Lecture 13:

Measuring Light:

Radiometry and Photometry

Computer Graphics and Imaging

UC Berkeley CS184/284A

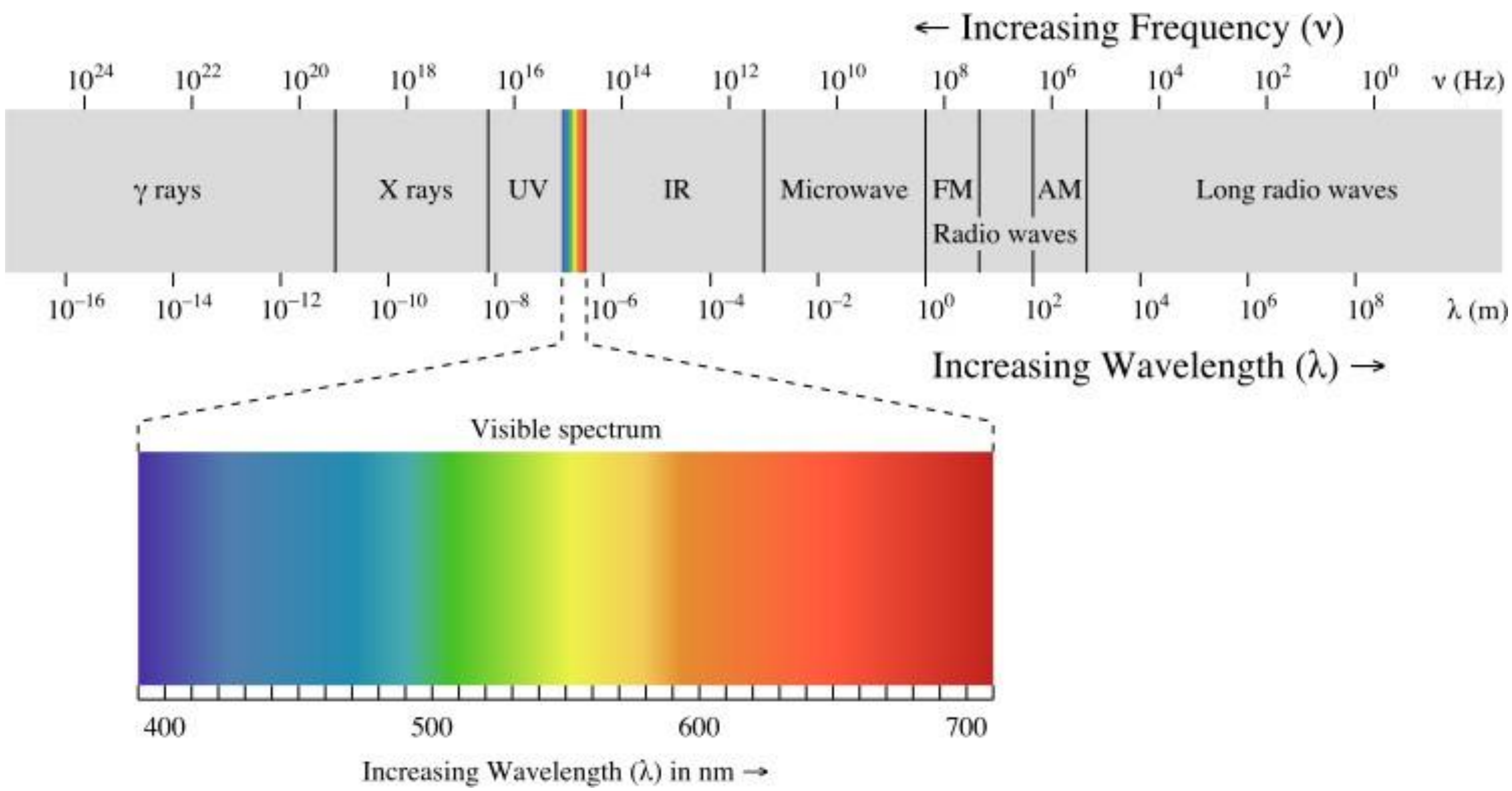
Radiometry

Measurement system for illumination

- **Specifically, measuring the spatial properties of light**
- **Allows lighting calculations using a physically based model**
- **Establishes SI units for illumination**
- **New terms:** Radiant Flux, Intensity, Irradiance, Radiance
- **Core assumption:**
  - **Start with a geometric optics model of light**
  - **Photons travel in straight lines** (represented by rays)

Light

Visible electromagnetic spectrum



Modern Lights: How Do They Work?

11 watt LED light bulb
(60w incandescent replacement)

Process converting energy into photons

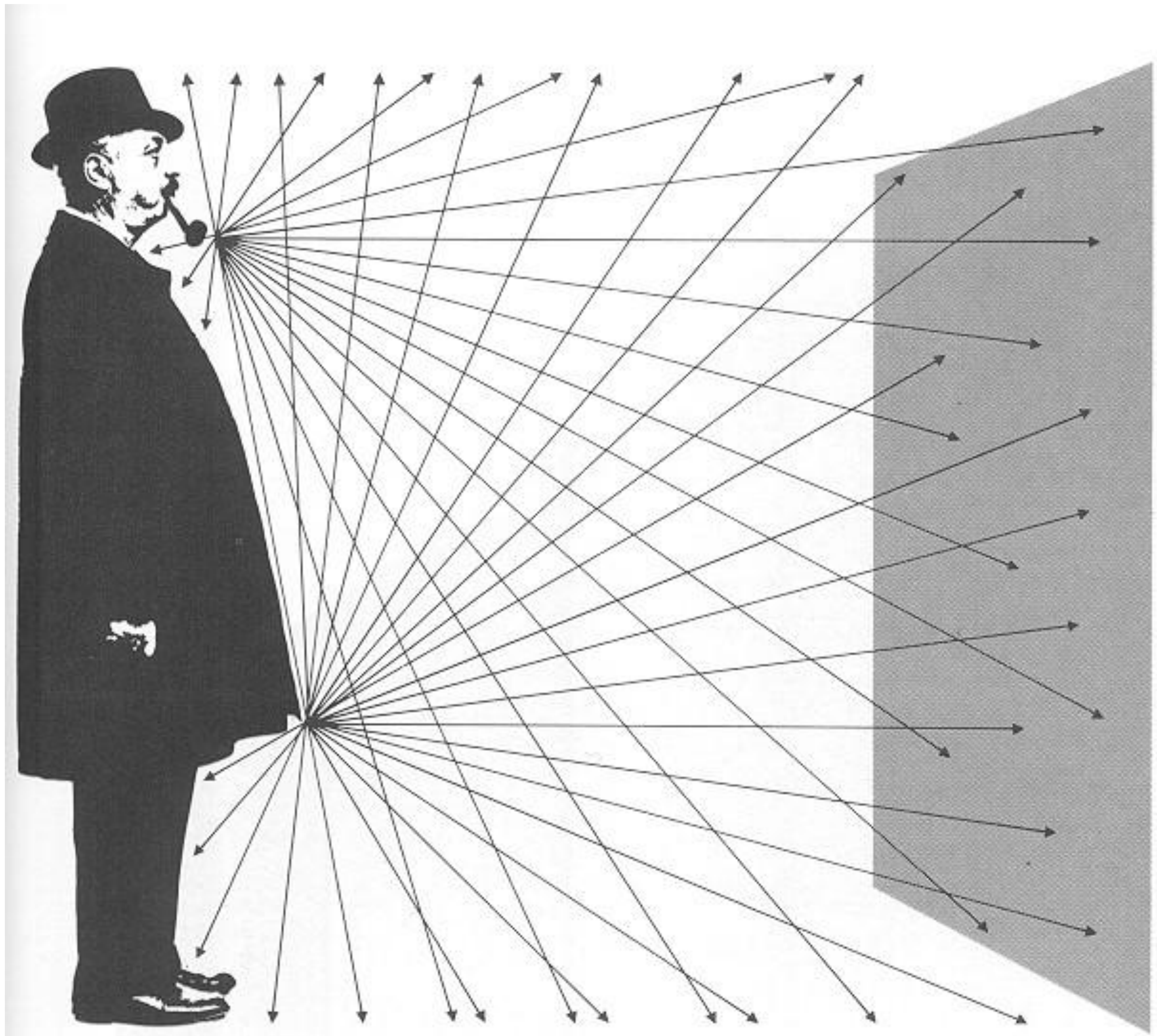
- Light source consumes energy (**Joules**) over a given time interval
- most energy turns into visible photons, Some turns into heat (thermal energy)
- Each photon carries a small amount of energy

Illuminating surrounding surfaces:

- Visible photons travel until they encounter a surface, illuminating it - exposure
- Film, sensors, microscopy, fluorescence,...
- Rate of energy consumption is constant, so **flux** (power) and energy are often interchangeable

Note: In Graphics we assume a **steady state flow**

Flux – What's the Density of Photons Flowing Through a Sensor?



