Lecture 5:

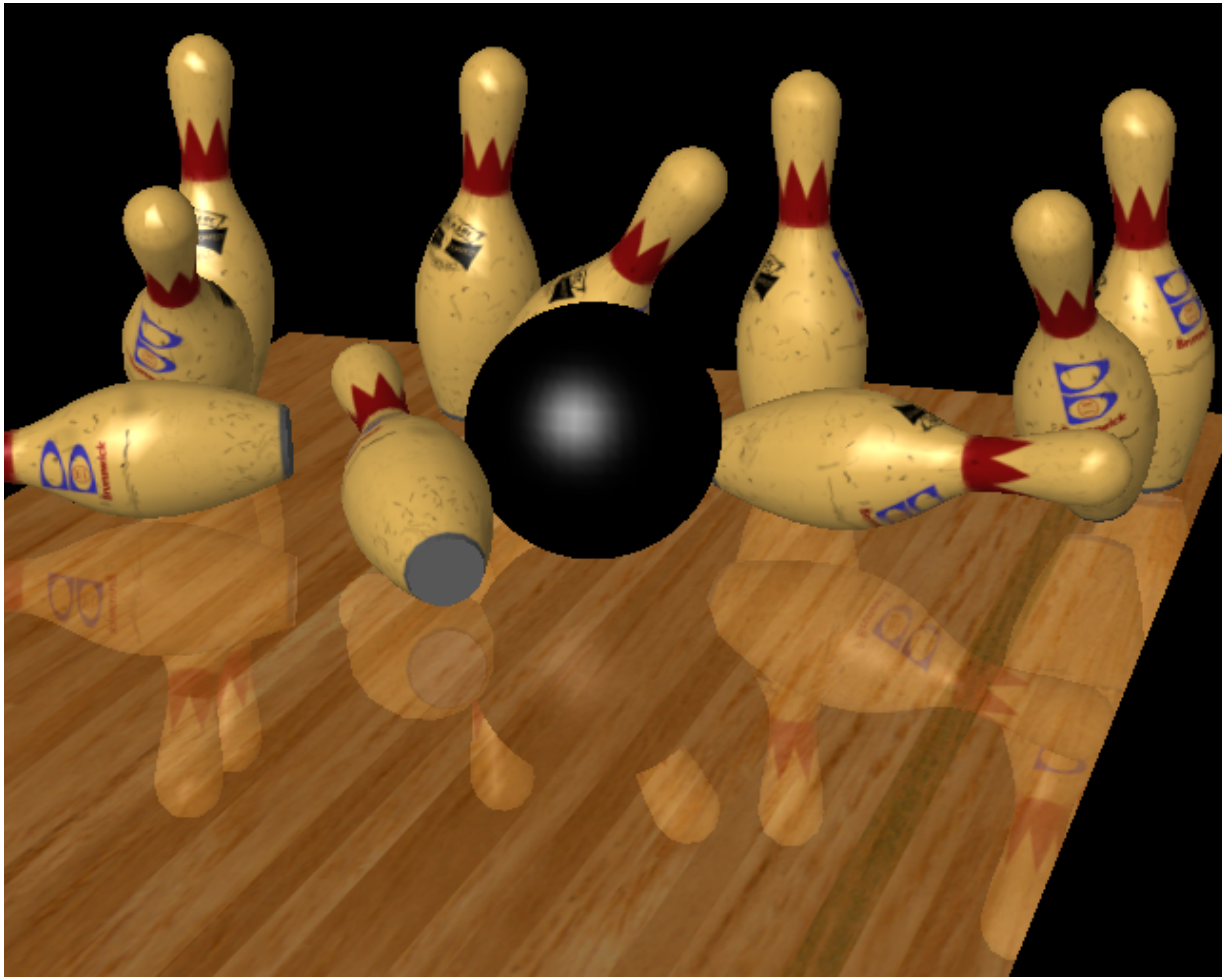
Texture Mapping

Computer Graphics and Imaging

UC Berkeley CS184/284A

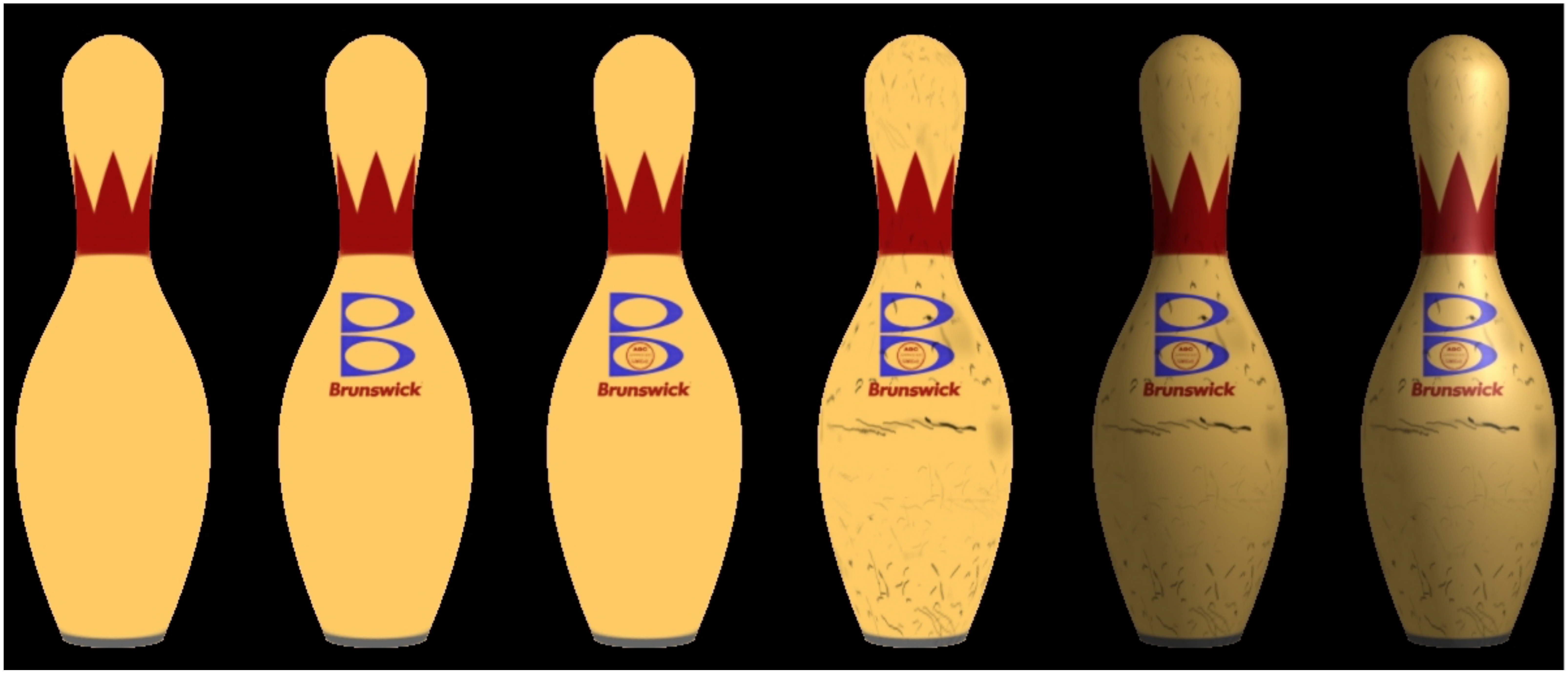
Texture Mapping Has Many Uses

Describe Surface Material Properties



Proudfoot et al.

Describe Surface Material Properties



Chan et al.

- Add details without raising geometric complexity
- Paste image onto geometry or define procedurally

Texture Coordinate Mappings

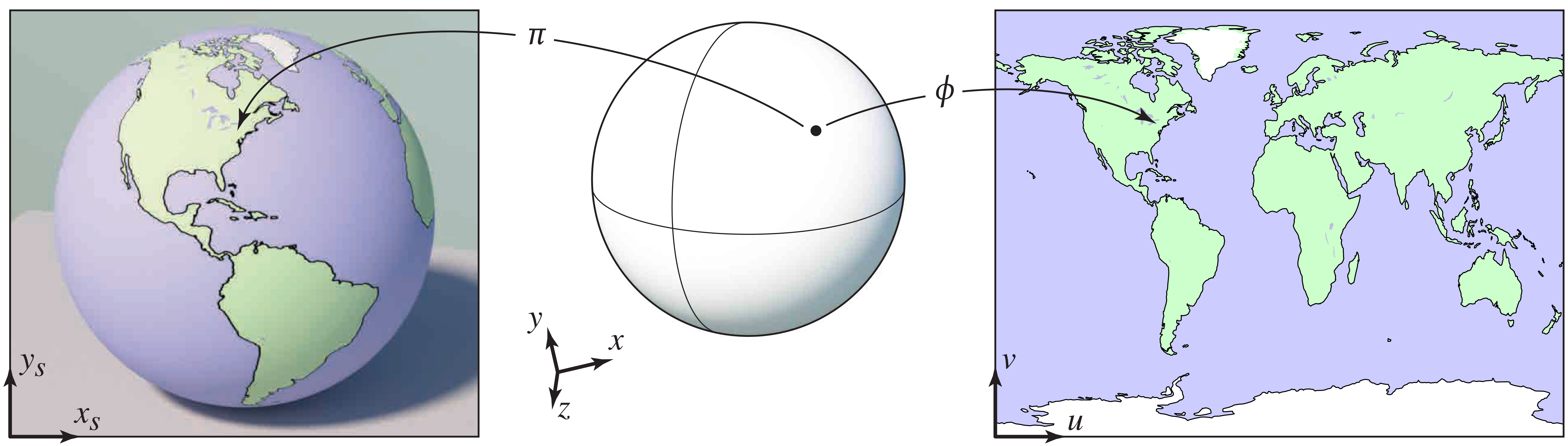
Think Chocolate Wrappers

Texture image

Three Spaces

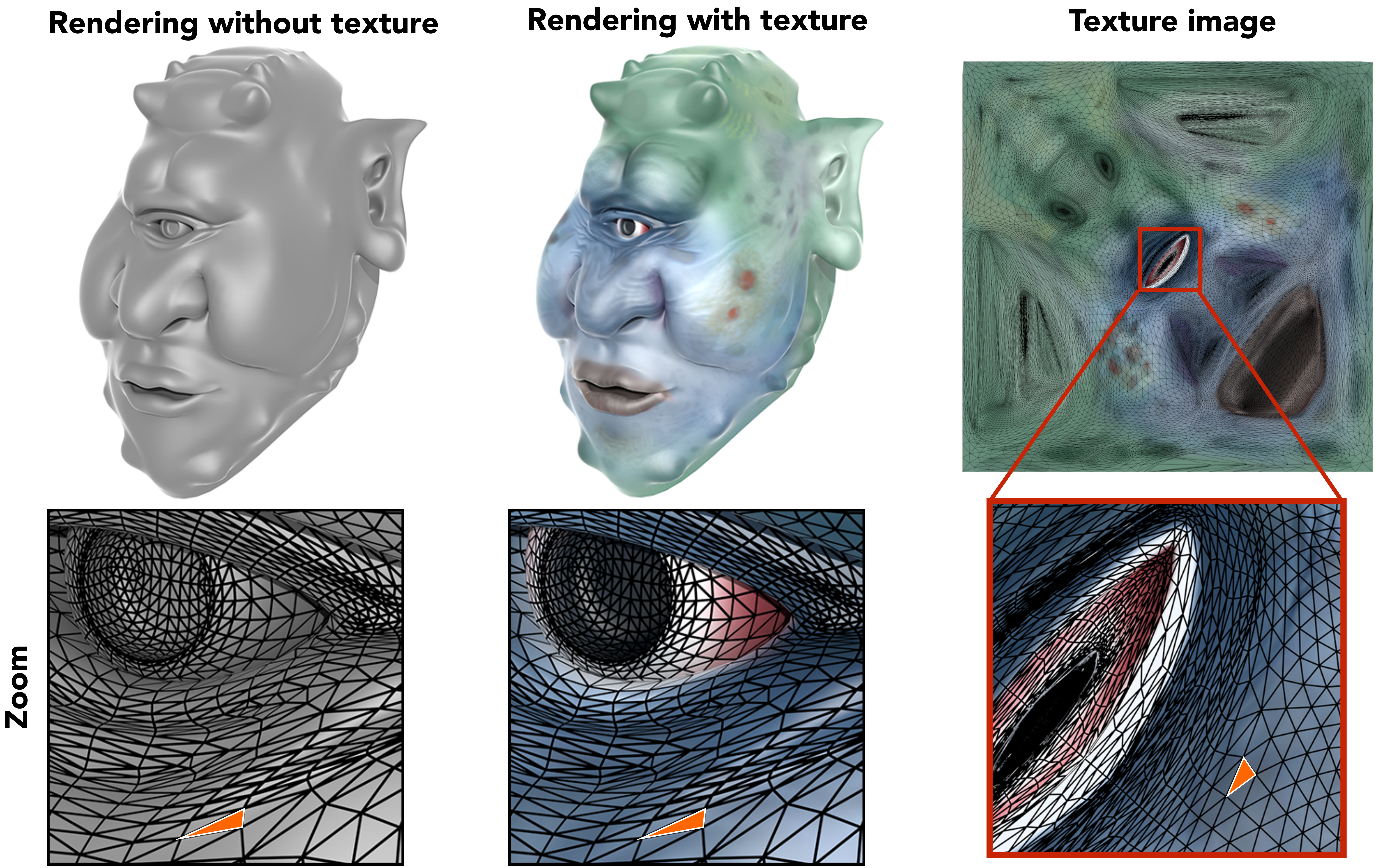
Surface lives in 3D world space

Every 3D surface point also has a place where it goes in the 2D image and in the 2D texture.



Screen space World space Texture space

Image Texture Applied to Surface

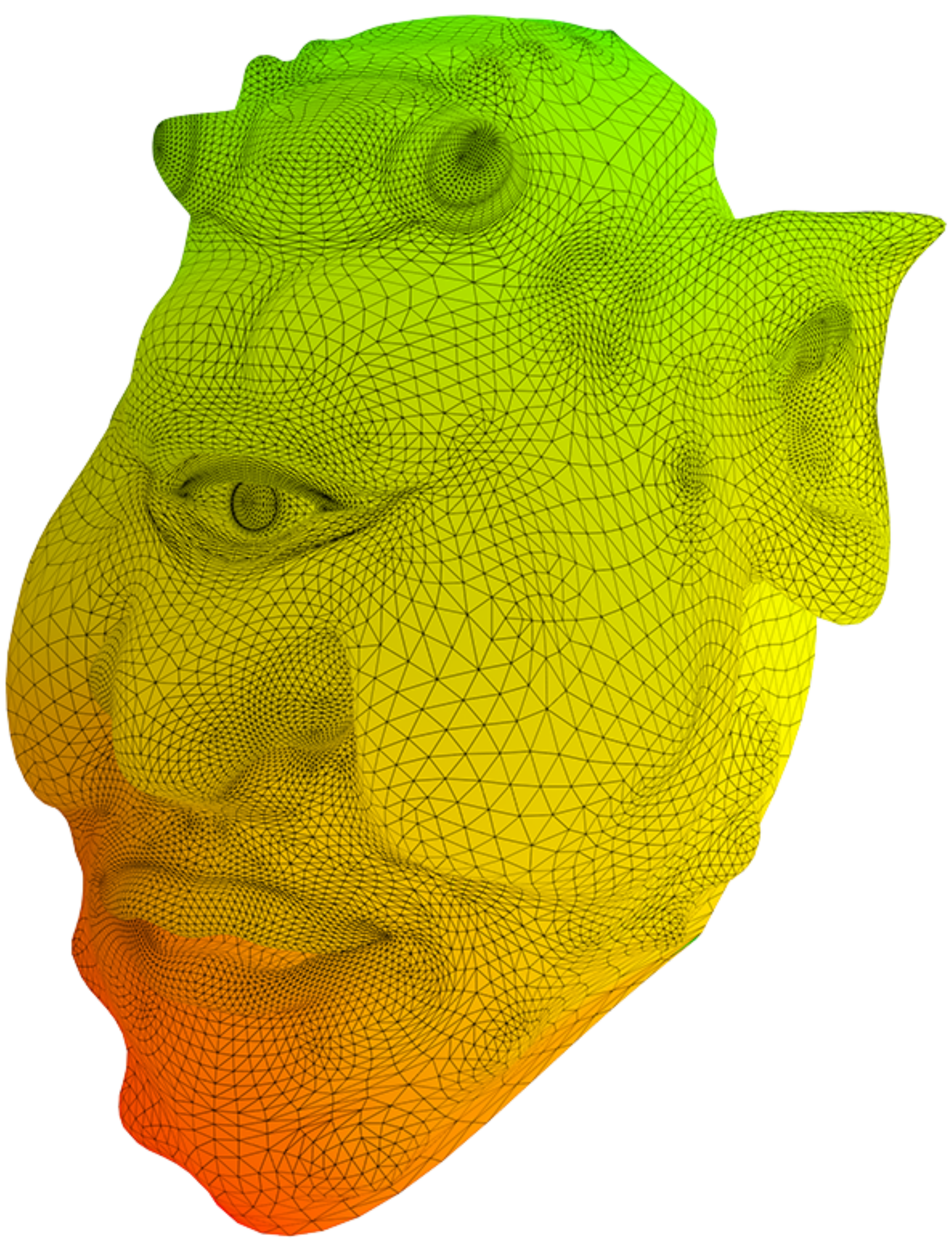


Each triangle “copies” a piece of the texture image back to the surface.

Visualization of Texture Coordinates

Each surface point is assigned a texture coordinate (u,v)

Visualization of texture coordinates



Triangle vertices in texture space

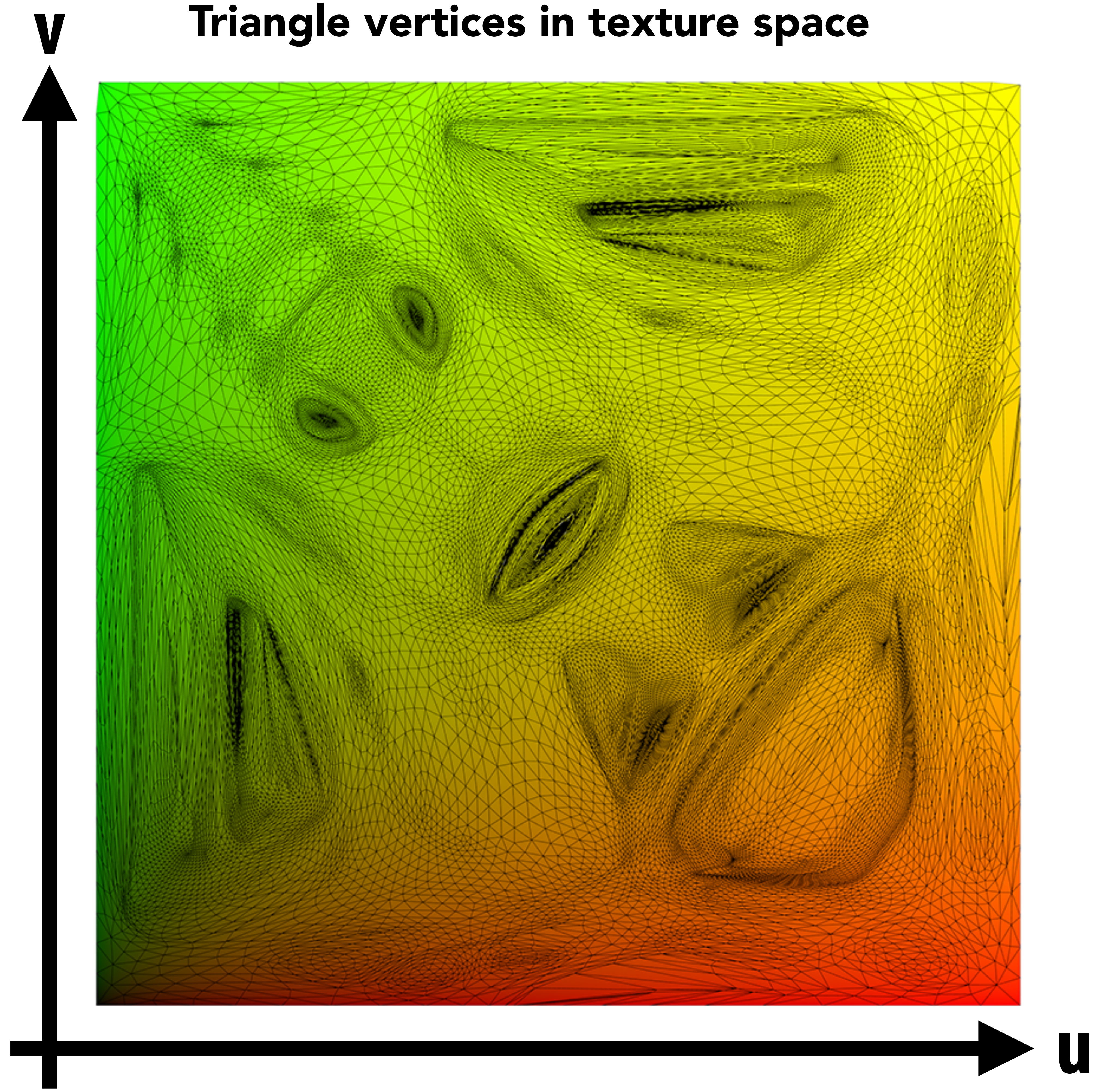
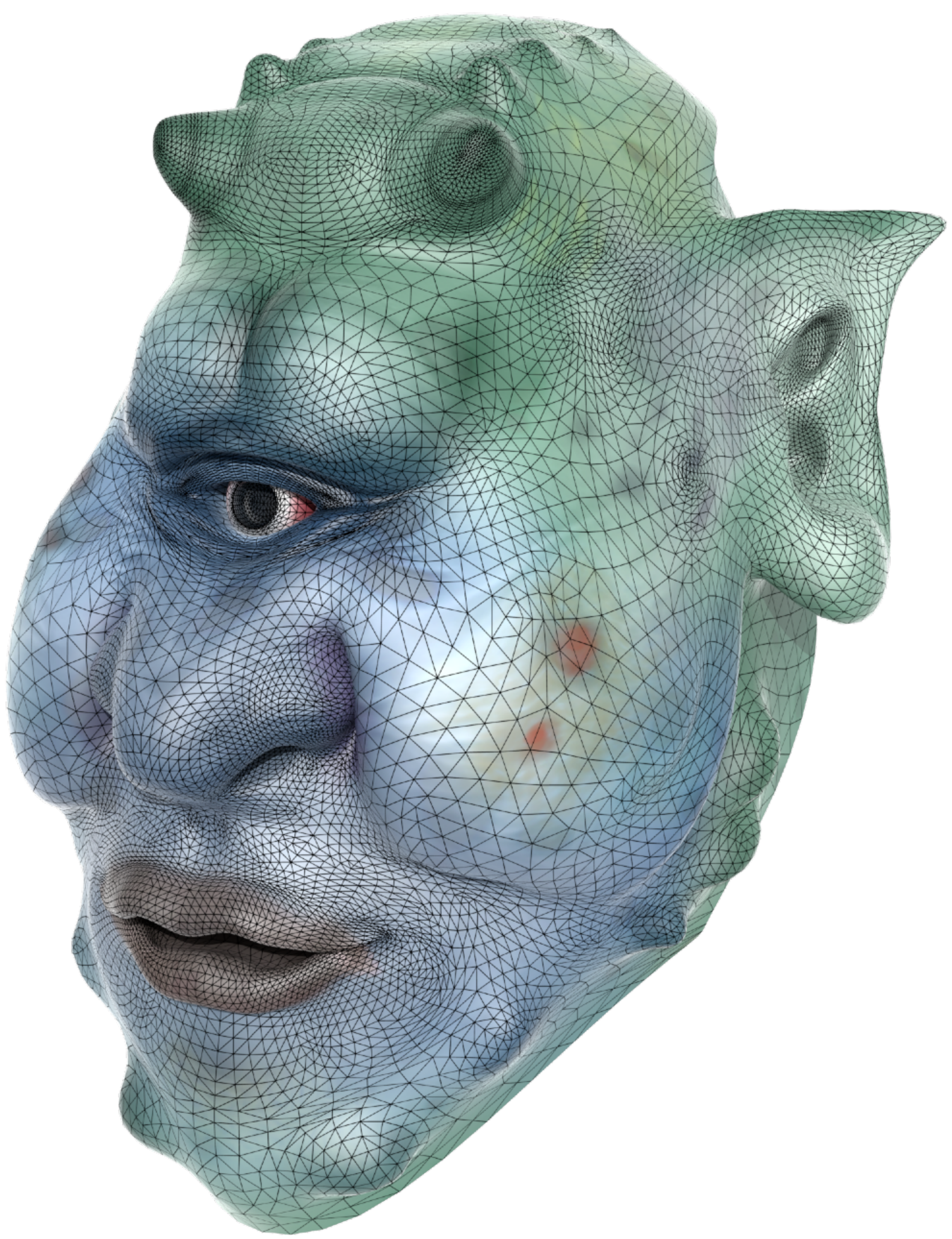


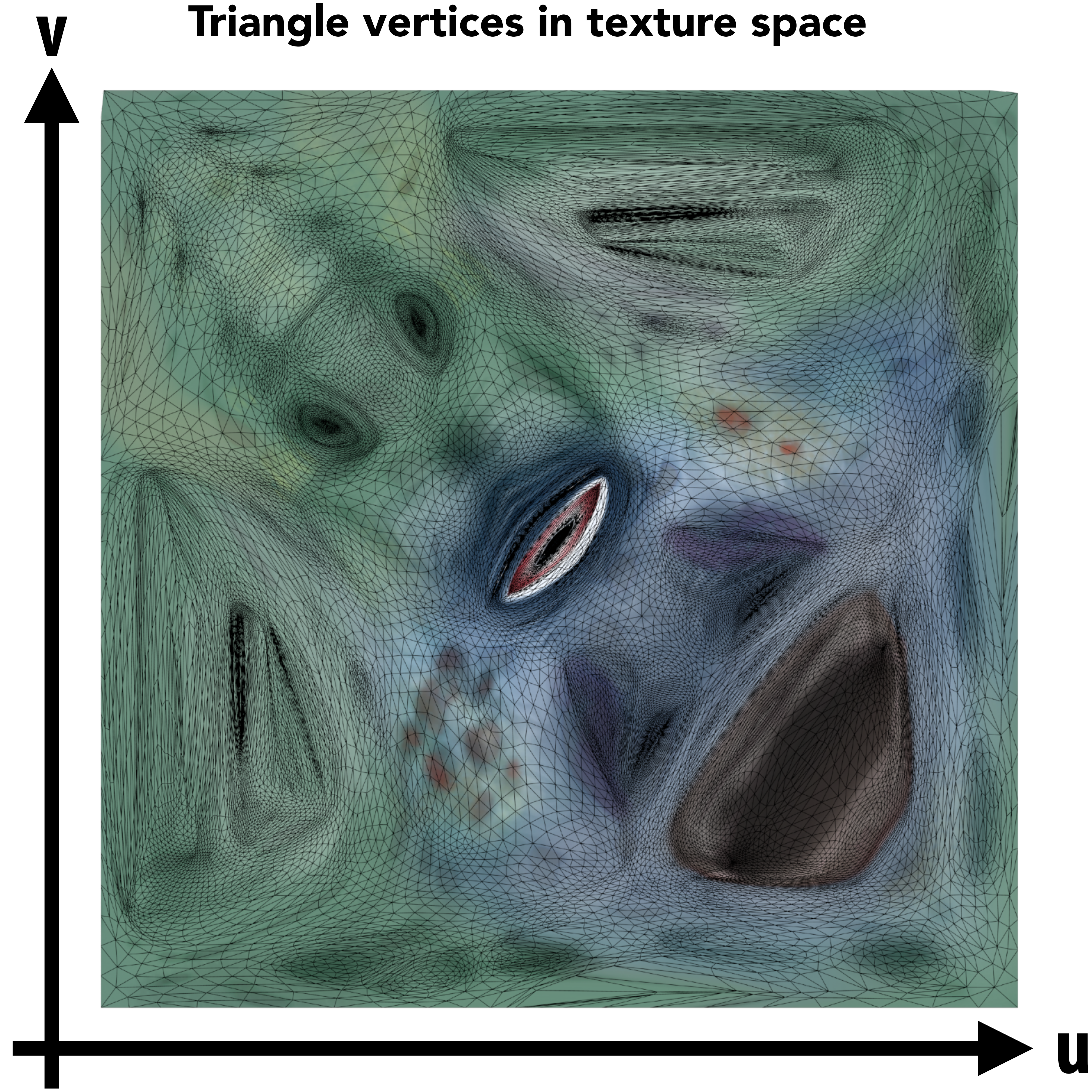
Image Texture Applied to Surface

Each surface point is assigned a texture coordinate (u,v)

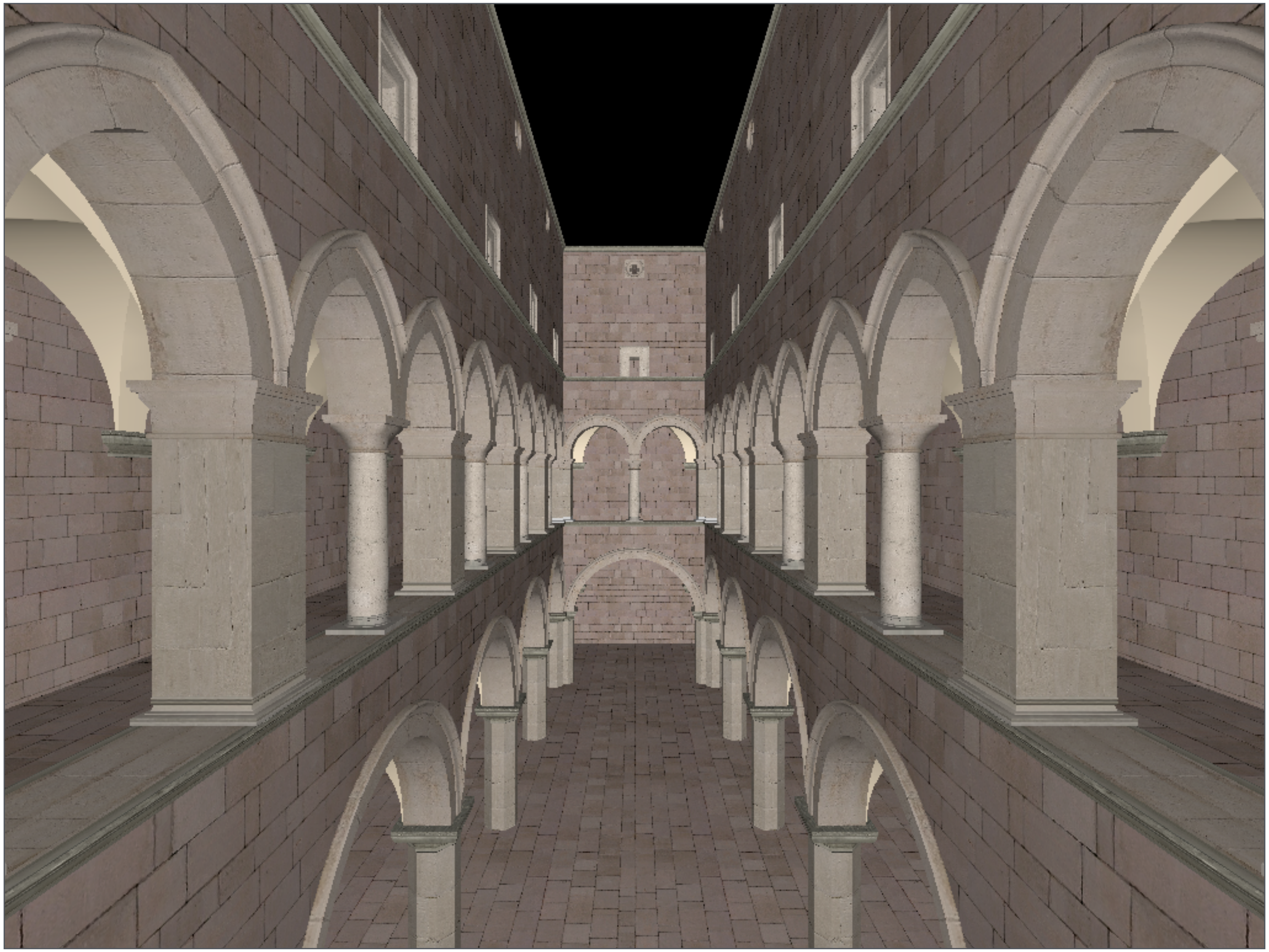
Rendered result



Triangle vertices in texture space

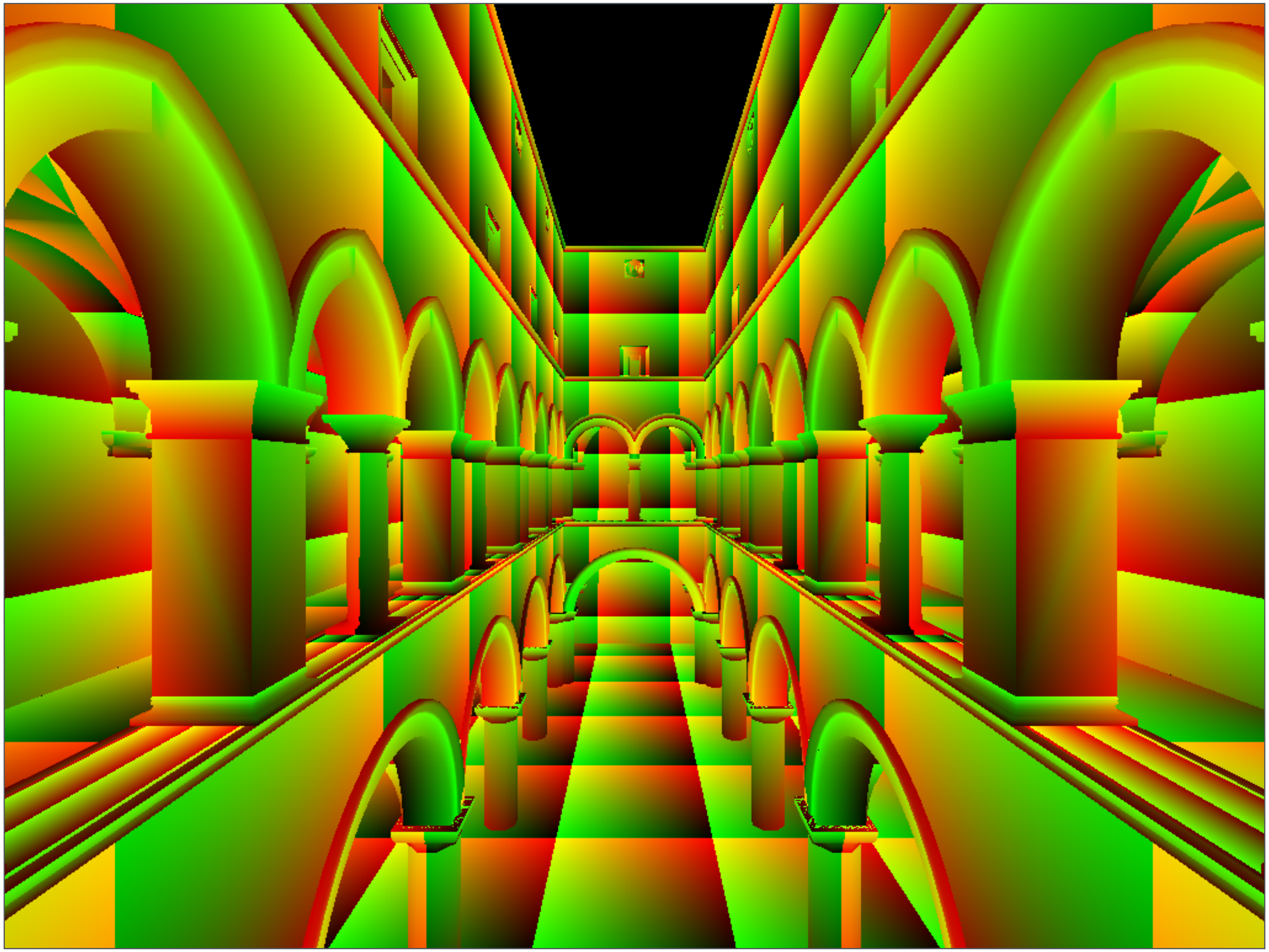


Sponza Palace Model



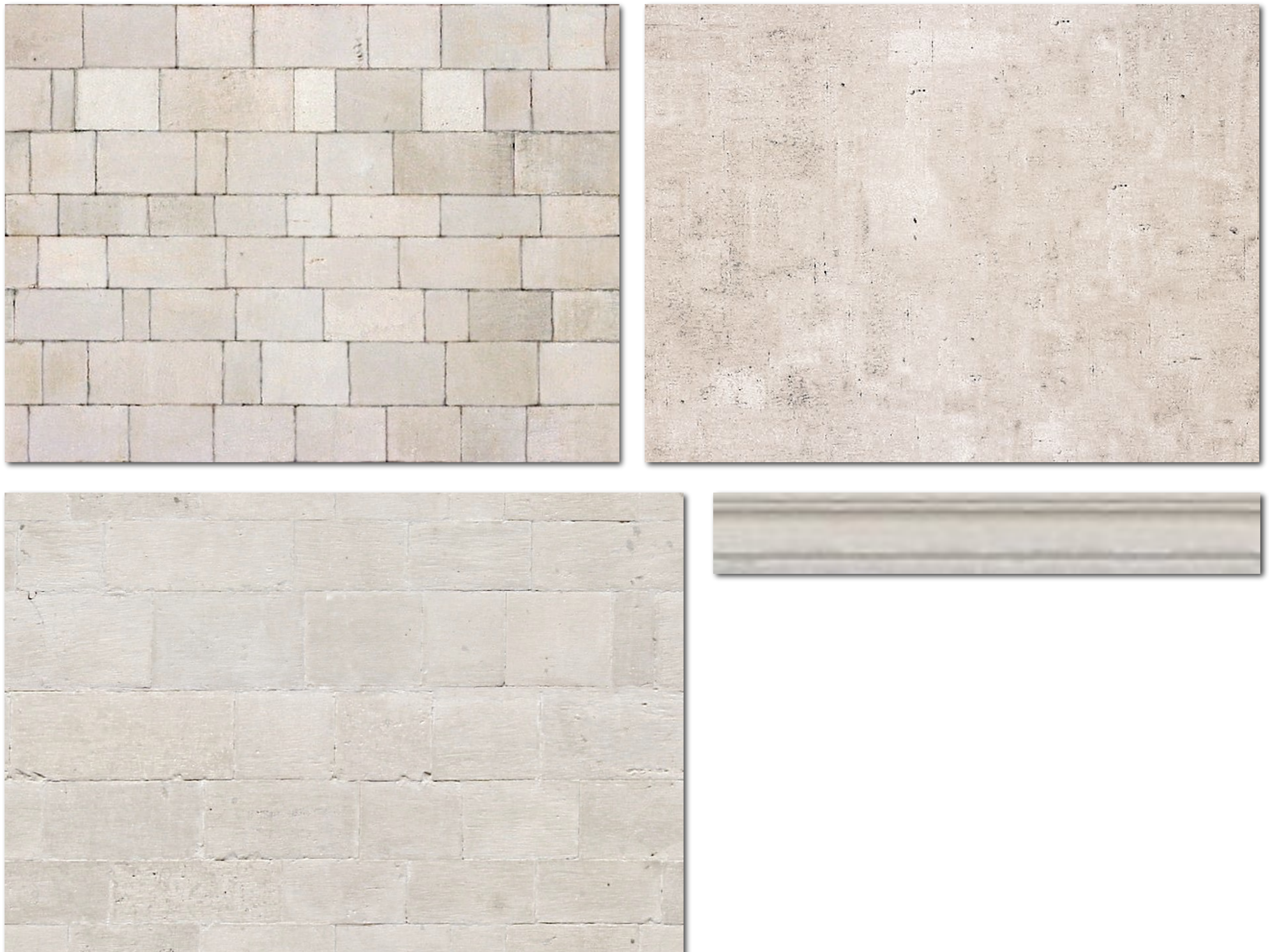
Textures applied to surfaces

Sponza Palace Model



Visualization of texture coordinates

Sponza Palace Model



Example textures used

Interpolation Across Triangles: Barycentric Coordinates

Interpolation Across Triangles

Why do we want to interpolate?

- Specify values (e.g. texture coordinates) at vertices, and obtain smoothly varying values across surface

What do we want to interpolate?

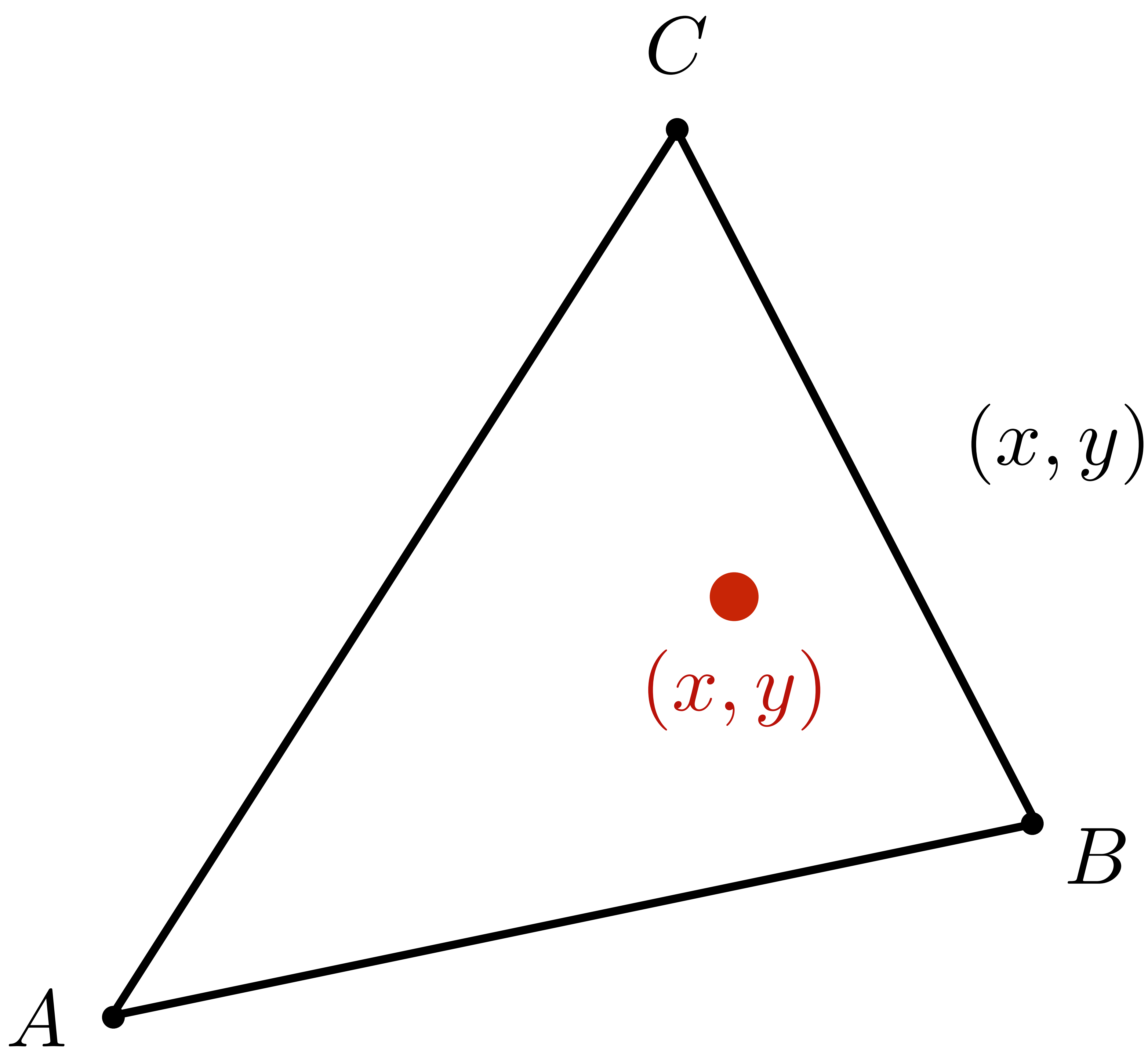
- Texture coordinates, colors, normal vectors, …

How do we interpolate?

