Geometry and Spline

CS 184

Summer 2020

Announcement

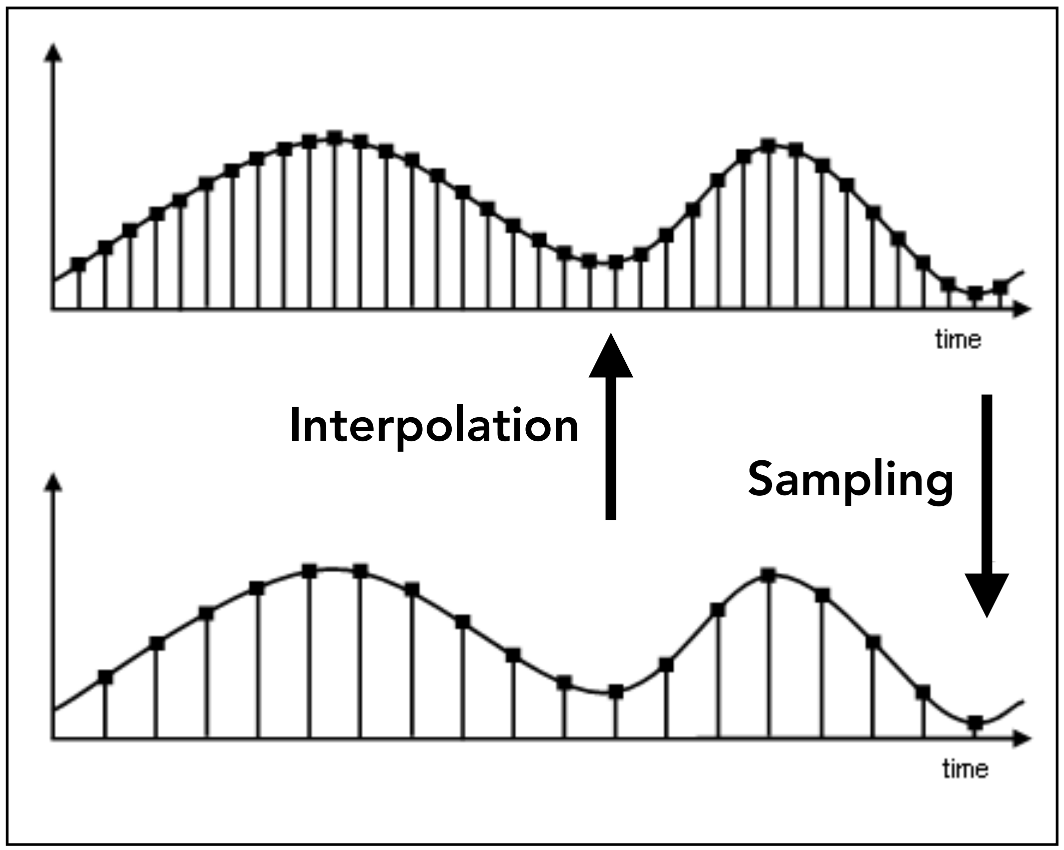
- Project party moved to Thursday (today)!
- Project 1 due Friday 11:59 PDT
- Project 2 released Friday

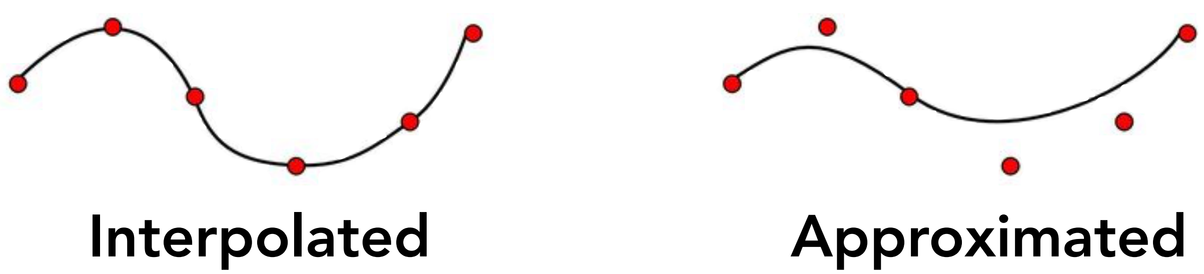
Agenda

- Different types of interpolation
  - Applications
- Polynomials for linear interpolation and their matrix intuition
- Bézier
  - Algebraic formula
  - de Casteljau recursive algorithm
- Demo