From London and Upton

Radiant Energy and Flux (Power)

Radiant Energy and Flux (Power)

Definition: Radiant (luminous*) energy is the energy of electromagnetic radiation. It is measured in units of joules, and denoted by the symbol:

$$Q \; [\mathrm{J} = \mathrm{Joule}]$$

Definition: Radiant (luminous*) flux is the energy emitted, reflected, transmitted or received, per unit time.

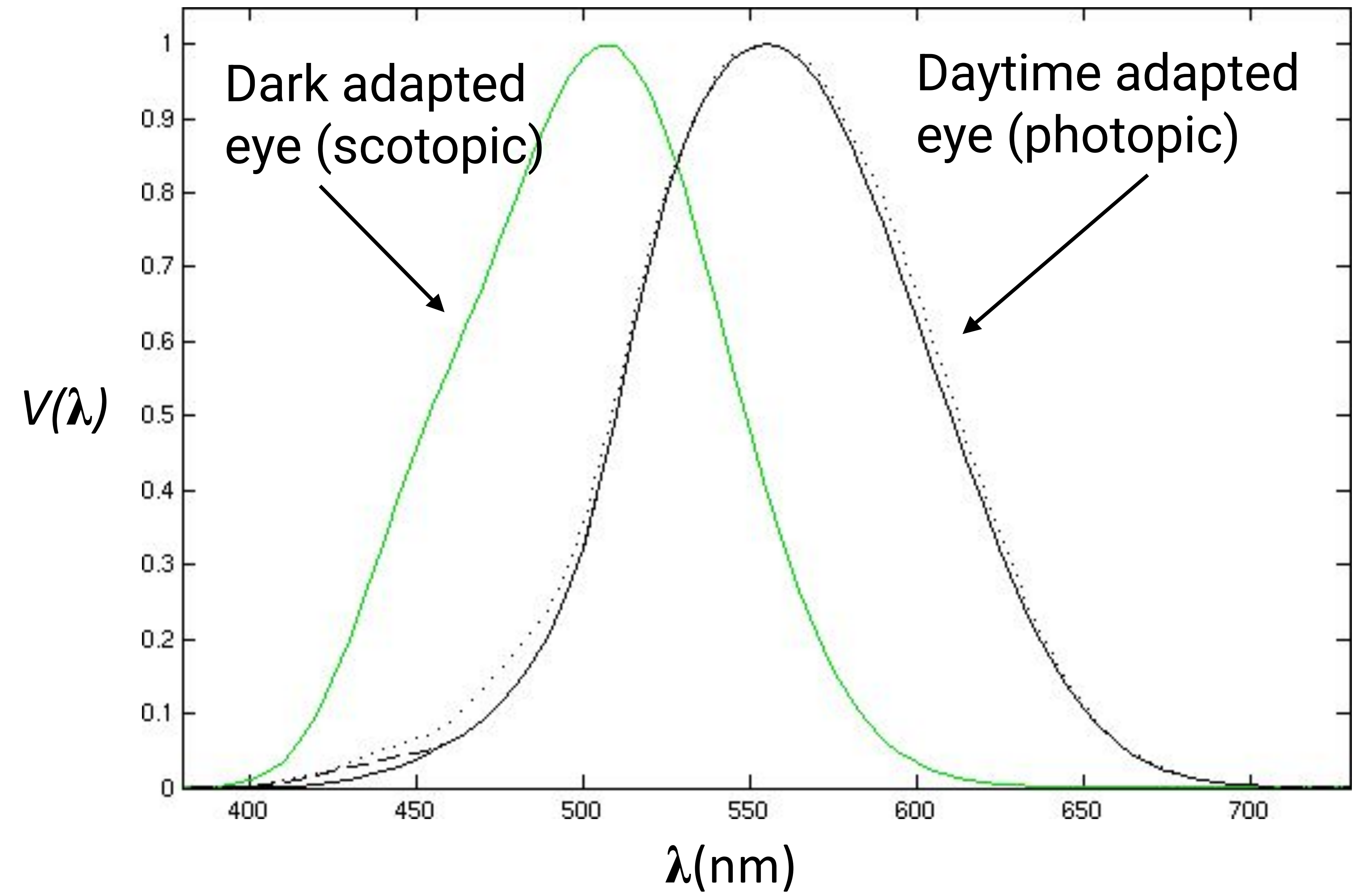
$$\phi_e \equiv \frac{\mathrm{d}Q}{\mathrm{d}t} \;\; [\mathrm{W} = \mathrm{Watt}][\mathrm{lm} = \mathrm{lumen}]^*$$

* Definition slides will provide photometric terms in parentheses and give photometric units

Photometry

All radiometric quantities have equivalents in photometry

- **Photometry**: accounts for response of human visual system
- Luminous flux $\mathbf{\Phi}_v$ is the photometric quantity that corresponds to radiant flux