- Barycentric coordinates

Barycentric Coordinates

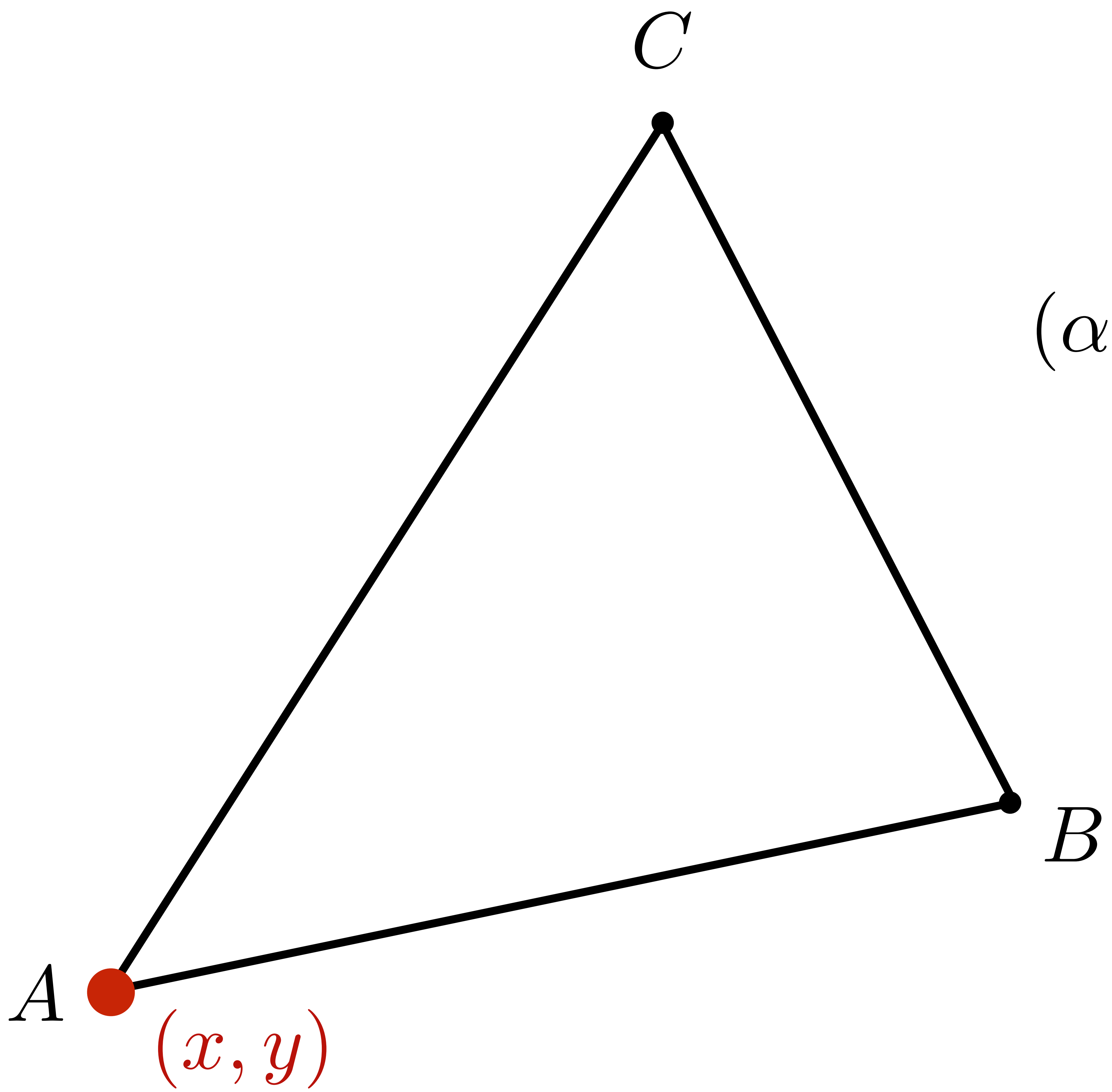
A coordinate system for triangles (α, β, γ)



$$(x, y) = \alpha\, A + \beta\, B + \gamma\, C$$
$$\alpha + \beta + \gamma = 1$$

Inside the triangle if all three coordinates are non-negative

Barycentric Coordinates - Examples

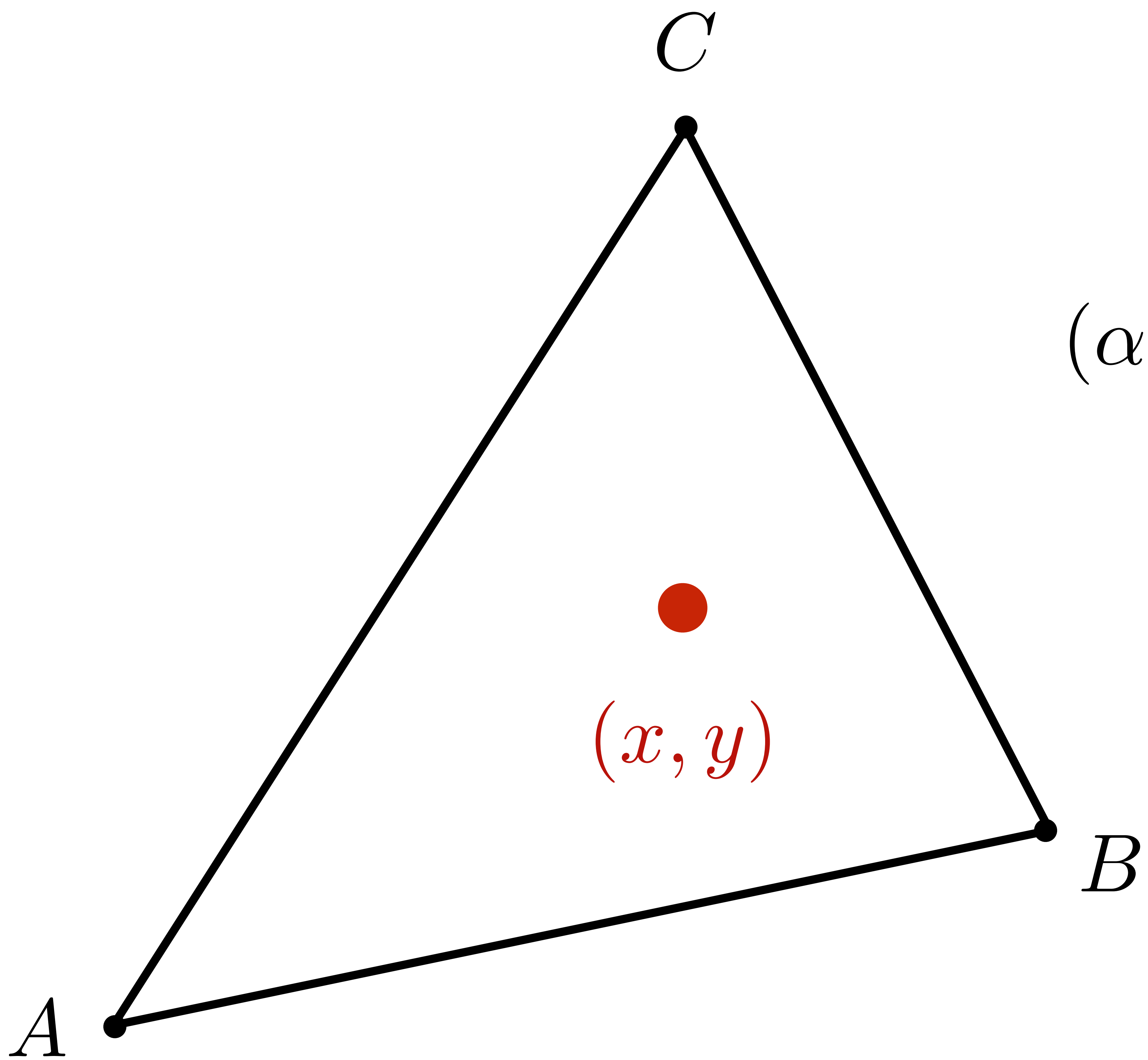


$$(\alpha, \beta, \gamma) = (1, 0, 0)$$

$$(x, y) = \alpha\, A + \beta\, B + \gamma\, C$$

$$= A$$

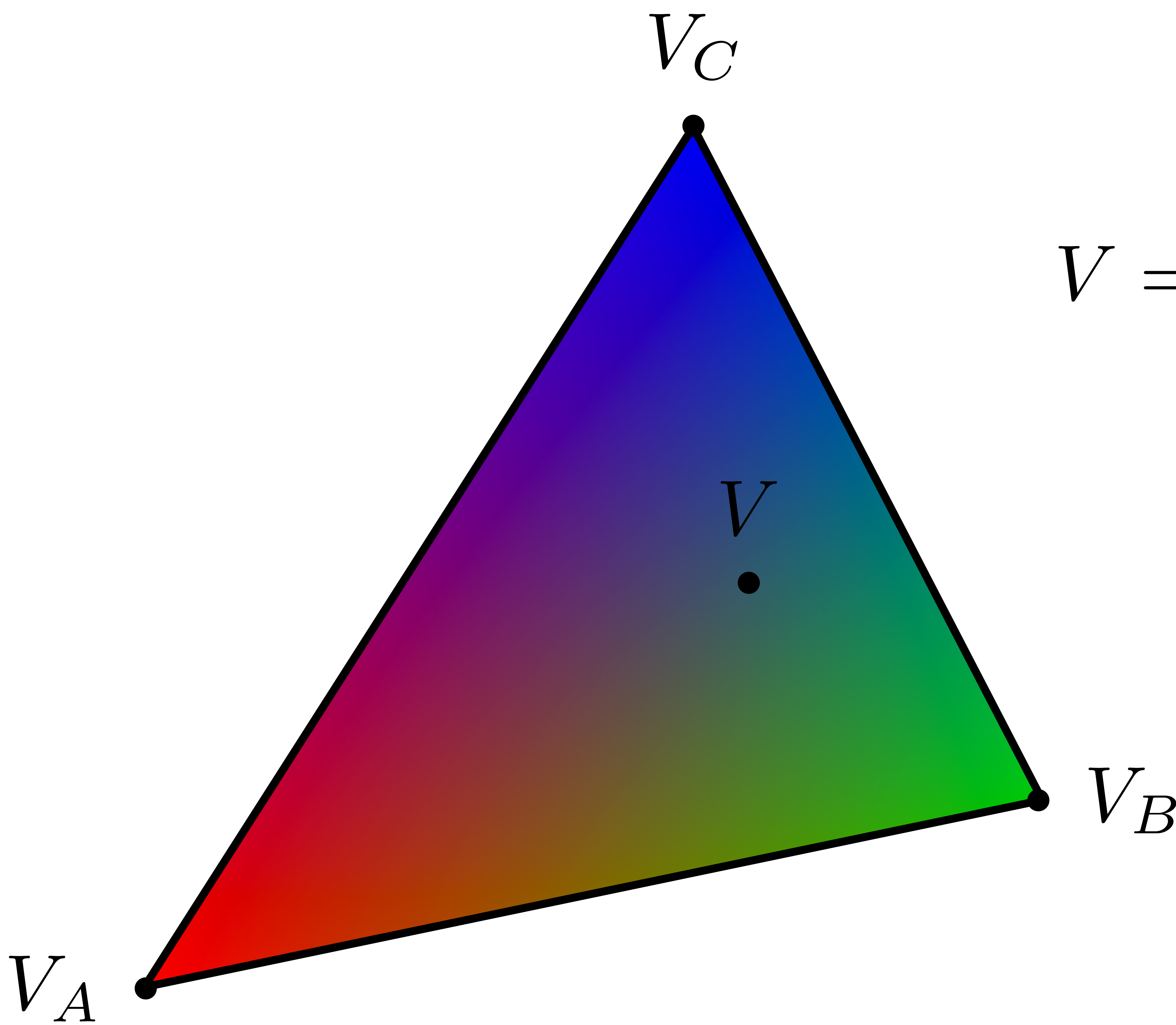
Barycentric Coordinates - Examples



$$(\alpha, \beta, \gamma) = \left(\frac{1}{3}, \frac{1}{3}, \frac{1}{3}\right)$$
$$(x, y) = \frac{1}{3}\,A + \frac{1}{3}\,B + \frac{1}{3}\,C$$

Linear Interpolation Across Triangle

Barycentric coords linearly interpolate values at vertices

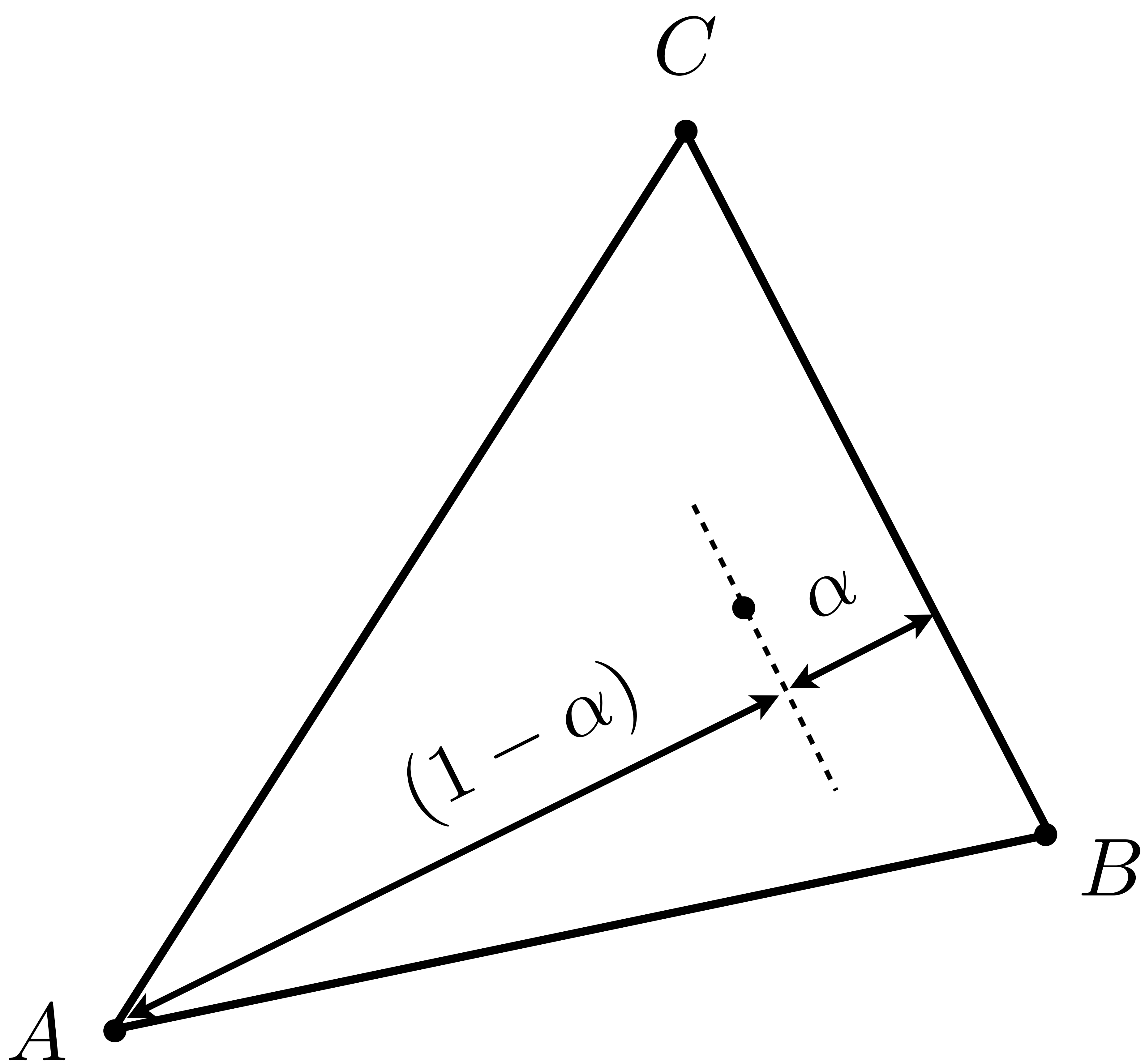


$$V = \alpha V_A + \beta V_B + \gamma V_C$$

V_A, V_B, V_C can be positions, texture coordinates, color, normal vectors, material attributes...

Barycentric Coordinates

Geometric viewpoint — proportional distances

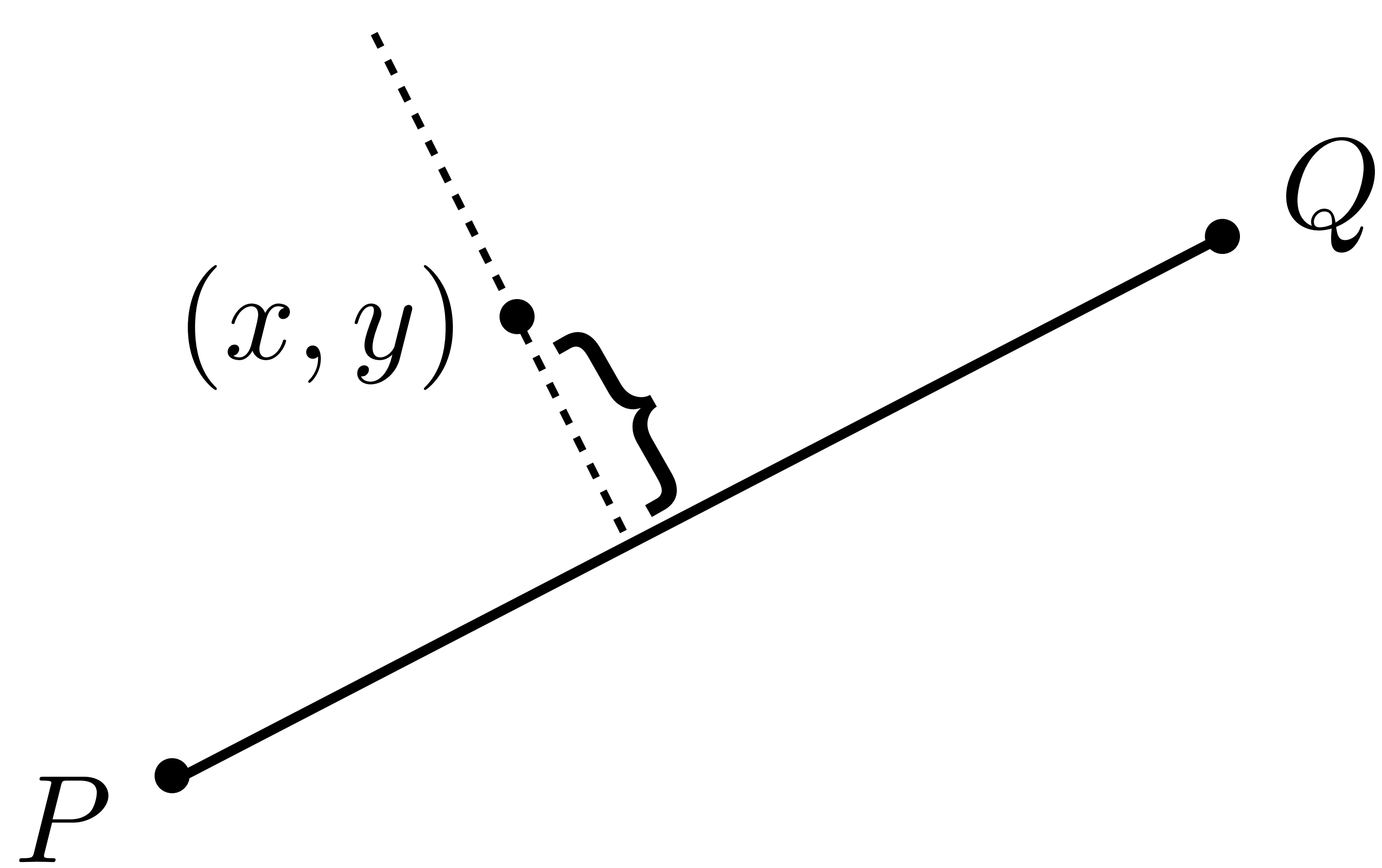


Similar construction for other coordinates

Computing Barycentric Coordinates

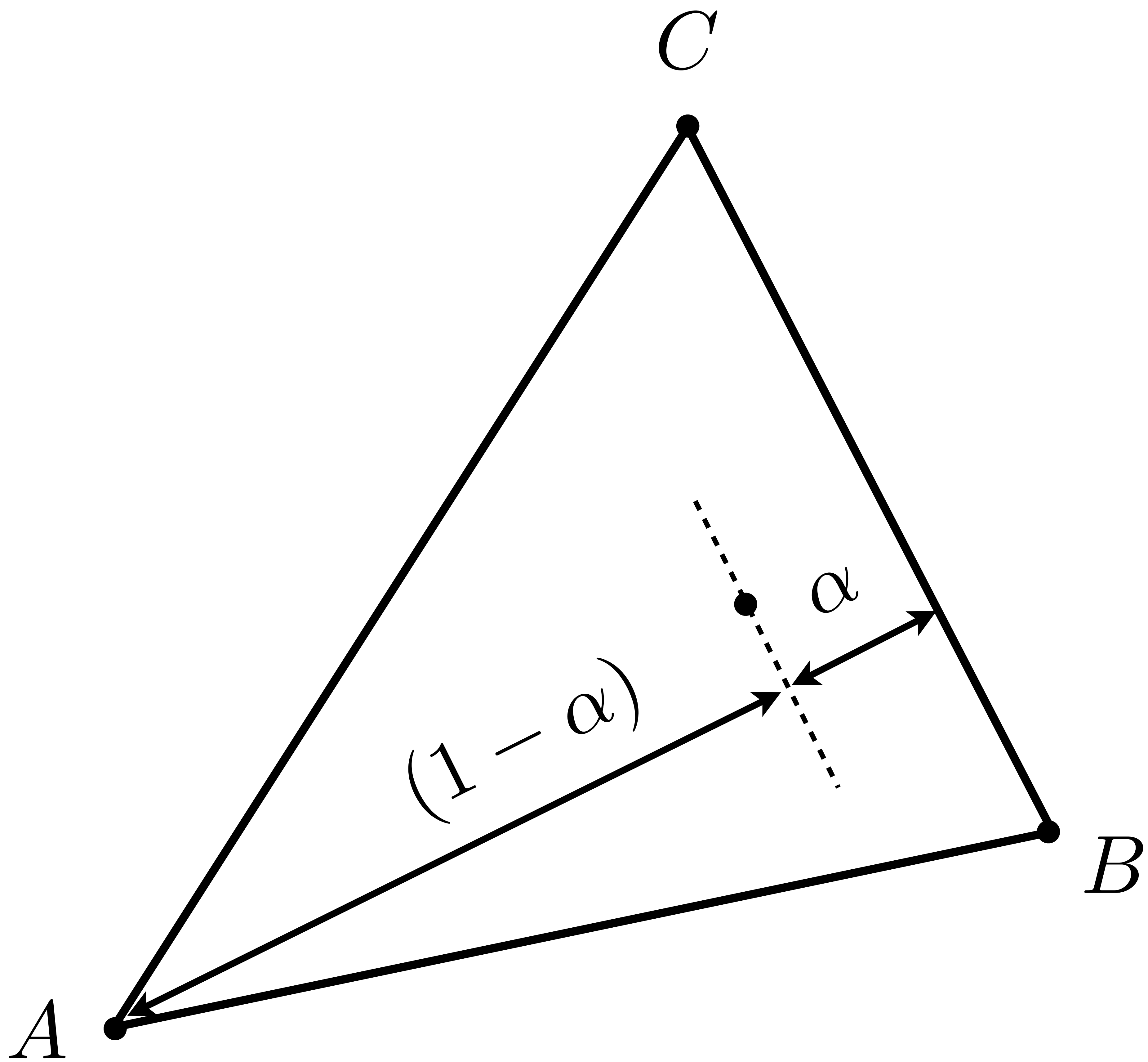
Recall the line equation we derived in Lecture 2.
$L_{PQ}(x,y)$ is proportional to the distance from line PQ.

$$L_{PQ}(x, y) = -(x - x_P)(y_Q - y_P) + (y - y_P)(x_Q - x_P)$$



Computing Barycentric Coordinates

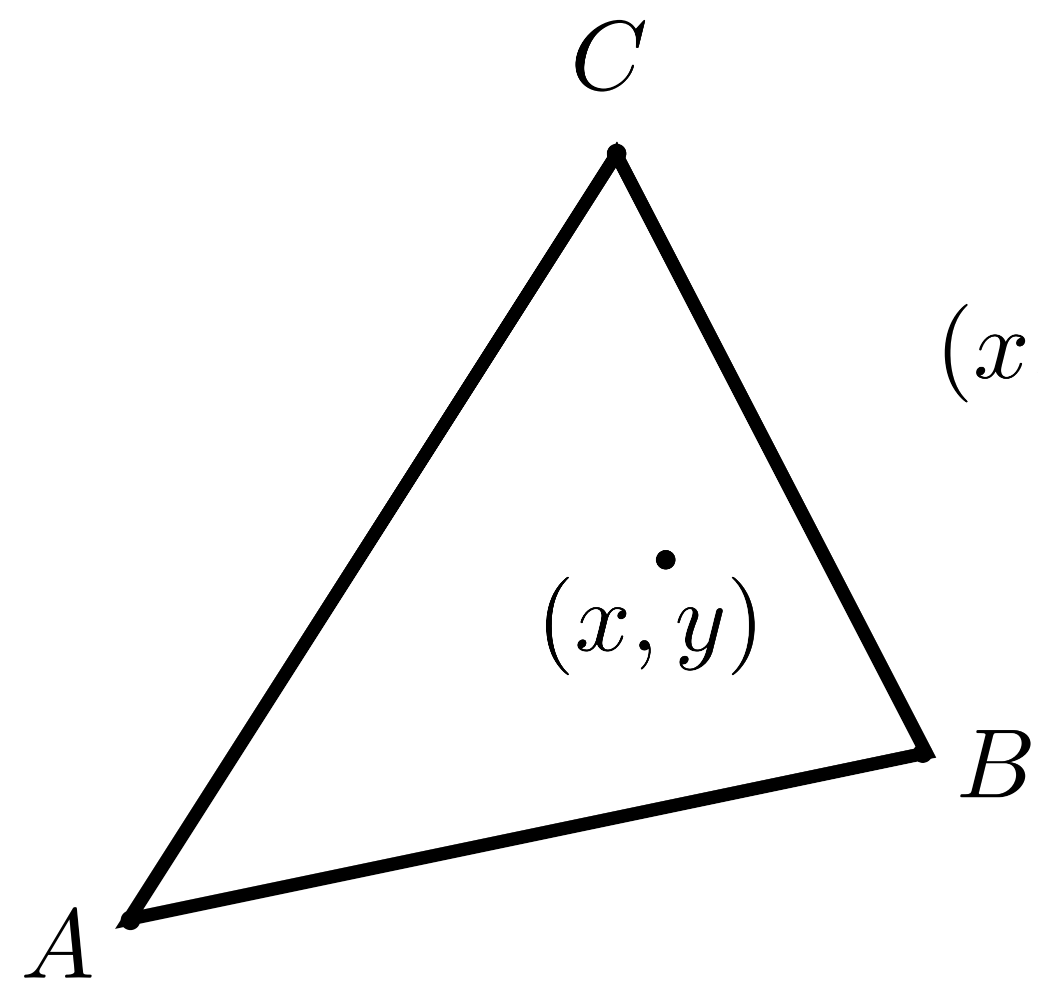
Geometric viewpoint — proportional distances



$$\alpha = \frac{L_{BC}(x, y)}{L_{BC}(x_A, y_A)}$$

Similar construction for other coordinates

Barycentric Coordinate Formulas



$$(x, y) = \alpha\, A + \beta\, B + \gamma\, C$$

$$\alpha + \beta + \gamma = 1$$

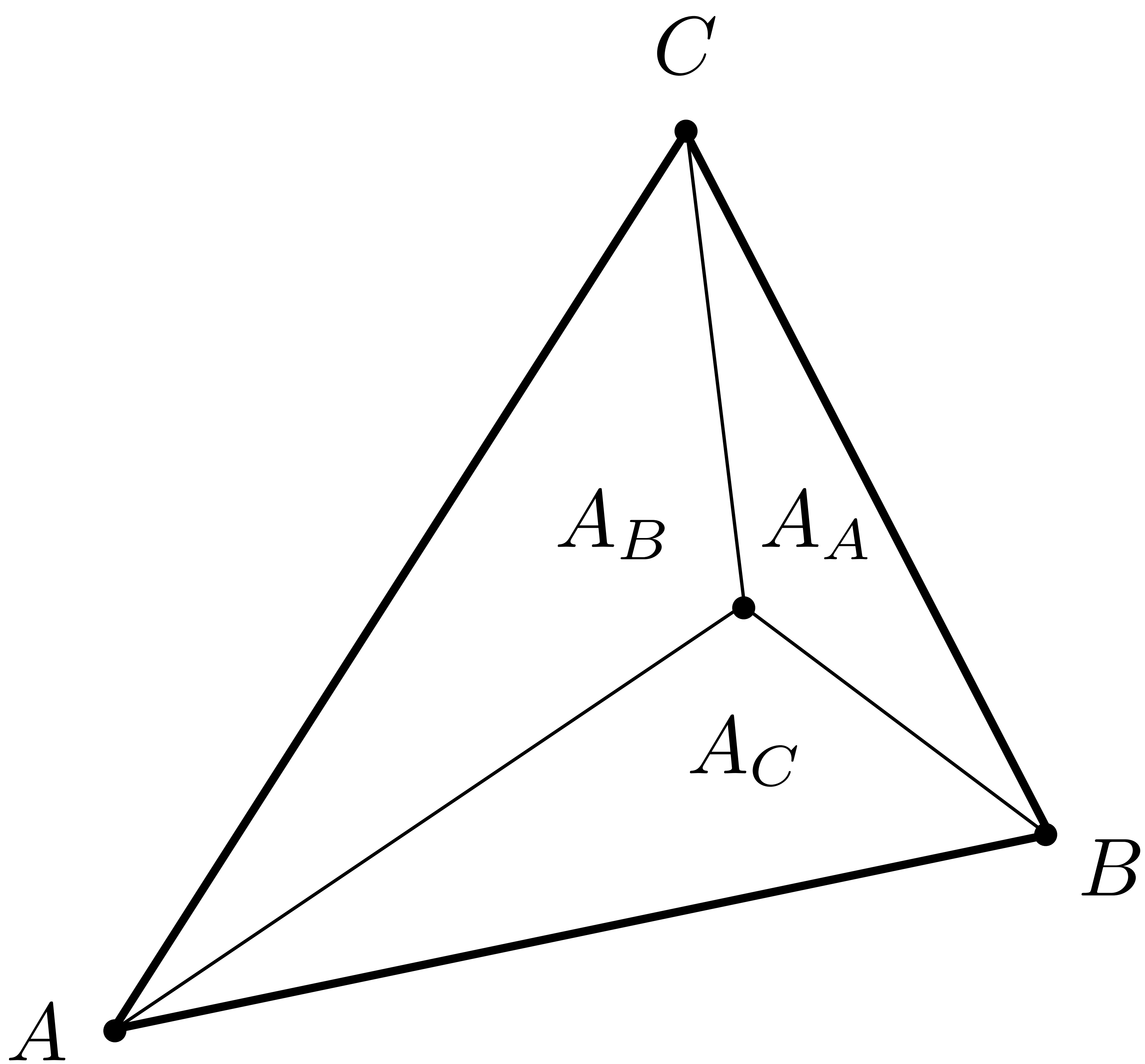
$$\alpha = \frac{-(x - x_B)(y_C - y_B) + (y - y_B)(x_C - x_B)}{-(x_A - x_B)(y_C - y_B) + (y_A - y_B)(x_C - x_B)}$$

$$\beta = \frac{-(x - x_C)(y_A - y_C) + (y - y_C)(x_A - x_C)}{-(x_B - x_C)(y_A - y_C) + (y_B - y_C)(x_A - x_C)}$$

$$\gamma = 1 - \alpha - \beta$$

Barycentric Coordinates

Alternative geometric viewpoint — proportional areas



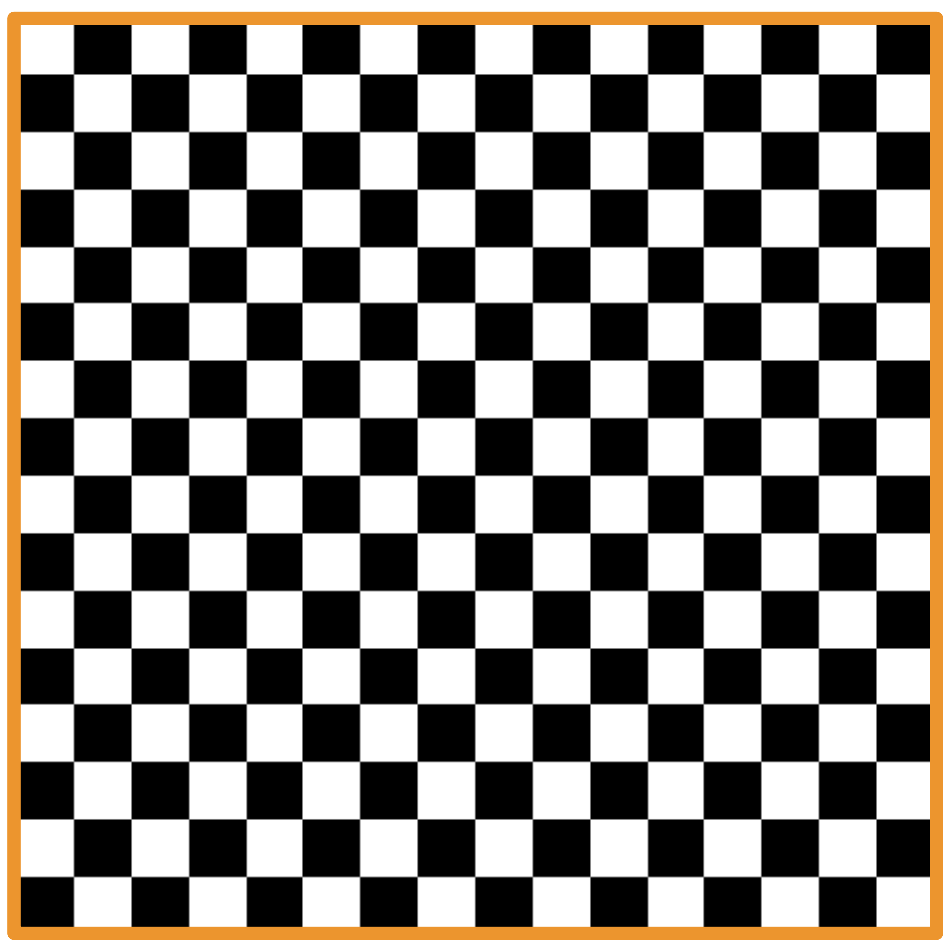
$$\alpha = \frac{A_A}{A_A + A_B + A_C}$$

$$\beta = \frac{A_B}{A_A + A_B + A_C}$$

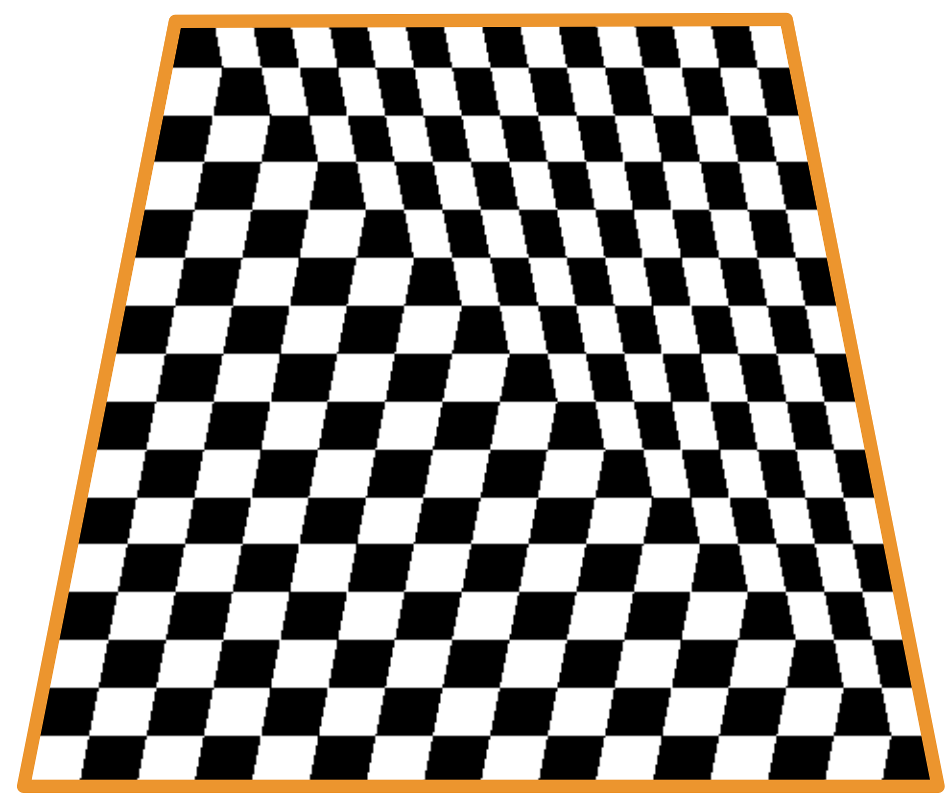
$$\gamma = \frac{A_C}{A_A + A_B + A_C}$$

Perspective Projection and Interpolation

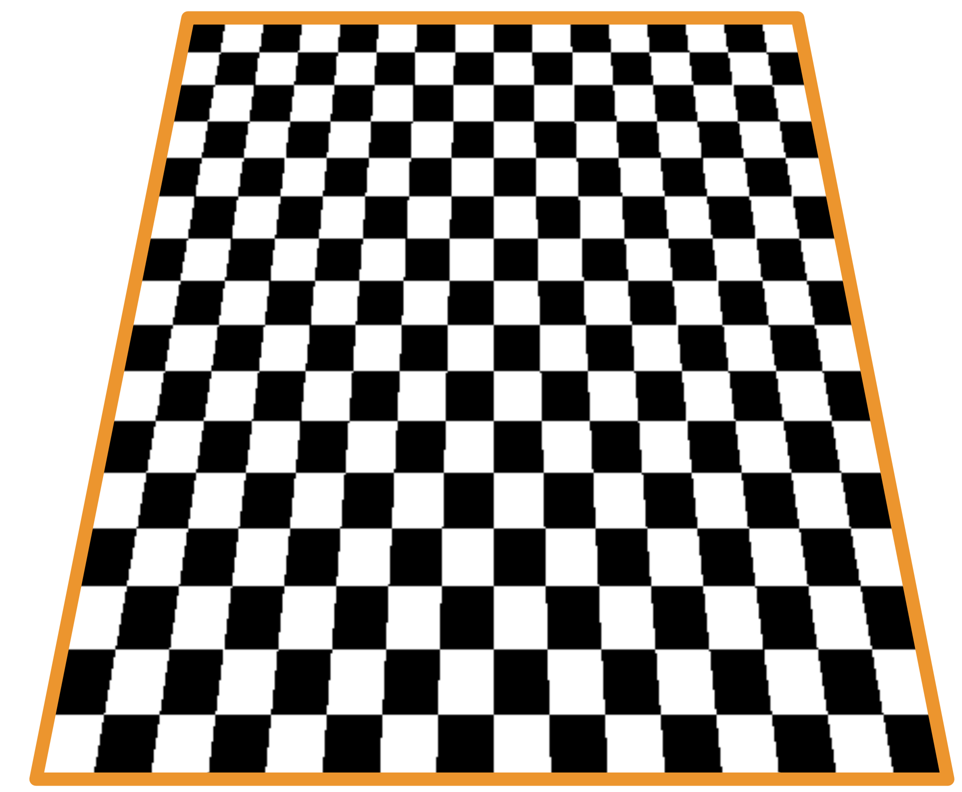
Perspective Projection and Interpolation



Texture

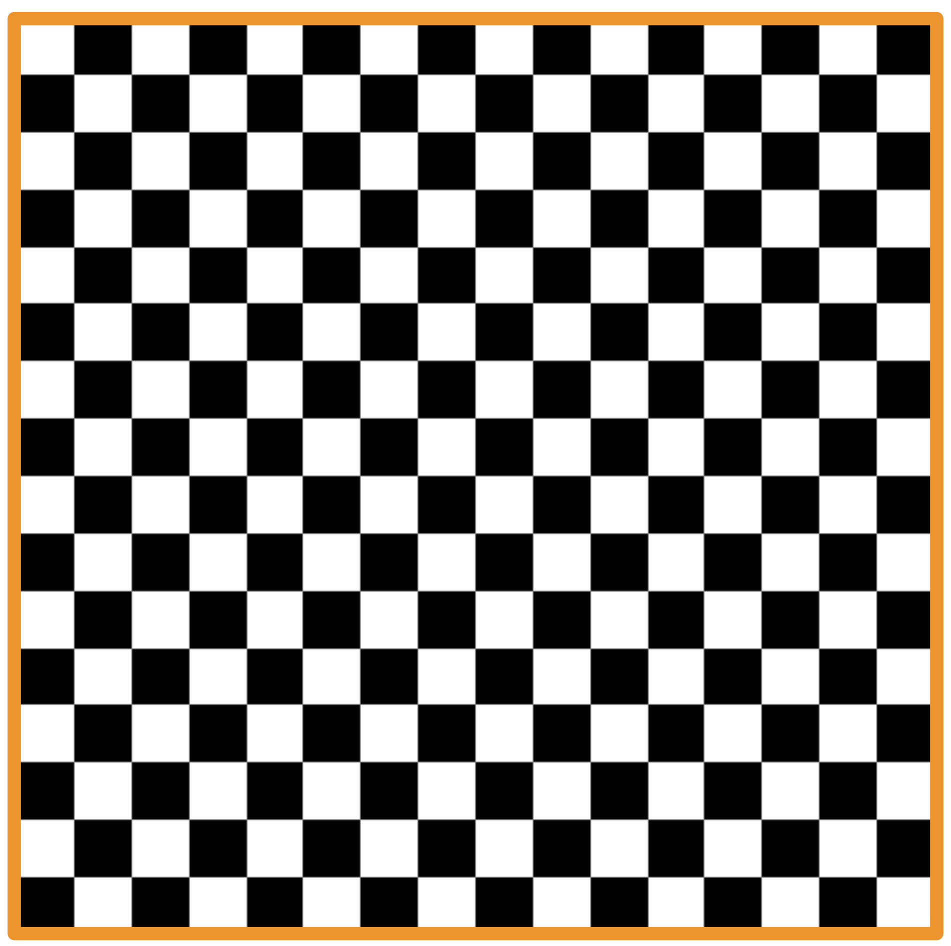


Plane tilted down with perspective projection — What’s wrong?

