COLORS 11

CS 184: FOUNDATIONS OF COMPUTER GRAPHICS

1 Metamer Magic

1. What is a metamer? Why are metamers useful?

2. Suppose you have a spectrum, $\boldsymbol{S}_{\text{ref}}$, that is metameric to another spectrum, $\boldsymbol{S}$, to an observer whose spectral sensitivities are represented by

$$\boldsymbol{C} = \begin{bmatrix} \text{——} & r_S & \text{——} \\ \text{——} & r_M & \text{——} \\ \text{——} & r_L & \text{——} \end{bmatrix}$$

   Write an equation in terms of $\boldsymbol{S}_{\text{ref}}$, $\boldsymbol{S}$, and $\boldsymbol{C}$ that represents this metameric relationship.

3. Show that $\boldsymbol{\Delta} = \boldsymbol{S} - \boldsymbol{S}_{\text{ref}}$ is in the null space of $\boldsymbol{C}$. That is, $\boldsymbol{C}\boldsymbol{\Delta} = 0$. What does this result imply about the differences between metameric spectra?

4. You have two matte (diffuse) prints of the same digital photo. At first, you look at the two photos under the sunlight, which has a spectrum $I(\lambda)$. The colors in the prints look identical. However, under the fluorescent lighting of your kitchen, with spectrum $F(\lambda)$, you notice that some of the colors in the two photos now look quite different. What is happening?

2 Hue Knew?

Answer true or false to the following statements. Provide an explanation for your answer.

1. Gamuts are device-dependent.

2. In the CIE chromaticity diagram, the colors are least saturated at the edge of the plot, and become more saturated towards the centroid at the plot.

3. The gamut of a display that uses combinations of three distinct colors (three primaries) is visualizable as a triangle in the CIE chromaticity diagram.

4. The CIE chromaticity diagram includes the color black.

5. In the CIELAB color space, equal distances in the color space correspond to roughly equal perceived differences in color.

6. In Maxwell's color matching experiment, the color appearance of any input light could be matched by adjusting the brightness of three primary lights.

3 Color Blindness Virus!

A terrible virus spreads rapidly through the world. It affects the M cone cells in our retinas, turning them into L cone cells. Now all humans only have S and L cones! You realize there's a silver lining: it might be possible to reproduce the appearance of "full" (which is now 2D) color using just the red and blue pixels on your phone.

Throughout this problem, assume that the spectral response curves of the remaining human cone cells, as functions of wavelength, are given by $r_S(\lambda)$ and $r_L(\lambda)$.

1. First, consider a target light with spectral power distribution (SPD) $I(\lambda)$ that we wish to reproduce. Write down expressions for the scalar response of each cone cell when exposed to $I(\lambda)$.

   $S_{\text{target}} =$

   $L_{\text{target}} =$

2. Now consider the red and blue pixels on your phone, with SPDs given by functions $s_R(\lambda)$ and $s_B(\lambda)$. If we set the brightness of these pixels by scalar values R and B, respectively, write down the scalar response from each cone cell type when exposed to the resulting light.

   $S_{\text{display}} =$

   $L_{\text{display}} =$

3. Note that we can re-write the result from part (ii) in matrix form:

$$\begin{bmatrix} S_{\text{display}} \\ L_{\text{display}} \end{bmatrix} = M \begin{bmatrix} R \\ B \end{bmatrix} = \begin{bmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{bmatrix} \begin{bmatrix} R \\ B \end{bmatrix} \tag{1}$$

   Write down expressions for the elements of this matrix:

   $m_{11} =$ $\qquad$ $m_{12} =$

   $m_{21} =$ $\qquad$ $m_{22} =$

4. Finally, to complete the color matching procedure for the color-blind humans using only red and blue pixels, determine how to choose values for R and B so that the display light matches the perceived color of the input SPD $I(\lambda)$. Assume that $I(\lambda)$ is within the gamut of the red and blue pixels.

   Write your answer as a one-line matrix expression for R and B. You may use any variables defined in previous parts of this question and standard matrix operations (e.g., transpose, inverse).

   Hint: Match the target and display responses!

   $R =$

   $B =$