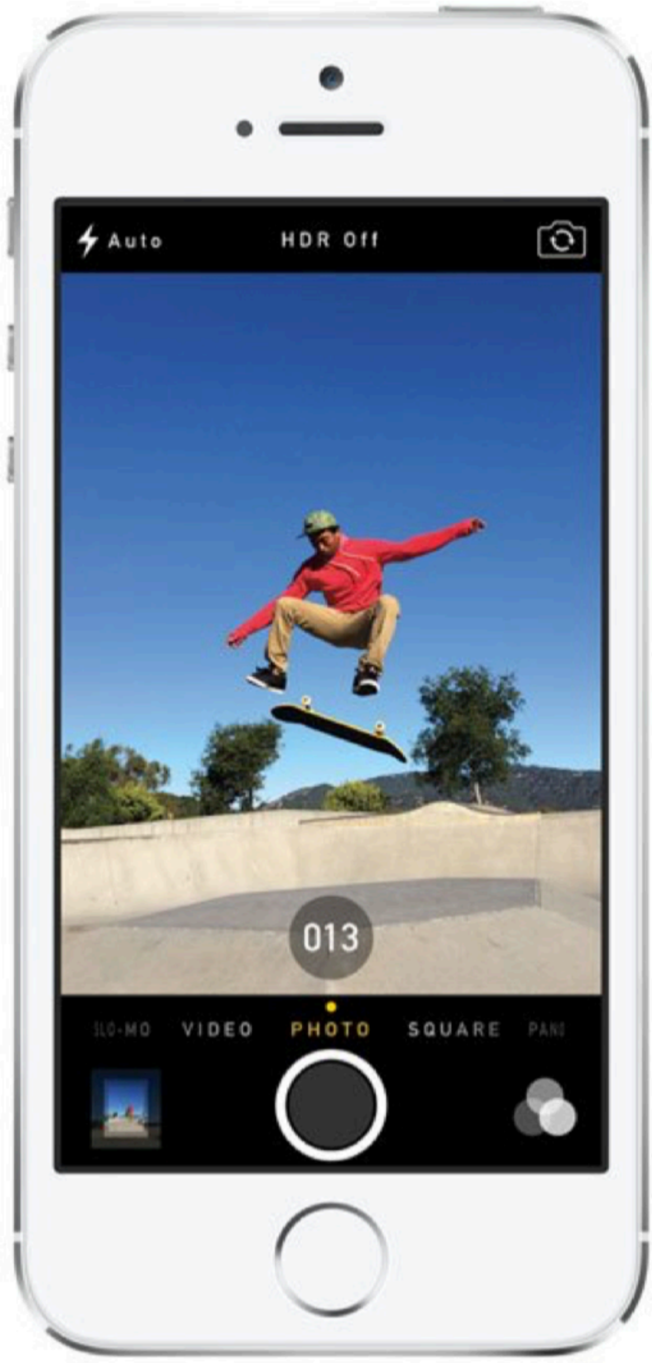
CS184/284A

Slide courtesy K. Breeden

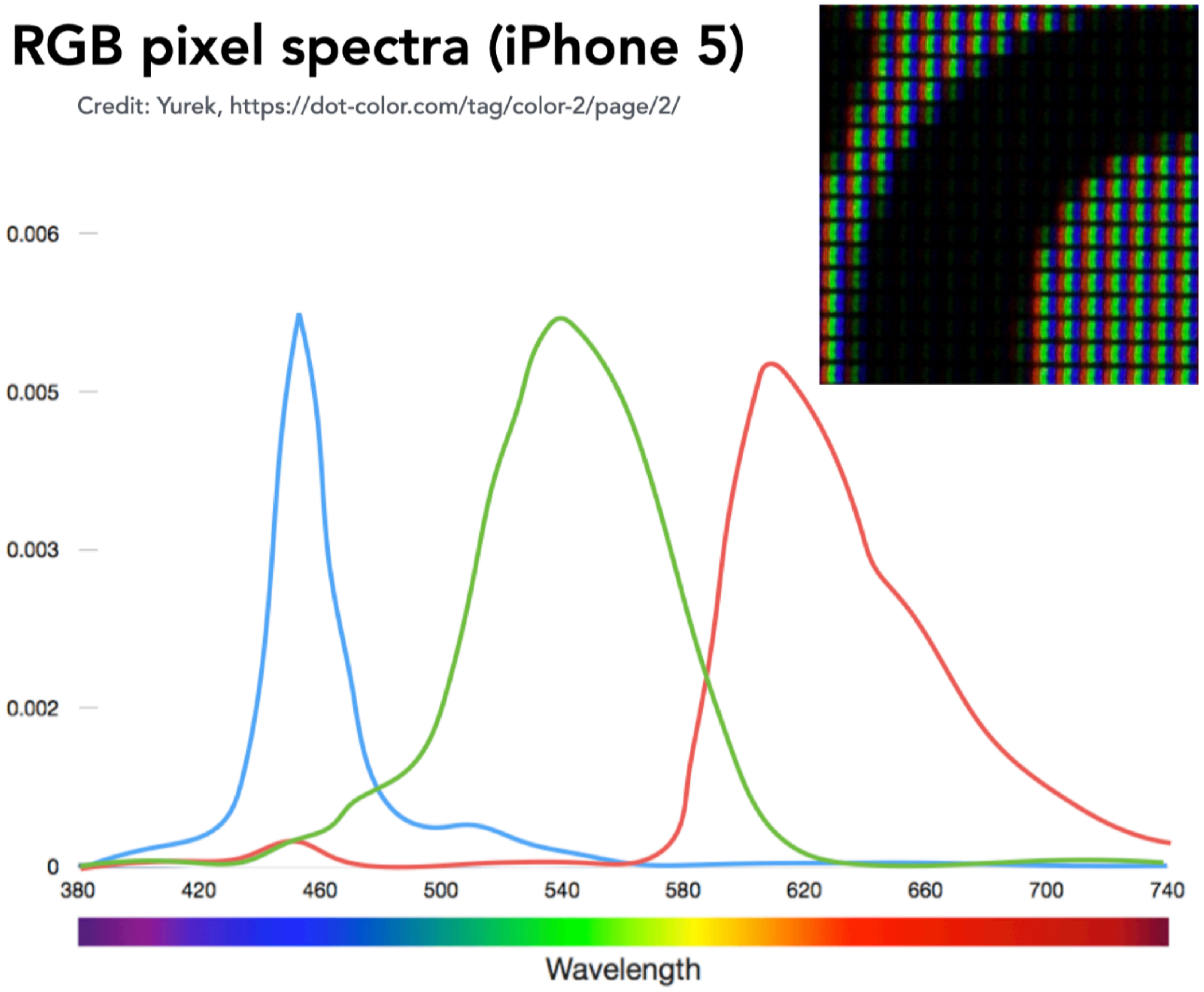
Ren Ng