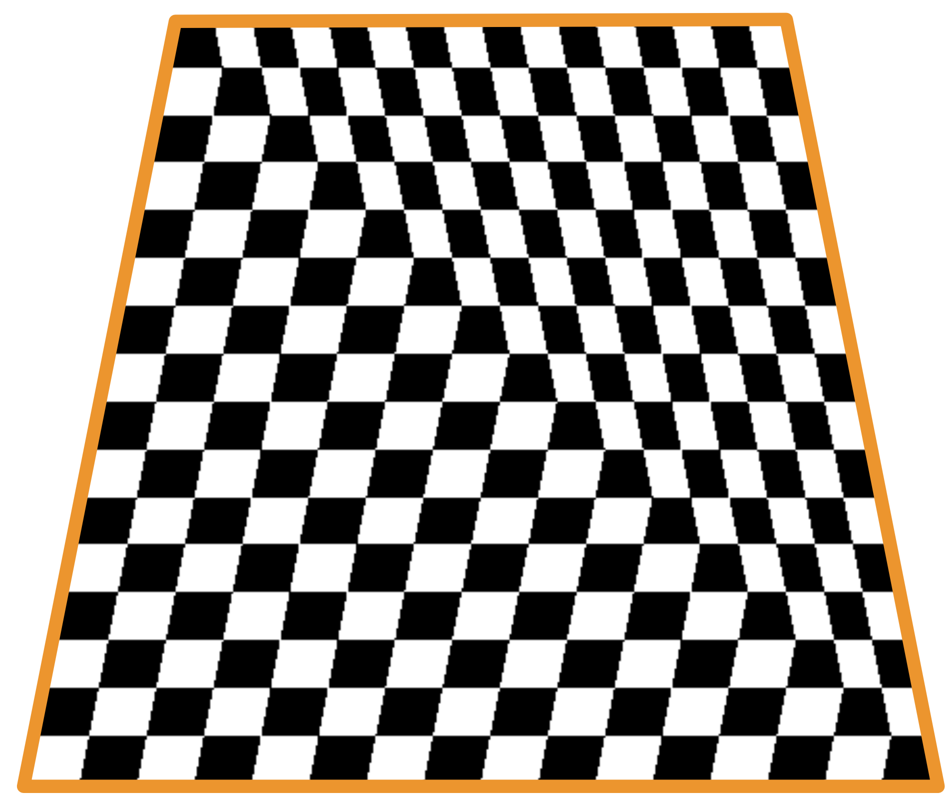


Correct image

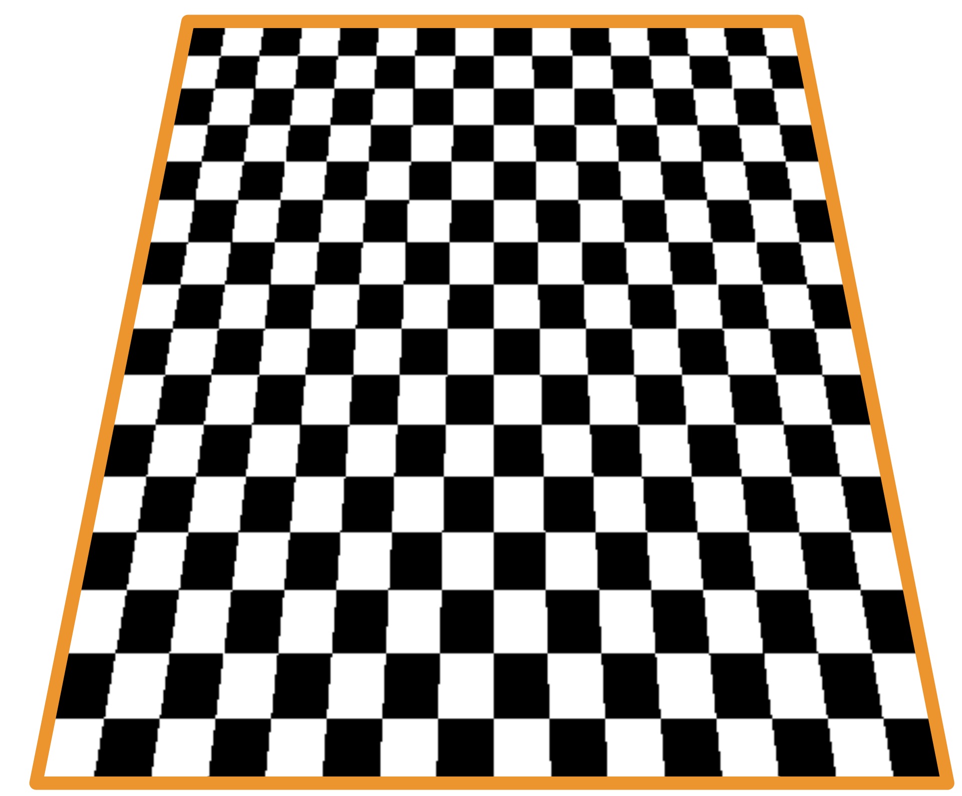
Perspective Projection and Interpolation



Texture



Barycentric interpolation of texture coordinates in screen-space

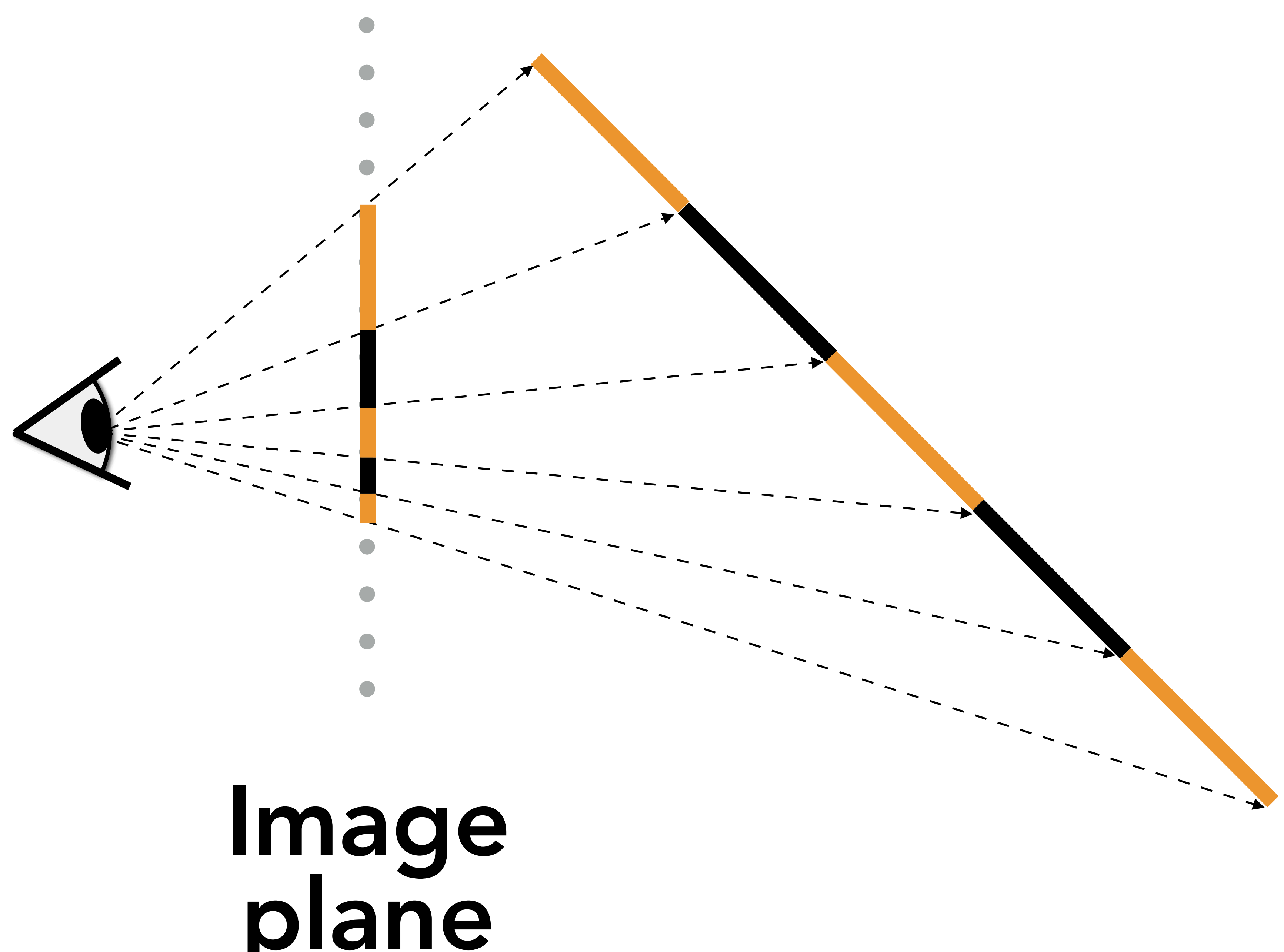


Correct image

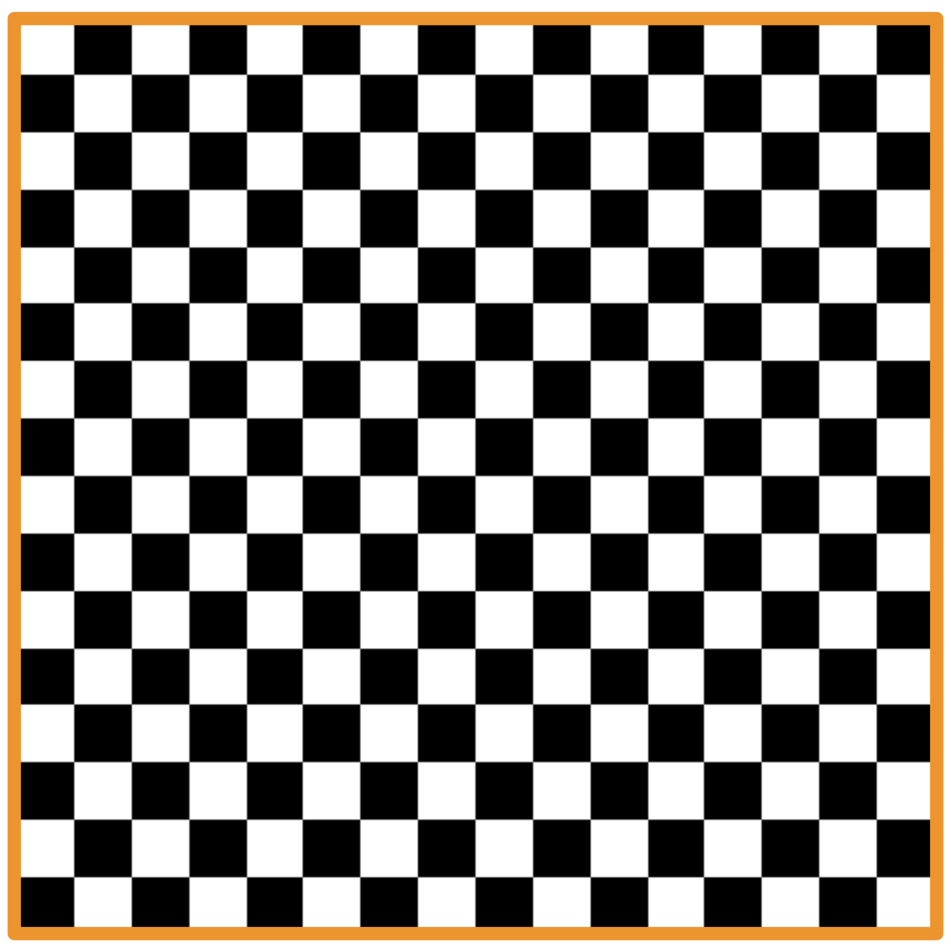
Perspective Projection Creates Non Linearity

Linear interpolation in world coordinates yields nonlinear interpolation in screen coordinates!

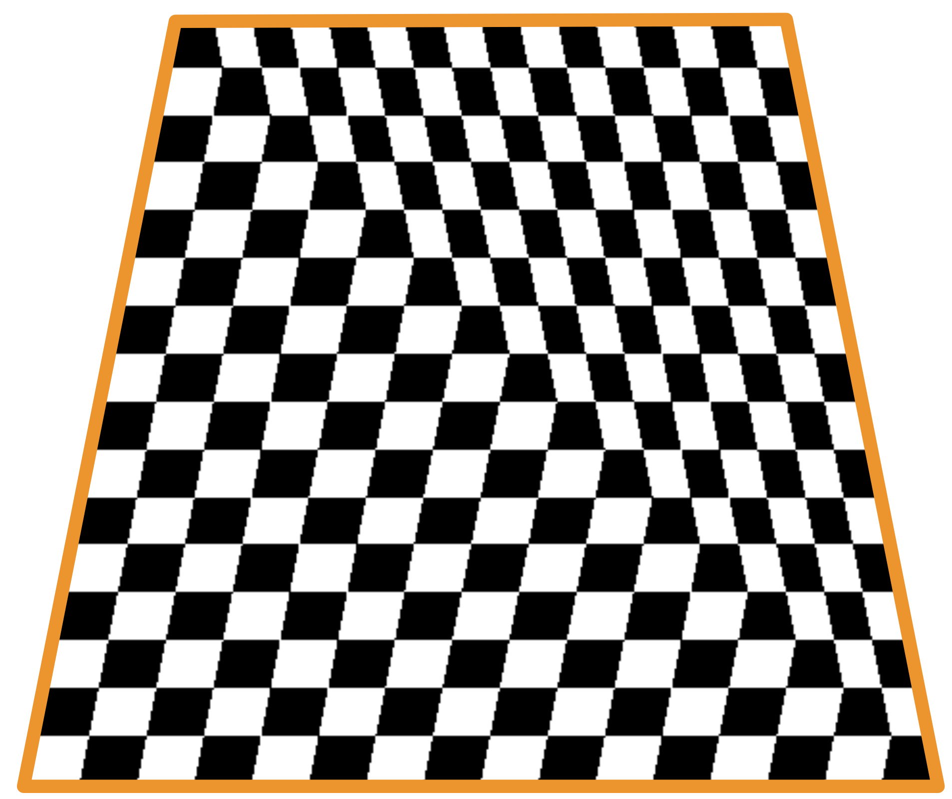
Perspective interpolation supported in GPU



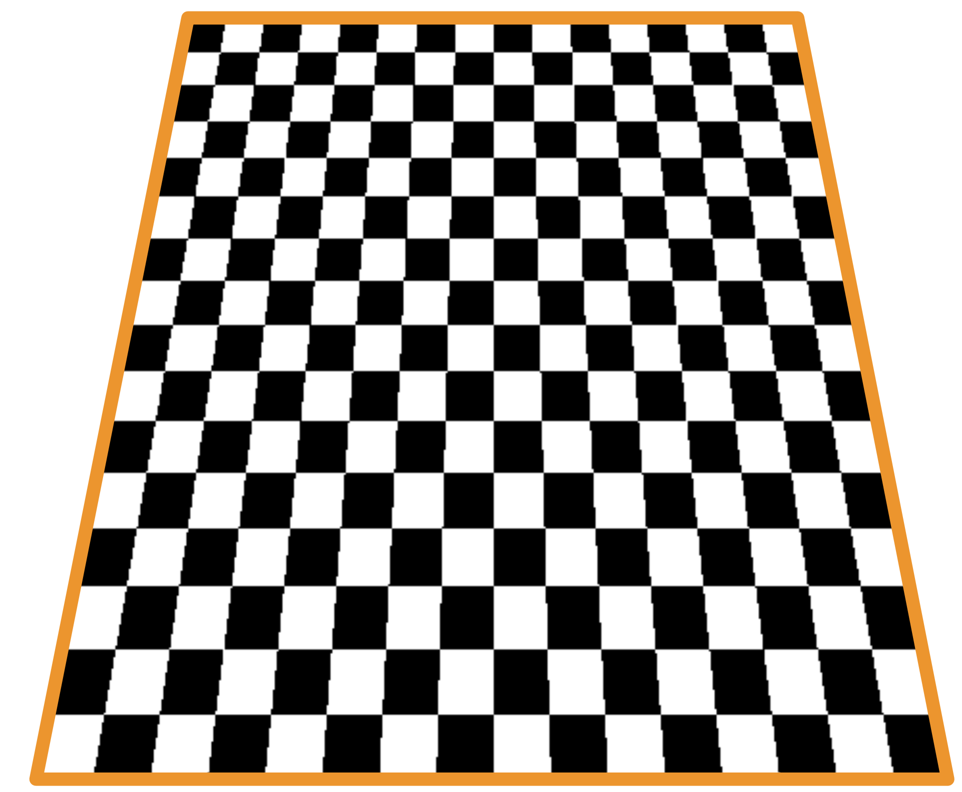
Perspective-Correct Interpolation



Texture



Affine screen-space interpolation



Perspective world-space interpolation

Applying Textures is Sampling!

Simple Texture Mapping Operation

```
for each rasterized screen sample (x,y):
    (u,v) = evaluate texcoord value at (x,y)
    float3 texcolor = texture.sample(u,v);
    set sample's color to texcolor;
```

Applying Textures is Sampling!

Actually "re-sampling"

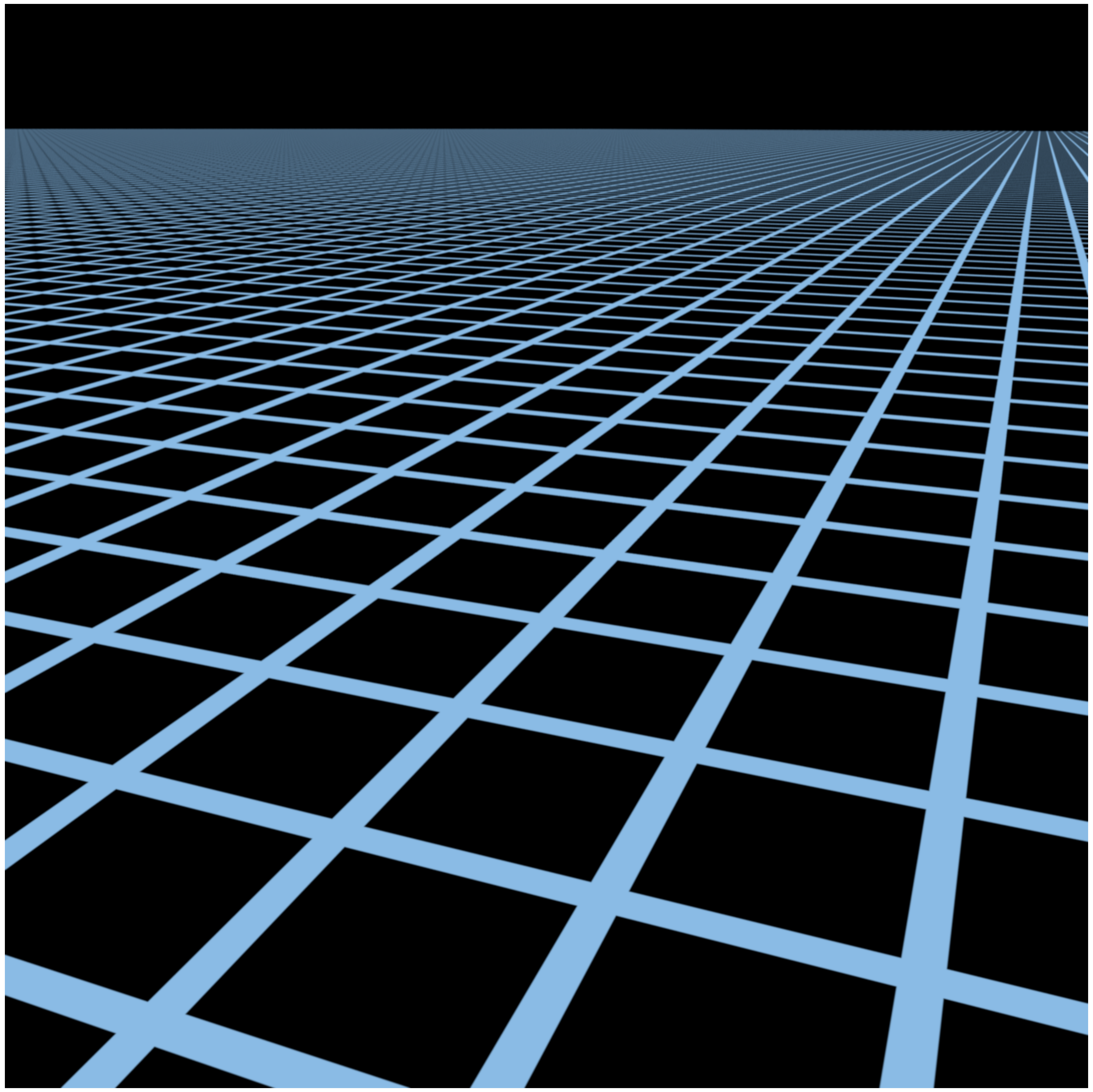
Mathematically, to draw a texture sample at (u,v):

- Start with discrete, sampled 2D function f(x,y). This function is only non-zero at sampled locations
- Reconstruct a continuous 2D function, $f_{cont}(x,y) = f(x,y) * k(x,y)$ by convolution with a reconstruction filter k(x,y)
- Draw the desired sample at (u,v) from the continuous 2D signal by function evaluation: $f_{cont}(u,v)$

Signal processing concepts that should come to mind for you:

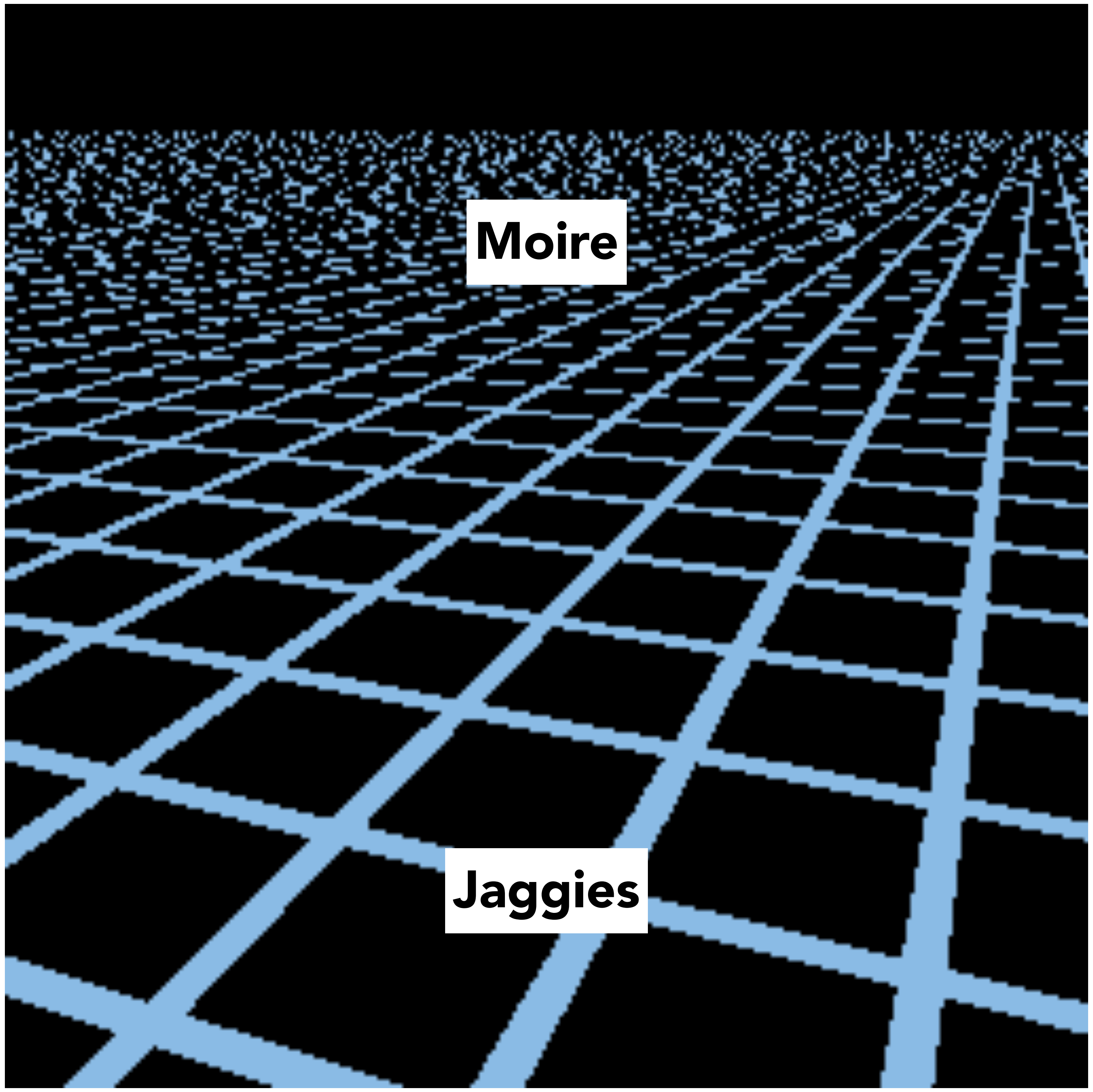
- Frequency spectrum, aliasing, Nyquist frequency, filtering, anti-aliasing…

Point Sampling Textures



High-res reference

Source image: 1280x1280 pixels

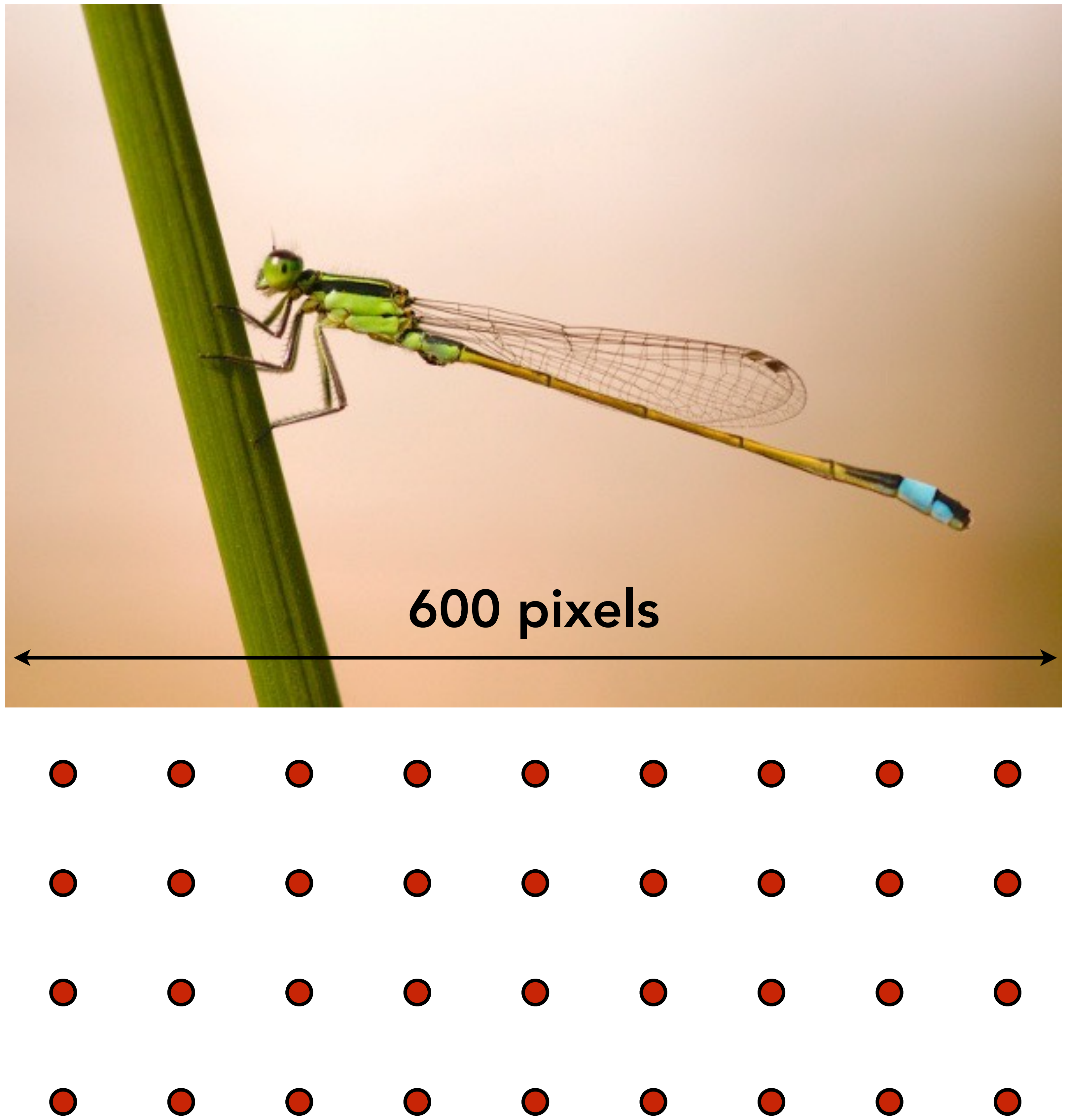


Point sampling

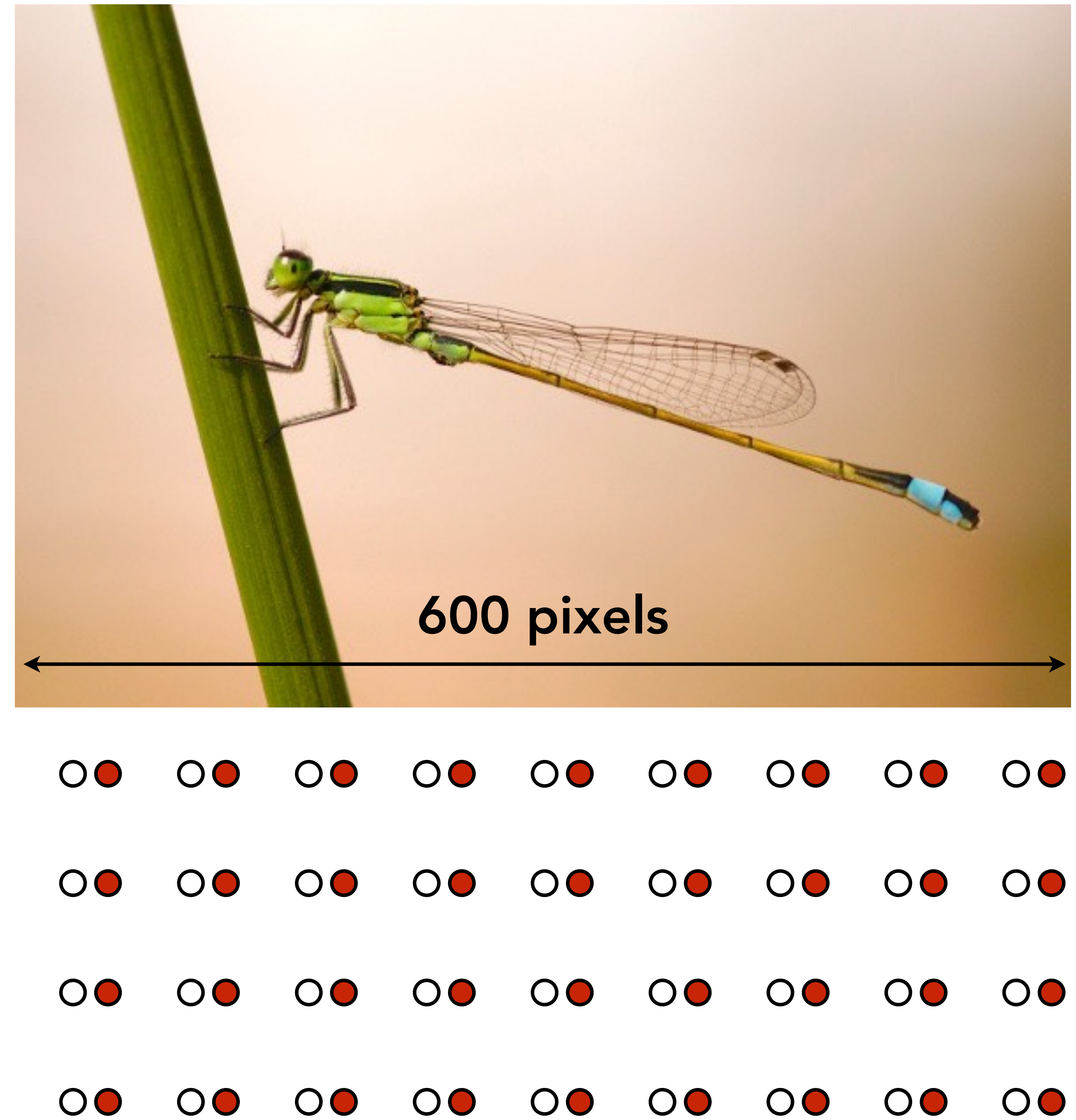
256x256 pixels

Texture Sampling Frequency

Sampling Rate on Screen vs Texture



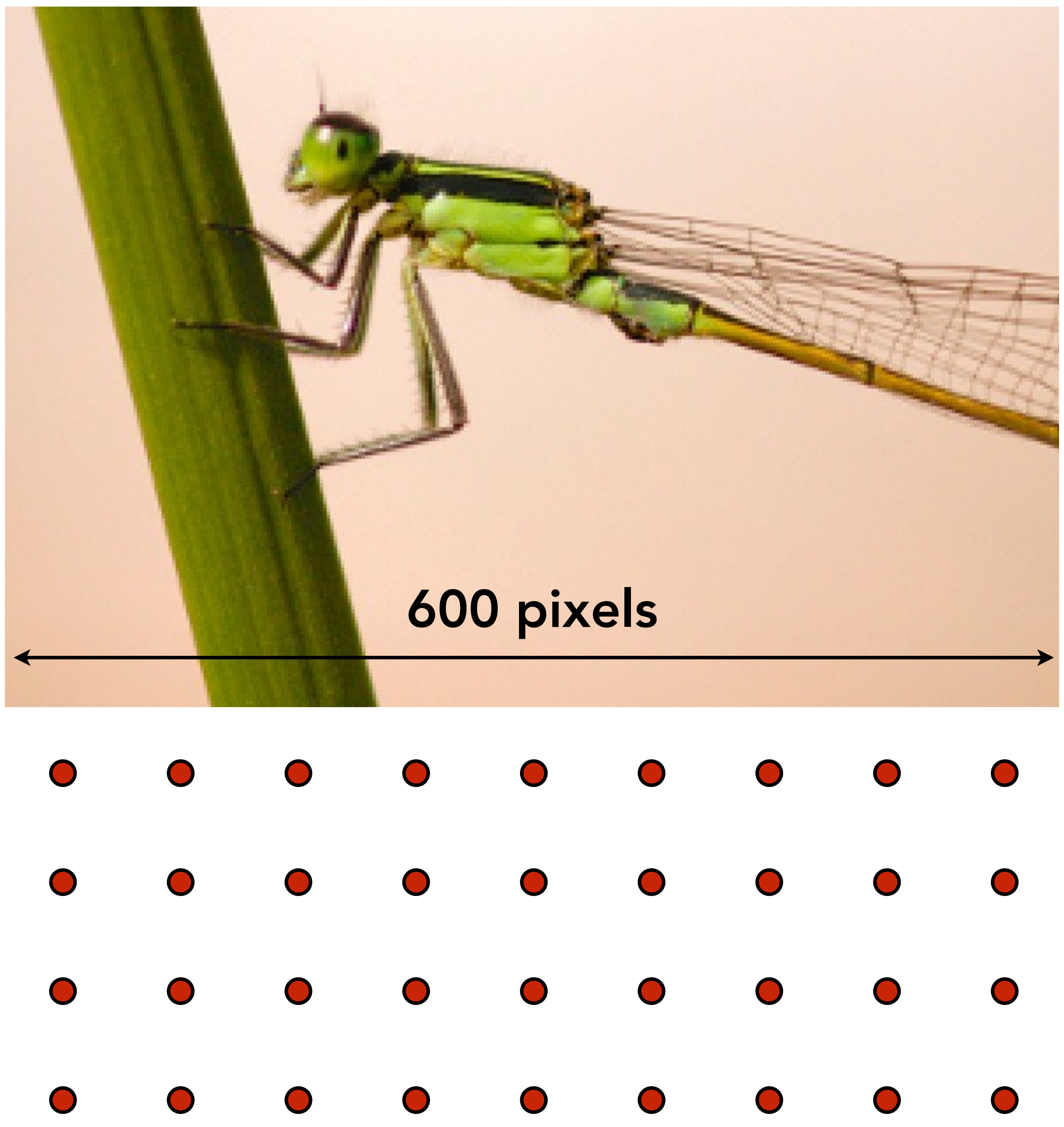
Screen space (x,y)



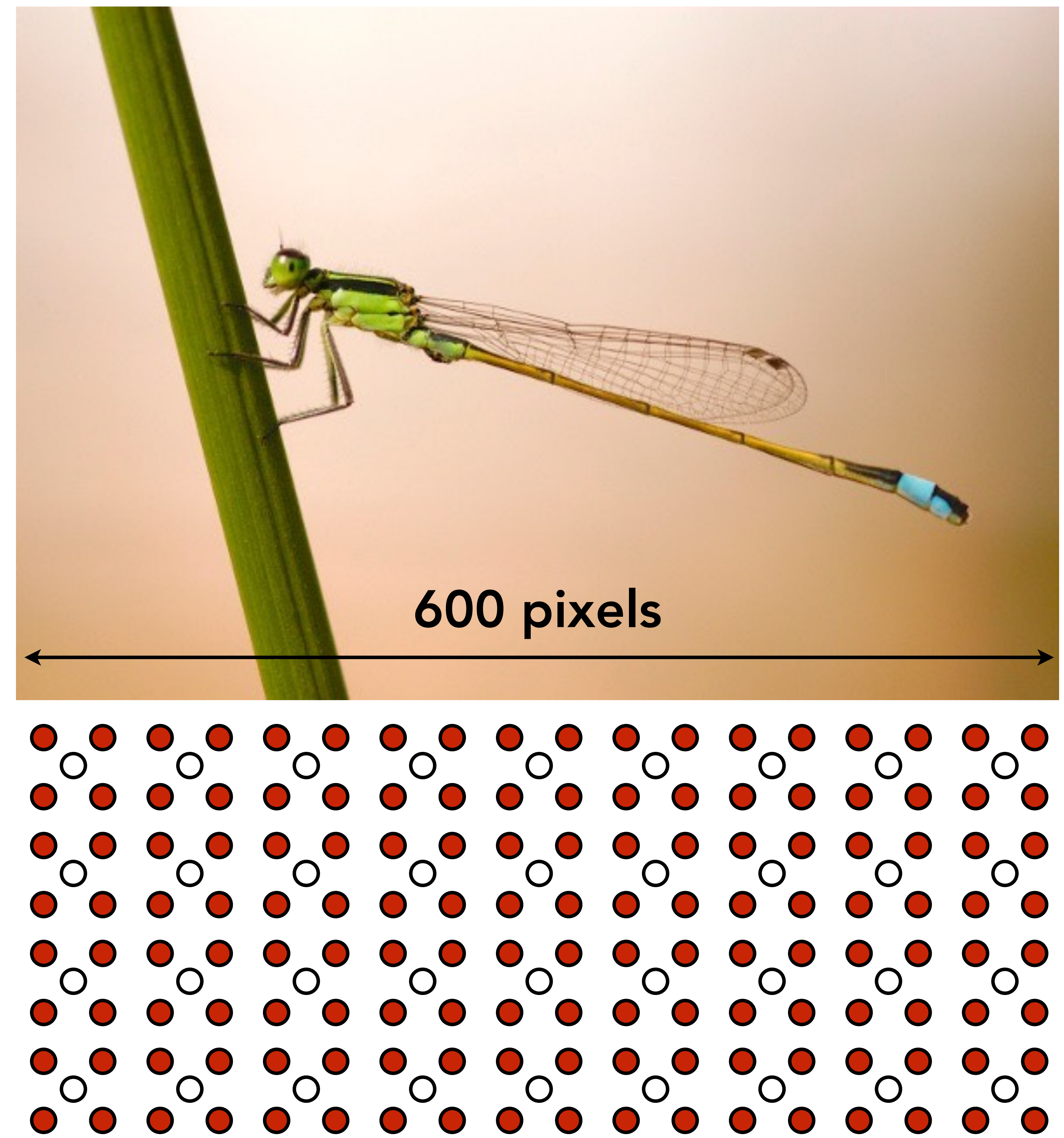
Texture space (u,v)

1:1 mapping

Sampling Rate on Screen vs Texture



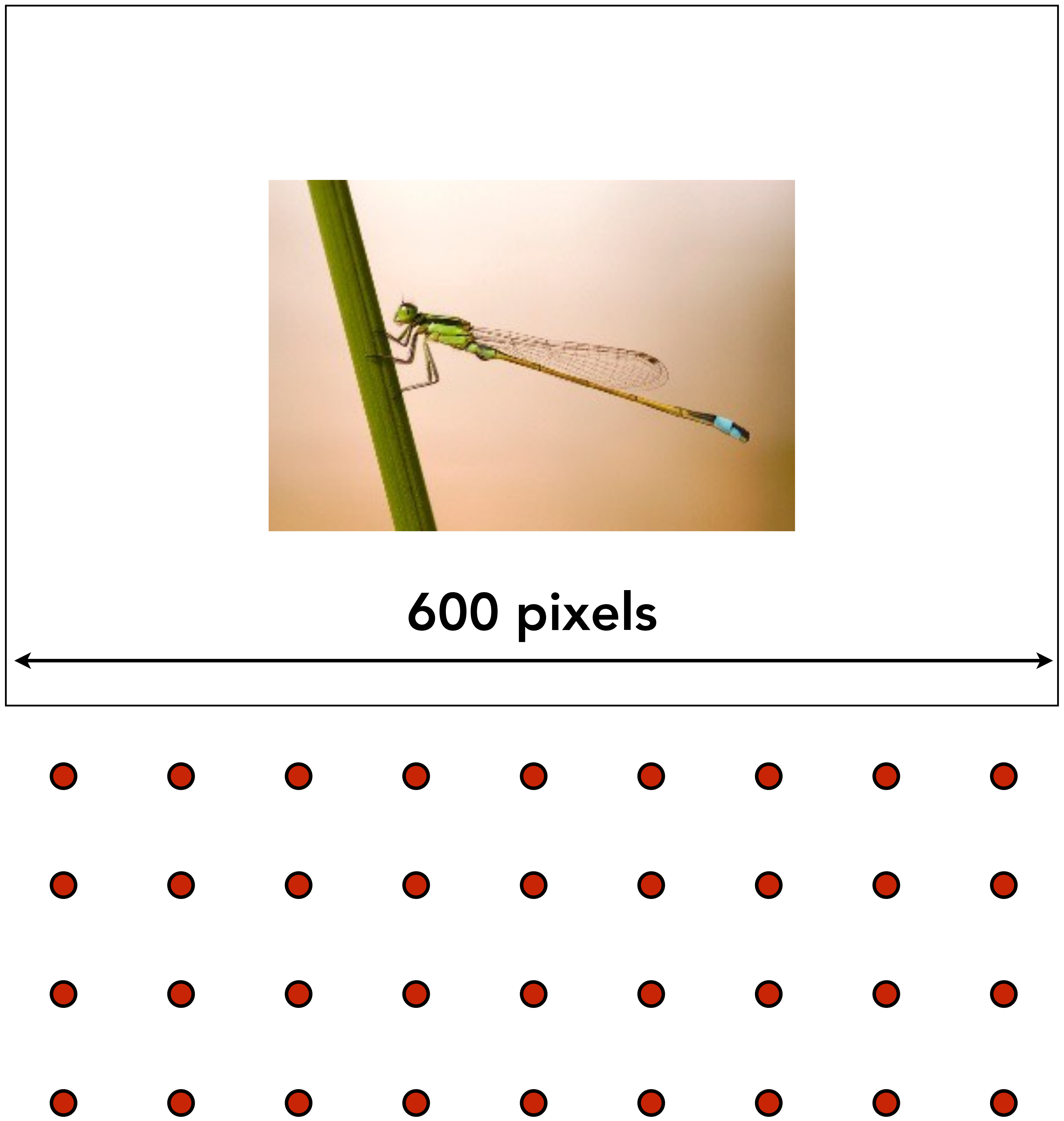
Screen space (x,y)



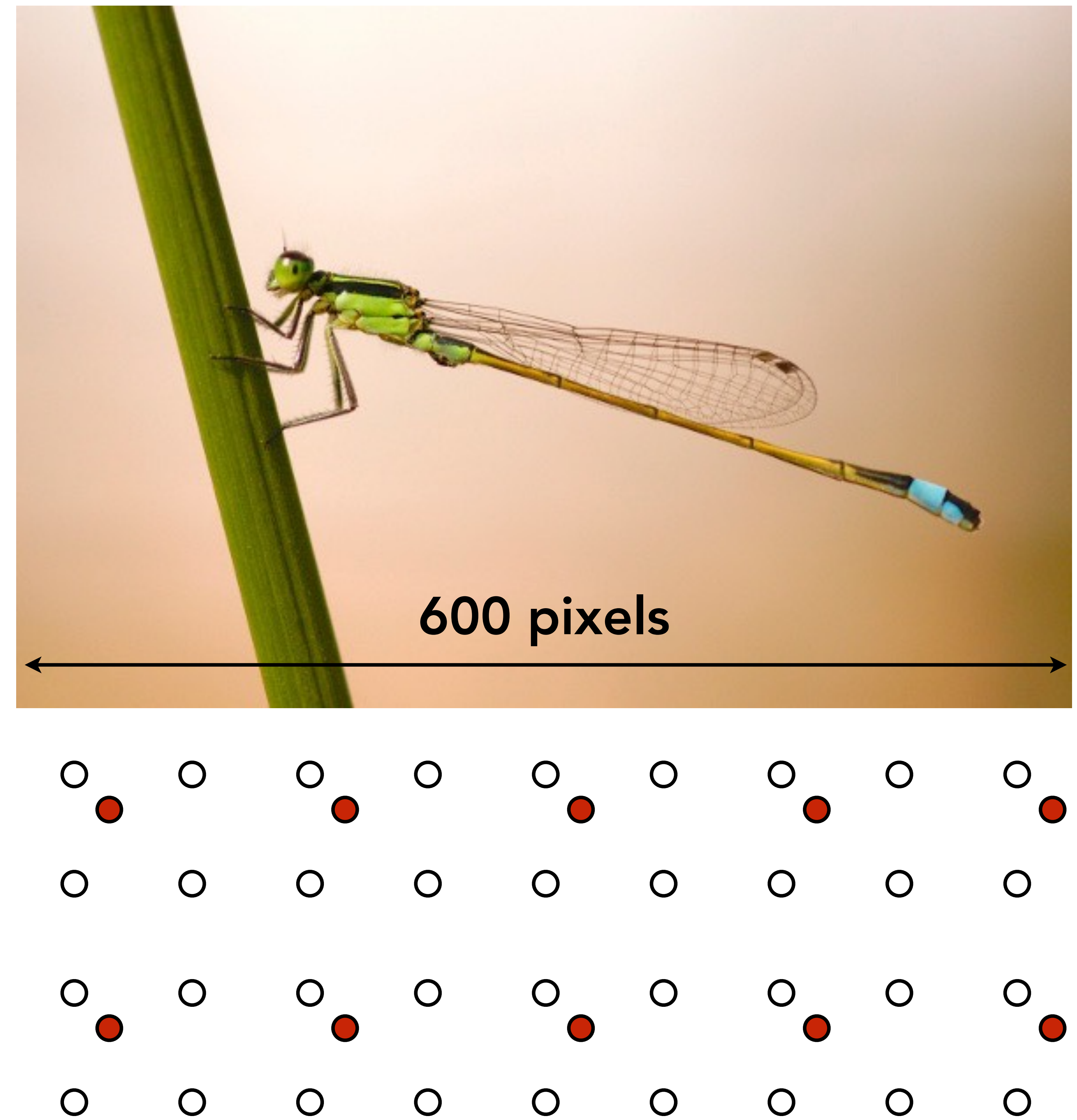
Texture space (u,v)

Magnified

Sampling Rate on Screen vs Texture



Screen space (x,y)



Texture space (u,v)

"Minified"

Texture Sampling Rate

The sampling frequency in screen space translates to a sampling frequency in texture space as determined by the mapping function.