Sampling and Interpolation

And approximation

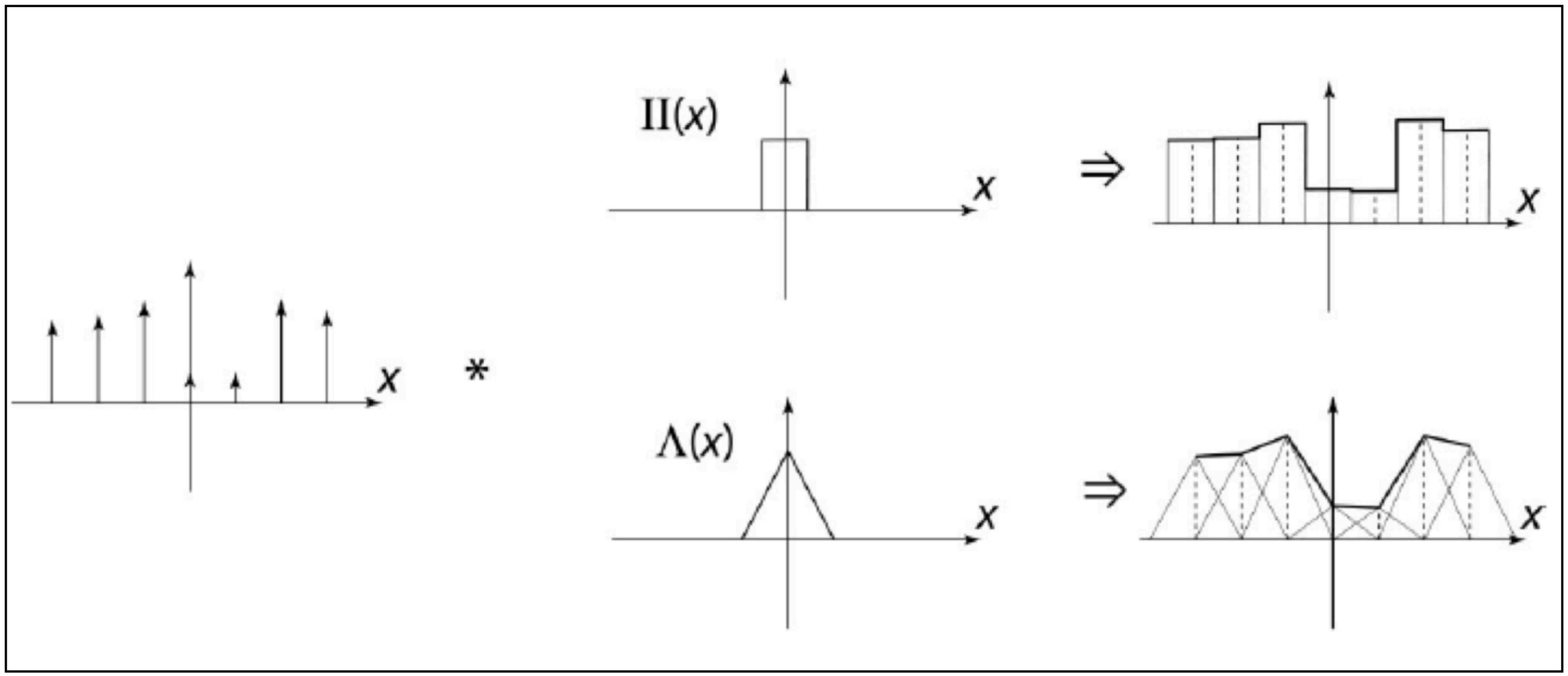




What should we do to make the approximated curve closer to the control points?

Interpolation

Interpolation and kernel

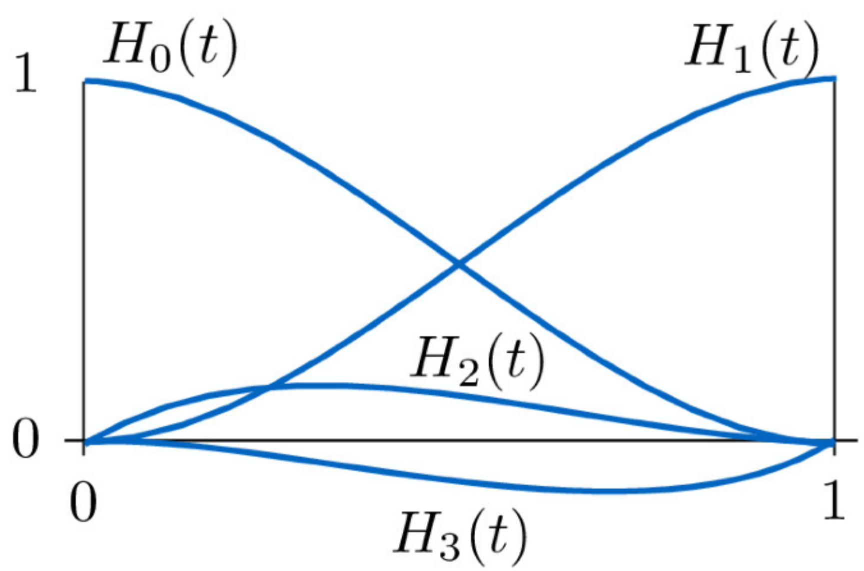


Nearest neighbor

?