Beam Colors and Additive Color





https://www.macrumors.com/roundup/iphone-5s/

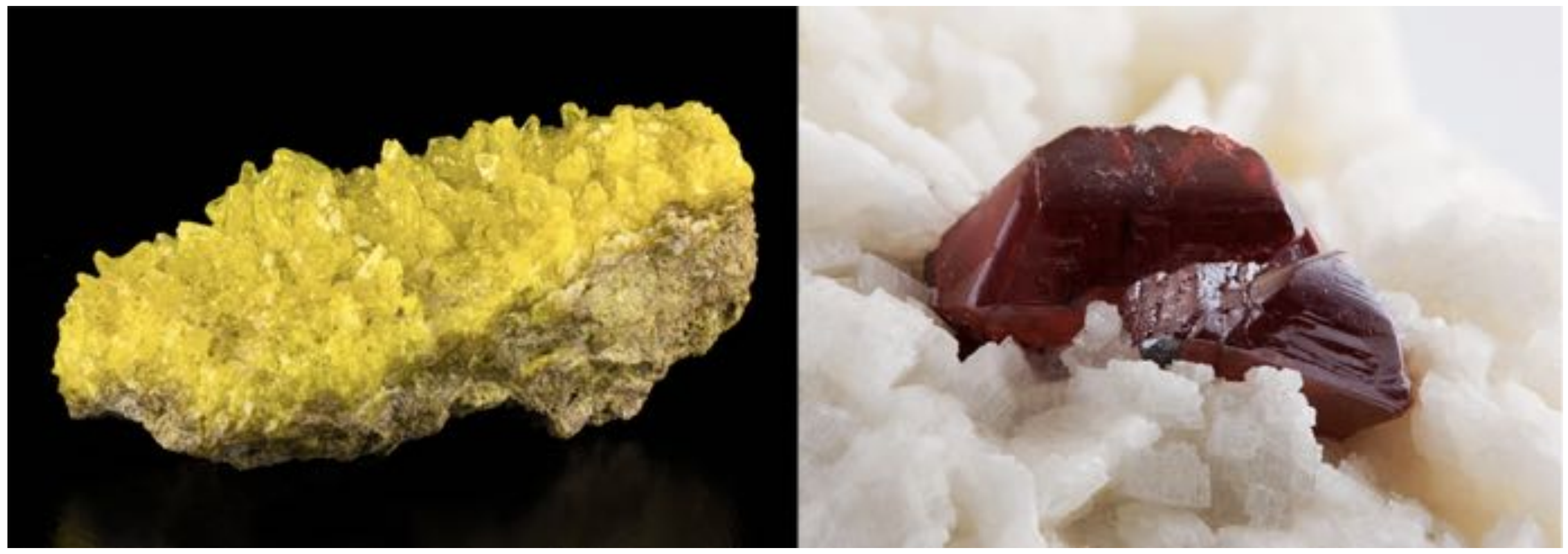


RGB pixel spectra (iPhone 5)