In general the frequency varies across the scene depending on geometric transforms, viewing transforms, and the texture coordinate function.

Screen Pixel Area vs Texel Area

At optimal viewing size:

- 1:1 mapping between pixel sampling rate and texel sampling rate
- Dependent on texture resolution! e.g. 512x512

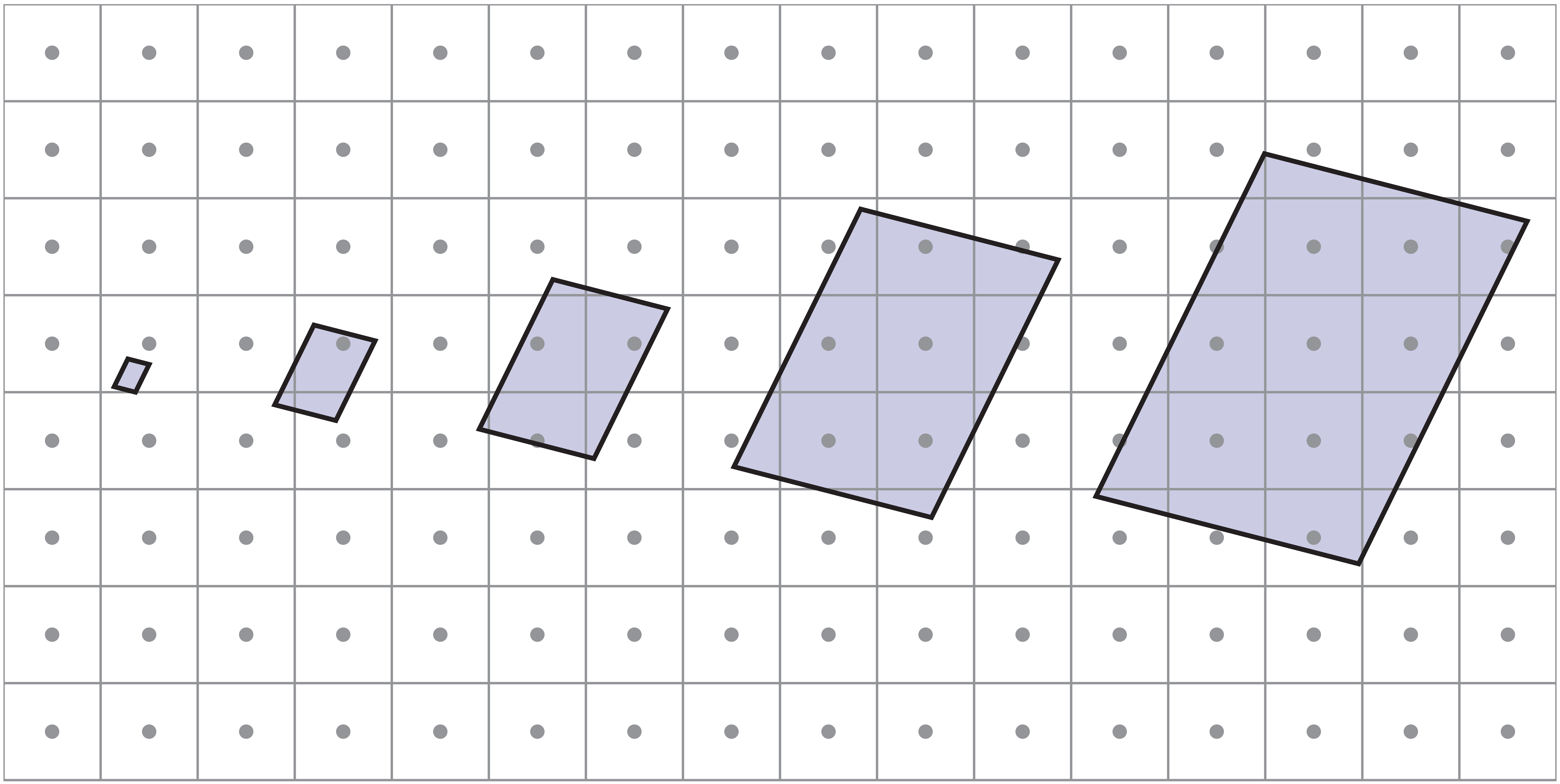
When larger (magnification)

- Multiple pixel samples per texel sample

When smaller (minification)

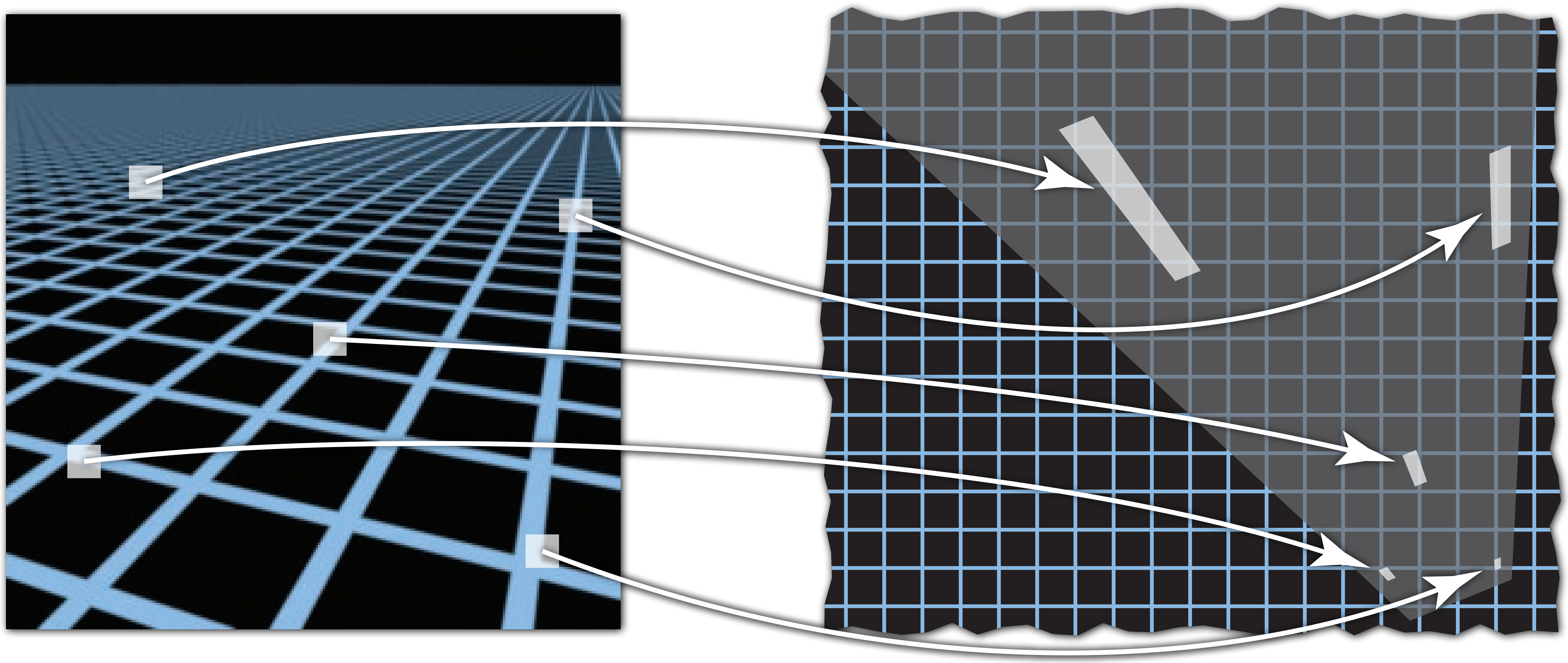
- One pixel sample per multiple texel samples

Screen Pixel Footprint in Texture



Upsampling (Magnification) ⟷ **Downsampling (Minification)**

Screen Pixel Footprint in Texture

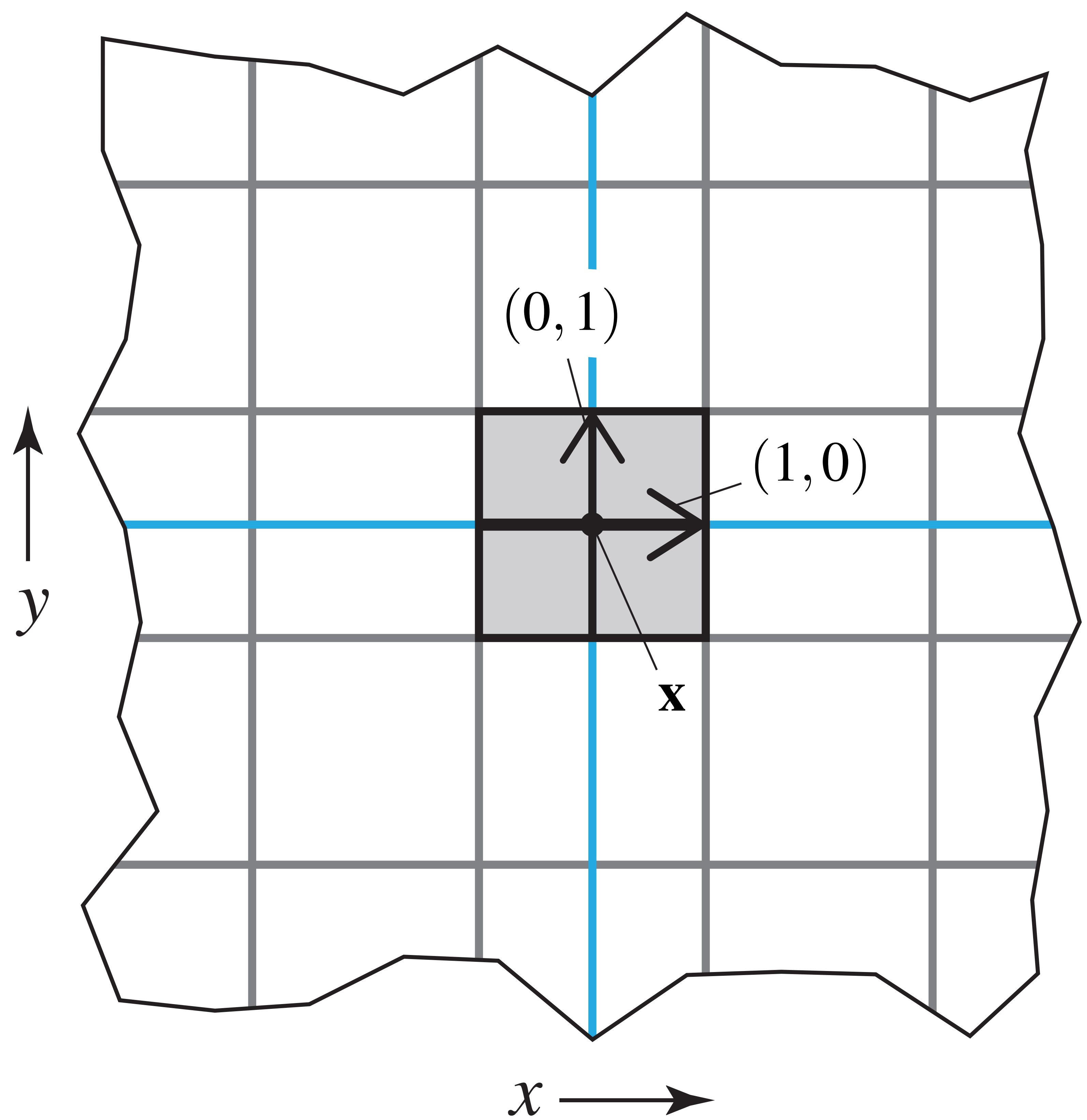


Screen space

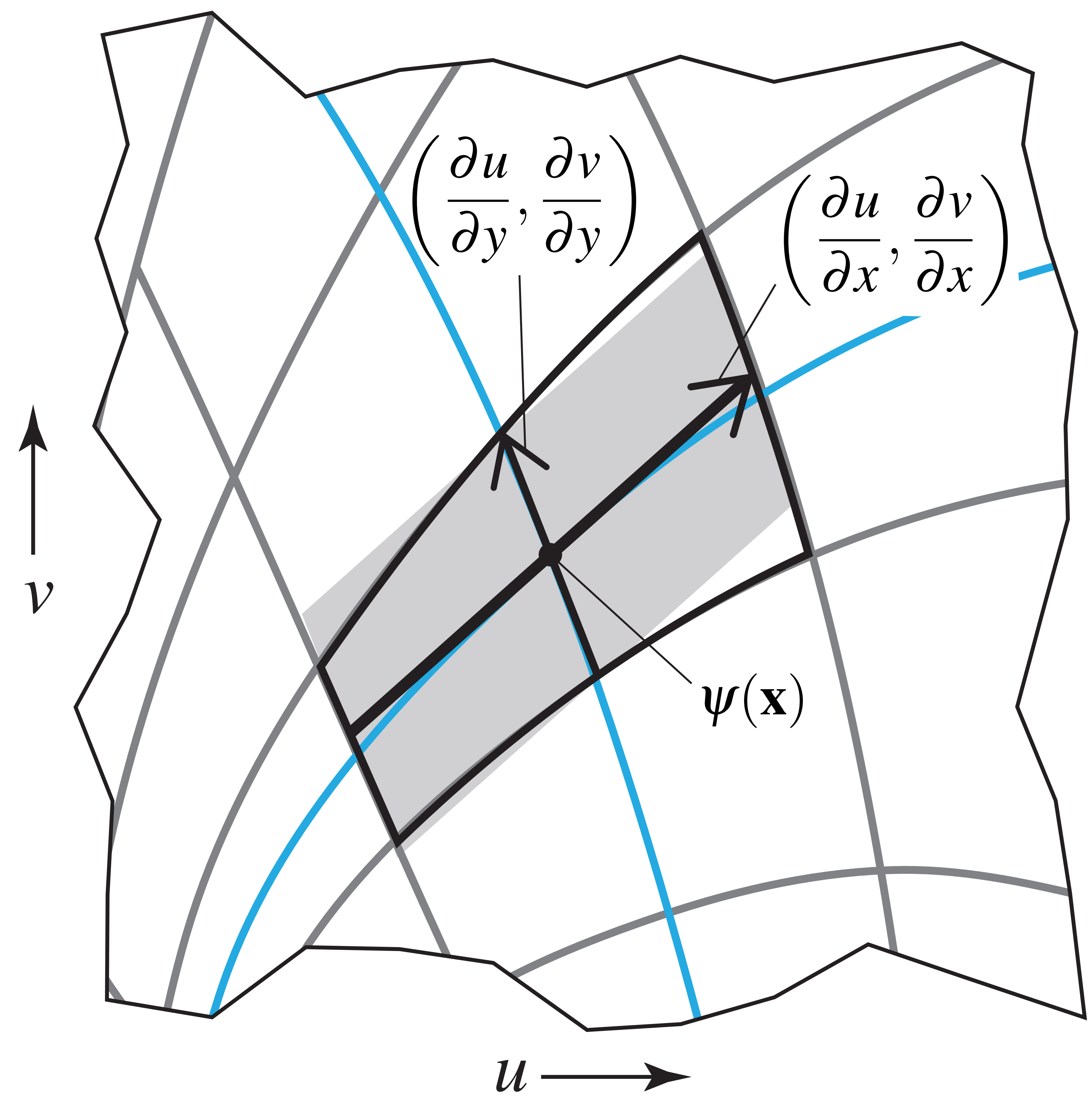
Texture space

NB: texture sampling pattern not rectilinear or isotropic

Estimating Footprint Area With Jacobian



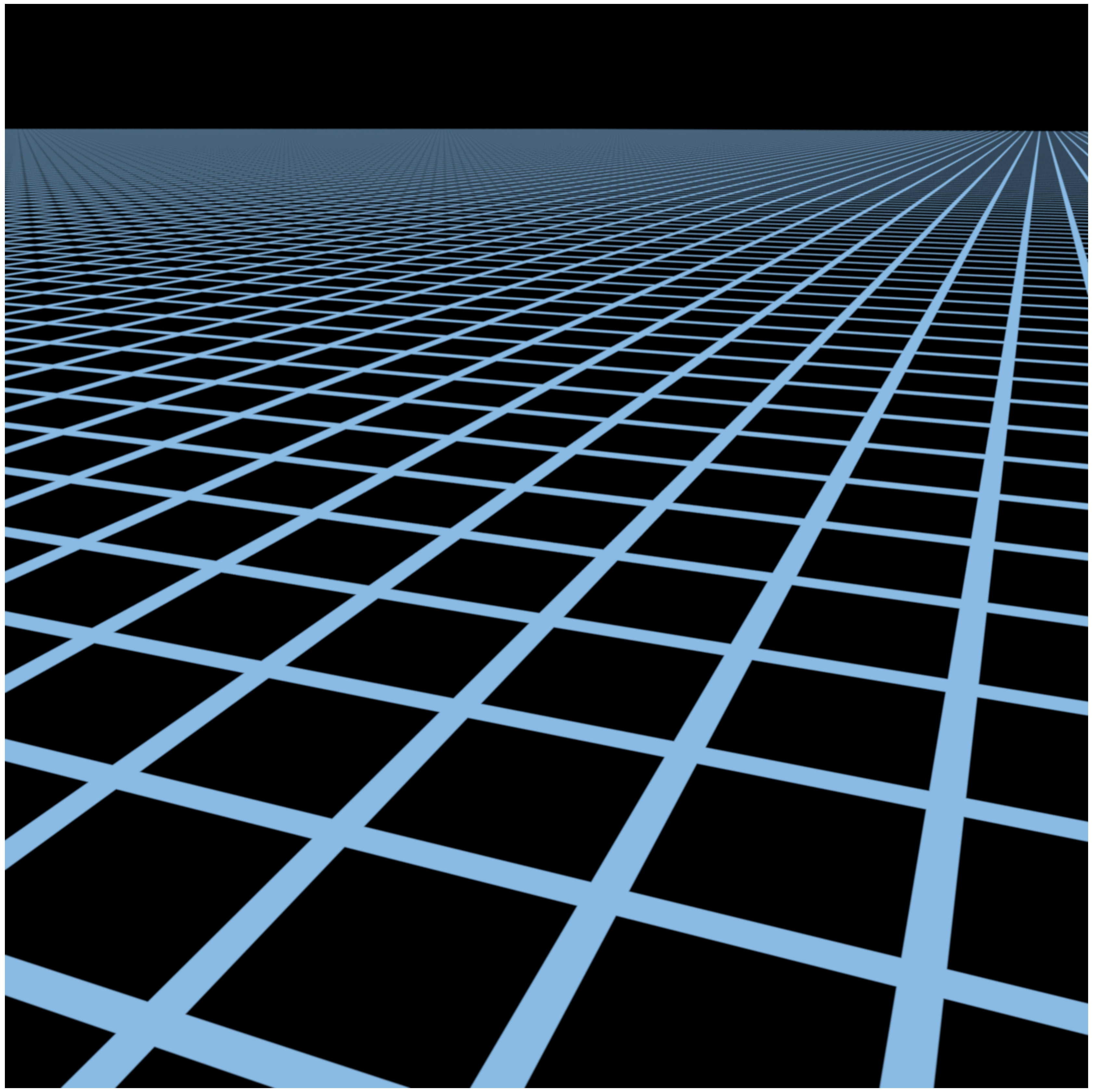
Screen space



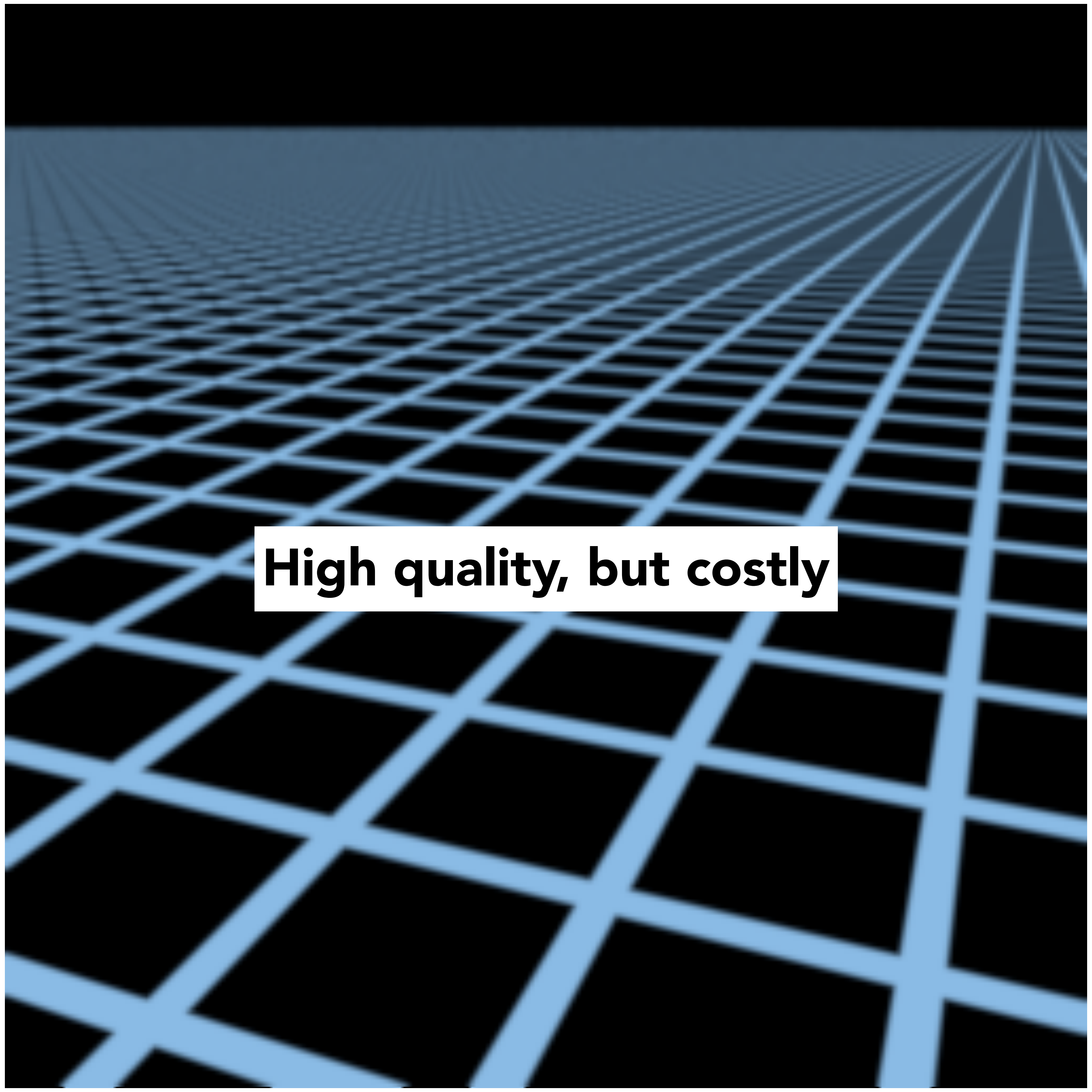
Texture space

Texture Antialiasing

Will Supersampling Antialias?



High-res reference



512x supersampling

Texture Antialiasing

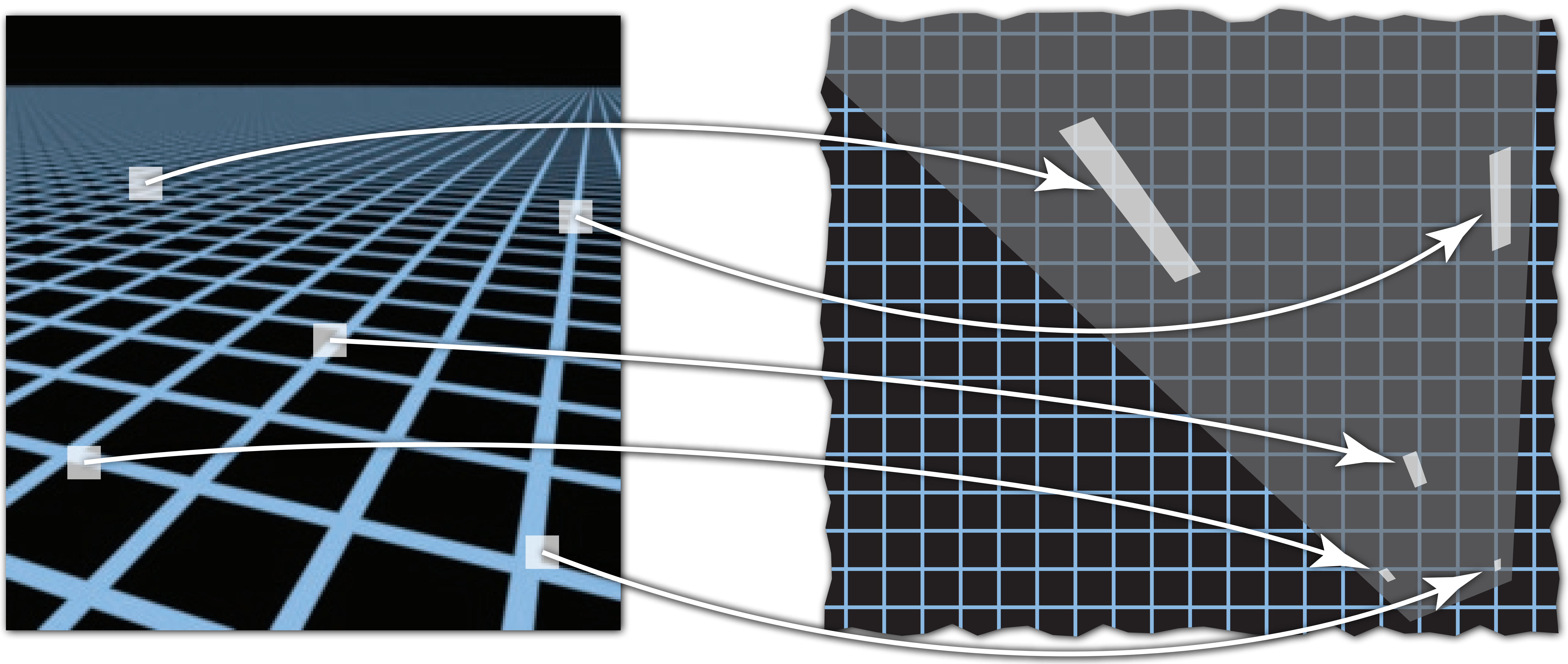
Will supersampling work?

- Yes, high quality, but costly
- When highly minified, many texels in pixel footprint

Goal: efficient texture antialiasing

- Want antialiasing with one/few texels per pixel
- How? Antialiasing = filtering before sampling!

Antialiasing: Signal, Sampling Rate, Nyquist Rate?



Screen space

Texture space

What signal are we sampling? What is the sampling frequency? What is the Nyquist frequency?

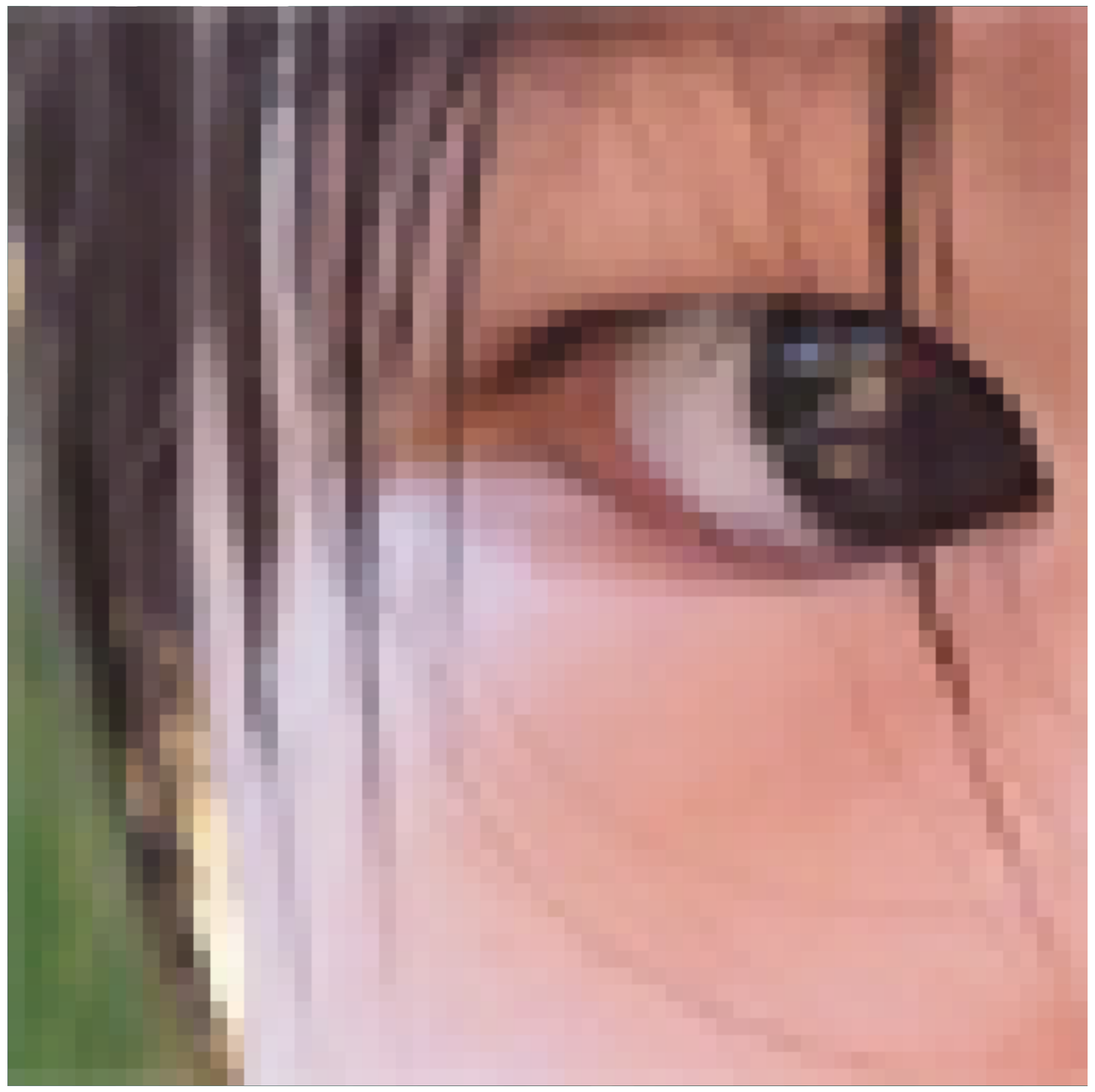
Texture Filtering

Texture Magnification

Texture Magnification - Easy Case

(Generally don't want this — insufficient resolution)

This is image interpolation (will see kernel function)

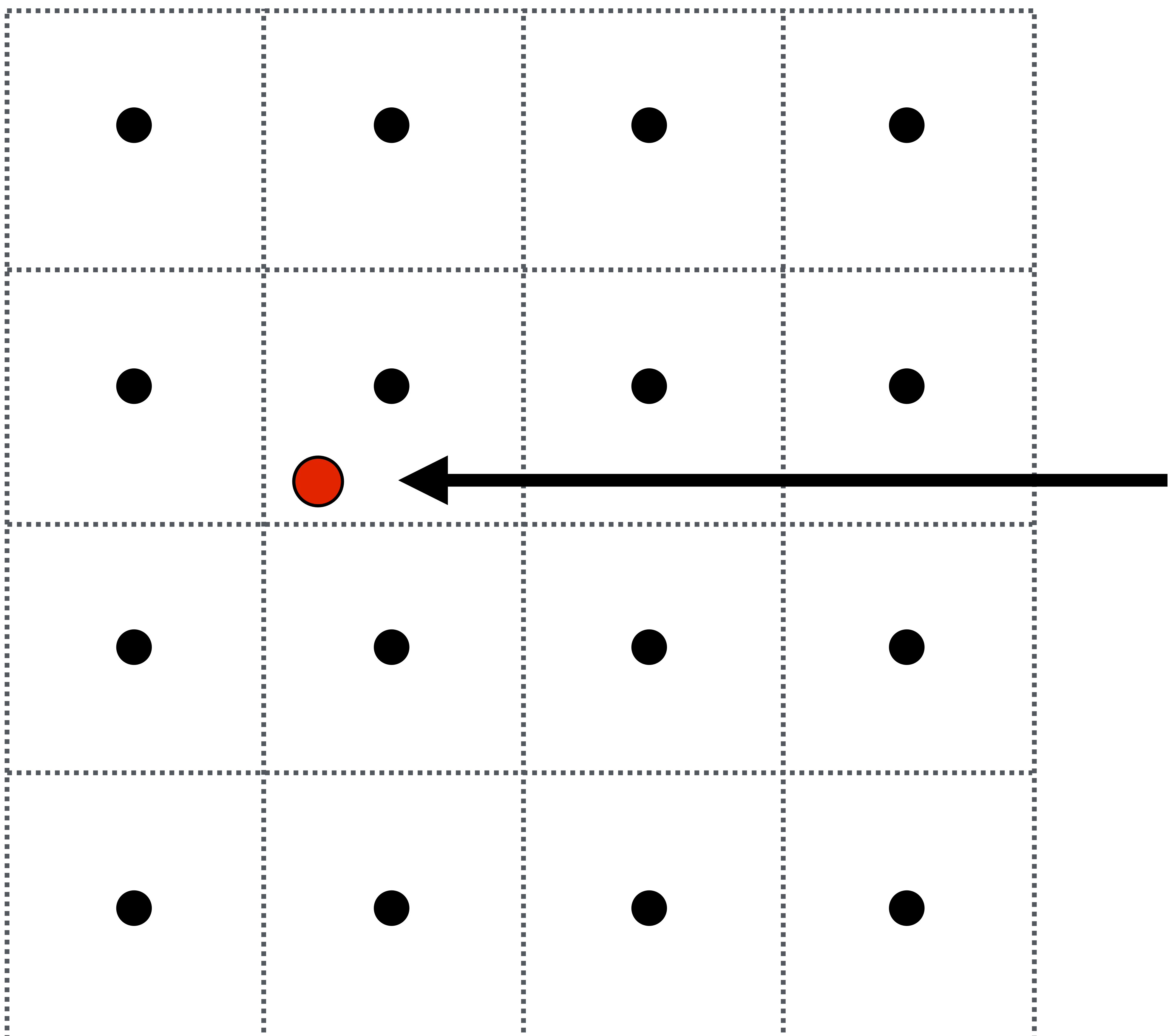


Nearest

Bilinear

Bicubic

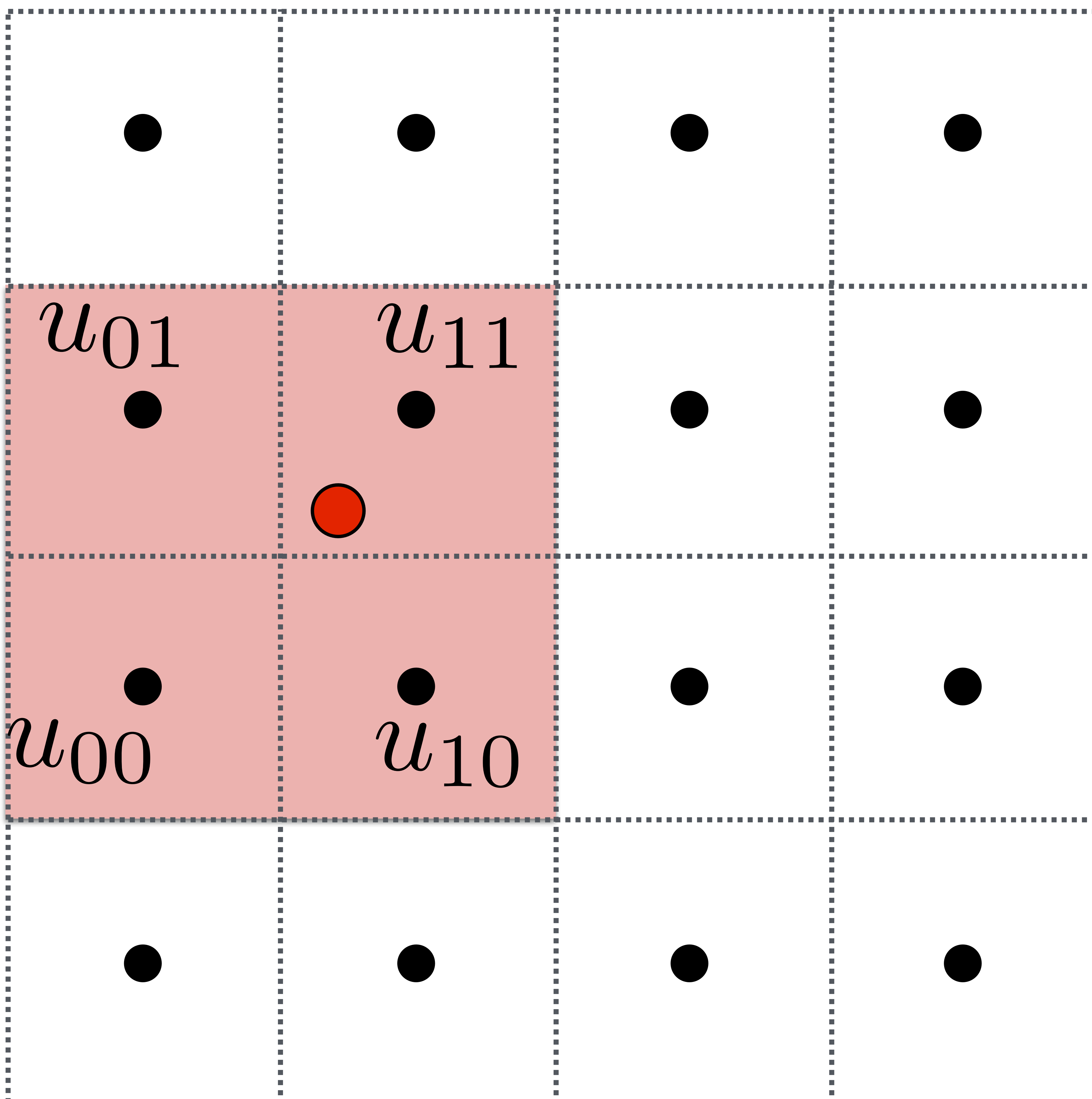
Bilinear Filtering



Want to sample texture value f(u,v) at red point

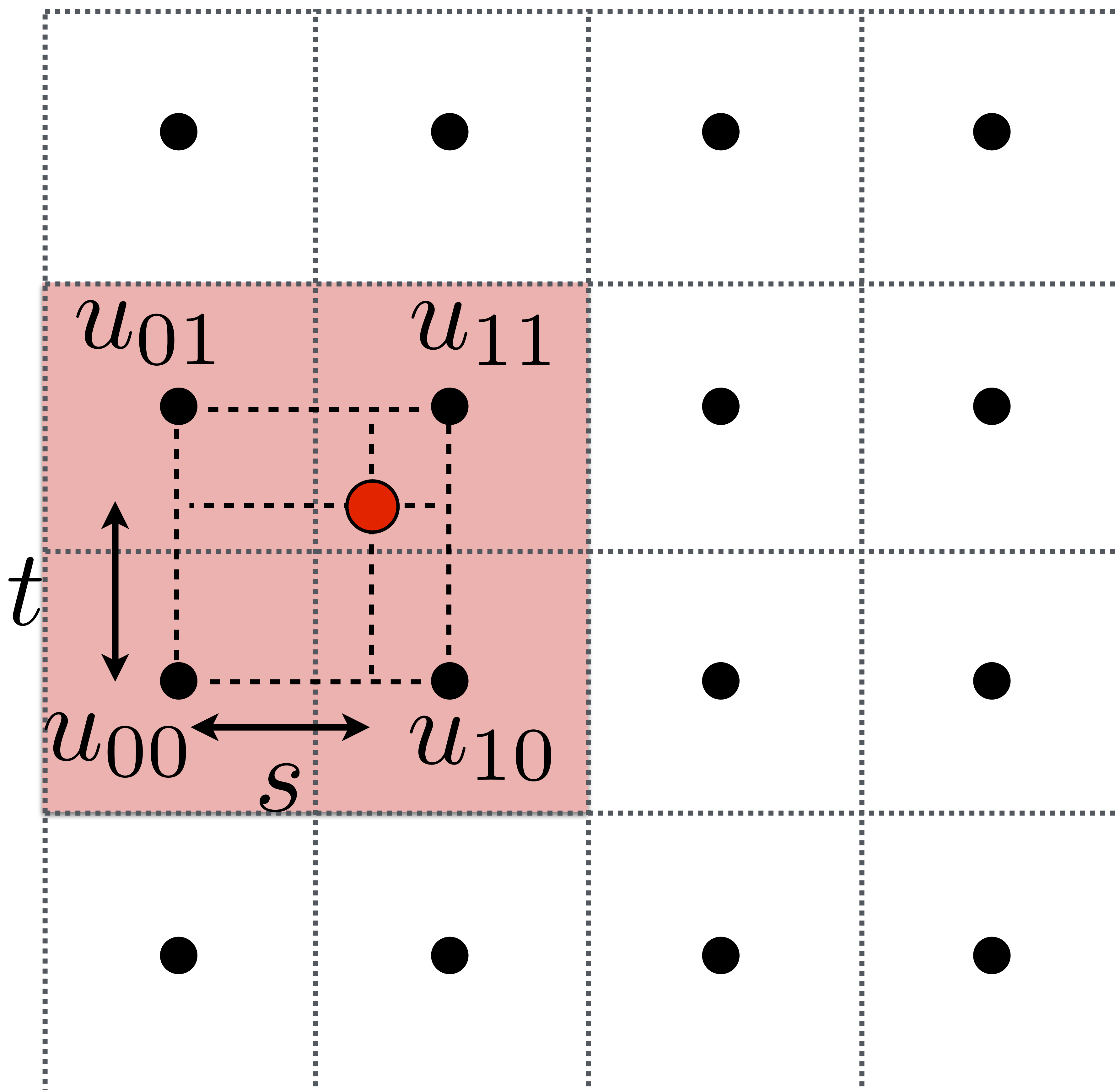
Black points indicate texture sample locations

Bilinear Filtering



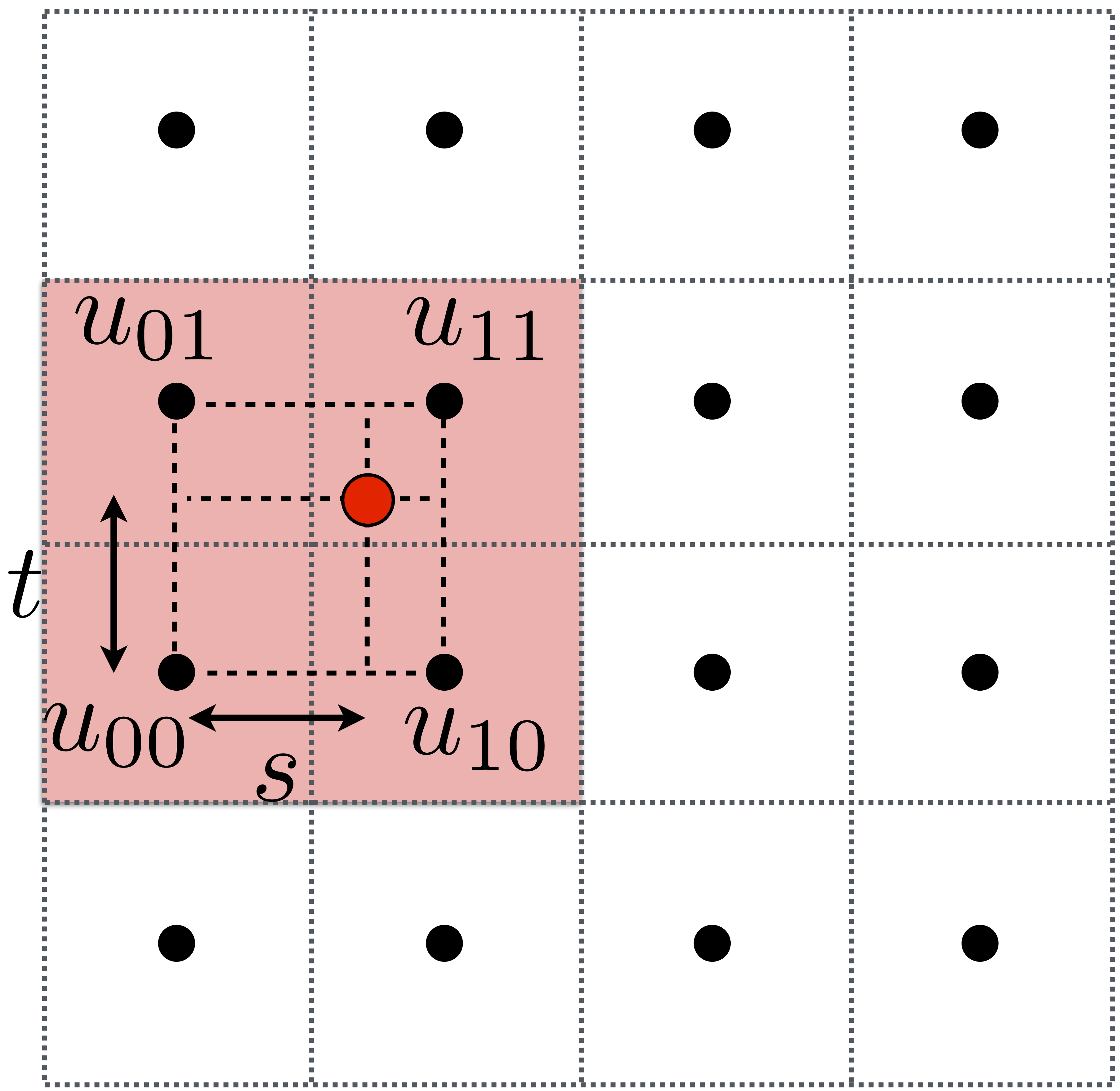
Take 4 nearest sample locations, with texture values as labeled.