Basis Functions

Hermite Basis Functions

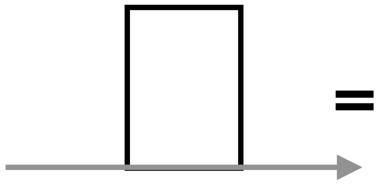


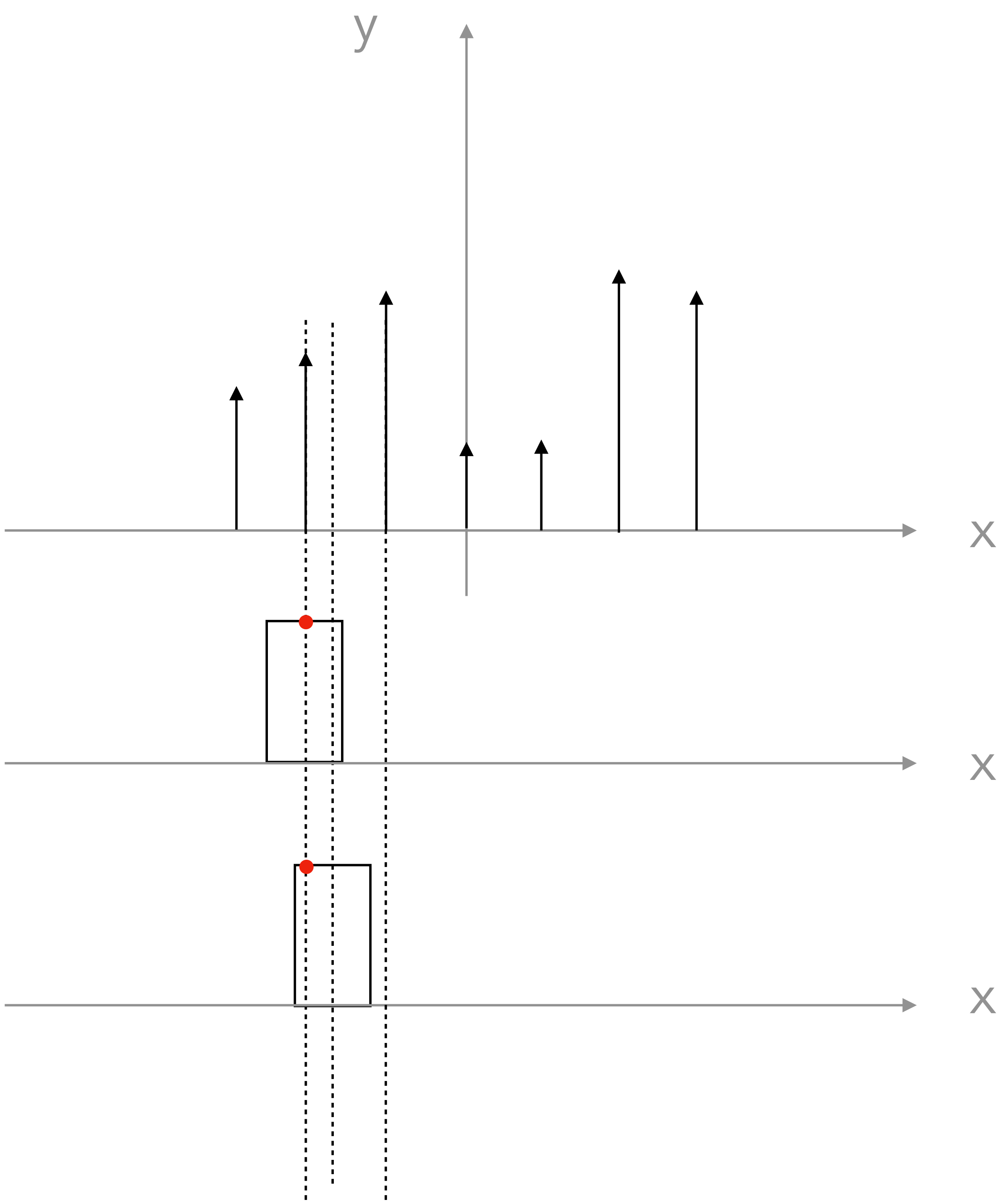
$$H_0(t) = 2t^3 - 3t^2 + 1$$
$$H_1(t) = -2t^3 + 3t^2$$
$$H_2(t) = t^3 - 2t^2 + t$$
$$H_3(t) = t^3 - t^2$$