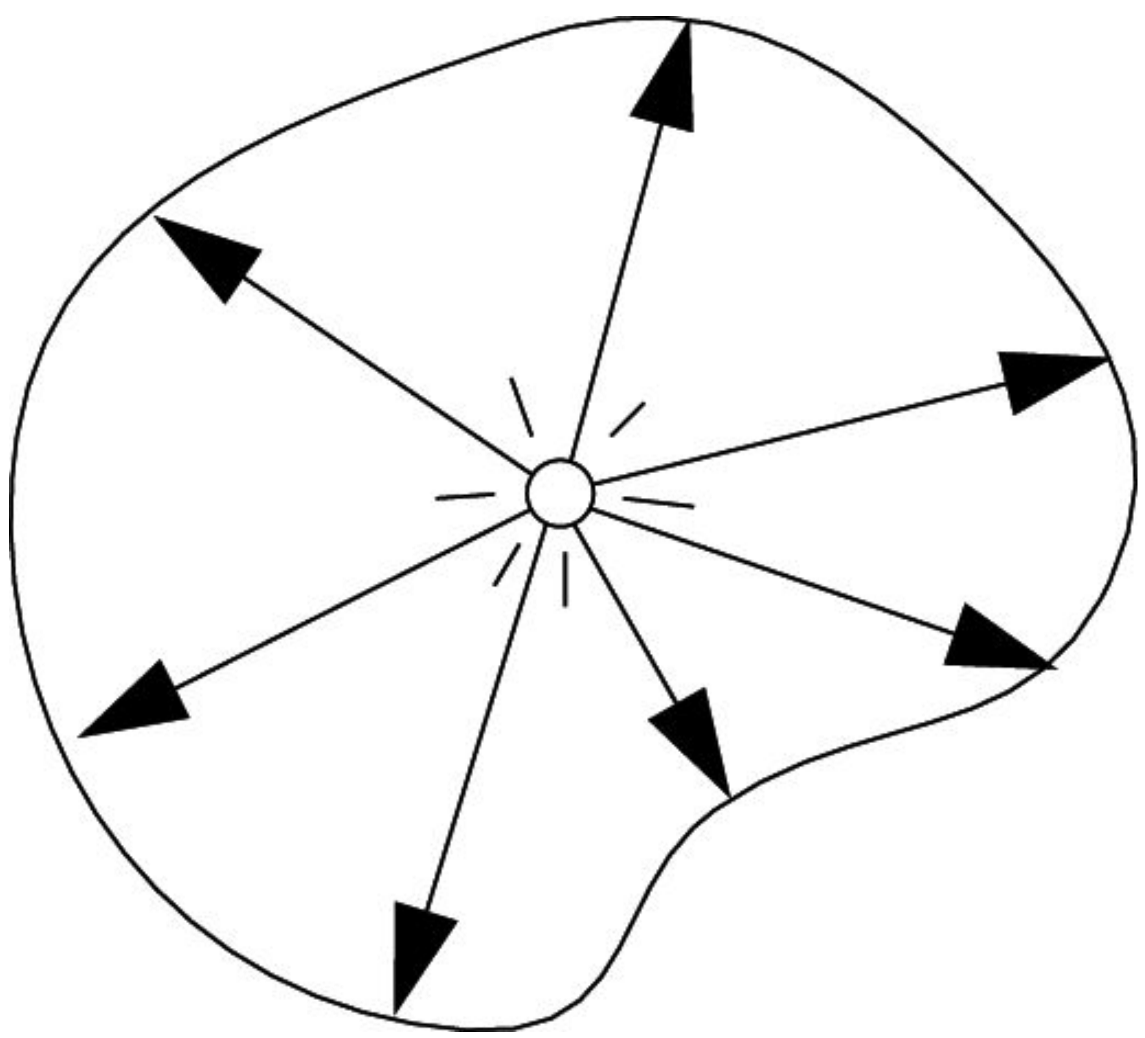


https://upload.wikimedia.org/wikipedia/commons/a/a0/Luminosity.png

- $\mathbf{\Phi}_v$: Integrate radiant flux over all wavelengths, weighted by eye's luminous efficiency curve $V(\lambda)$

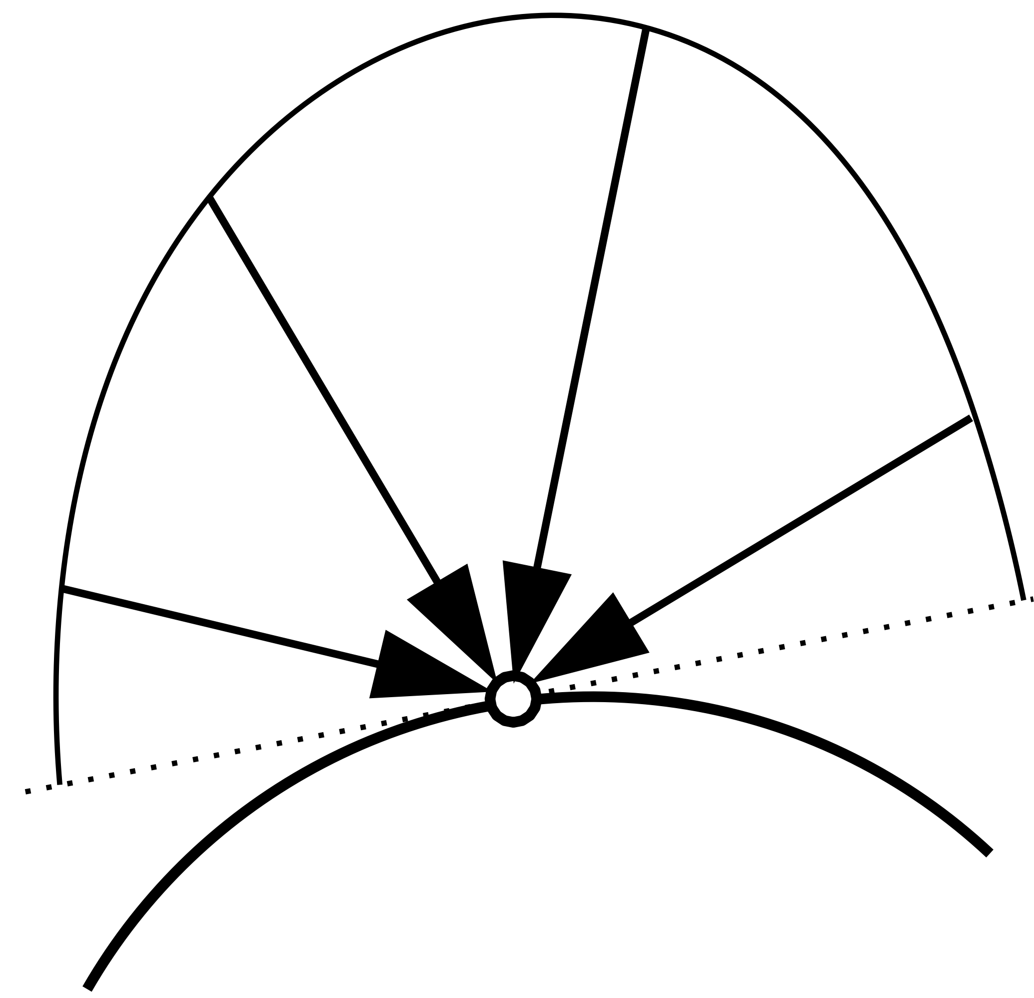
$$\Phi_v = \int_0^\infty \Phi_e(\lambda) V(\lambda)\, d\lambda$$

Example Light Measurements



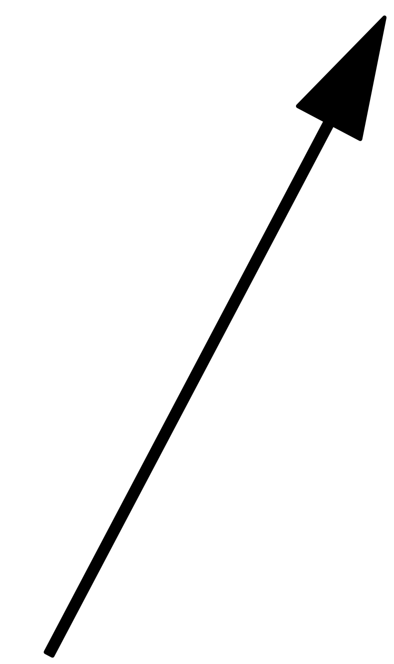
Light Emitted
From A Source

"Radiant Intensity"



Light Falling
On A Surface

"Irradiance"



