Photometry & Monte Carlo 8

CS 184: Foundations of Computer Graphics

1 Lumens and Joules and Nits — Oh My!

1. Fill in the table below. In the right-most column, R denotes the distance to the light source.

Symbol/Name	**Radiometry Unit/Name**	**Photometry Unit/Name**	**Effect of Increased R**
Q: Energy	Radiant Energy Joules (W·s)	Luminous Energy Lumen·sec	$\uparrow$ (=) $\downarrow$
Φ: Flux (Power)			$\uparrow$ = $\downarrow$
I: Angular Flux Density			$\uparrow$ = $\downarrow$
E: Spatial Flux Density			$\uparrow$ = $\downarrow$
L: Spatio-Angular Flux Density			$\uparrow$ = $\downarrow$

2. For these questions, feel free to use infinitesimal quantities, e.g. dA, to represent a small area around a point, and θ to represent the angle between the light direction and surface normal.

   (a) What is Φ in terms of Q and time t?

   (b) What is $I(p, \omega)$ at a point p in terms of flux and solid angle?

   (c) What is irradiance $E(p)$ at a point p in terms of flux and area?

   (d) What is surface radiance $L(p, \omega)$ at a point p in a direction ω in terms of flux, area, and solid angle?

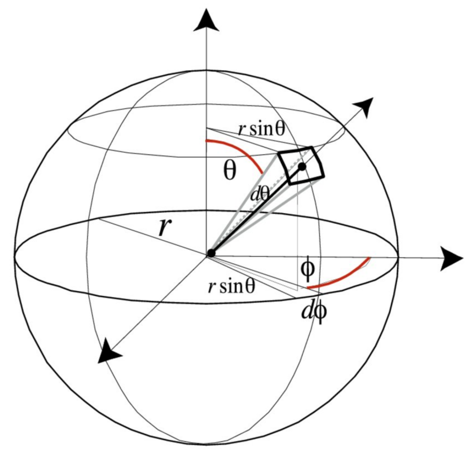
(e) How can surface radiance $L(p, \omega)$ be expressed in terms of irradiance $E(p)$?

(f) How can surface radiance $L(p, \omega)$ be expressed in terms of intensity $I(p, \omega)$?

2 Shedding Some Light

1. Suppose we use (θ, ϕ)-parameterization of directions. Recall that the solid angle represents the ratio of the subtended area on a sphere to the radius squared, $\Omega = \frac{A}{r^2}$. Estimate the solid angle subtended by a patch that covers $\theta \in [\pi/6 - \pi/12, \pi/6 + \pi/12]$ and $\phi \in [\pi/5 - \pi/24, \pi/5 + \pi/24]$?

   (Hint: you may assume that the patch is small enough. Recall or derive the differential solid angle $d\omega$, then use the values given.)



2. A point light at position $(6, 0, 8)$ (in meters) with radiant flux (power) of 100 watts directs all its light uniformly into the hemisphere directly below it. Some of this light falls on a flat, tilted surface passing through the origin, with surface normal vector $(1, 1, 1)$.

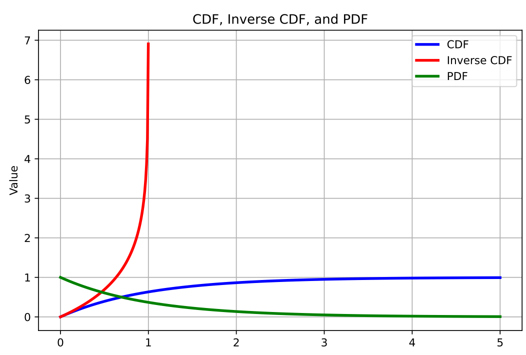
   What is the irradiance at the origin? Show your work and use the correct units.

3 Inversion Method

Given a uniform random variable U in the interval $[0, 1]$, we can generate a random variable from any other one dimensional distribution using its cumulative distribution function: $X = F^{-1}(U)$. This is how we choose sample points when running a ray tracing algorithm.

1. What function of U will return a sample from the exponential distribution (with parameter λ)? This distribution has density $p_\lambda(x) = \lambda e^{-\lambda x}$, and is defined for $x \geq 0$.

2. For $\lambda = 1$, the plot below shows the CDF, Inverse CDF, and PDF. If we sample $U = 0.75$, determine the corresponding sampled value using the inverse method. Mark the corresponding point on the CDF curve in the graph.



3. What does the x-axis represent for the blue CDF curve?

4 Unbiased Estimators

1. Let $f : [-2, 2] \times [-2, 2] \to \mathbb{R}$ be a function. You have a machine that allows you to sample $2n$ values independently and uniformly from the interval $[-2, 2]$. Construct an unbiased Monte Carlo estimator for

$$F = \int_{-2}^{2} \int_{-2}^{2} f(x, y) dx dy.$$