} A set of cubic polynomial basis

Every cubic Hermite spline is a linear combination (blend) of these 4 functions

NN Interpolation — Square basis

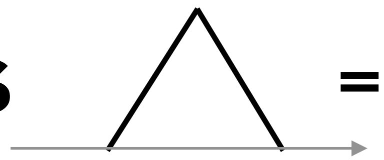




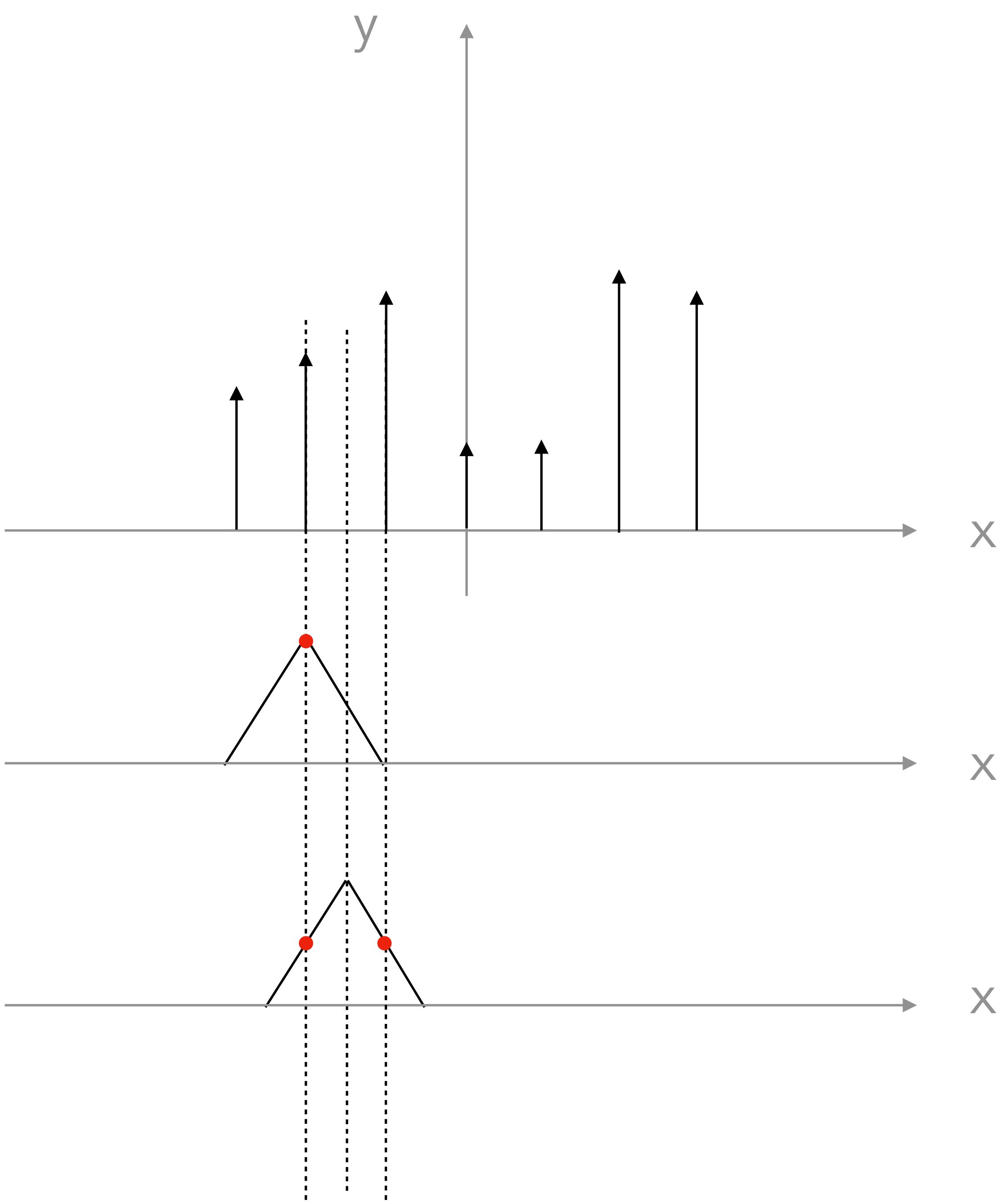
Evaluate on existing sample location

Evaluate on new sample location

Linear Interpolation — Triangle basis



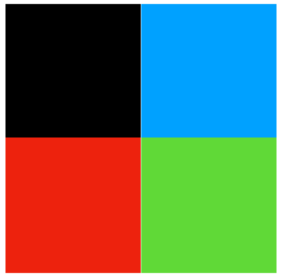
= "tent filter"