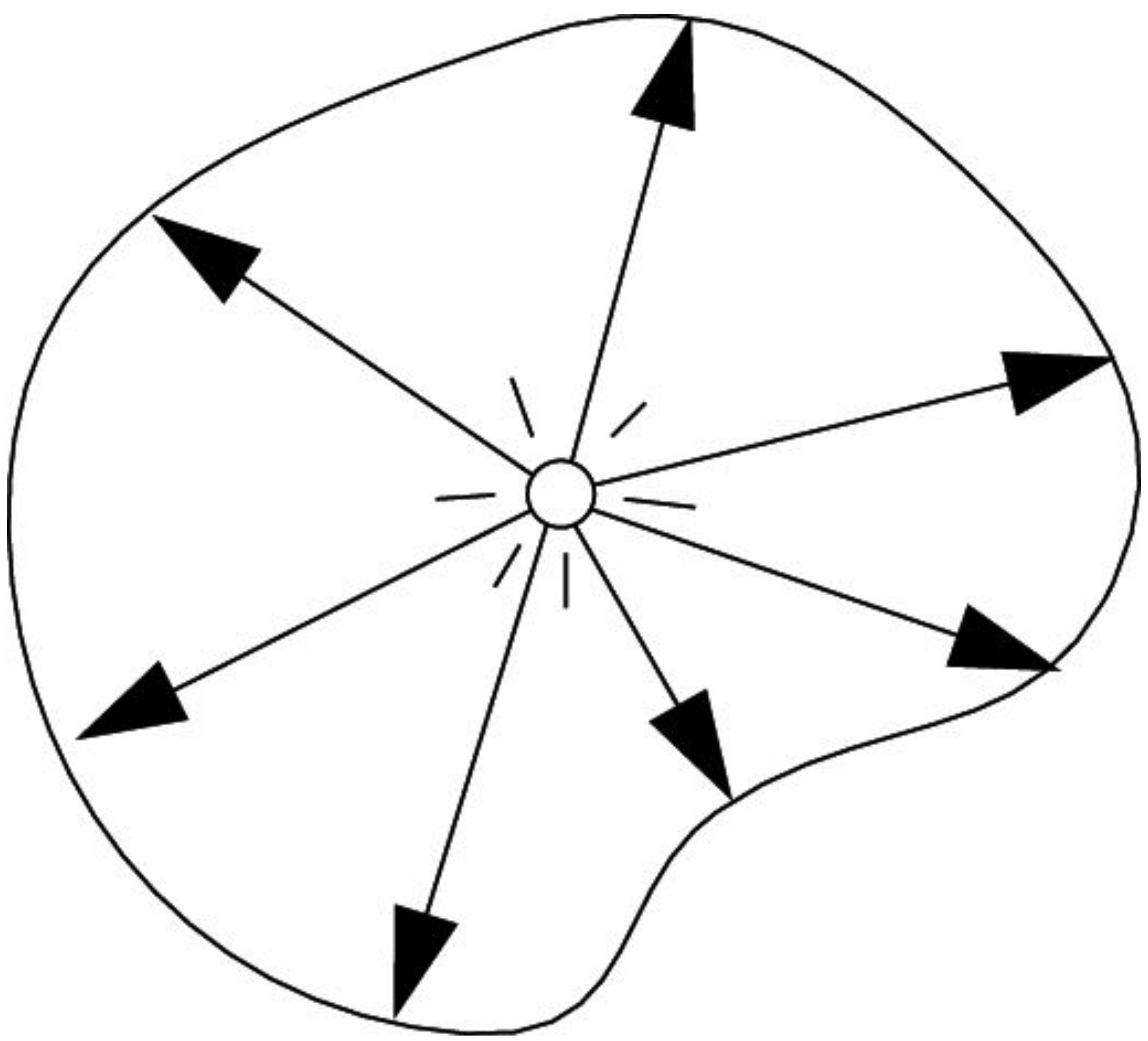
Light Traveling
Along A Ray

"Radiance"

Radiant Intensity

Radiant Intensity

Definition: The radiant (luminous) intensity is the power per unit solid angle (steradians) emitted by a point light source.



$$I(\omega) \equiv \frac{\mathrm{d}\Phi}{\mathrm{d}\omega}$$

radiant flux

unit of solid angle

$$\left[\frac{\mathrm{W}}{\mathrm{sr}}\right] \left[\frac{\mathrm{lm}}{\mathrm{sr}} = \mathrm{cd} = \mathrm{candela}\right]$$

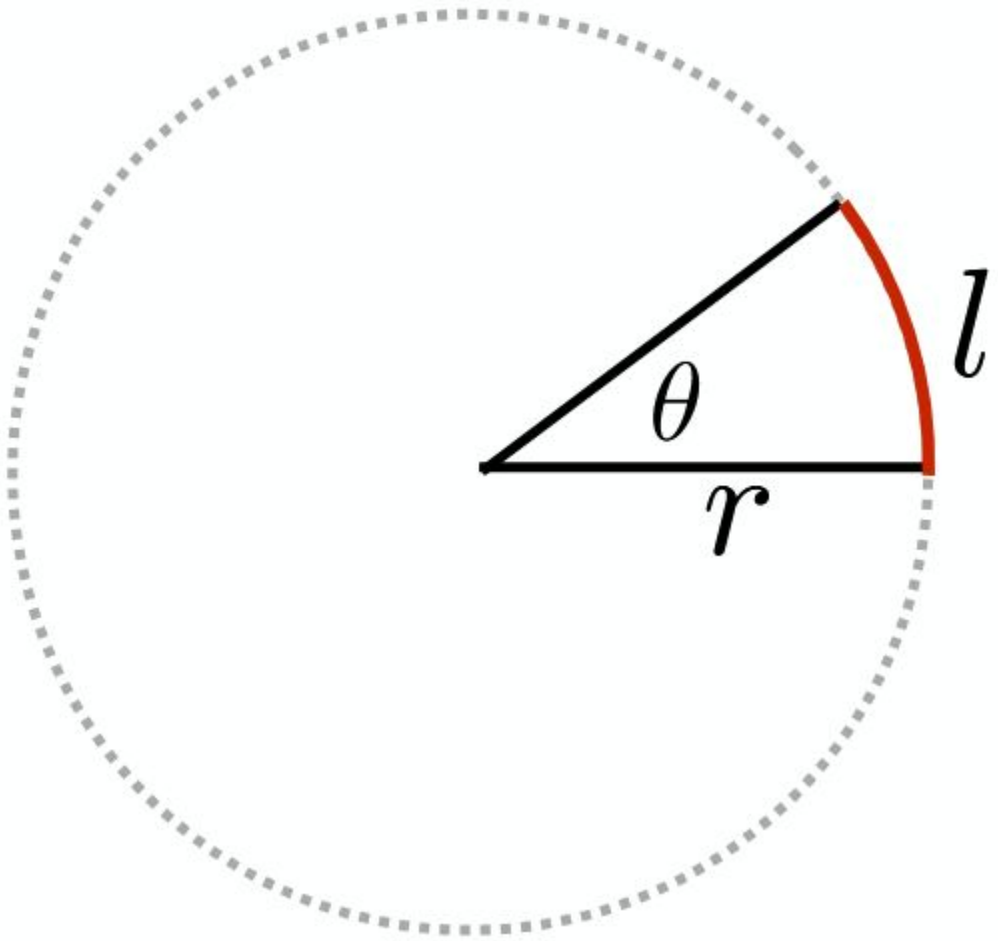
** The candela is one of the seven SI base units: (m, s, mole, A, K, cd, kg)*

