

2 Tying Shoelaces

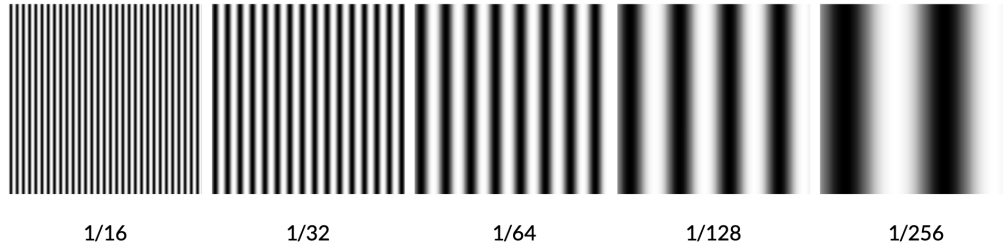
The cross product between two vectors, \mathbf{a} and \mathbf{b} , in three-dimensional space is given by:

$$\mathbf{a} \times \mathbf{b} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

1. Given two vectors, $\mathbf{a} = \begin{pmatrix} 10 \\ 7 \\ 0 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} -2 \\ -11 \\ 0 \end{pmatrix}$, calculate $\mathbf{a} \times \mathbf{b}$. Next, calculate $\mathbf{b} \times \mathbf{a}$.
2. Draw the triangle given by points $(2, -1)$, $(12, 6)$, $(10, -5)$. What is the *winding order* of the triangle? In other words, are the points given in clockwise or counter-clockwise order?
3. What are the three vectors defined by the edges of this triangle? Assume the triangle is lying in the xy -plane. Select two of the three vectors. Calculate their cross product.
4. In general, given triangle vertices $\mathbf{p}_0, \mathbf{p}_1, \mathbf{p}_2$, how can winding order be determined?

3 Nyquist Theorem

1. We have five examples of spatial frequencies in cycles per pixel (given by the number below the image). What frequency should each image be sampled at to avoid aliasing?



2. We have two wheels that rotate at different speeds with different numbers of spokes that we want to record with our camera.
- **Wheel A** has 4 spokes and rotates at a rate of 6 rotations per second.
 - **Wheel B** has 6 spokes and rotates at a rate of 5 rotations per second.
- What frame rate (in frames per second) would our camera need to avoid aliasing effects?

4 Ashley vs. Helicopter

1. Ashley's video camera records at 128 frames per second. The helicopter she is filming has eight blades that rotate. At what rotation rate(s) of the helicopter rotor will Ashley's video suffer from aliasing effects? Give your answer in rotations per second.