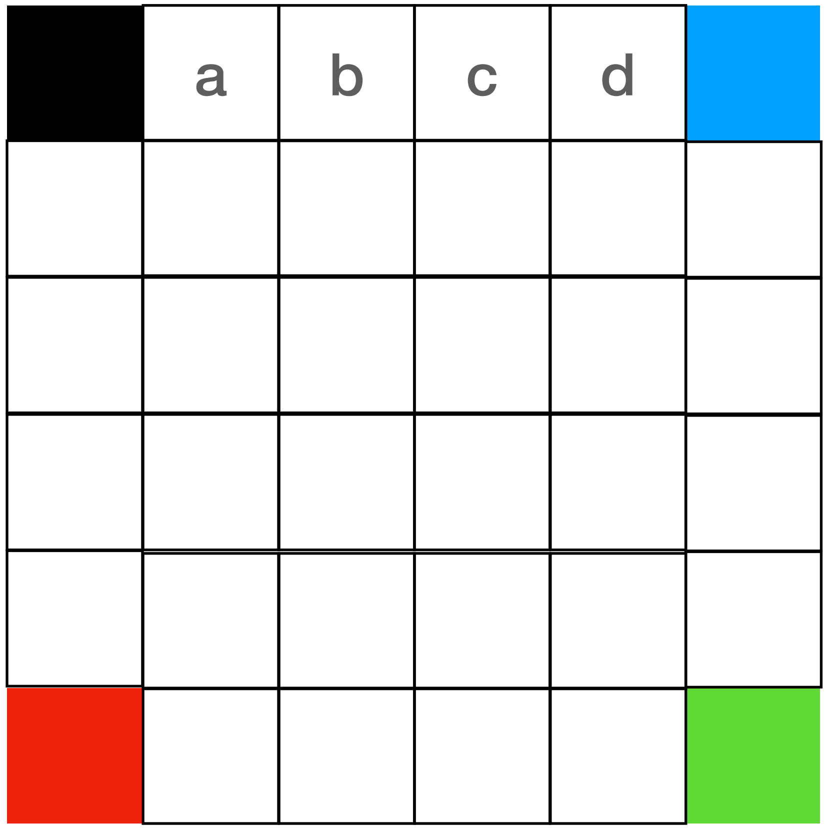


Evaluate on existing sample location

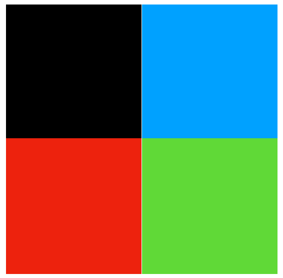
Evaluate on new sample location

Linear Interpolation in 2D



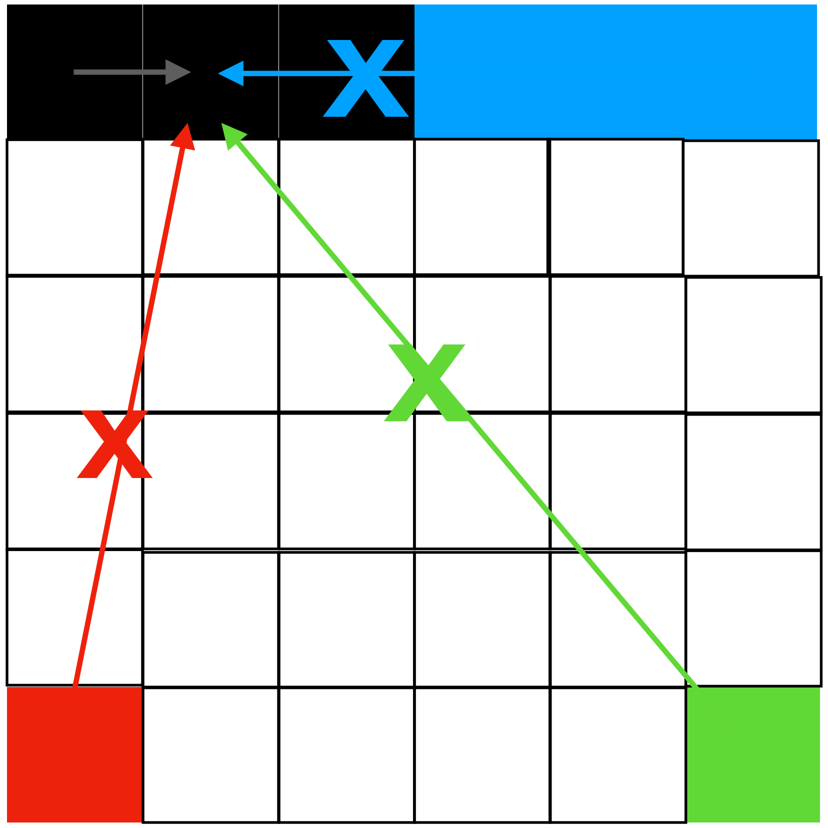


Linear Interpolation in 2D

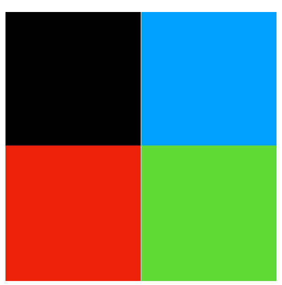


Nearest neighbor

Only one contributes