Angles and Solid Angles

Measures the *Field of View* a patch has from a given point

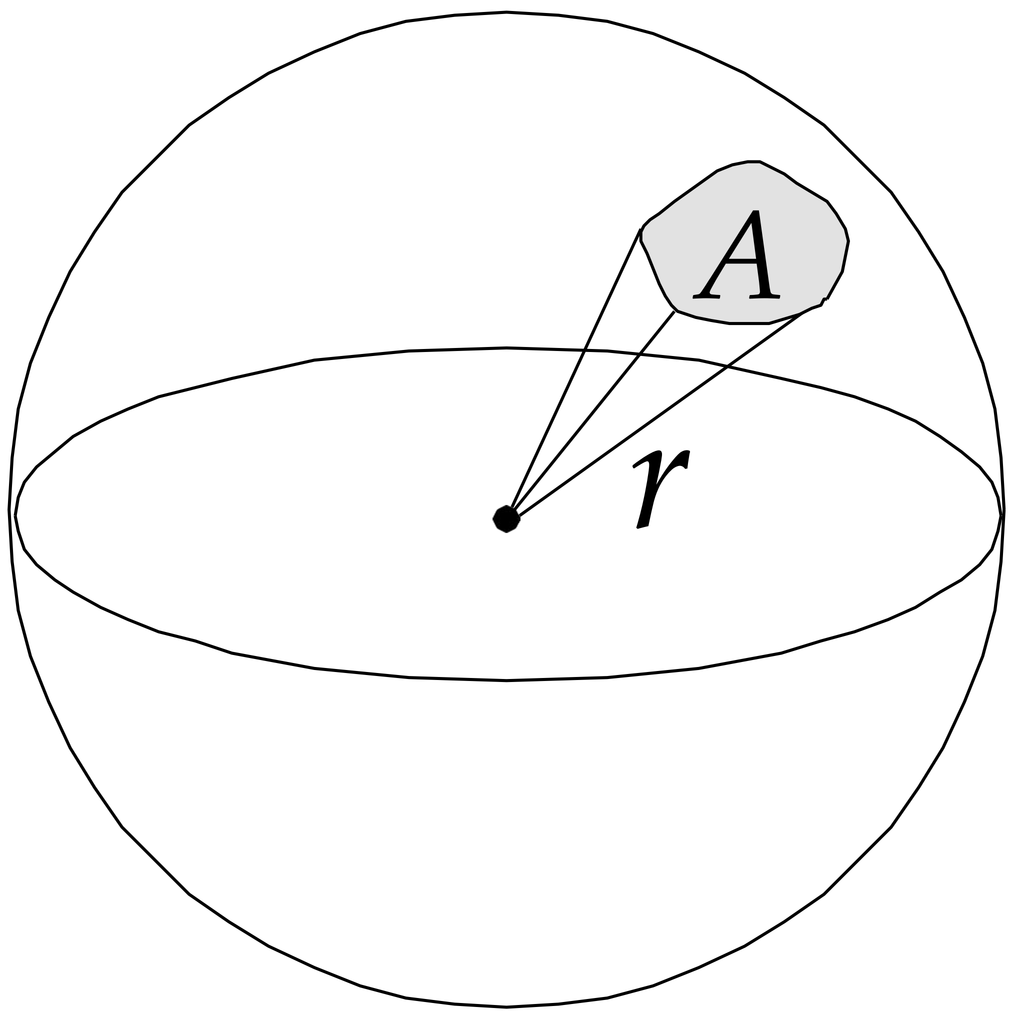
Angle (2D): ratio of subtended arc length on circle to radius: $\theta = l/r$

- Circle has 2π radians

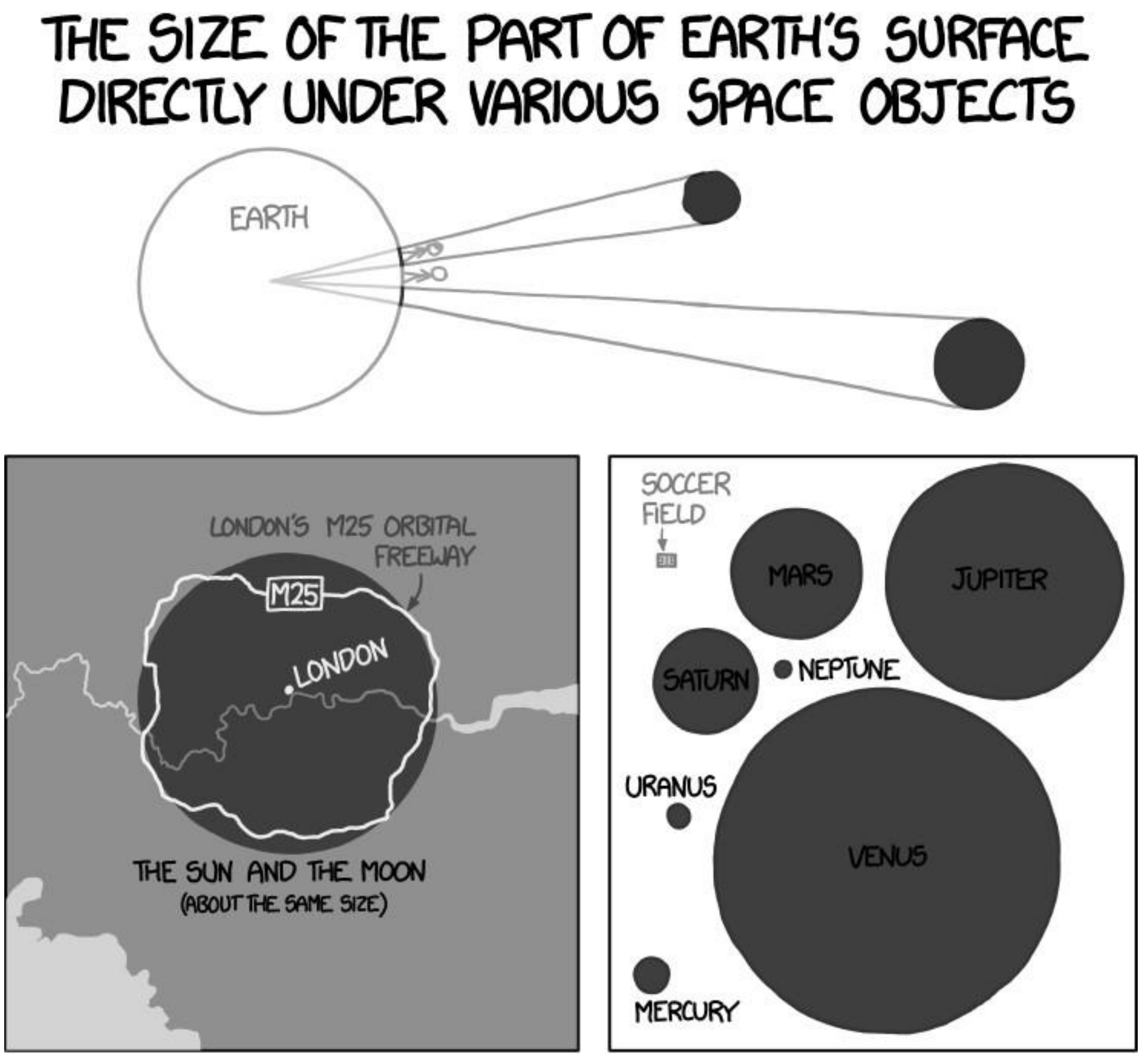


Solid angle (3D) : ratio of subtended area on sphere to radius squared: $\Omega = A/r^2$