Bilinear Filtering



And fractional offsets, (s,t) as shown

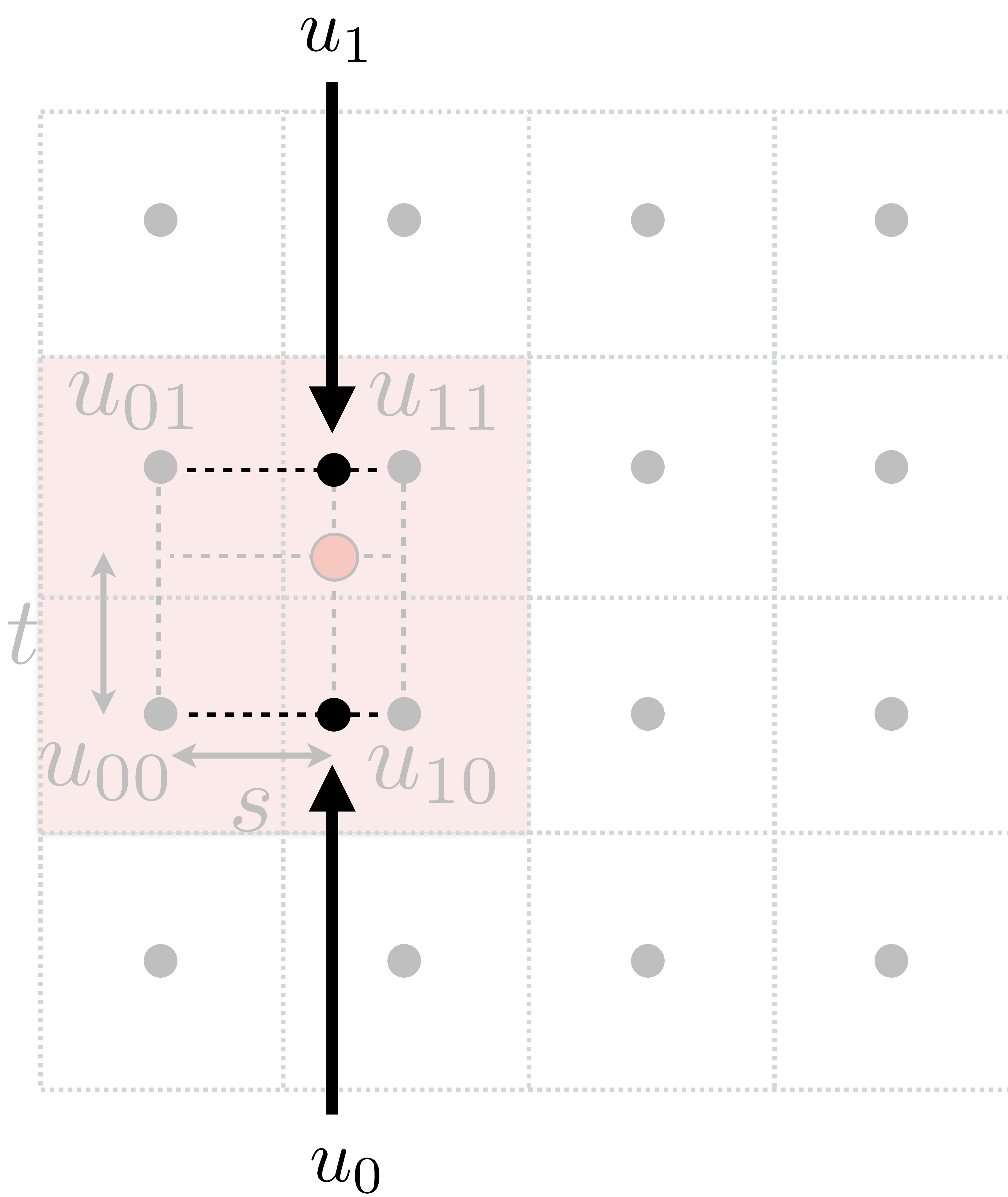
Bilinear Filtering



Linear interpolation (1D)

$$\text{lerp}(x, v_0, v_1) = v_0 + x(v_1 - v_0)$$

Bilinear Filtering



Linear interpolation (1D)

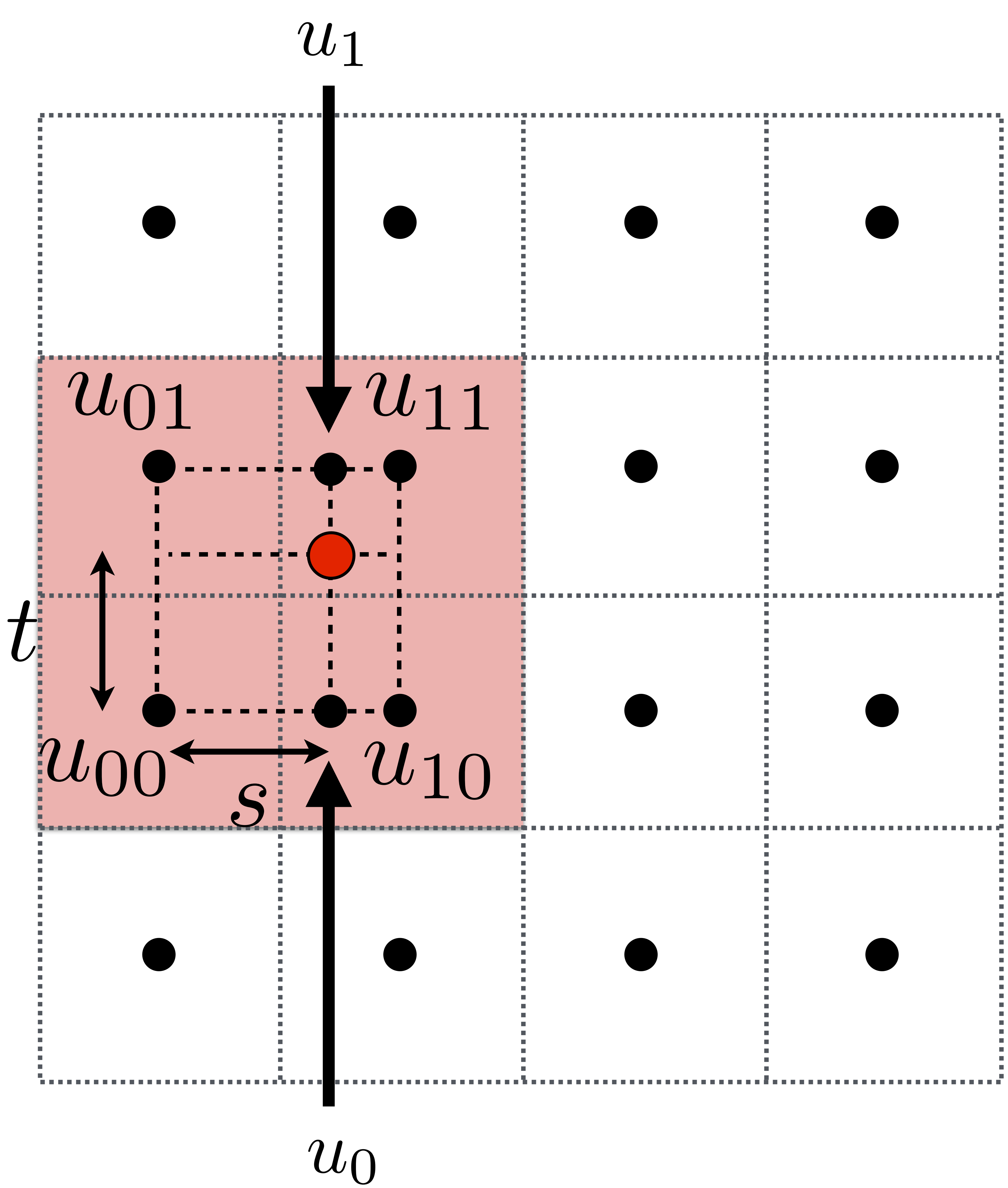
$$\text{lerp}(x, v_0, v_1) = v_0 + x(v_1 - v_0)$$

Two helper lerps (horizontal)

$$u_0 = \text{lerp}(s, u_{00}, u_{10})$$

$$u_1 = \text{lerp}(s, u_{01}, u_{11})$$

Bilinear Filtering



Linear interpolation (1D)

$$\text{lerp}(x, v_0, v_1) = v_0 + x(v_1 - v_0)$$

Two helper lerps

$$u_0 = \text{lerp}(s, u_{00}, u_{10})$$

$$u_1 = \text{lerp}(s, u_{01}, u_{11})$$

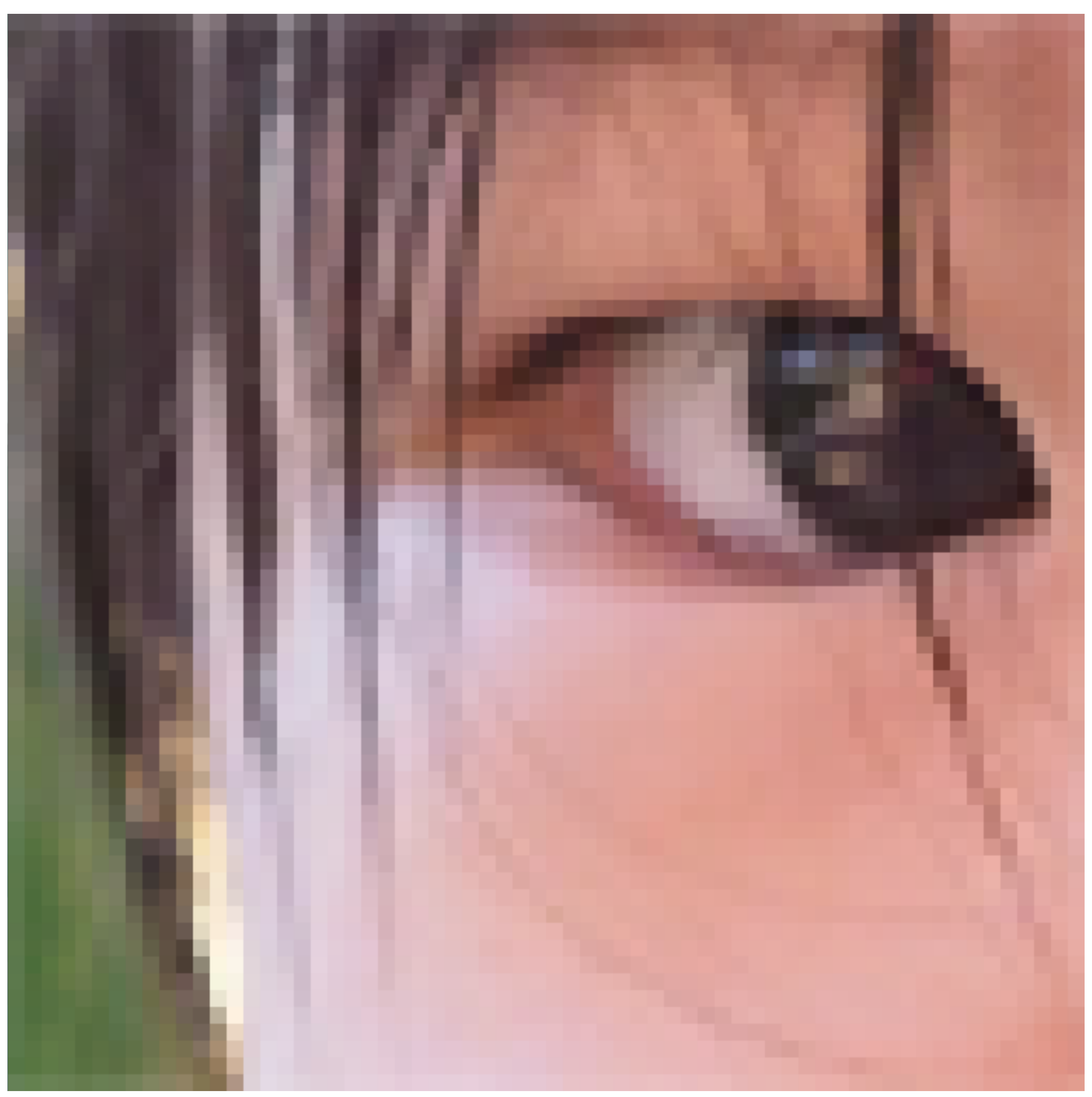
Final vertical lerp, to get result:

$$f(x, y) = \text{lerp}(t, u_0, u_1)$$

Reconstruction Filter Function

Test your understanding:

- What is the reconstruction filter k(x,y) for bilinear interpolation? Nearest? What is a theoretically ideal filter? What are the pros/cons of each?



Nearest

Bilinear

Bicubic

Texture Minification

Texture Minification - Hard Case

Challenging

- Many texels can contribute to pixel footprint
- Shape of pixel footprint can be complex

Idea:

- Take texture image file, then low-pass filter it (i.e. filter out high frequencies) and downsample it (i.e. sample at a lower resolution) texture file. Do this recursively, and store successively lower resolution, each with successively lower maximum signal frequency.
- For each sample, use the texture file whose resolution approximates the screen sampling rate

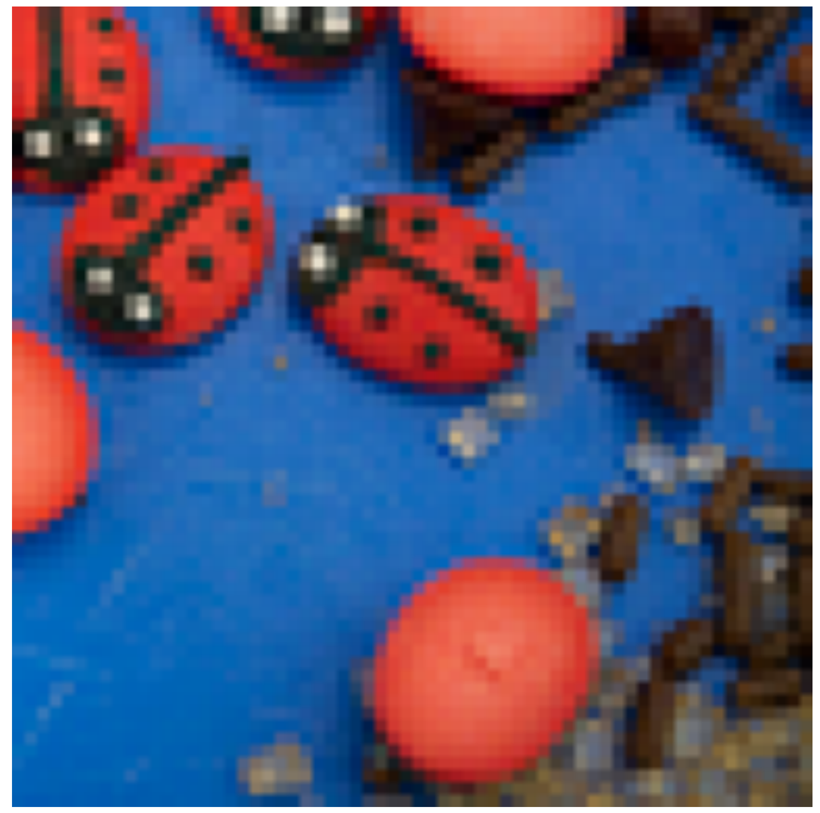
Level 0 - Full Resolution Texture

Level 2 - Downsample 4x4

Level 4 - Downsample 16x16

Mipmap (L. Williams 83)

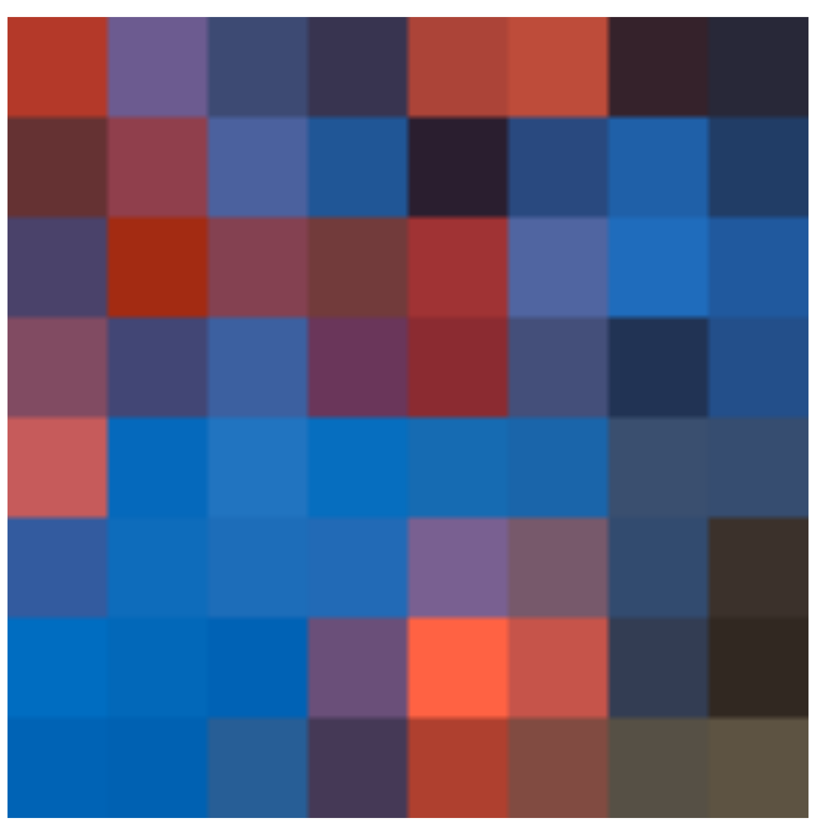
Level 0 = 128x128



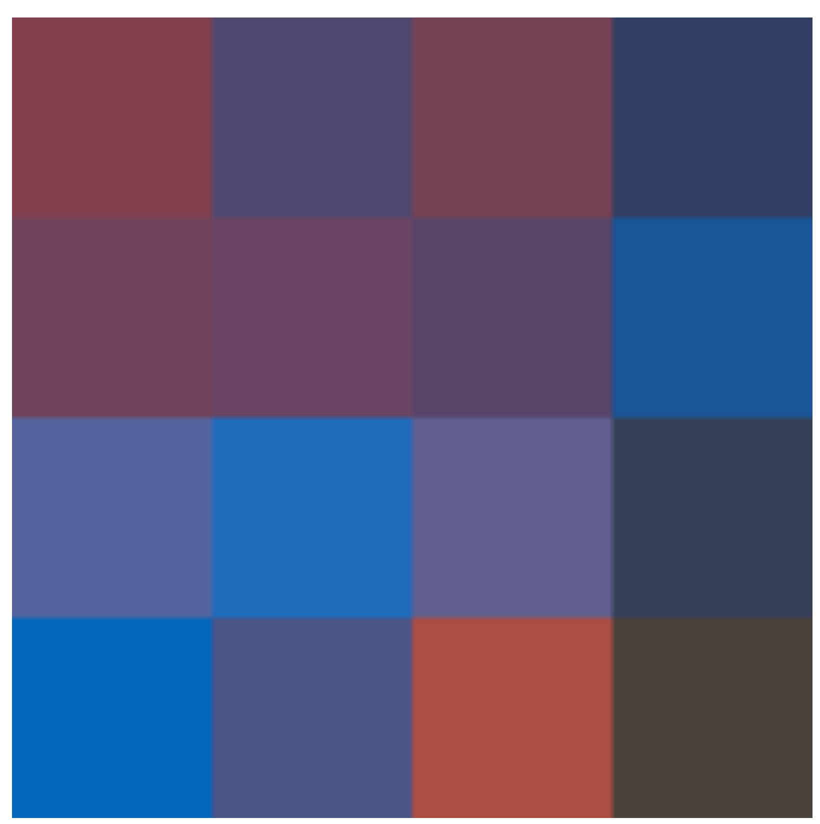
Level 1 = 64x64

Level 2 = 32x32

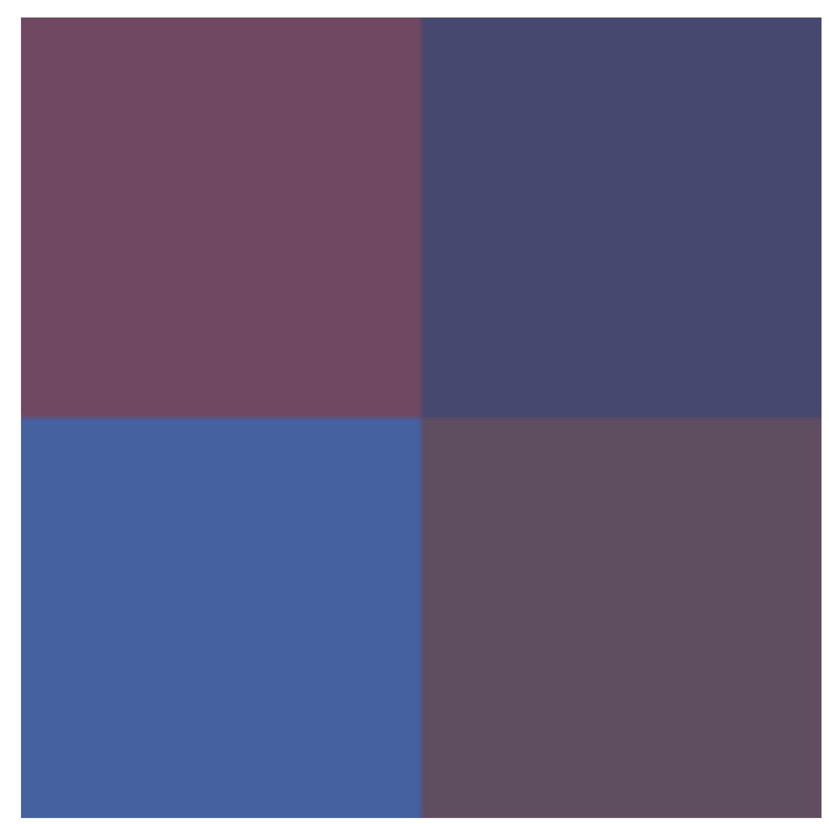
Level 3 = 16x16



Level 4 = 8x8



Level 5 = 4x4

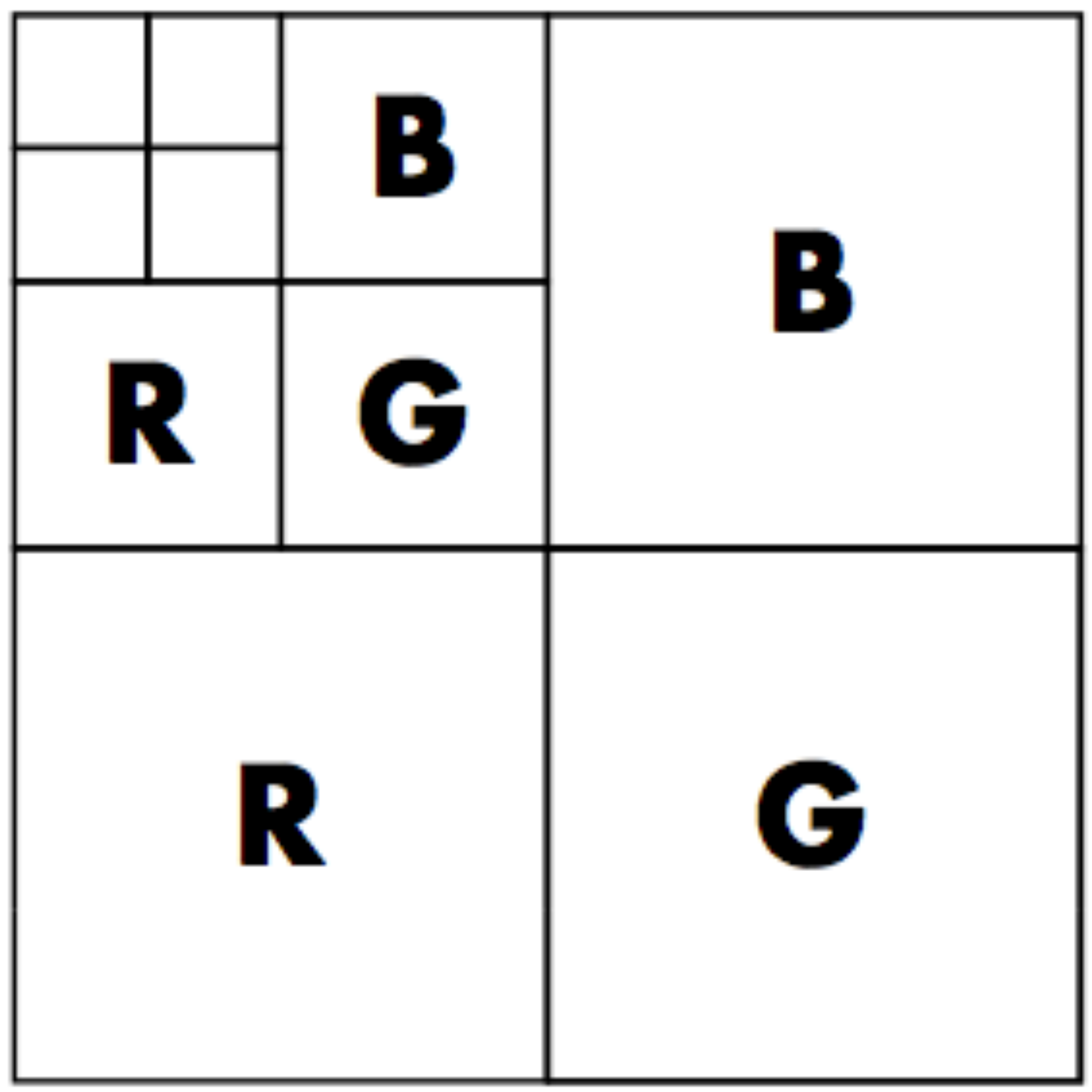


Level 6 = 2x2

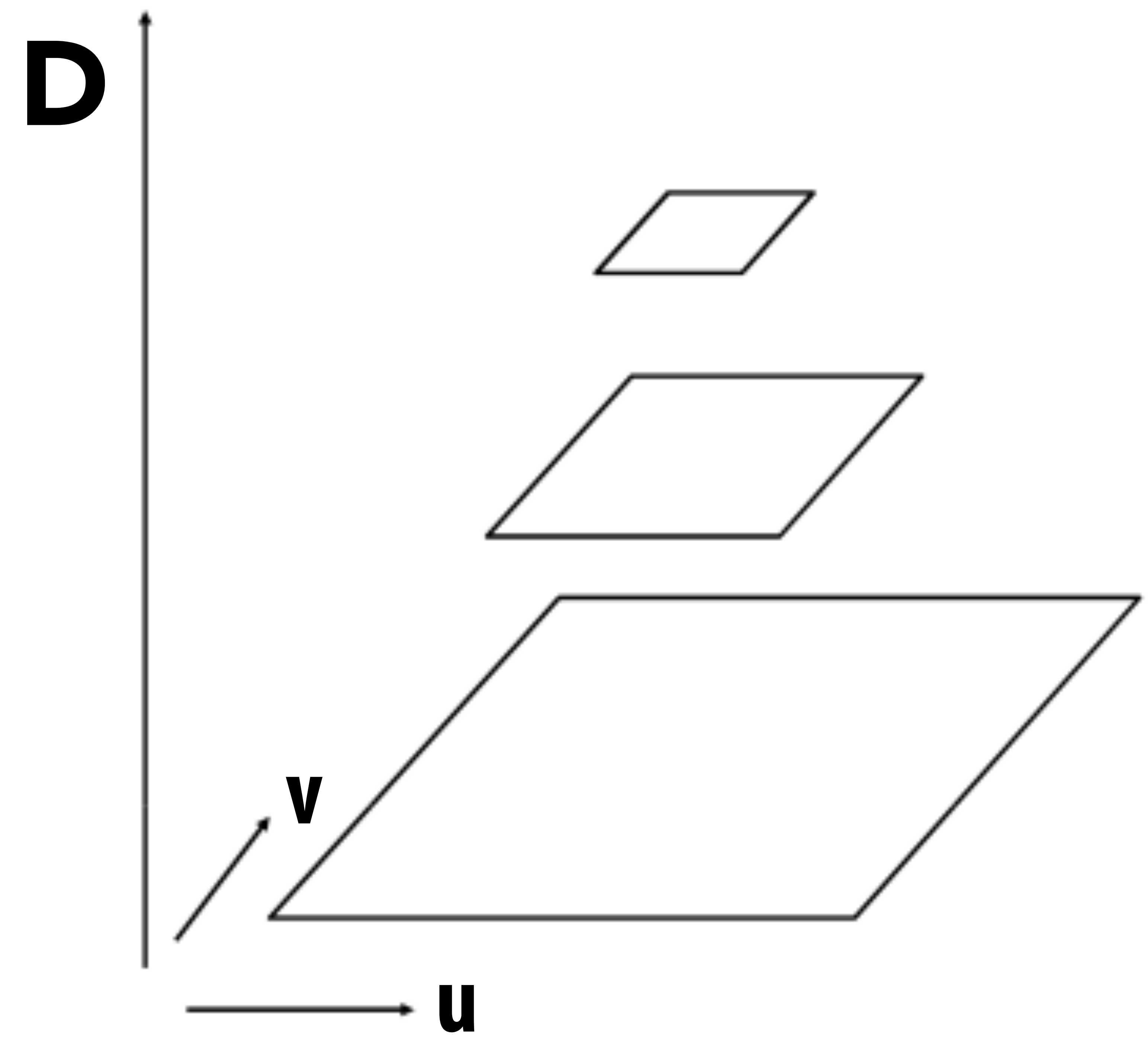
Level 7 = 1x1

“Mip” comes from the Latin “multum in parvo", meaning a multitude in a small space

Mipmap (L. Williams 83)



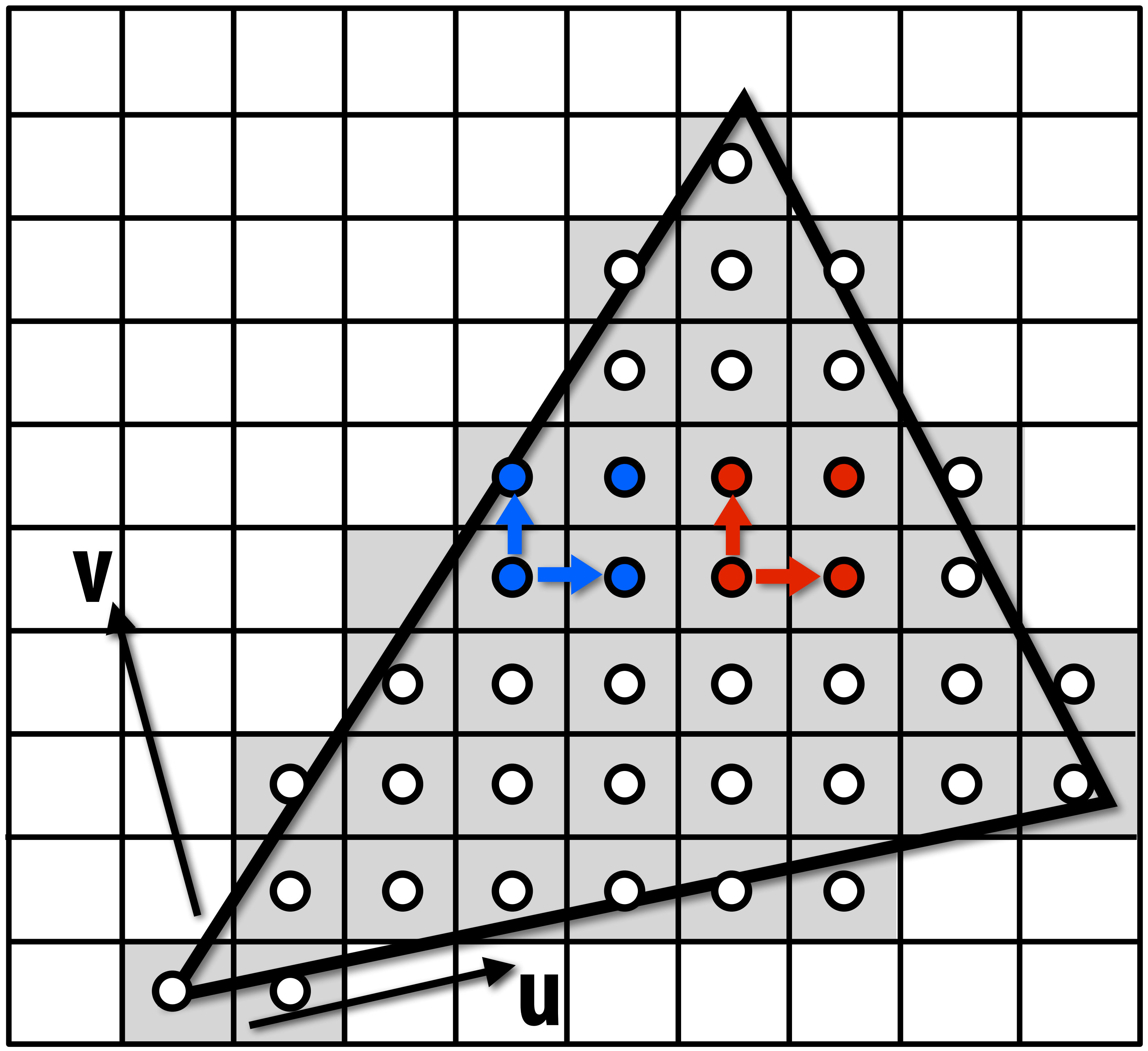
Williams' original proposed mipmap layout



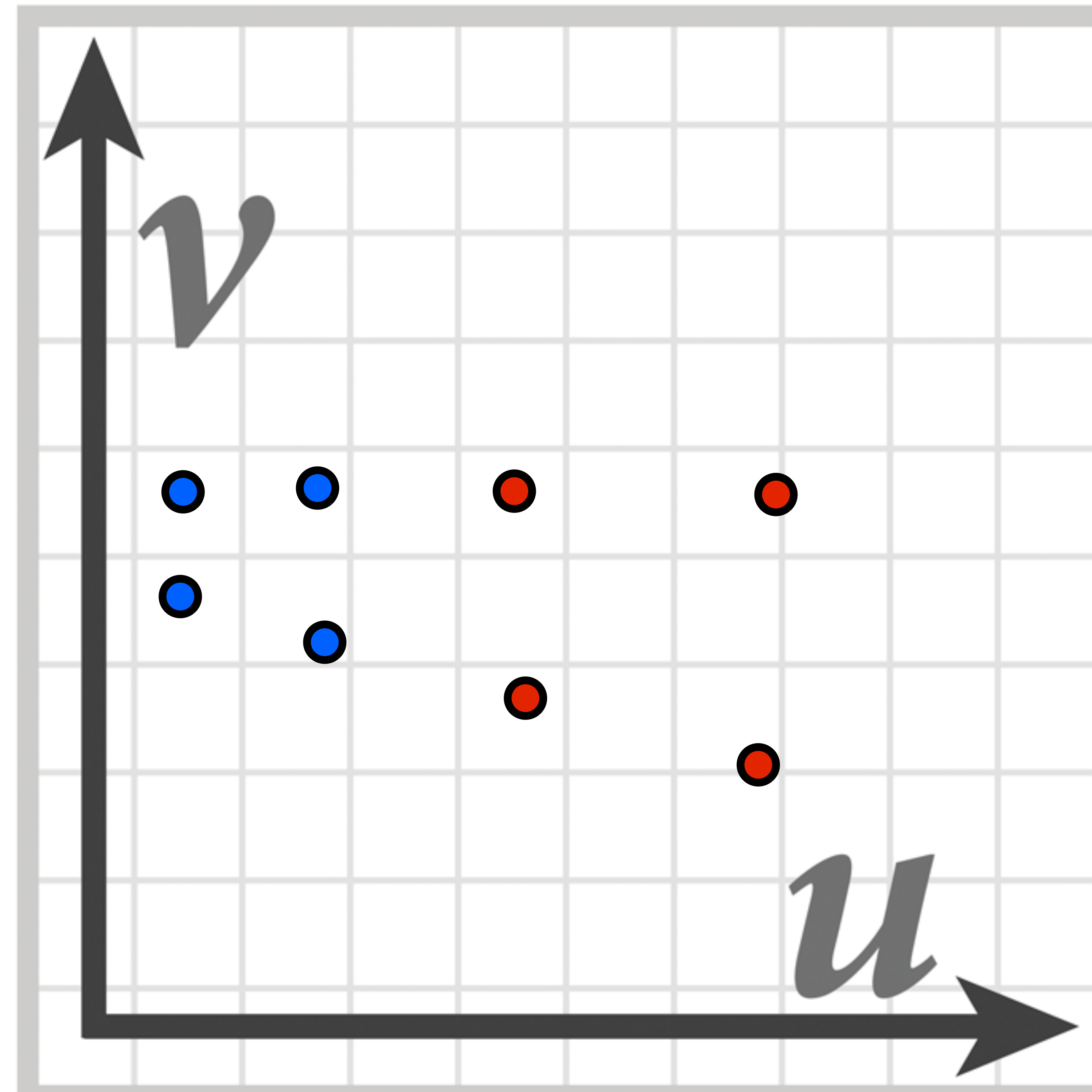
"Mip hierarchy" level = D

What is the storage overhead of a mipmap?