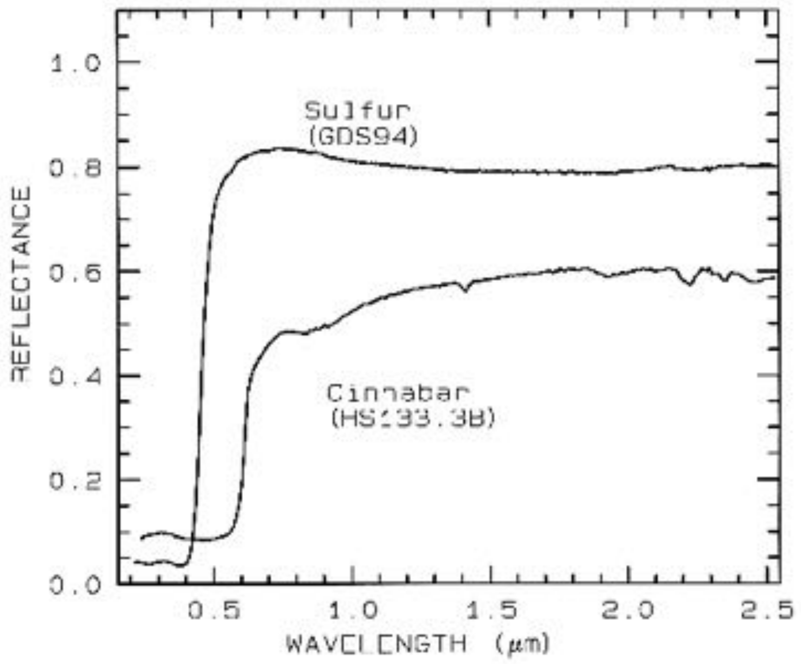
Credit: Yurek, https://dot-color.com/tag/color-2/page/2/

Object Colors - Multiplicative Color

The "Seven Sisters", Sussex. Such white chalk cliffs are the primordial objects.



Sulphur crystals (the element, bright yellow) and cinnabar (a deep red mercury(II) sulphide) on Dolomite.



Reflection (range 0–100%) spectra of sulphur and cinnabar. The wavelength range involves the infrared, the visual range is about 0.4–0.75µm. Notice that these spectra are roughly of an all-or-none type. There are no signs of anything special at some "yellow or red wavelength" as many naive persons are wont to think.

Koenderink, Colour in the Wild. 2018

Things to Remember

Physics of Light

- Spectral power distribution (SPD)
- Superposition (linearity)

Tristimulus theory of color

- Spectral response of human cone cells (S, M, L)
- Metamers - different SPDs with the same perceived color
- Color reproduction mathematics
- Color matching experiment, per-wavelength matching functions

Color spaces

- CIE RGB, XYZ, xy chromaticity, LAB (and DeltaE), HSV
- Gamut

Acknowledgments

Many thanks and credit for slides to Steve Marschner, Kayvon Fatahalian, Brian Wandell, Marc Levoy, Katherine Breeden, Austin Roorda, James O'Brien, Maureen Stone, Atsunobu Kotani.

Credit to

Michael S. Brown, "Understanding the In-Camera Image Processing Pipeline for Computer Vision", IEEE Computer Vision and Pattern Recognition - Tutorial, June 26, 2016.

Mark D. Fairchild, "Color appearance, color order, & other color systems," ISCC-AIC Munsell Centennial Color Symposium, Boston (2018).


CALVIN AND HOBBES
by WATTERSON
WOW, HONEY, YOU'RE MISSING A BEAUTIFUL SUNSET OUT HERE!
I'LL COUNT TO 10, AND THEN... POW!
DAD, HOW COME OLD PHOTOGRAPHS ARE ALWAYS BLACK AND WHITE? DIDN'T THEY HAVE COLOR FILM BACK THEN?
SURE THEY DID. IN FACT, THOSE OLD PHOTOGRAPHS ARE IN COLOR. IT'S JUST THE WORLD WAS BLACK AND WHITE THEN.
REALLY?
YEP. THE WORLD DIDN'T TURN COLOR UNTIL SOMETIME IN THE 1930s, AND IT WAS PRETTY GRAINY COLOR FOR A WHILE, TOO.
THAT'S REALLY WEIRD.
WELL, TRUTH IS STRANGER THAN FICTION.
BUT THEN WHY ARE OLD PAINTINGS IN COLOR?! IF THE WORLD WAS BLACK AND WHITE, WOULDN'T ARTISTS HAVE PAINTED IT THAT WAY?
NOT NECESSARILY. A LOT OF GREAT ARTISTS WERE INSANE.
BUT... BUT HOW COULD THEY HAVE PAINTED IN COLOR ANYWAY? WOULDN'T THEIR PAINTS HAVE BEEN SHADES OF GRAY BACK THEN?
OF COURSE, BUT THEY TURNED COLORS LIKE EVERYTHING ELSE DID IN THE '30s.
SO WHY DIDN'T OLD BLACK AND WHITE PHOTOS TURN COLOR TOO?
BECAUSE THEY WERE COLOR PICTURES OF BLACK AND WHITE, REMEMBER?
THE WORLD IS A COMPLICATED PLACE, HOBBES.
WHENEVER IT SEEMS THAT WAY, I TAKE A NAP IN A TREE AND WAIT FOR DINNER.