- Sphere has 4π steradians



Solid Angles in Practice

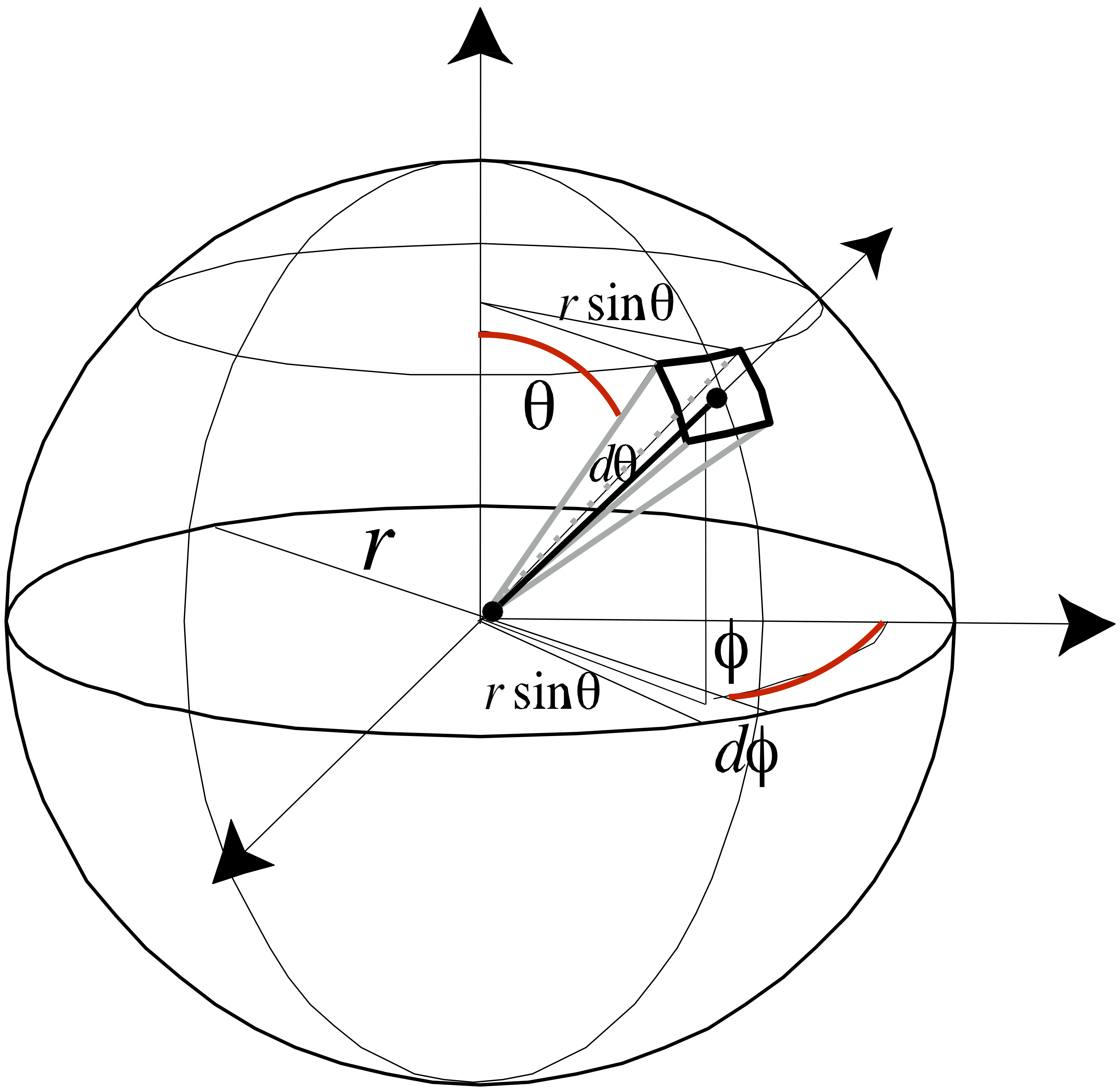


http://xkcd.com/1276/

- Sun and moon both subtend ~60μ sr as seen from earth
- Surface area of earth: ~510M km^2
- Projected area:

$$510\text{Mkm}^2\,\frac{60\mu\text{sr}}{4\pi\text{sr}} = 510\,\frac{15}{\pi}$$
$$\approx 2400\text{km}^2$$

Differential Solid Angles



Sphere: S^2

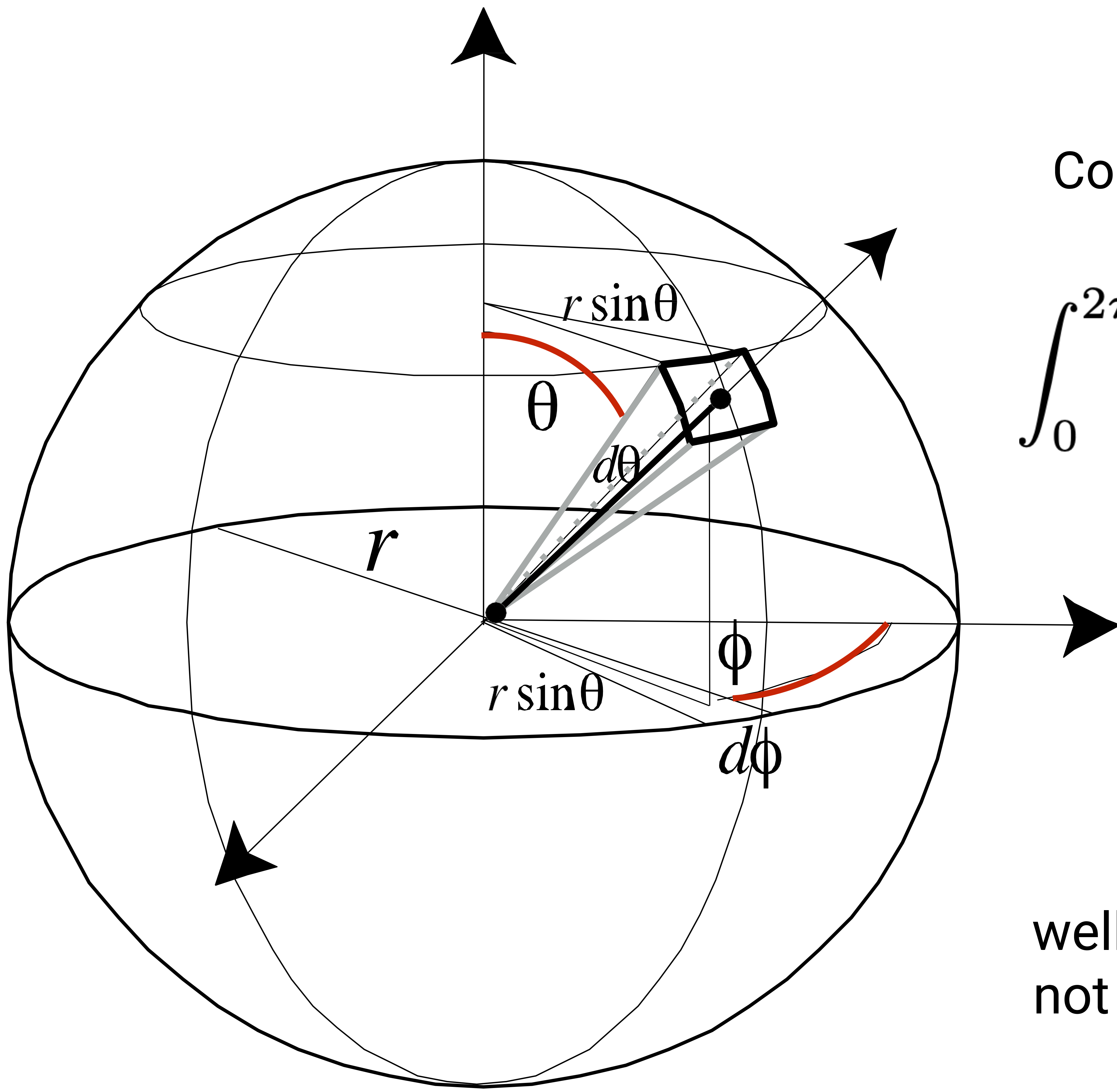
$$\Omega = \int_{S^2} \mathrm{d}\omega$$

$$= \int_0^{2\pi} \int_0^{\pi} \sin\theta \, \mathrm{d}\theta \, \mathrm{d}\phi$$

$$= 4\pi$$

what??

Differential Solid Angles



Consider if $d\omega = 1 d\theta d\phi$

$$\int_0^{2\pi} \int_0^{\pi} 1\, d\theta\, d\phi = \int_0^{2\pi} \theta\Big|_0^{\pi} d\phi$$