Computing Mipmap Level D



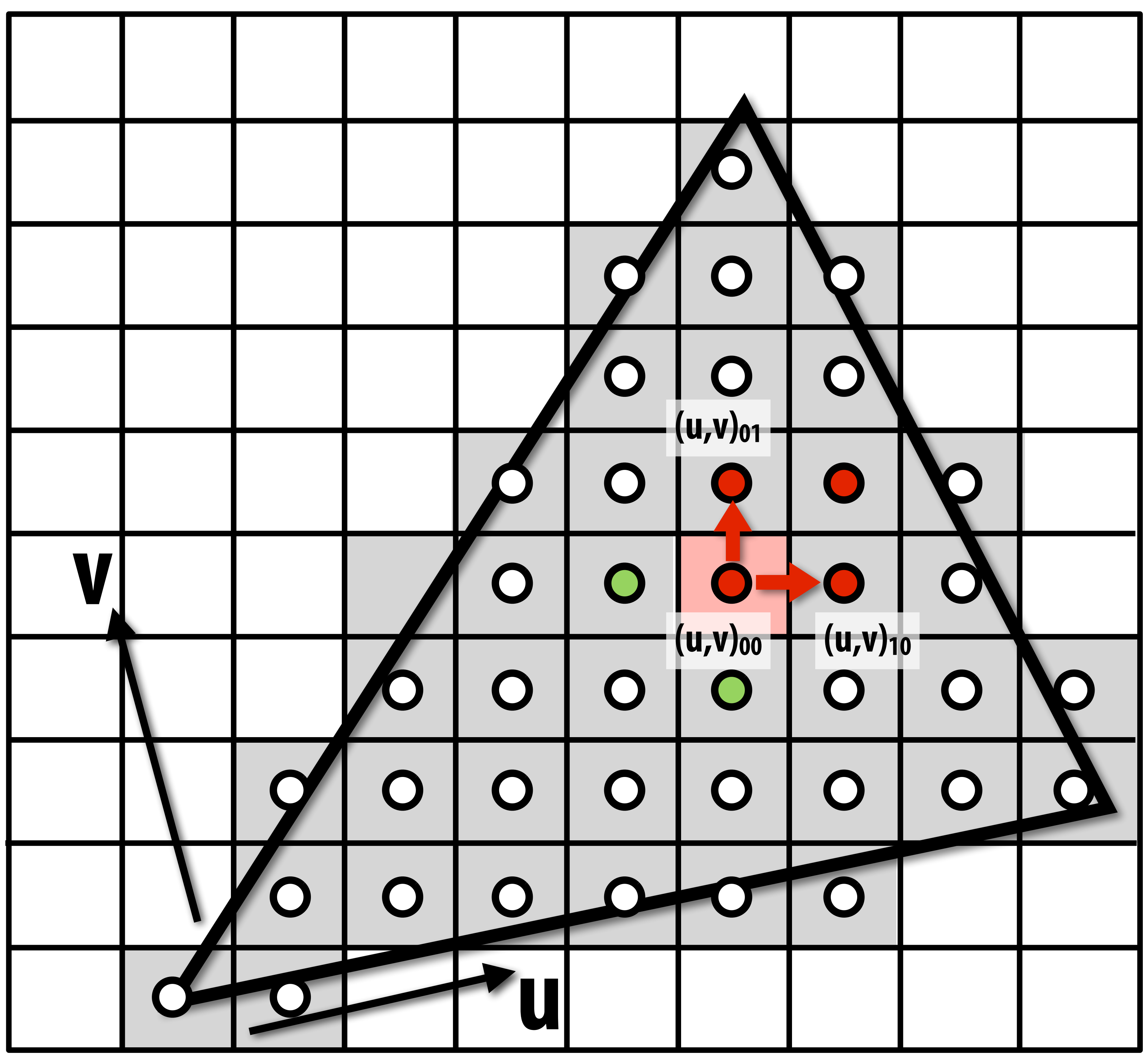
Screen space (x,y)

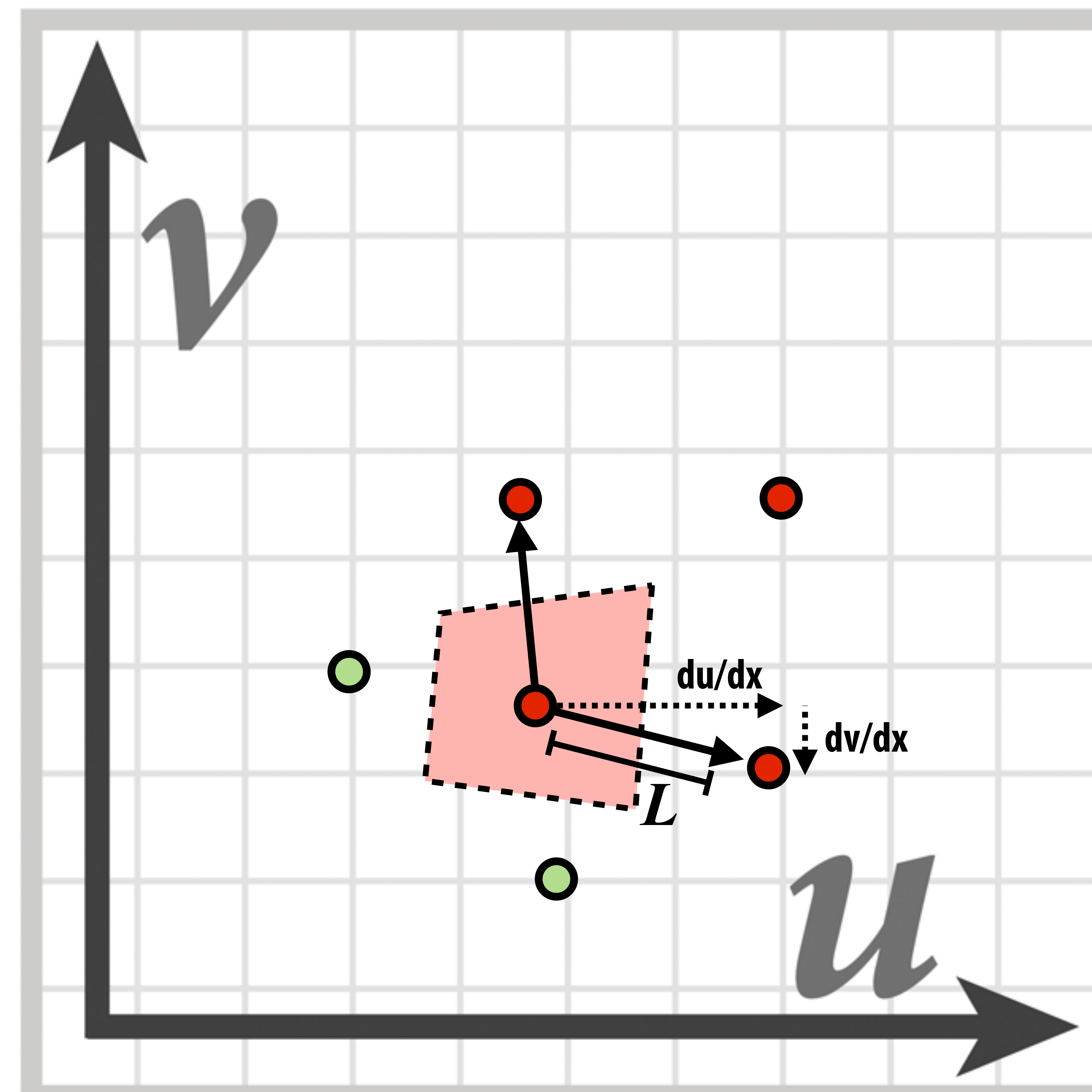


Texture space (u,v)

Estimate texture footprint using texture coordinates of neighboring screen samples

Computing Mipmap Level D

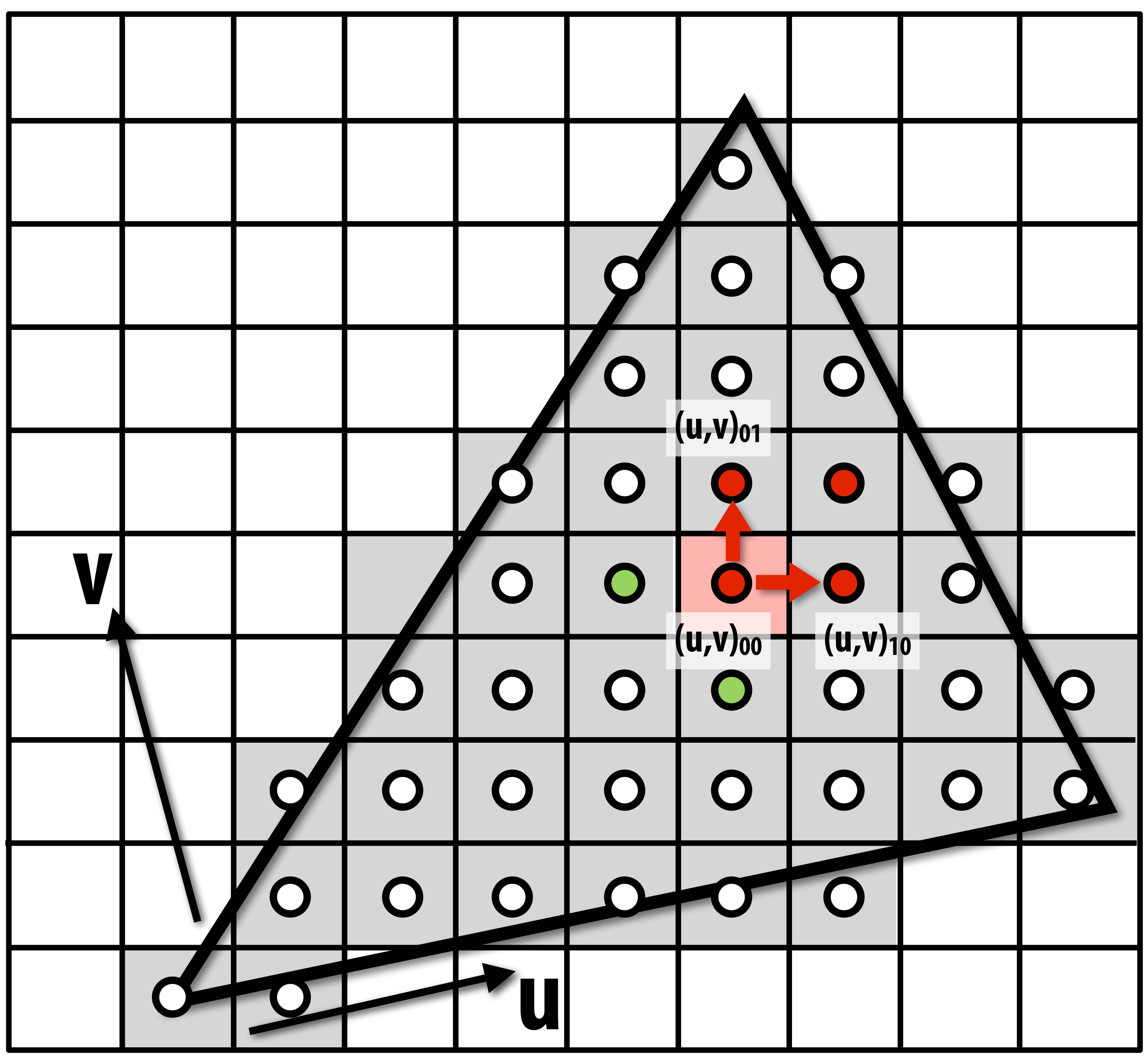


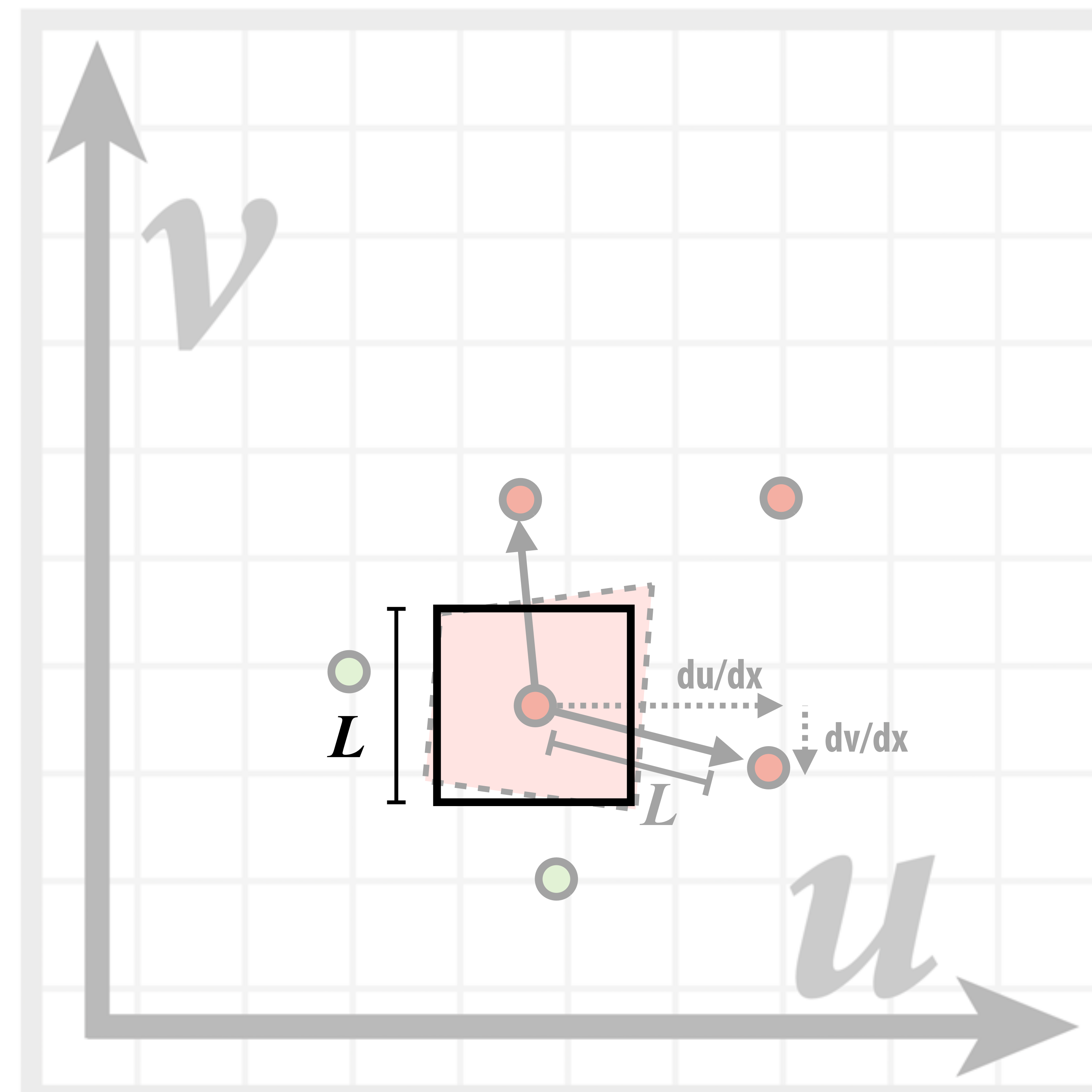


$$D = \log_2 L$$

$$L = \max\left(\sqrt{\left(\frac{du}{dx}\right)^2 + \left(\frac{dv}{dx}\right)^2}, \sqrt{\left(\frac{du}{dy}\right)^2 + \left(\frac{dv}{dy}\right)^2}\right)$$

Computing Mipmap Level D

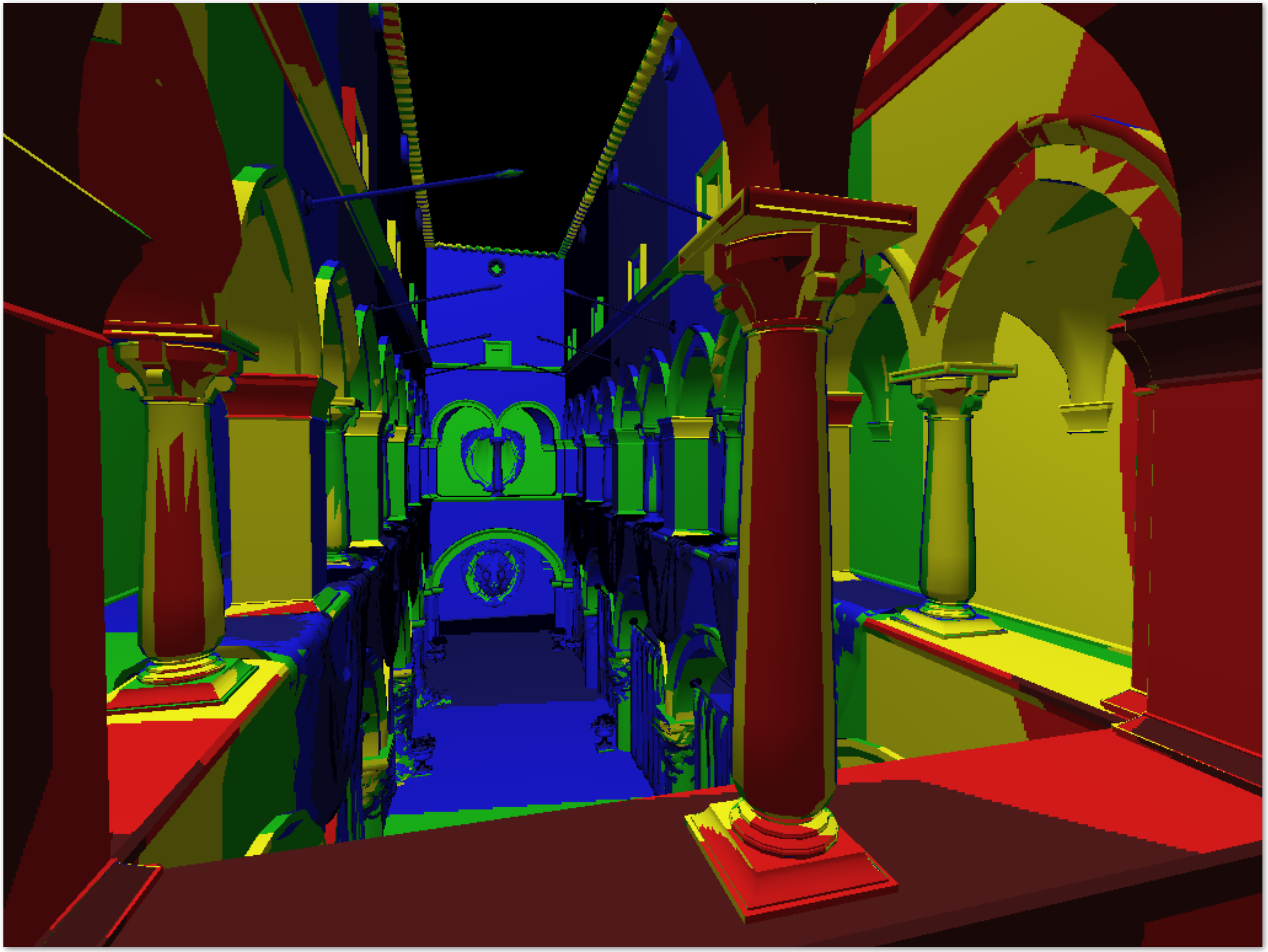




$$D = \log_2 L$$

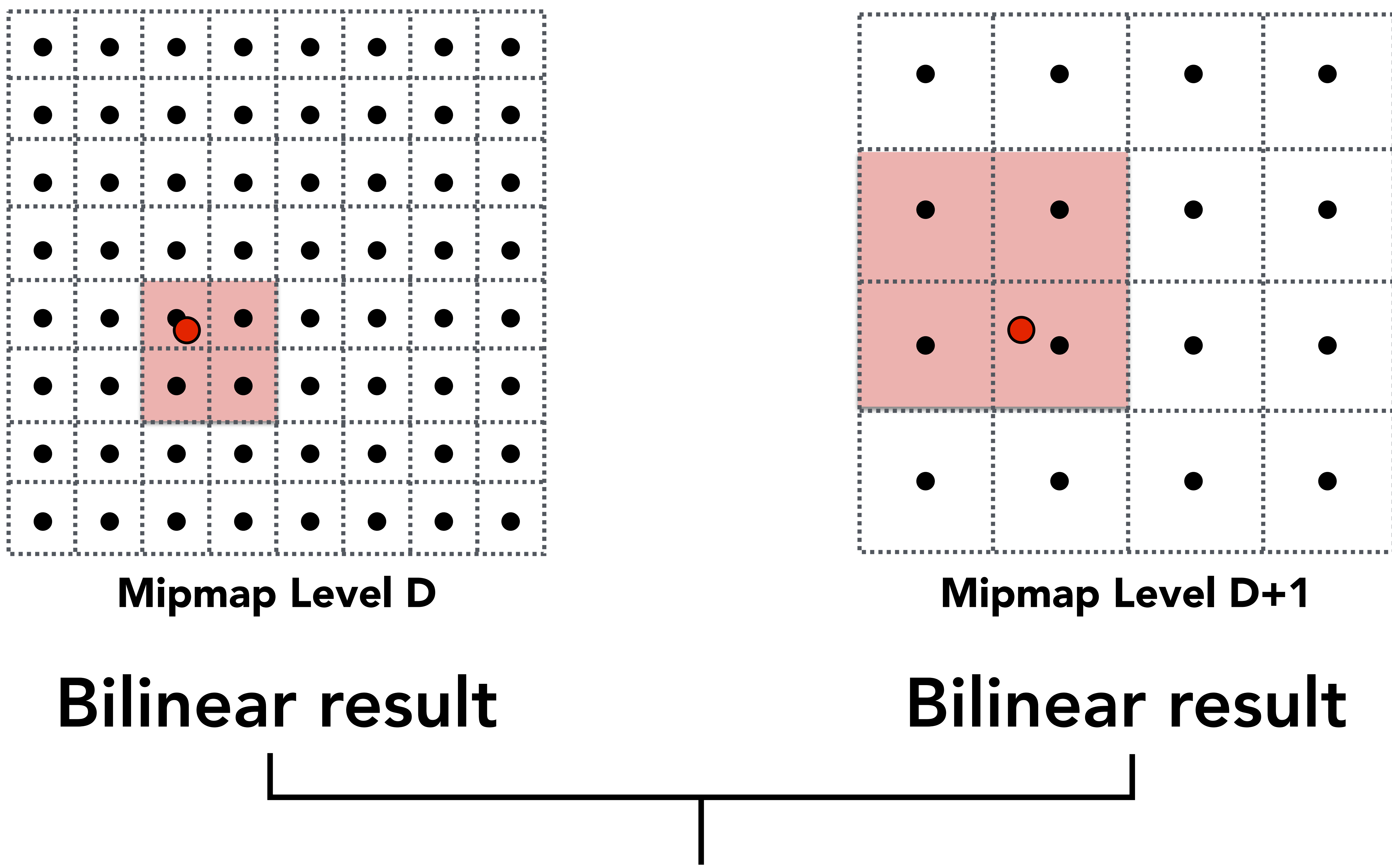
$$L = \max \left(\sqrt{\left(\frac{du}{dx}\right)^2 + \left(\frac{dv}{dx}\right)^2}, \sqrt{\left(\frac{du}{dy}\right)^2 + \left(\frac{dv}{dy}\right)^2} \right)$$

Visualization of Mipmap Level



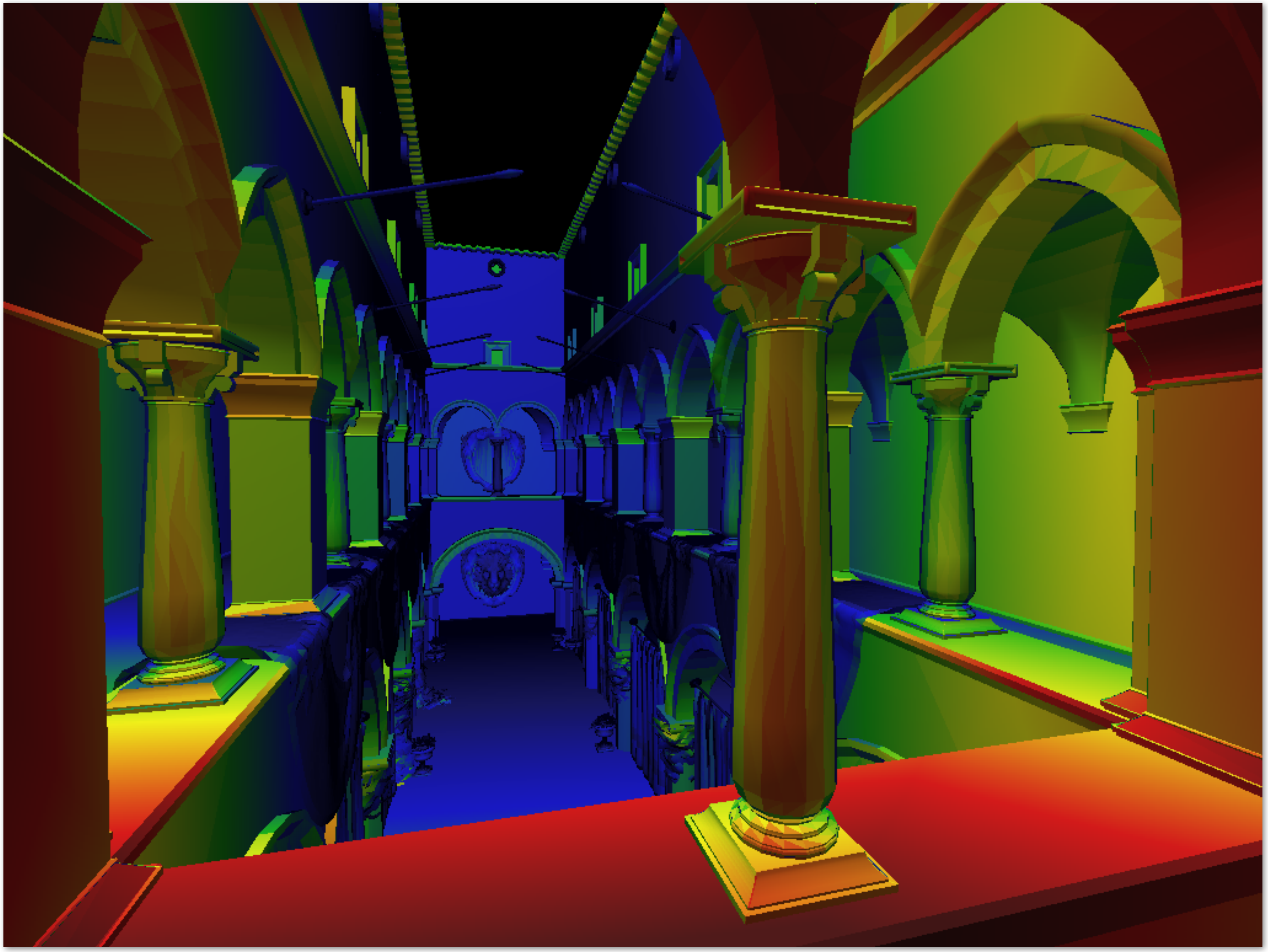
D rounded to nearest integer level

Trilinear Filtering



Linear interpolation based on continuous D value

Visualization of Mipmap Level



Trilinear filtering: visualization of continuous D

Bilinear vs Trilinear Filtering Cost

Bilinear resampling:

- 4 texel reads
- 3 lerps (3 mul + 6 add)

Trilinear resampling:

- 8 texel reads
- 7 lerps (7 mul + 14 add)

Texture Filtering in Assignment

Image resampling choices

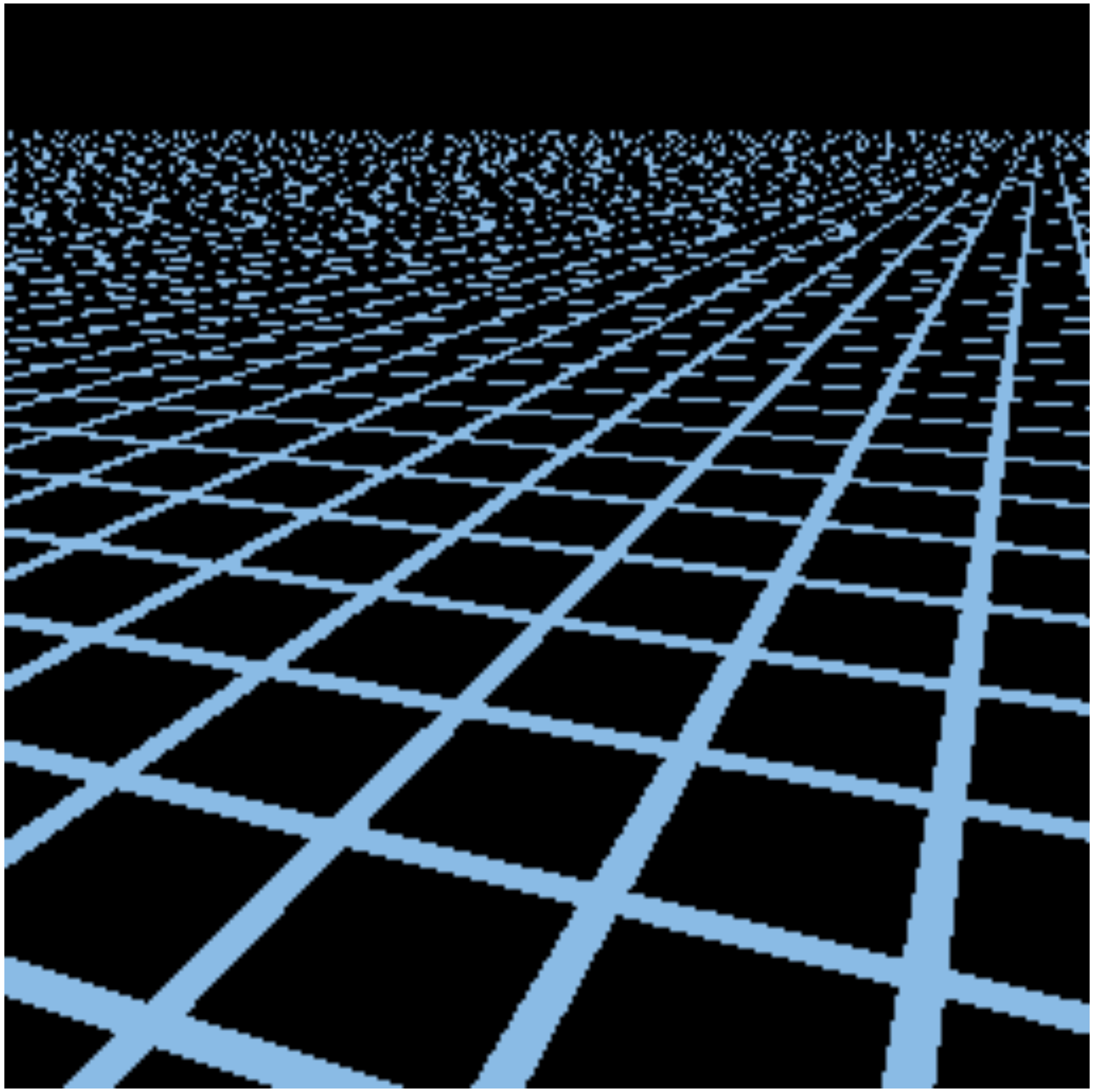
- Nearest
- Bilinear interpolation

Mipmap level resampling choices

- Always level 0
- Nearest D
- Linear interpolation

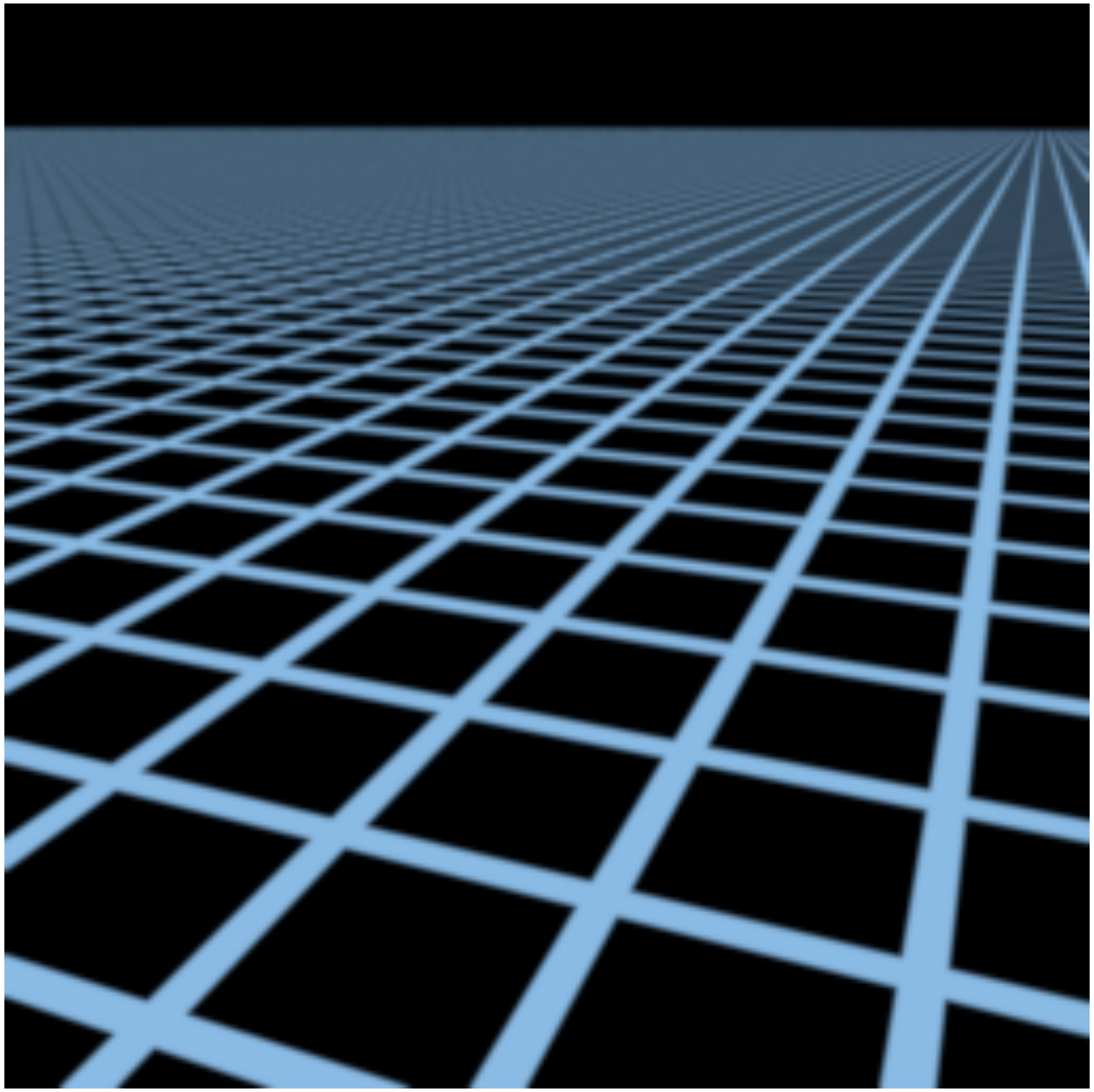
2 x 3 = 6 choices

Mipmap Limitations



Point sampling

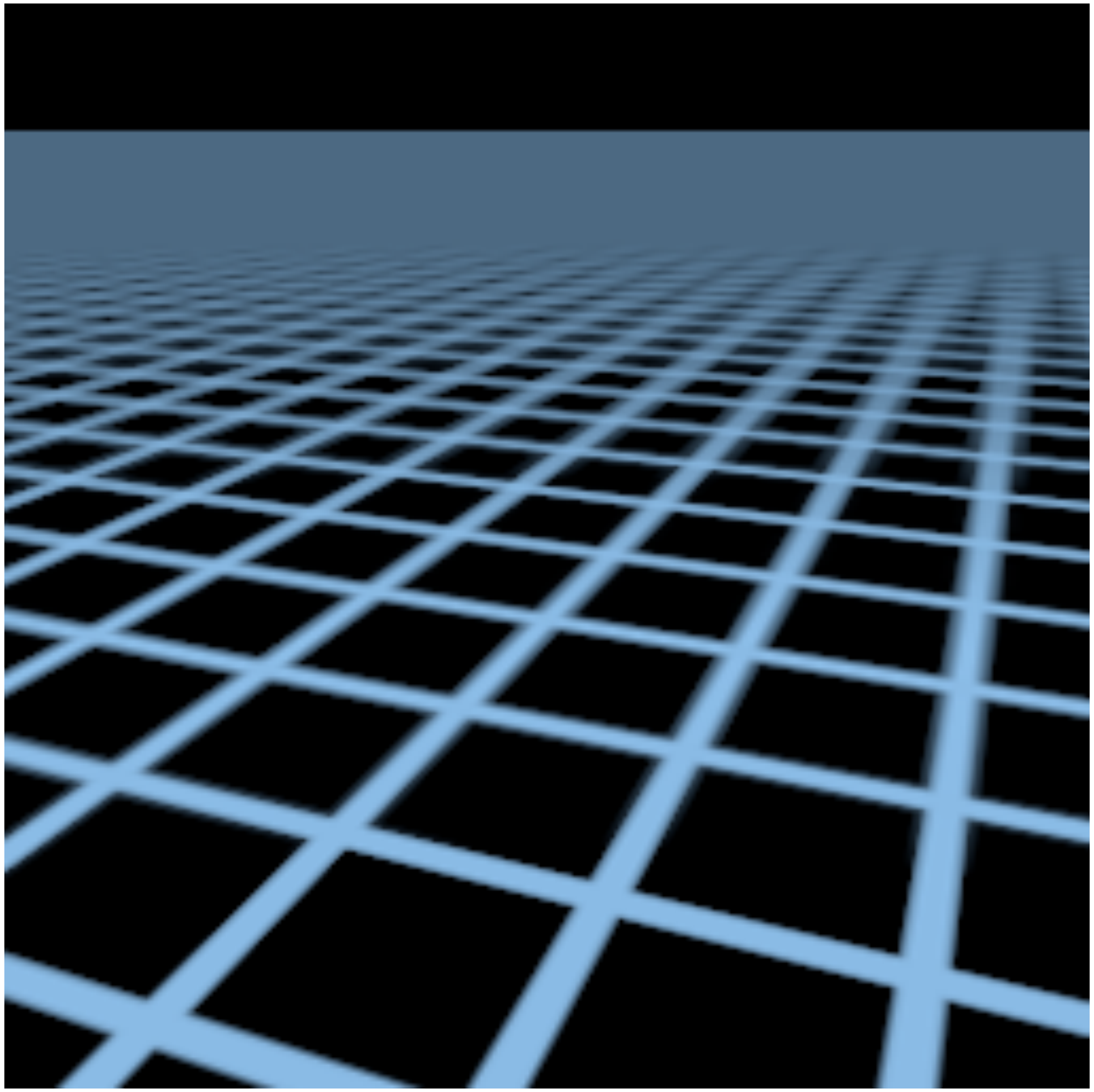
Mipmap Limitations



Supersampling 512x

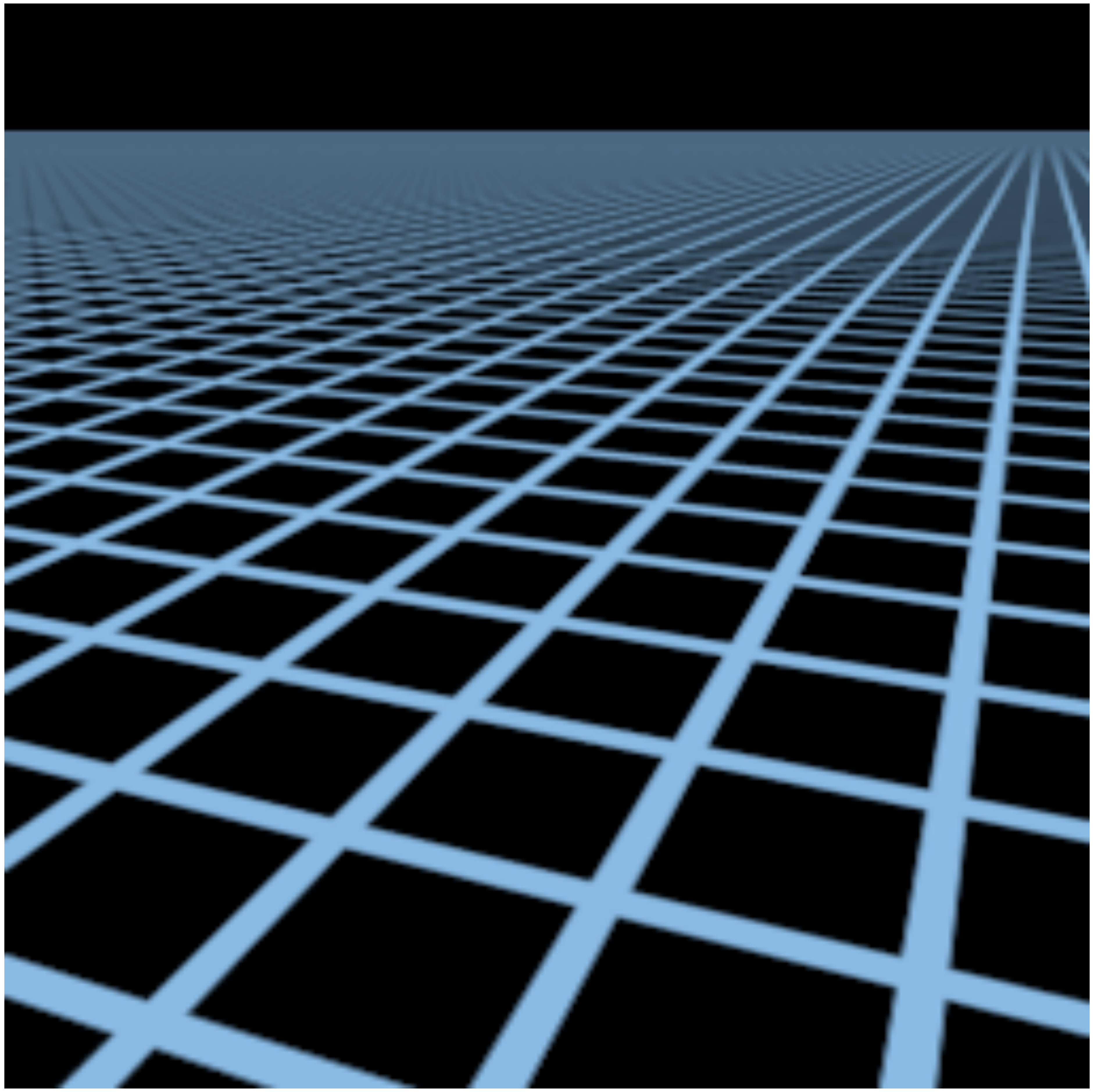
Mipmap Limitations

Overblur
Why?