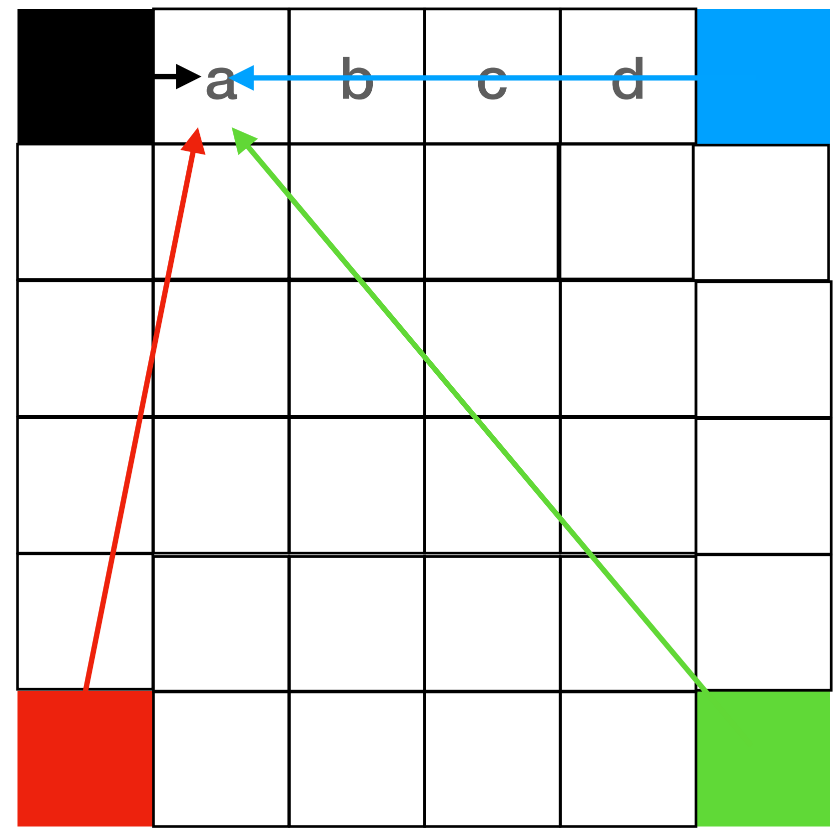


Linear Interpolation in 2D



Upsample
3X

Bilinear
All four contribute



Application: Interpolation for digital zoom

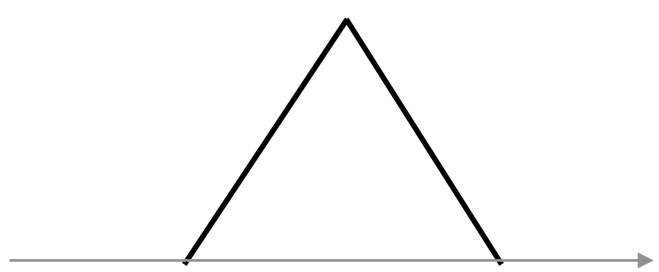
4X

More complicated / Smoother Splines

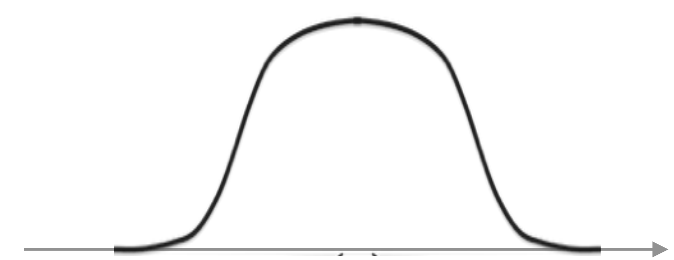
Bicubic or higher order polynomials

General form of cubic polynomial:

$$f(x) = ax^3 + bx^2 + cx + d$$



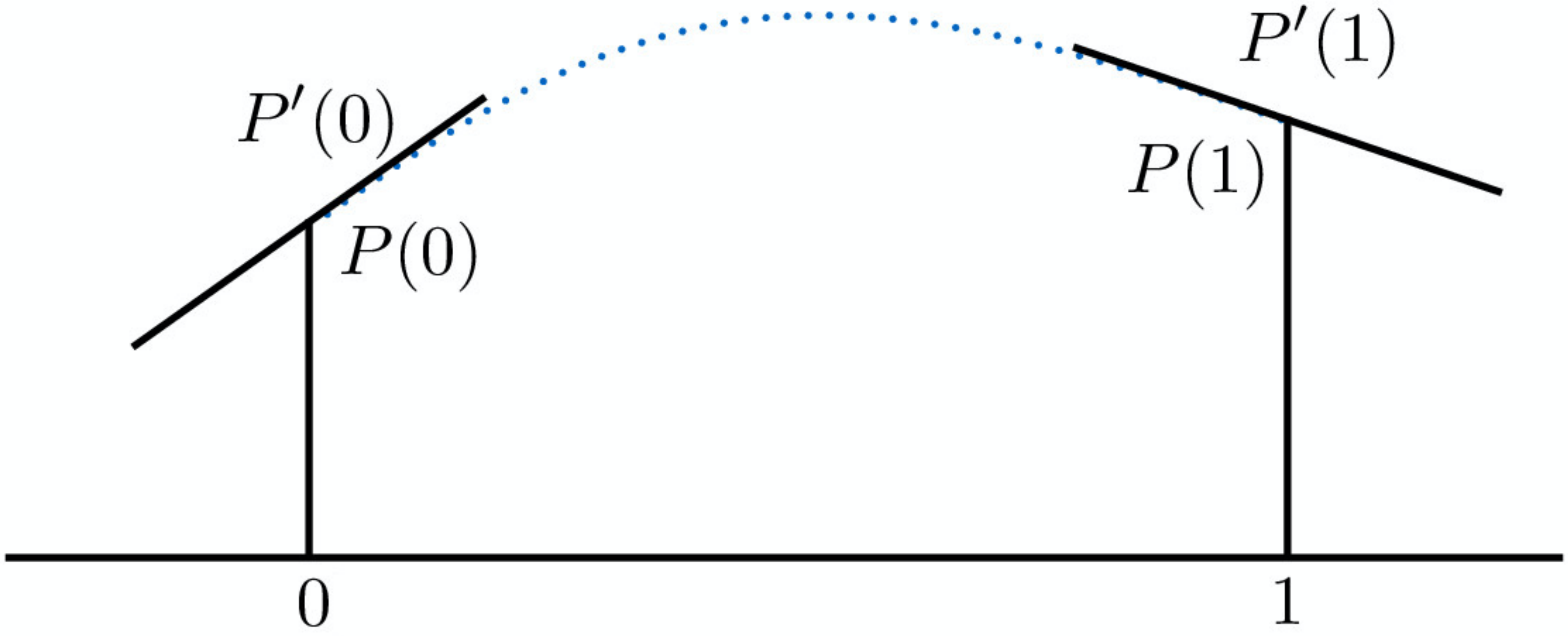
Two neighboring points are needed



? points are needed

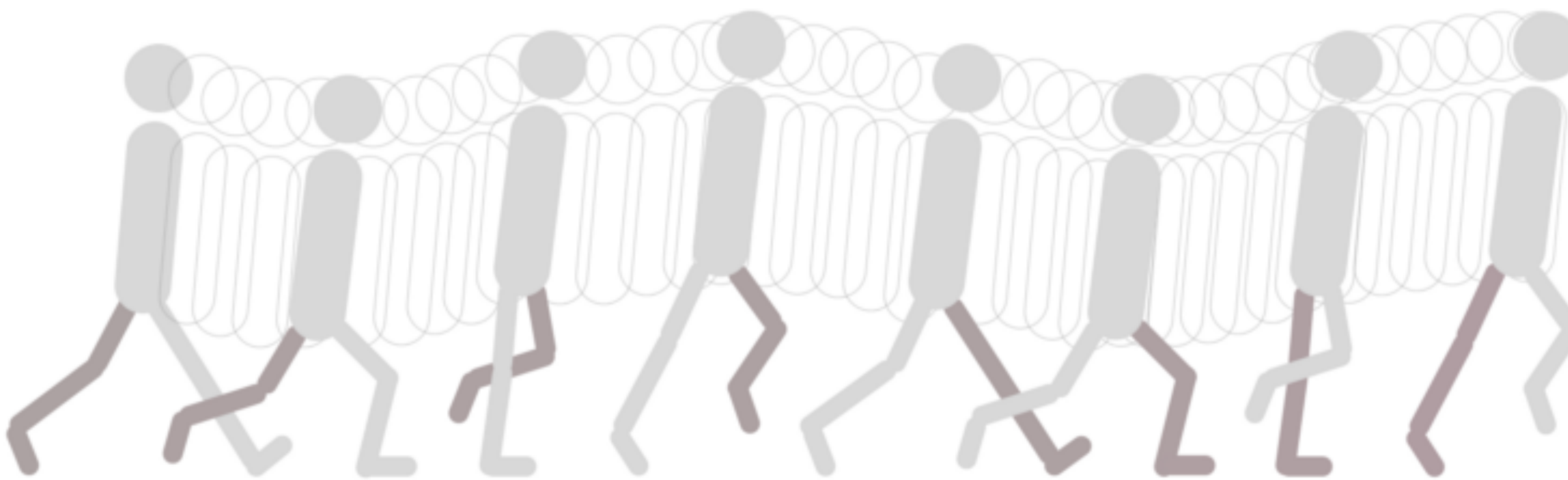
? points are needed for a Nth order polynomial

Cubic spline



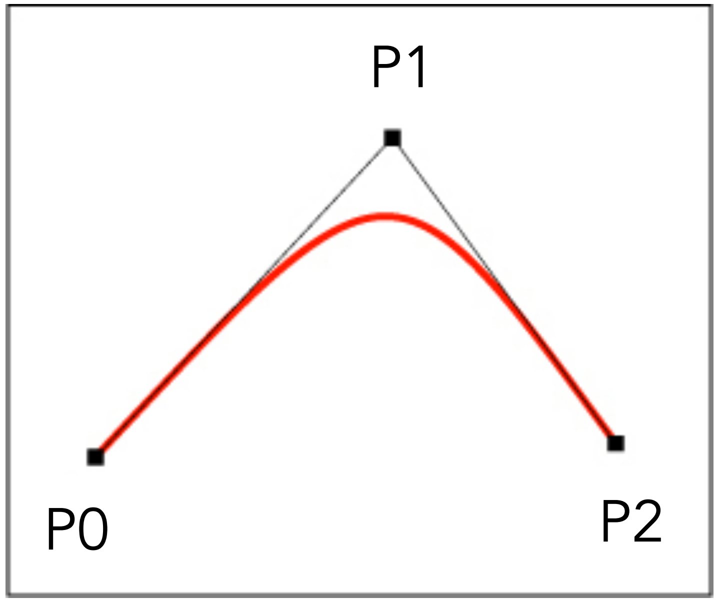
Inputs: values and derivatives at endpoints

Application: Animation Interpolation



Bézier Curve

$$f(t) = P(1) \cdot (1-t)^2 + P(2) \cdot 2t(1-t) + P(3) \cdot t^2$$



P(t) is a weighted combination of the 3 control points with weights of the three basis function

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The control points are the "coordinates" of the curve in the basis functions — like 2D points (x, y)

Bézier Curve

All about linear interpolation

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Matrix