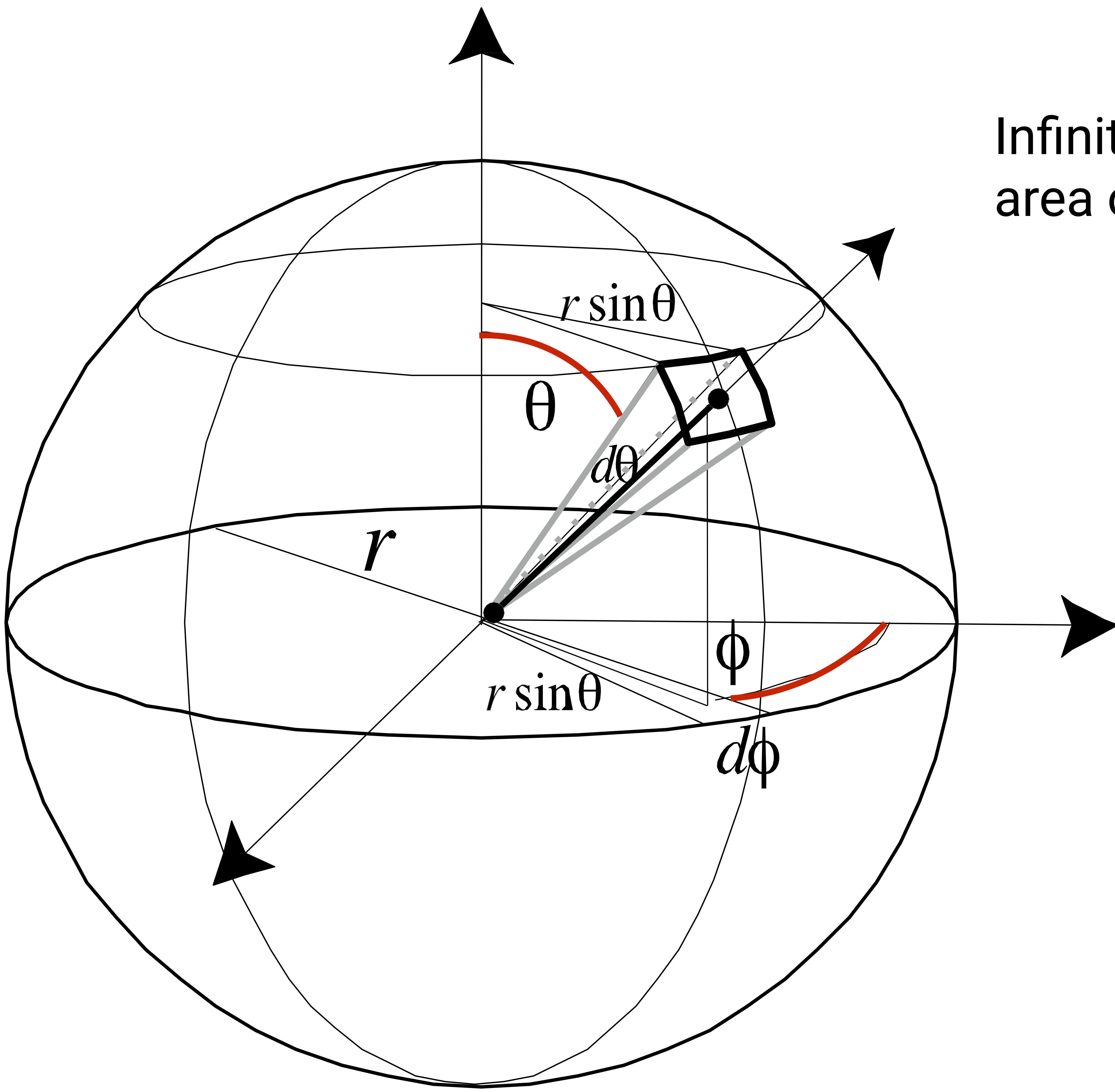
$$= \int_0^{2\pi} \pi\, d\phi$$

$$= \pi \left(\phi\Big|_0^{2\pi} \right)$$

$$= 2\pi^2$$

well that's
not right...

Differential Solid Angles

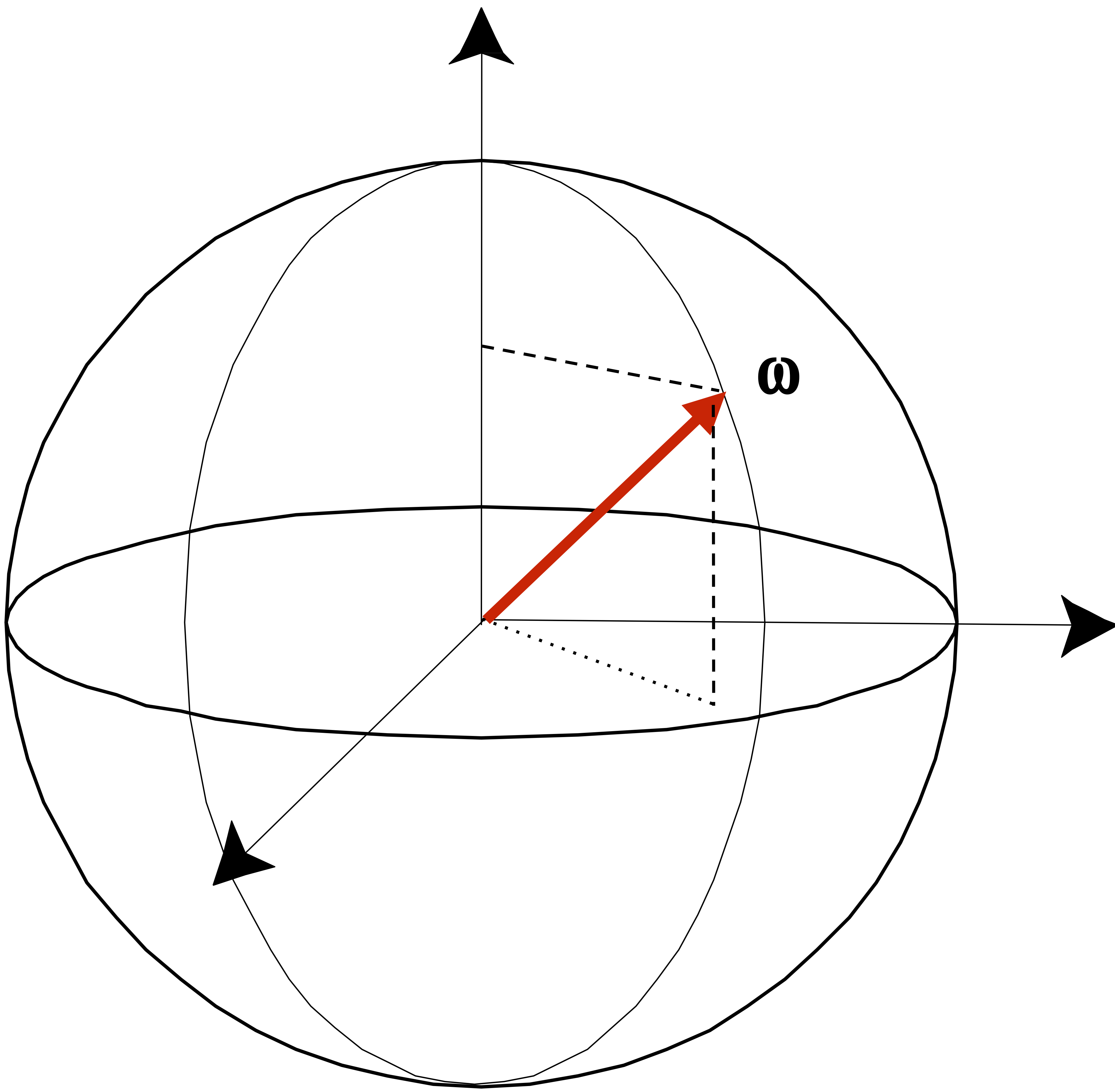


Infinitesimally small patch of area on a sphere should be

$$\begin{aligned} \mathrm{d}A &= (r\,\mathrm{d}\theta)(r\sin\theta\,\mathrm{d}\phi) \\ &= r^2\sin\theta\,\mathrm{d}\theta\,\mathrm{d}\phi \end{aligned}$$