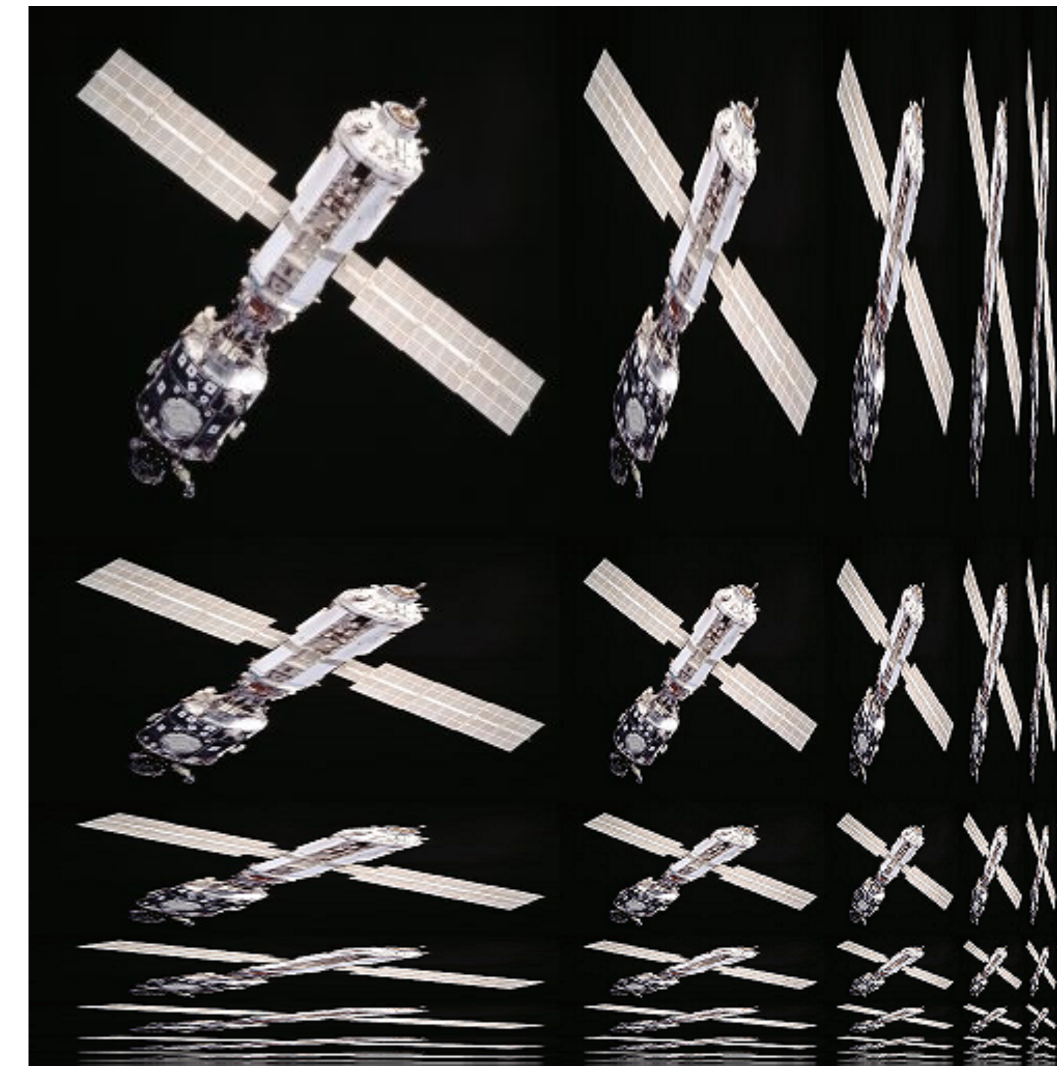
Mipmap trilinear sampling

Anisotropic Filtering



Elliptical weighted average (EWA) filtering

Anisotropic Filtering



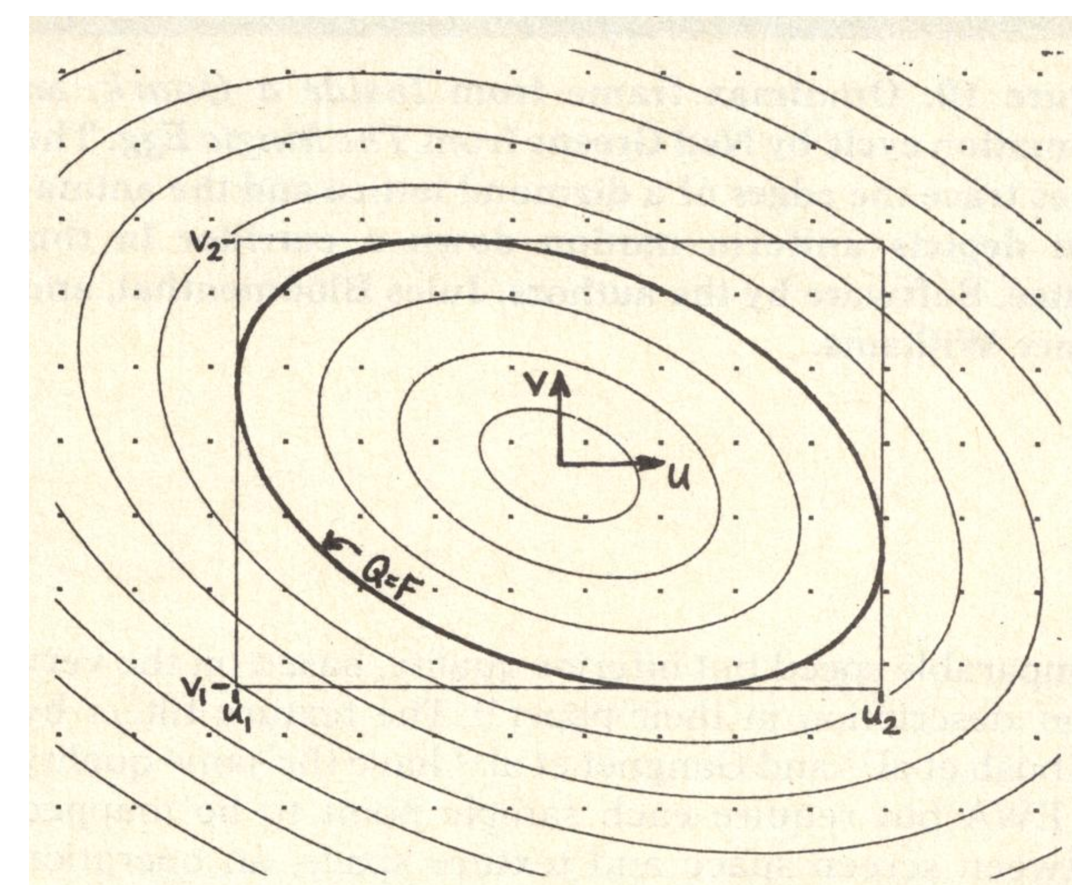
Wikipedia

Ripmaps and summed area tables

- Can look up axis-aligned rectangular zones
- Diagonal footprints still a problem

EWA filtering

- Use multiple lookups
- Weighted average
- Mipmap hierarchy still helps



Greene & Heckbert '86

Advanced Texturing Methods

Many, Many Uses for Texturing

In modern GPUs, texture = memory + filtering

- General method to bring data to fragment calculations

Many applications

- Environment lighting
- Store microgeometry
- Procedural textures
- Solid modeling
- Volume rendering
- …

Environment Map

A function from the sphere to colors, stored as a texture.

Lat / long texture map

Reflection vector indexes into texture map

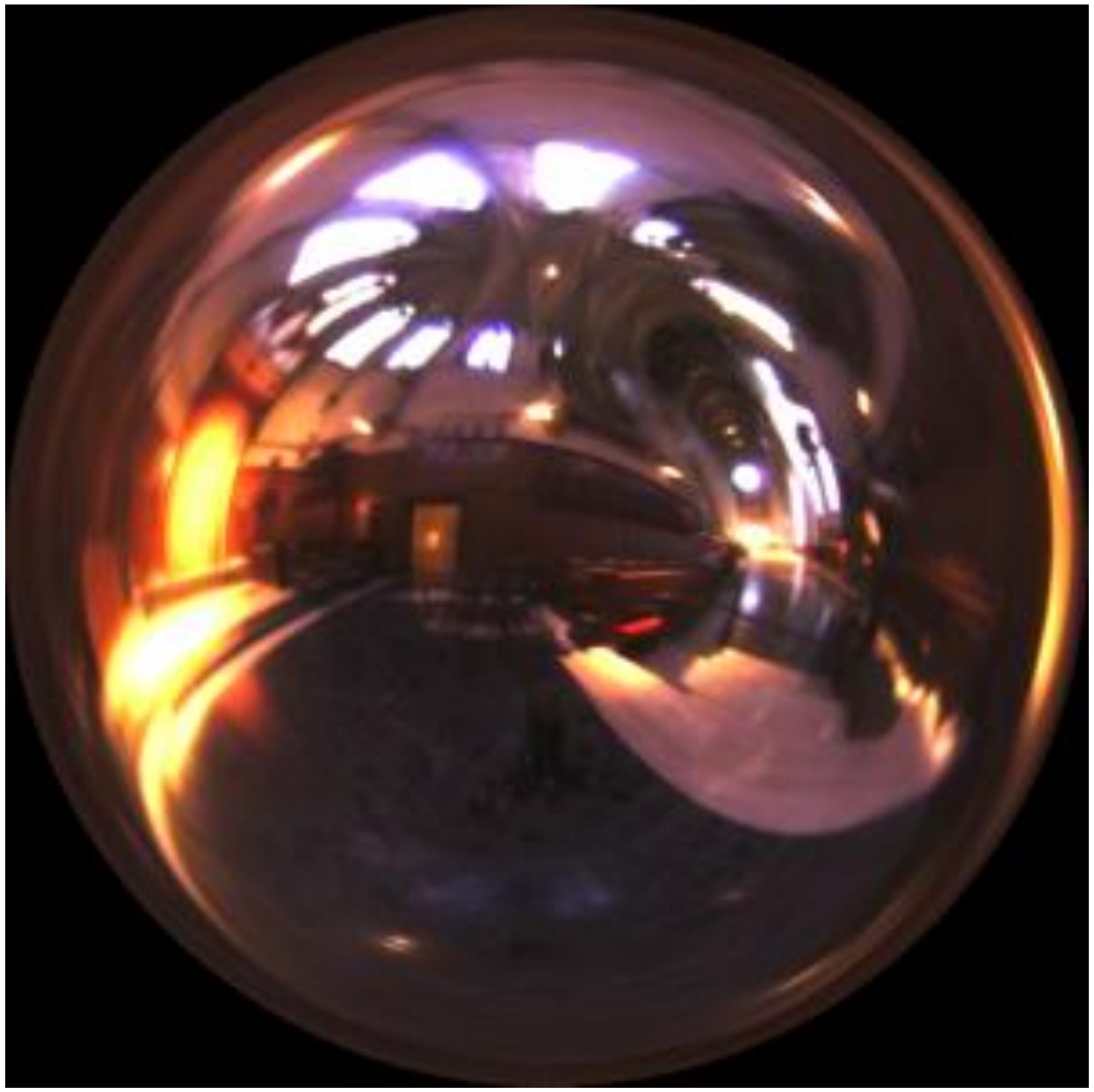
[Blinn & Newell 1976]

Spherical Environment Map

Hand with Reflecting Sphere. M. C. Escher, 1935. lithograph

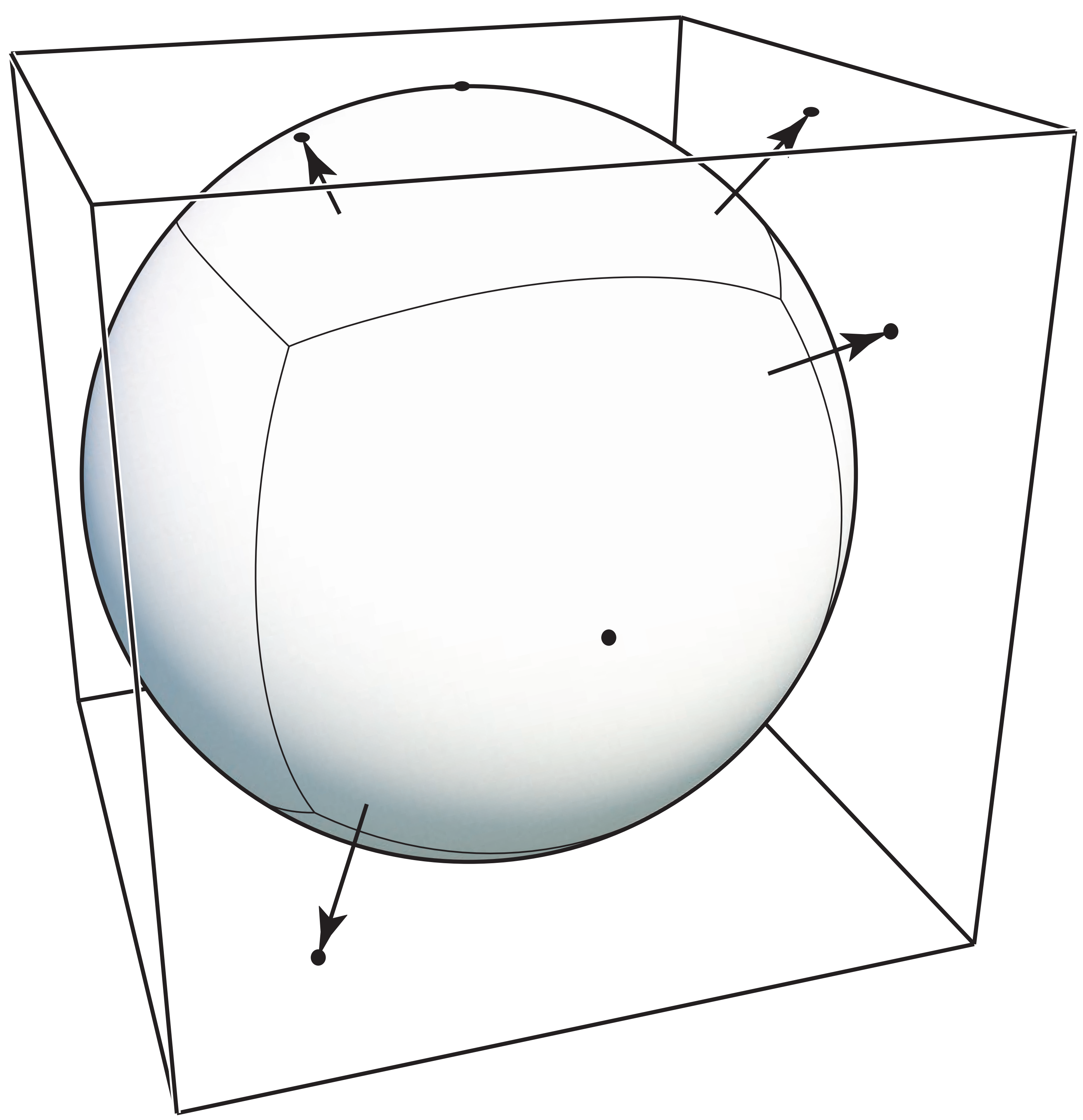
Light Probes, Paul Debevec

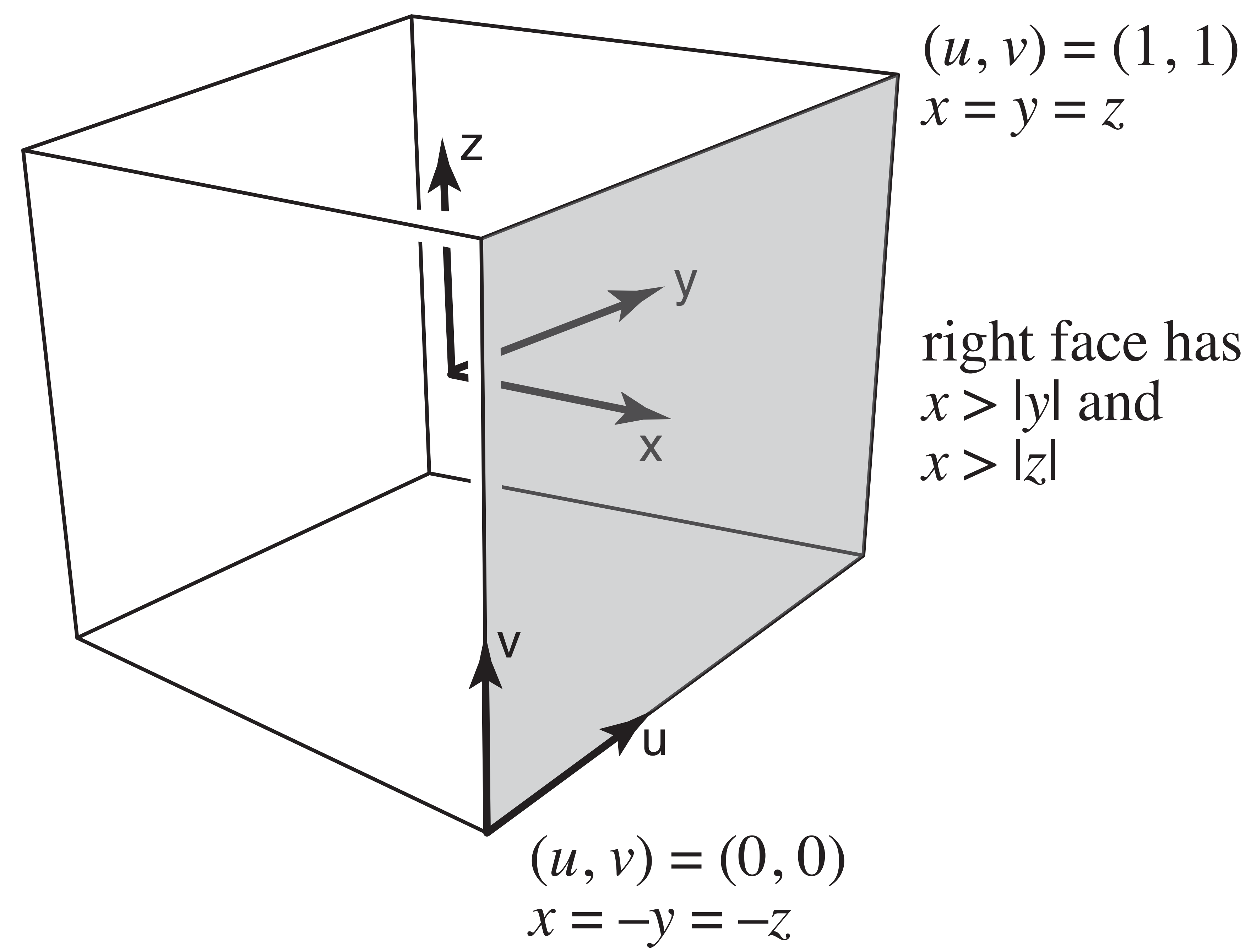
Environmental Lighting



Environment map (left) used to render realistic lighting

Cube Map





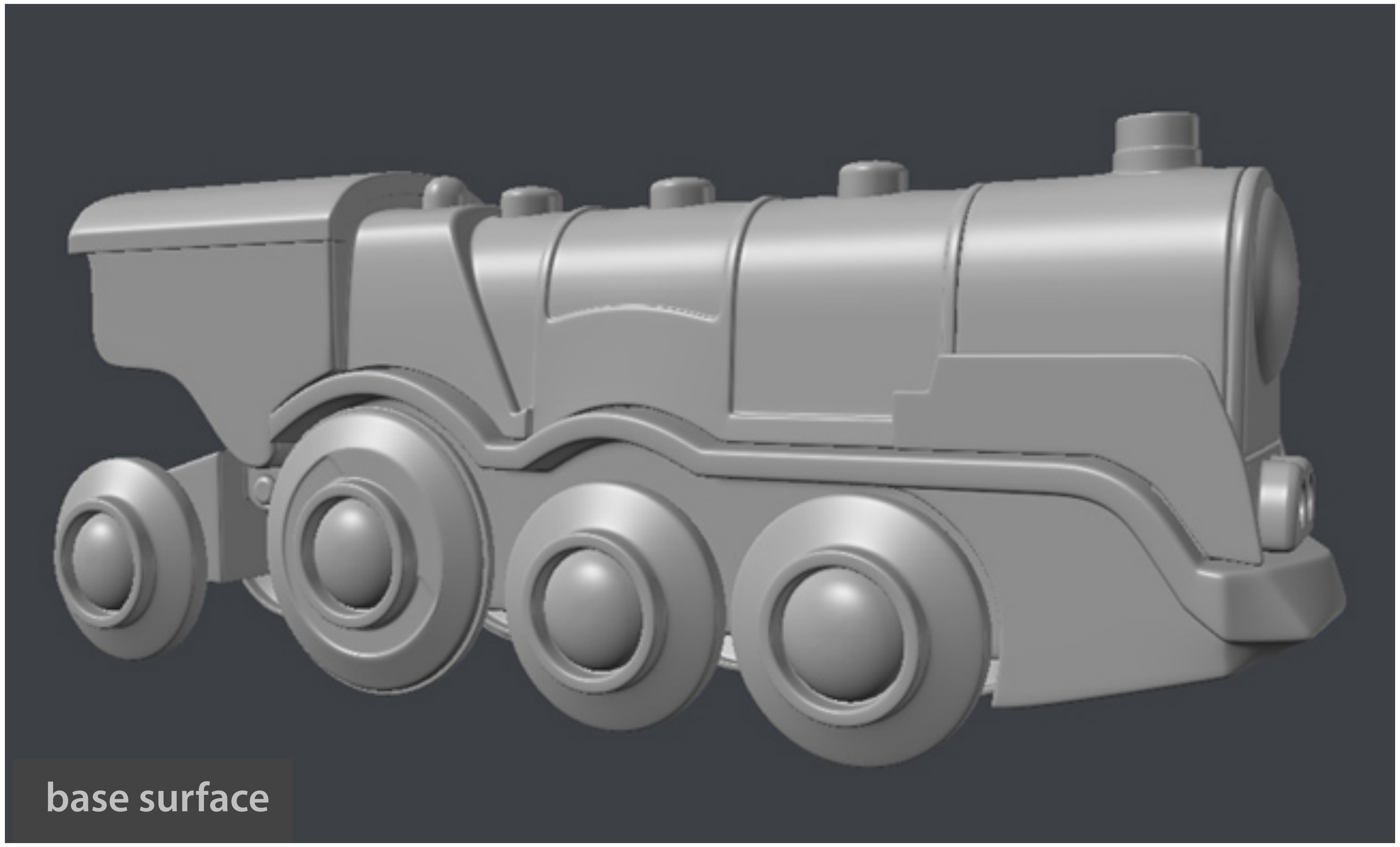
A vector maps to cube point along that direction.

The cube is textured with 6 square texture maps.

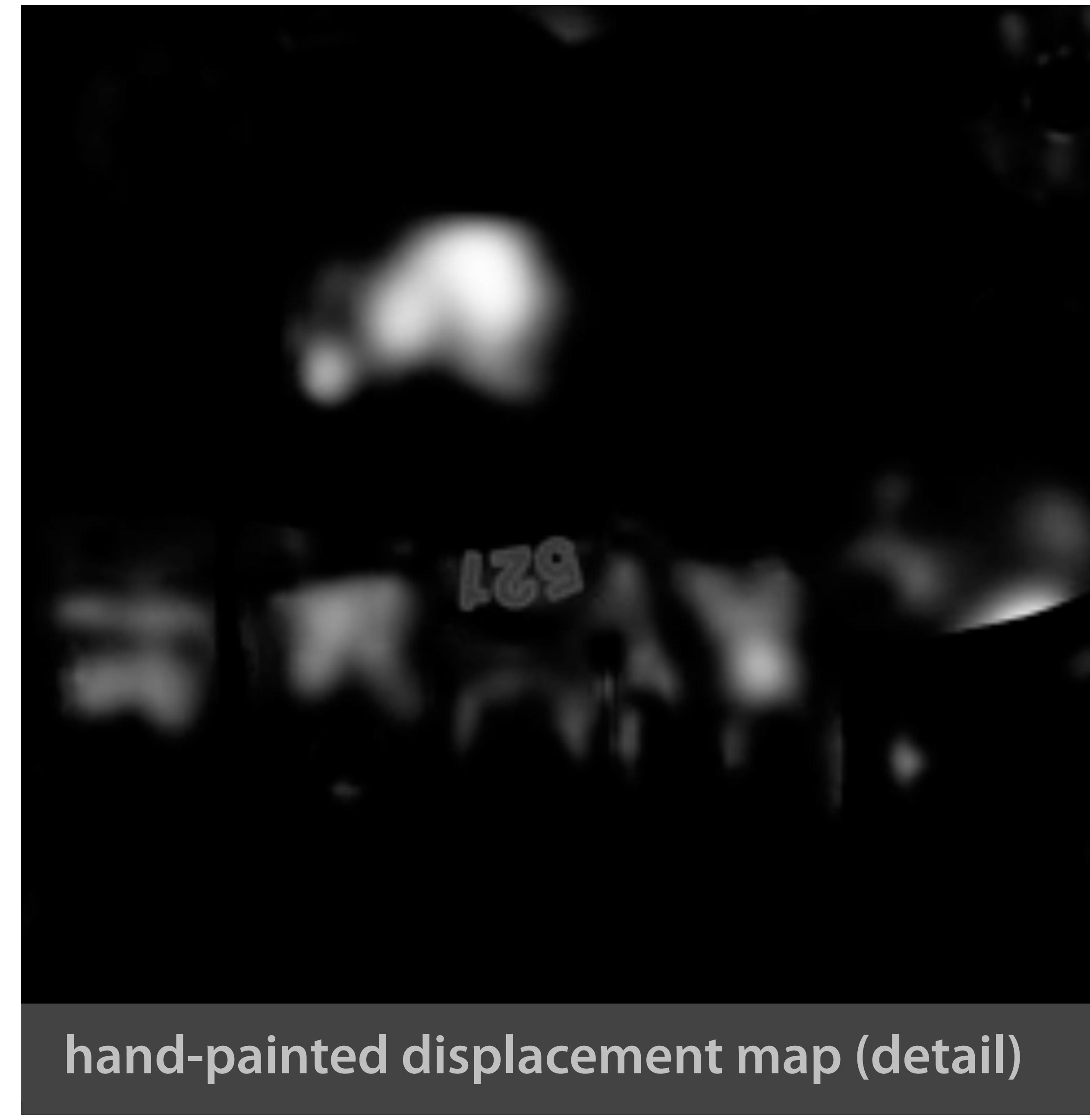
[Emil Persson]

Displacement Mapping

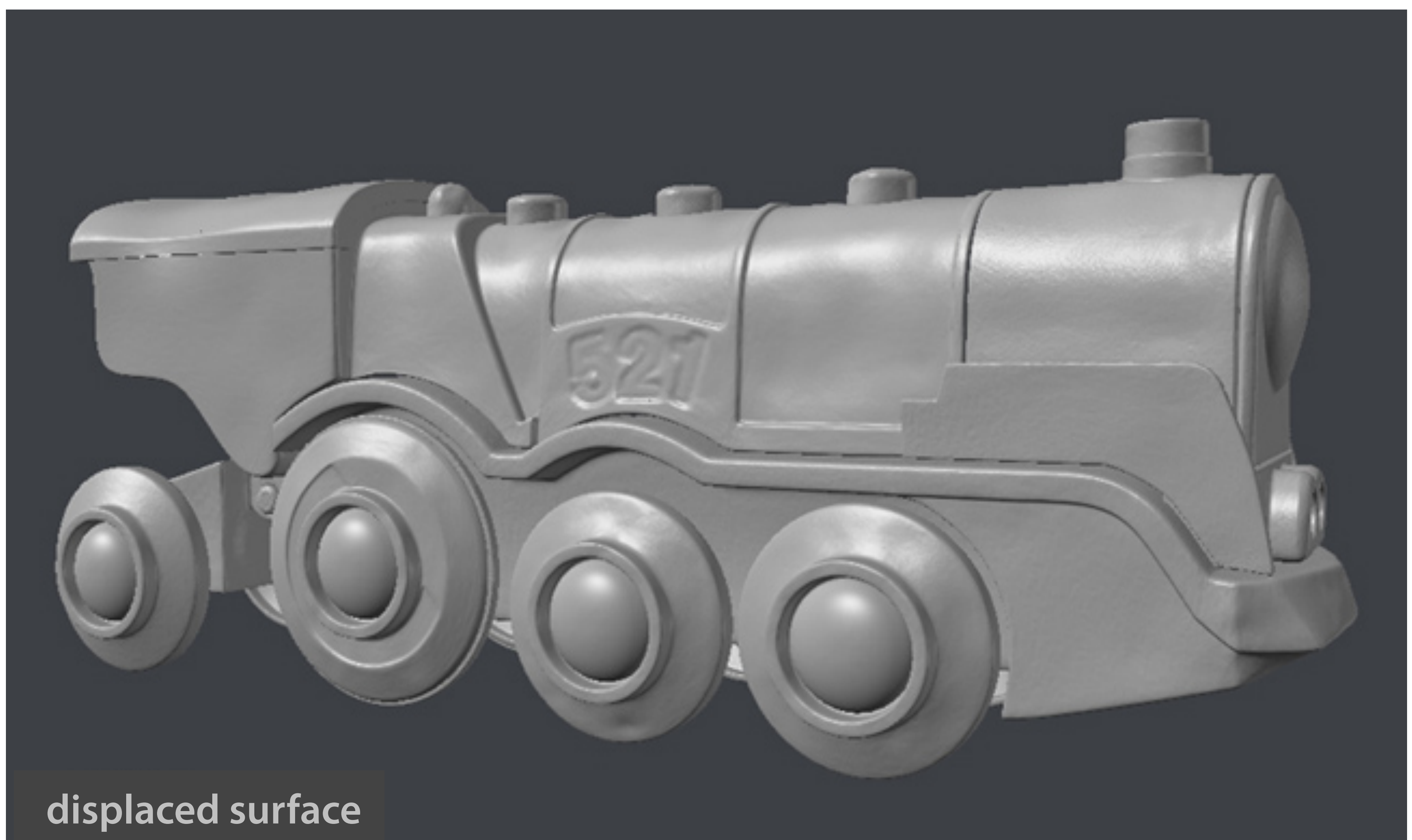
Texture stores perturbation to surface position



base surface



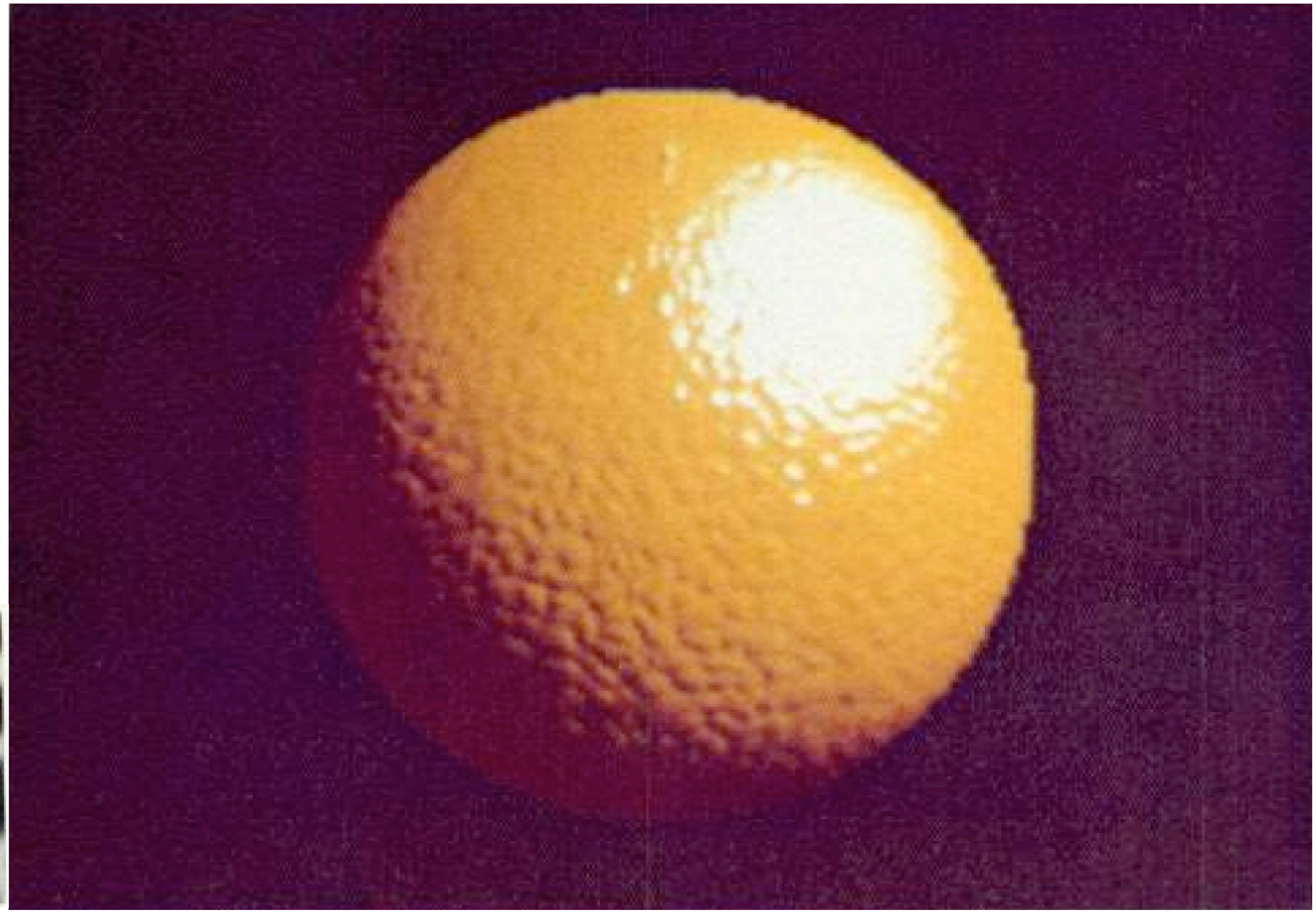
hand-painted displacement map (detail)



displaced surface

Paweł Filip
tolas.wordpress.com

Bump Mapping

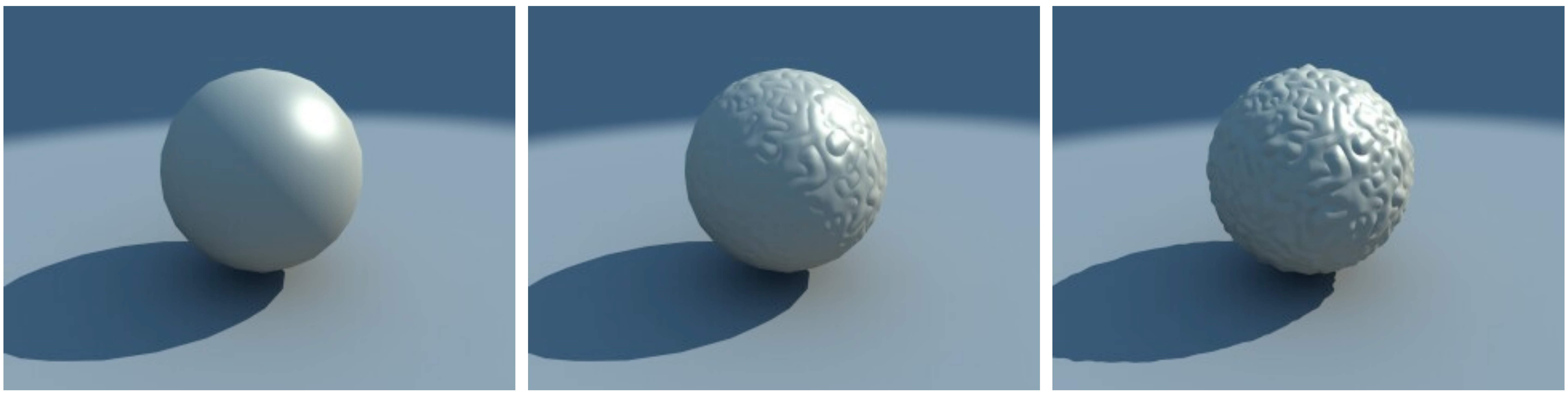


[Blinn 1978]

Texture stores perturbation to surface normal

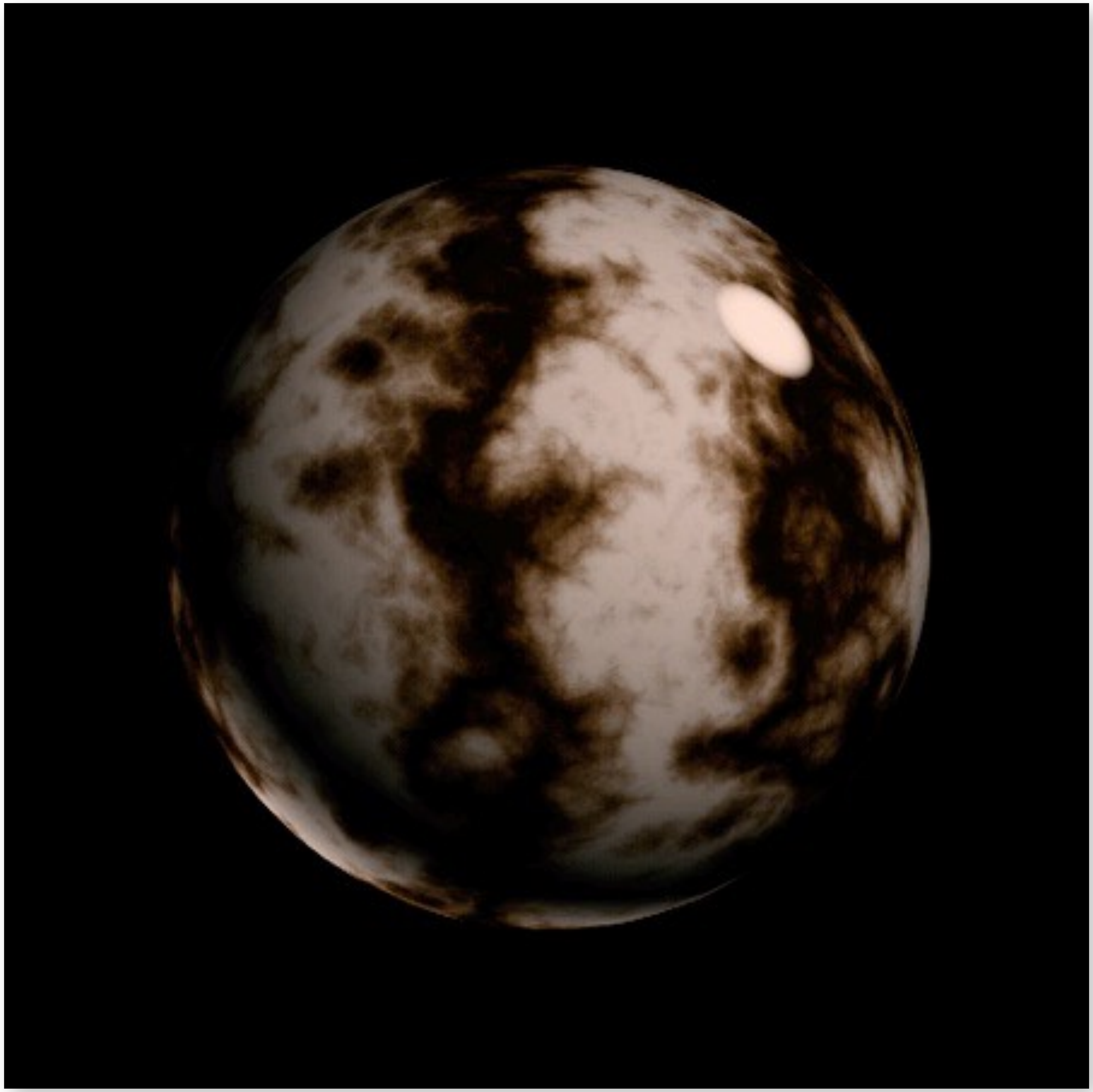
Bump Mapping

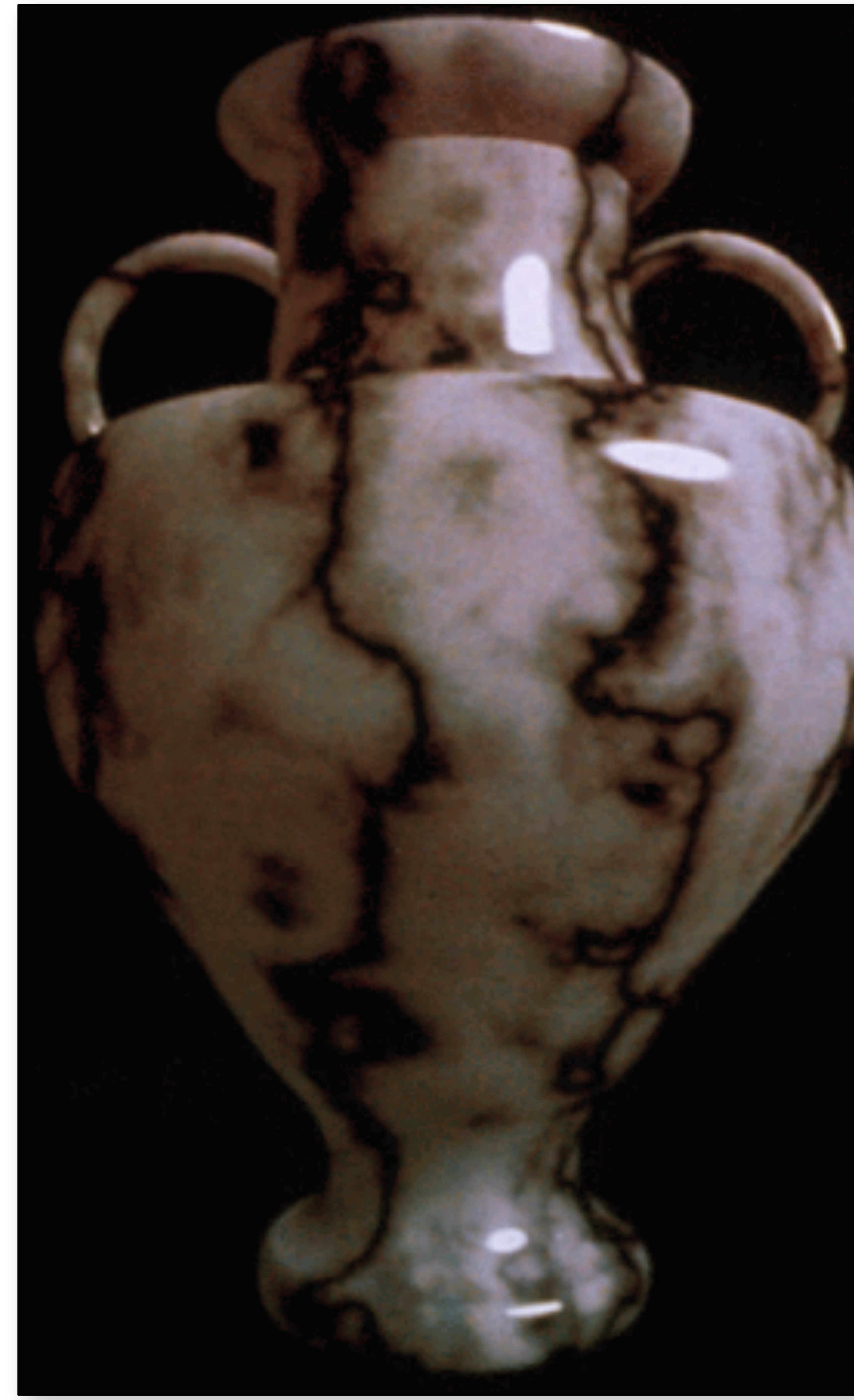
What is missing?



Geometry	Bump mapping	Displacement mapping
	Perturbs normals	Perturbs positions

3D Procedural Noise + Solid Modeling





Perlin noise, Ken Perlin

Provide Precomputed Shading

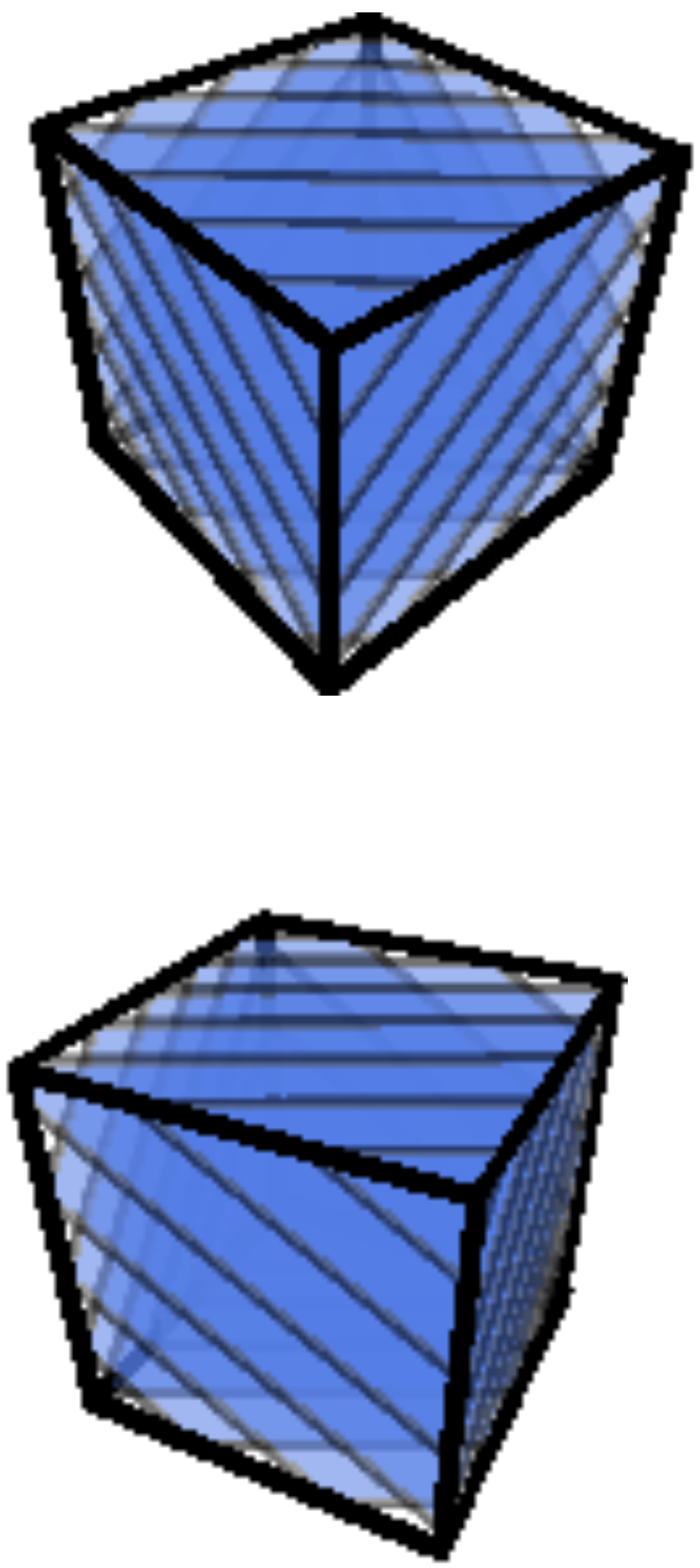
Simple shading

Ambient occlusion texture map

With ambient occlusion

Autodesk

3D Textures and Volume Rendering



Marc Levoy

Things to Remember

Many uses of texturing

- Bring high-resolution data to fragment calculations
- Colors, normals, lighting on sphere, volumetric data, …

How does texturing work?