$$f(t) = P(1)\cdot(1-t)^2 + P(2)\cdot 2t(1-t) + P(3)\cdot t^2$$

$$P(1)\cdot(1-t)^2 \qquad P(2)\cdot 2t(1-t) \qquad P(3)\cdot t^2$$

$$1t^0 - 2t^1 + t^2 \qquad 0t^0 + 2t^1 - 2t^2 \qquad 0t^0 + 0t^1 + t^2$$

$$f(t) = \begin{bmatrix} t^0 & t^1 & t^2 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ -2 & 2 & 0 \\ 1 & -2 & 1 \end{bmatrix} \begin{bmatrix} P(1) \\ P(2) \\ P(3) \end{bmatrix}$$

Bézier Matrix

What if we swap t-matrix and the p-matrix?

Bézier Matrix — General Form

Algebraic formulation

Bernstein polynomials:

$$= \binom{n}{i} t^{n-i}(1-t)^i$$

$$= \binom{n}{i} t^{n-i} \sum_{k=0}^{i} \binom{i}{k} 1^k(-t)^{i-k}$$

$$= \binom{n}{i} t^{n-i} \sum_{k=0}^{i} \binom{i}{k} (-1)^{i-k}(-t)^{i-k}$$

$$= \sum_{k=0}^{i} \binom{n}{i} \binom{i}{k} (-1)^{i-k} t^{n-k}$$

Separate out "t"

Demo