- Texture coordinate parameterization
- Barycentric interpolation of coordinates
- Texture sampling pattern and frequency
- Mipmaps: texture filtering hierarchy, level calculation, trilinear interpolation
- Anisotropic sampling

Acknowledgments

Thanks to Kayvon Fatahalian, Steve Marschner, Mark Pauly and Angjoo Kanazawa for presentation resources.

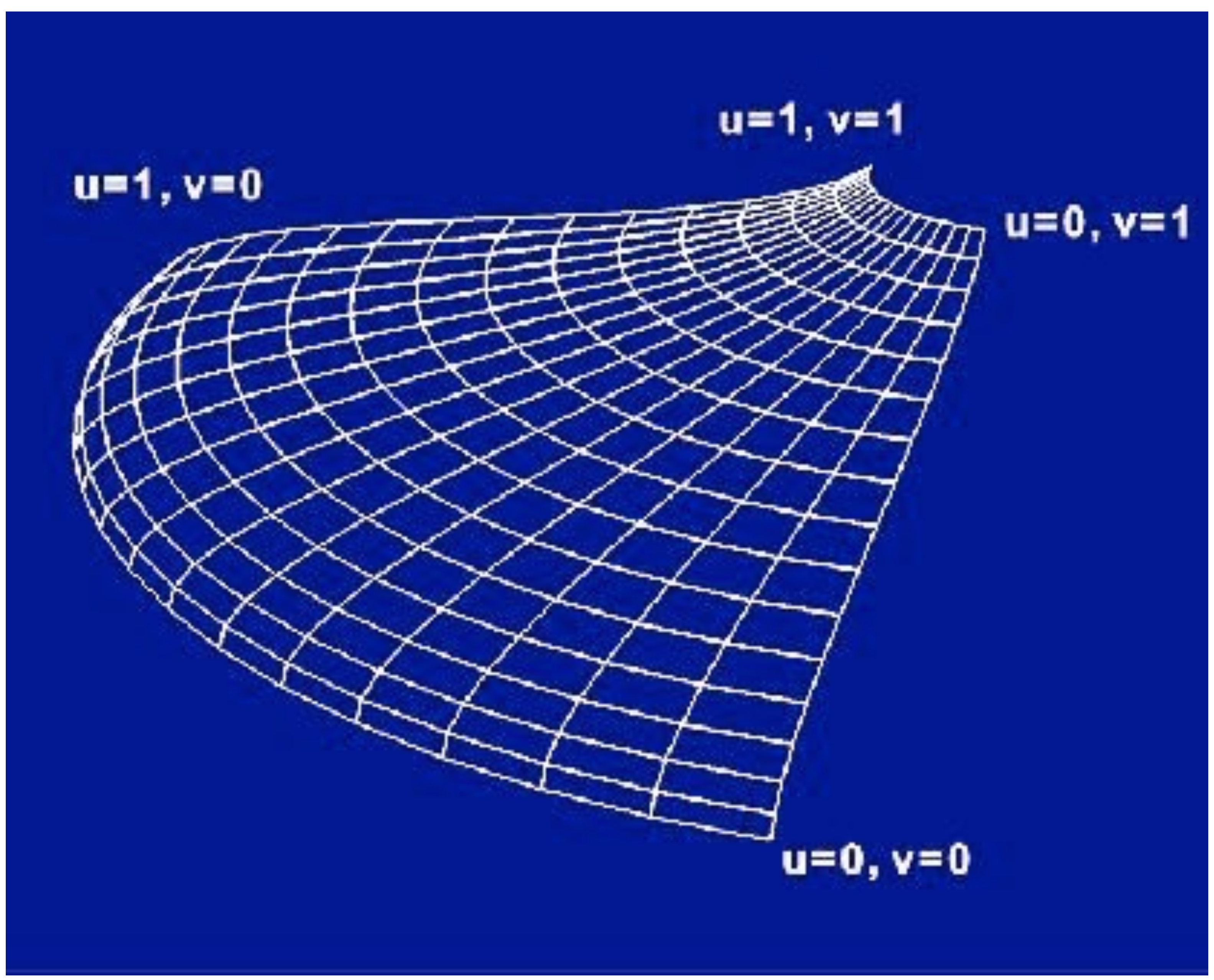
Bonus Slides

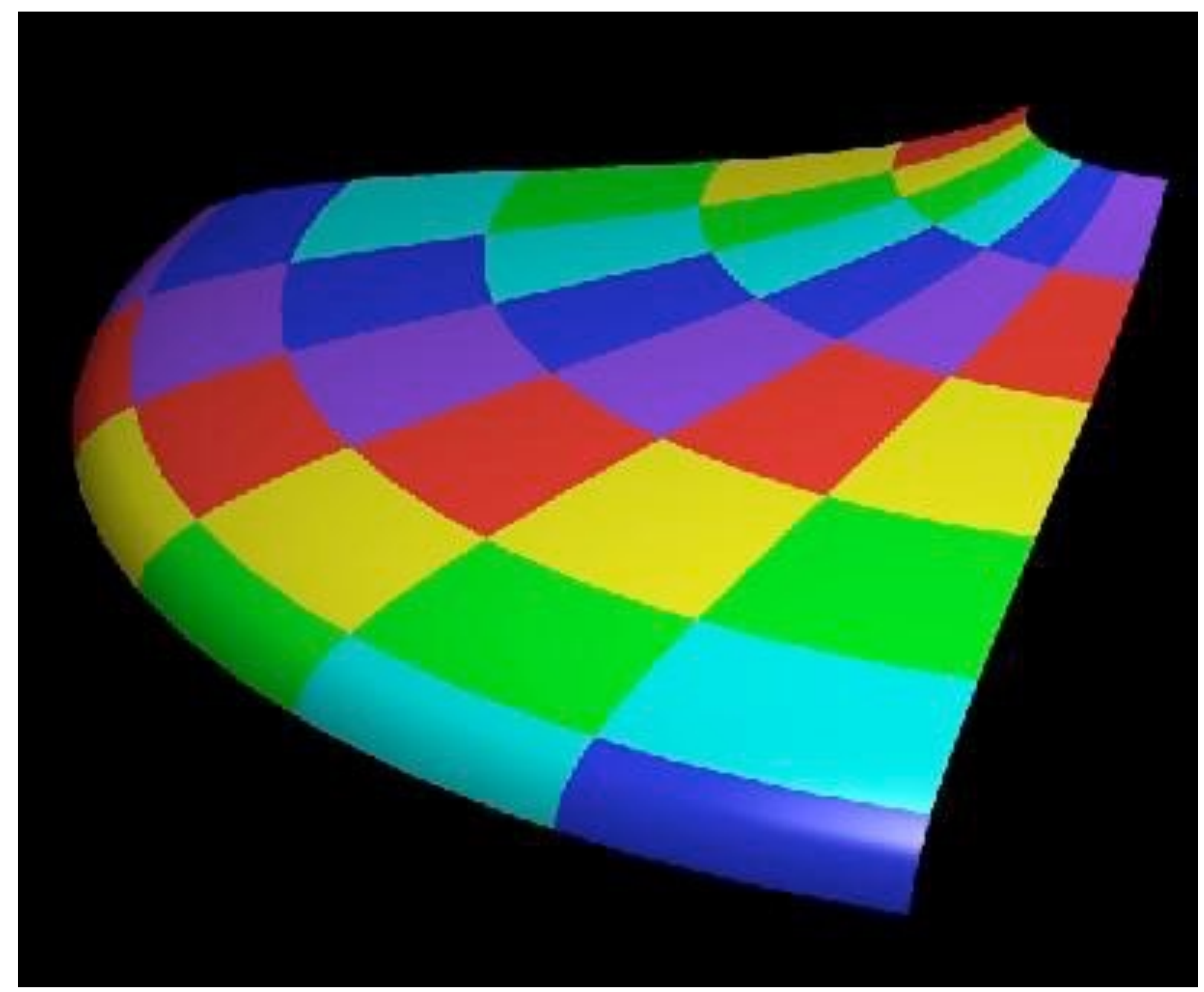
Examples of Texture Coordinate Functions

Examples of Texture Coordinate Functions

A parametric surface (e.g. spline patch)

- Use parameter space coordinates as texture coordinates directly

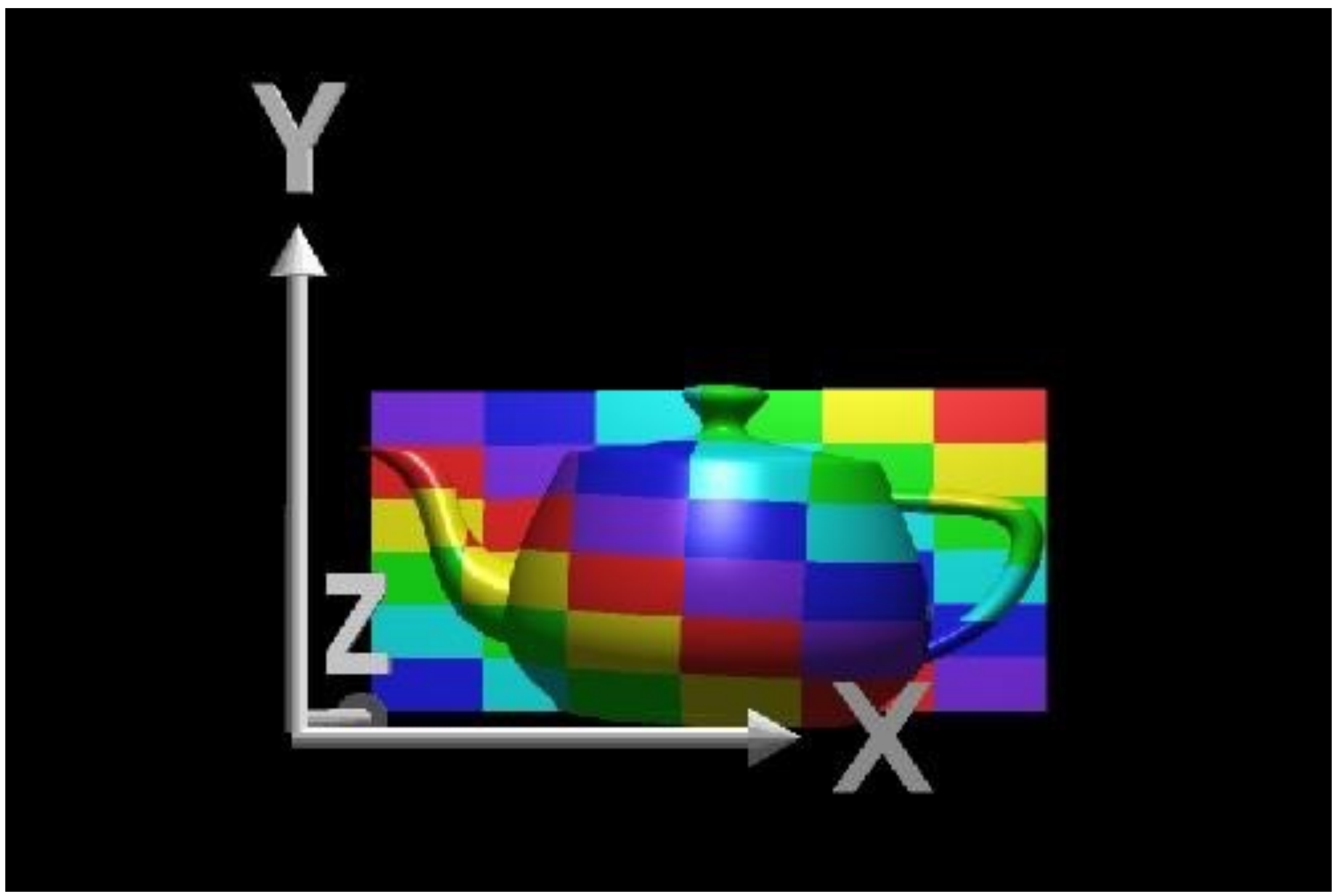


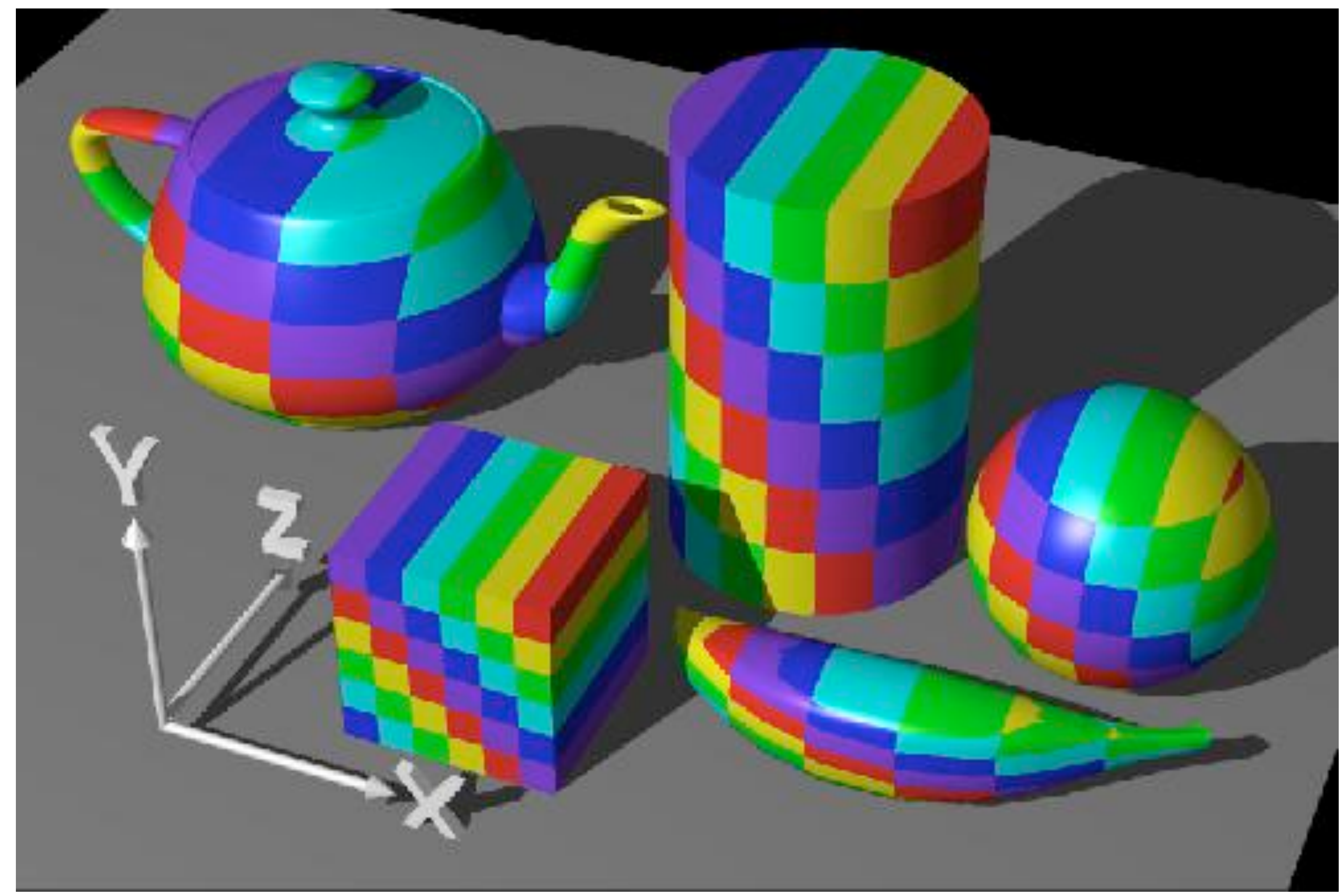


[Wolfe / SG97 Slide set]

Examples of Texture Coordinate Functions

Planar projection



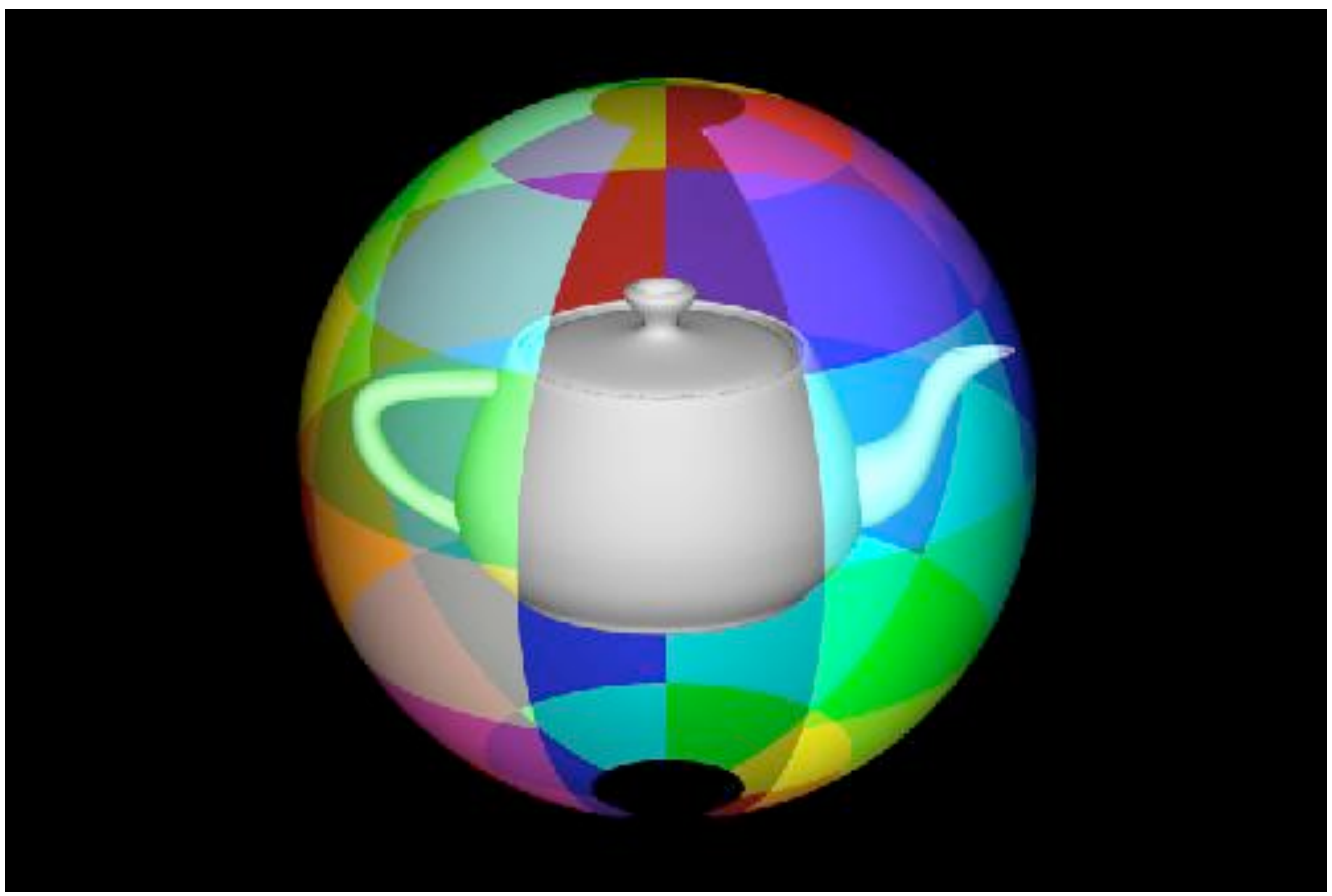


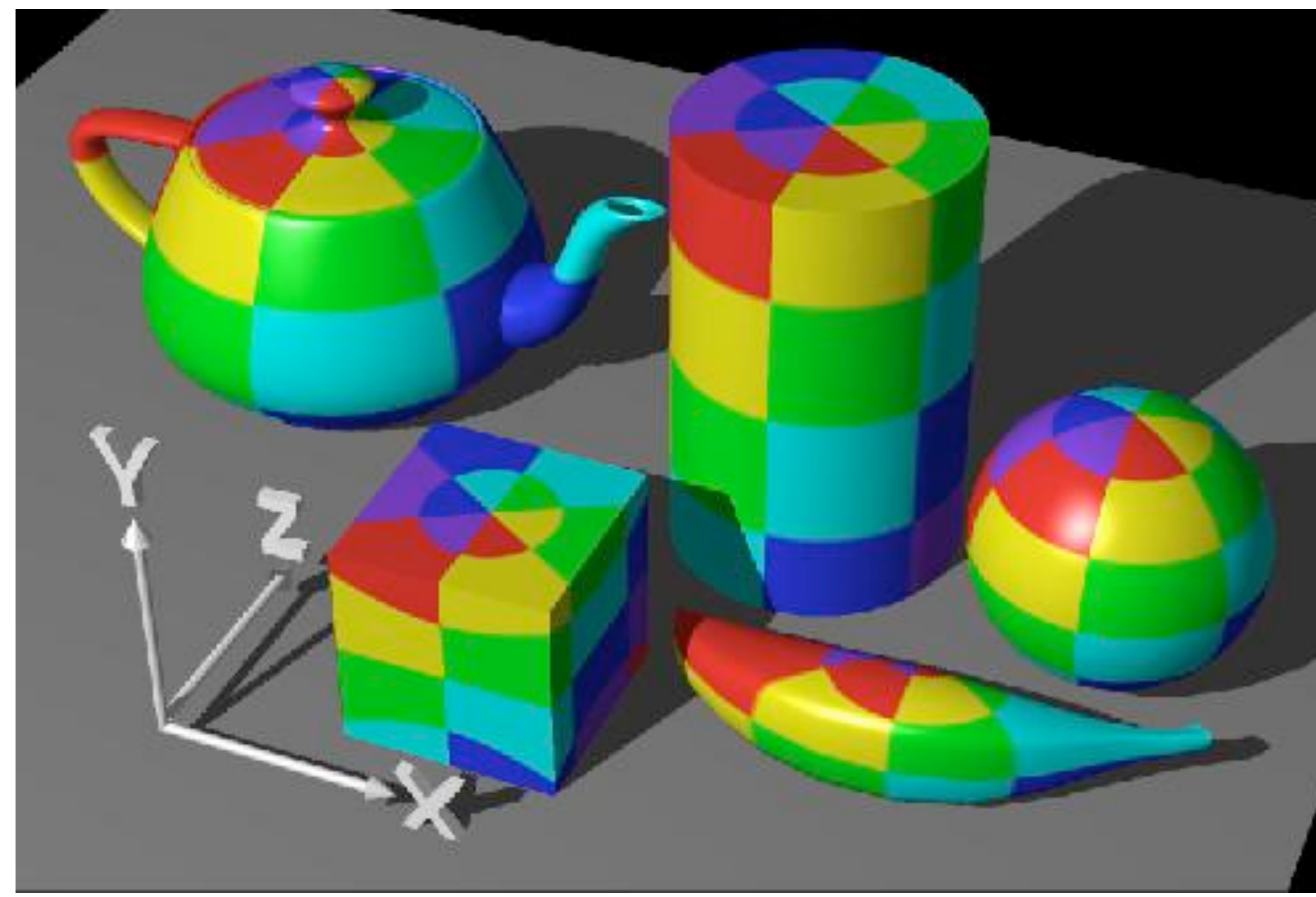
Rosalee Wolfe

http://www.siggraph.org/education/materials/HyperGraph/mapping/r_wolfe/r_wolfe_mapping_1.htm

Examples of Texture Coordinate Functions

Spherical projection



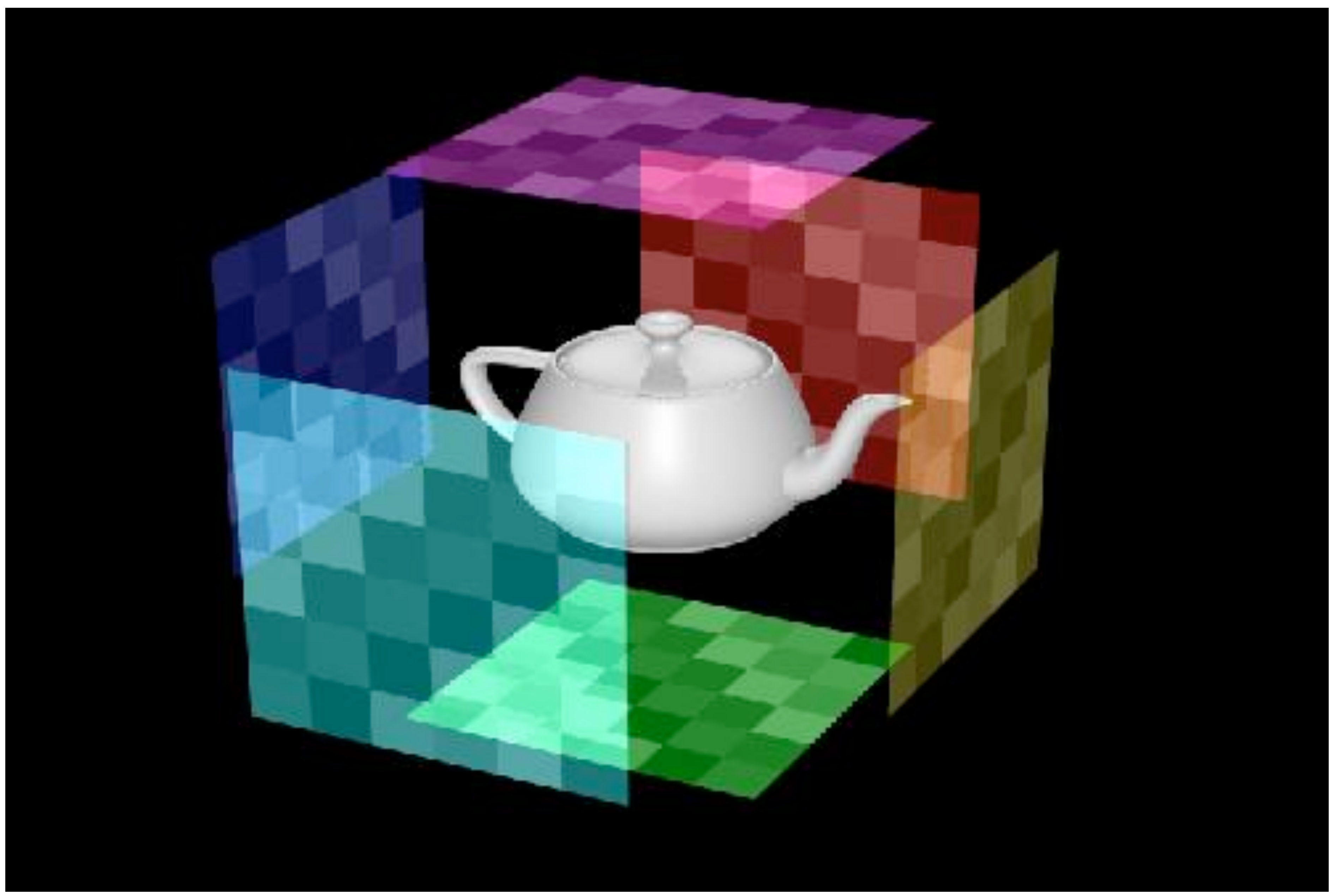


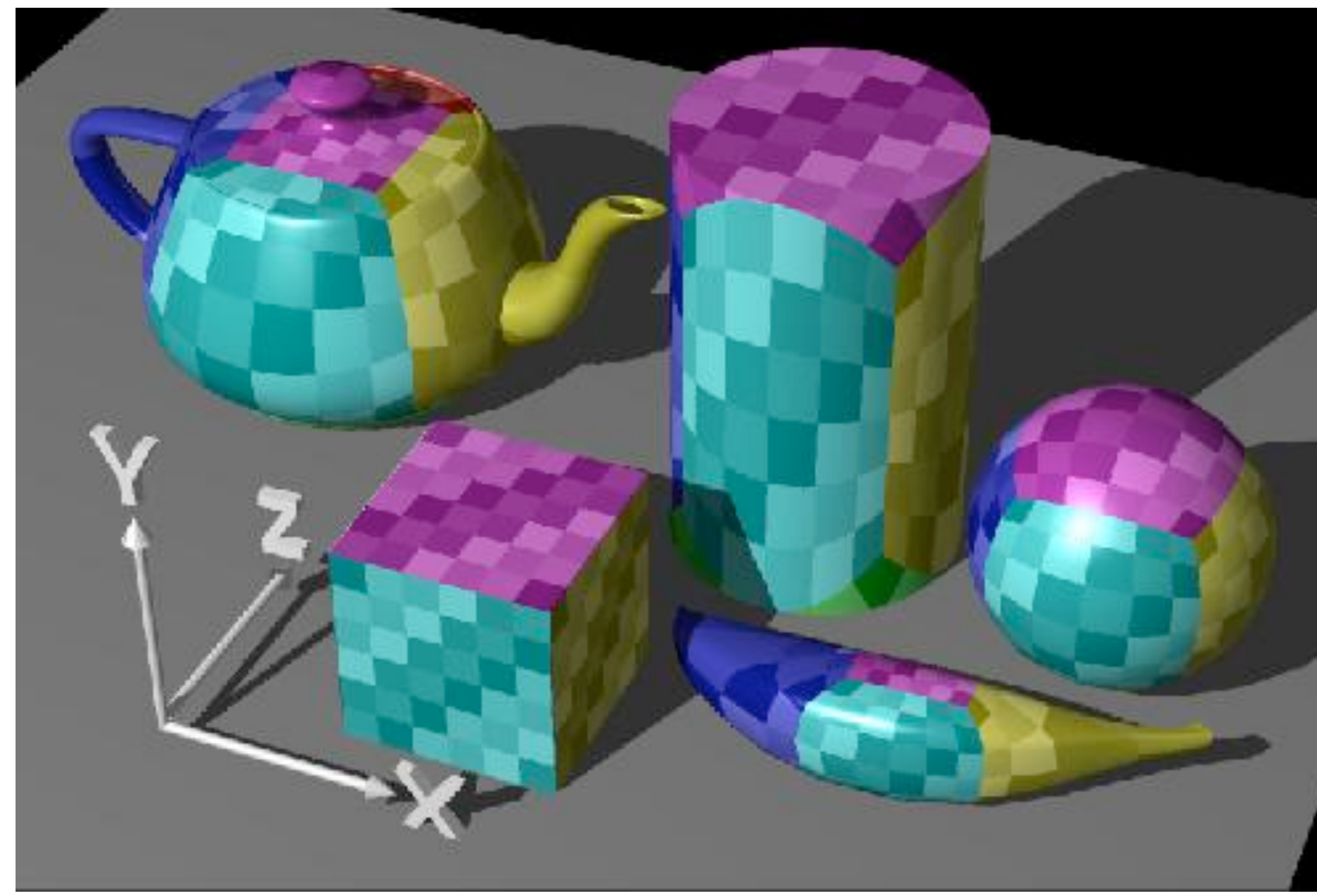
Rosalee Wolfe

http://www.siggraph.org/education/materials/HyperGraph/mapping/r_wolfe/r_wolfe_mapping_1.htm

Examples of Texture Coordinate Functions

Cube map projection



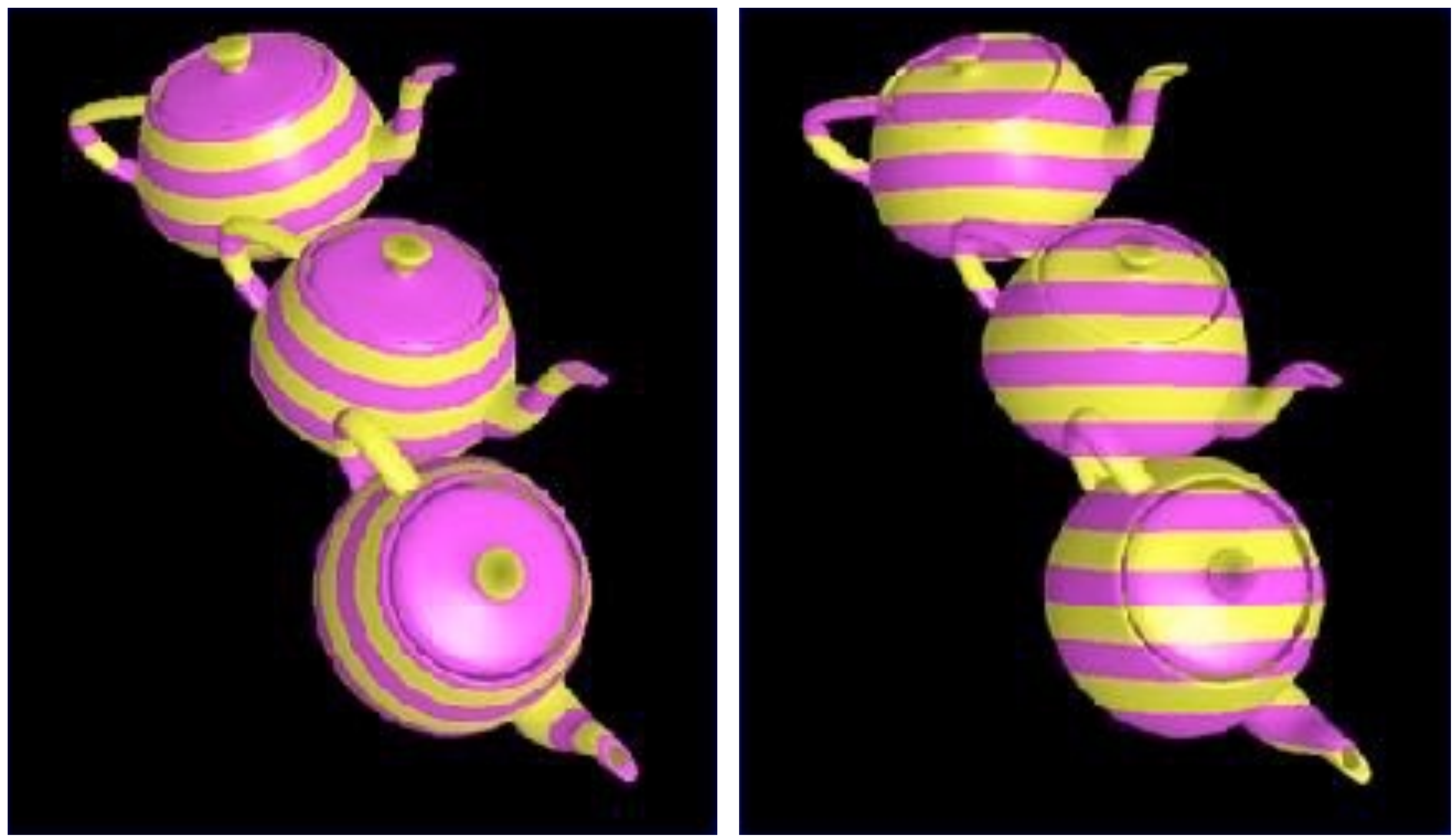


Rosalee Wolfe

http://www.siggraph.org/education/materials/HyperGraph/mapping/r_wolfe/r_wolfe_mapping_1.htm

Examples of Texture Coordinate Functions

Function of object or world coordinates?

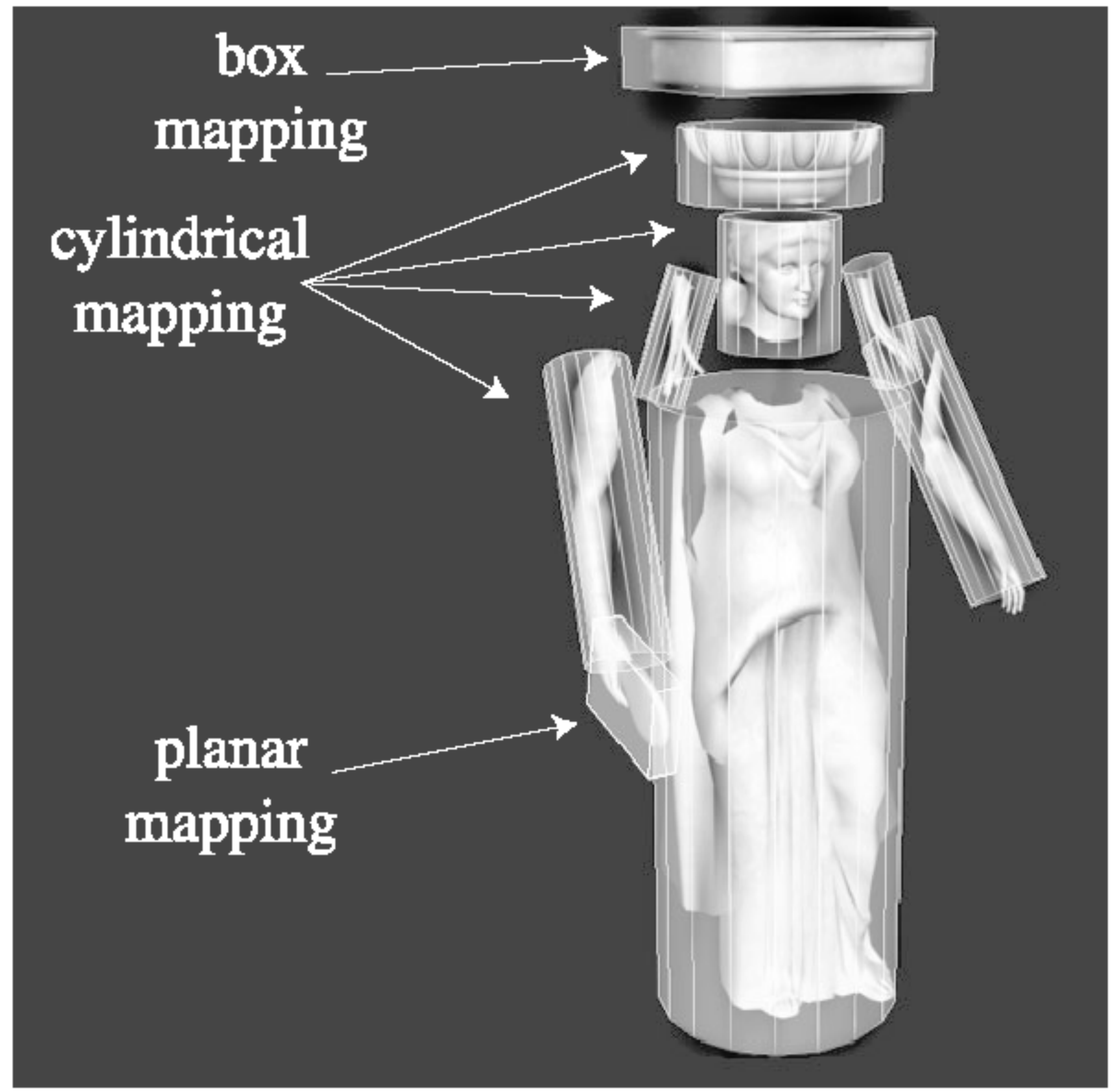


Rosalee Wolfe

http://www.siggraph.org/education/materials/HyperGraph/mapping/r_wolfe/r_wolfe_mapping_1.htm

Examples of Texture Coordinate Functions

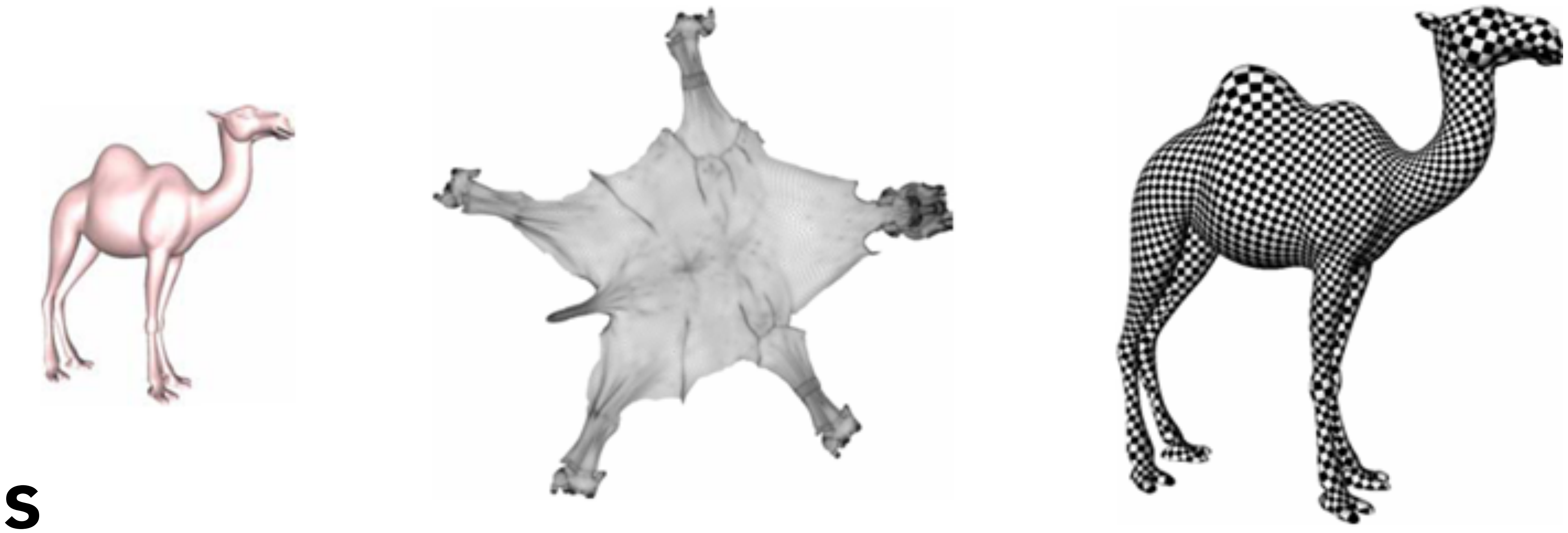
Complex surfaces: project parts to parametric surfaces



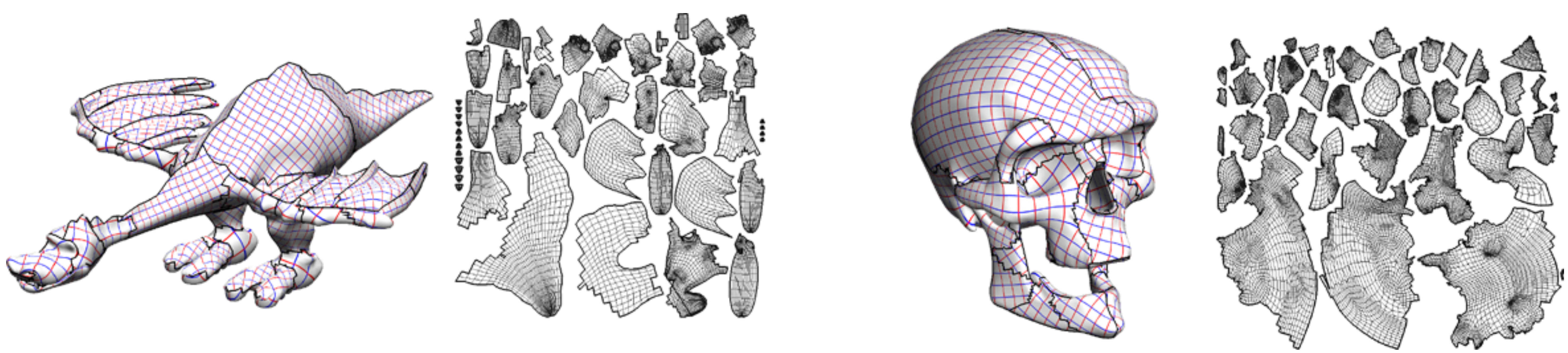
[Tito Pagan]

Creating Good Surface Coordinates is Hard

Finding cuts



Texture atlases



Levy et al: Least Squares Conformal Maps
for Automatic Texture Atlas Generation, SIGGRAPH, 2002