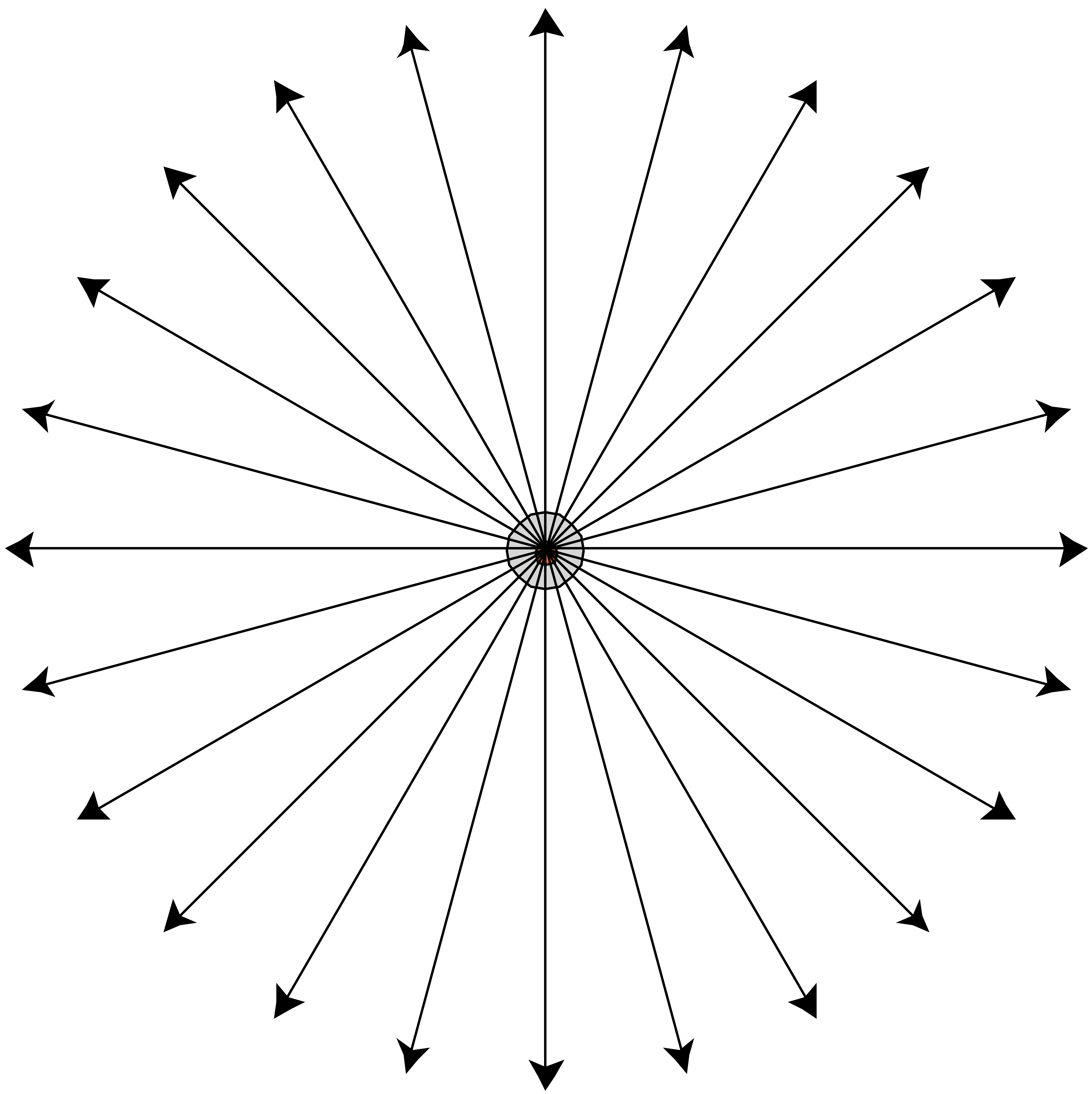
$$\mathrm{d}\omega = \frac{\mathrm{d}A}{r^2} = \sin\theta\,\mathrm{d}\theta\,\mathrm{d}\phi$$

Omega (ω) as a direction vector



We will use ω to denote a direction vector (unit length)

Isotropic Point Source



$$\Phi = \int_{S^2} I \, \mathrm{d}\omega$$
$$= 4\pi I$$

$$I = \frac{\Phi}{4\pi}$$

Modern LED Light

Output: 815 lumens (luminous flux)
(11W LED replacement for 60W incandescent)

What is the Luminous intensity?

Assuming isotropic:
Intensity = **815** lumens / **4π** sr
= **65** candelas

If focused into 20° diameter cone. Intensity = ?

Spectral Power Distribution

Describes distribution of energy by wavelength

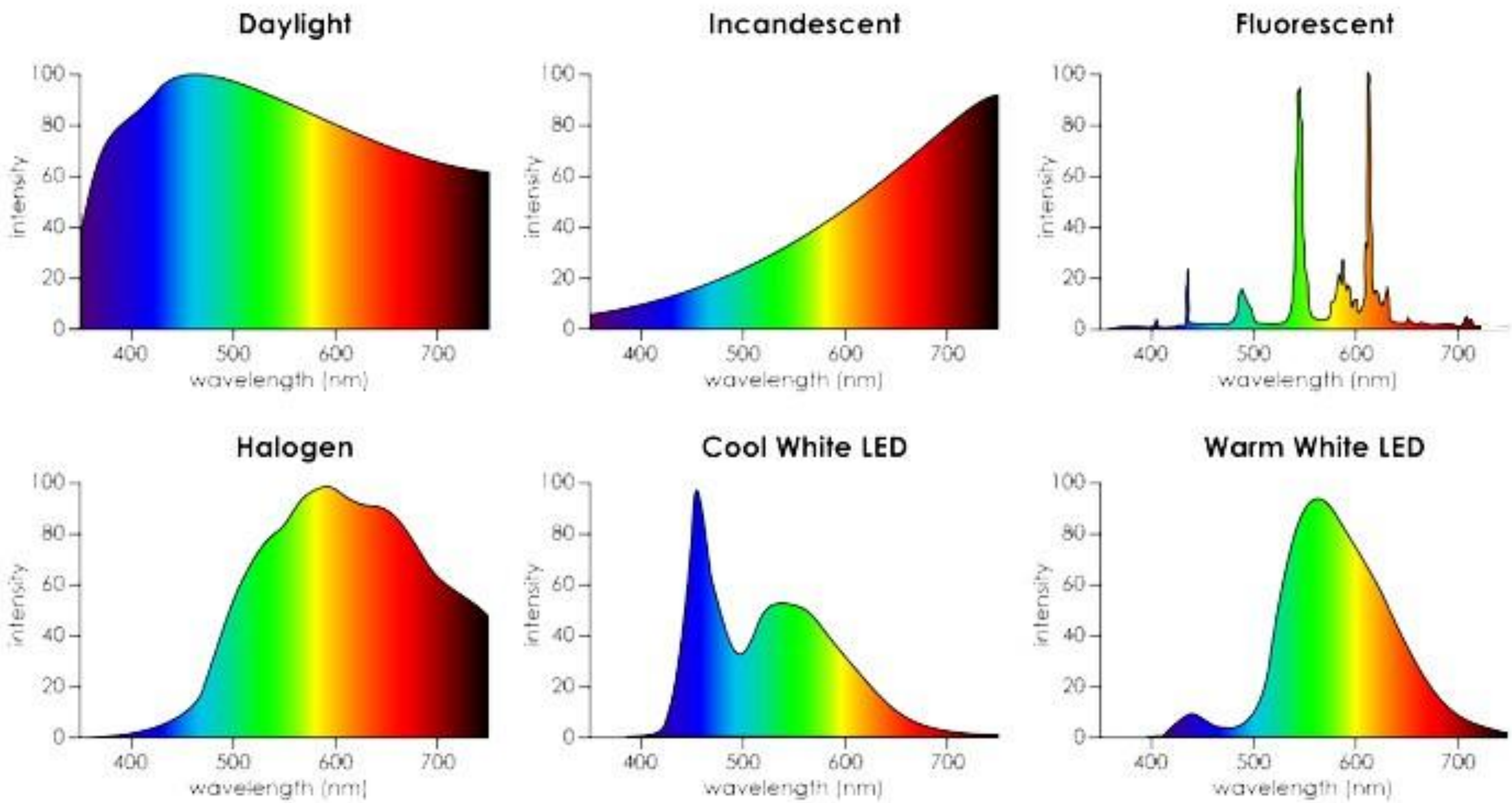
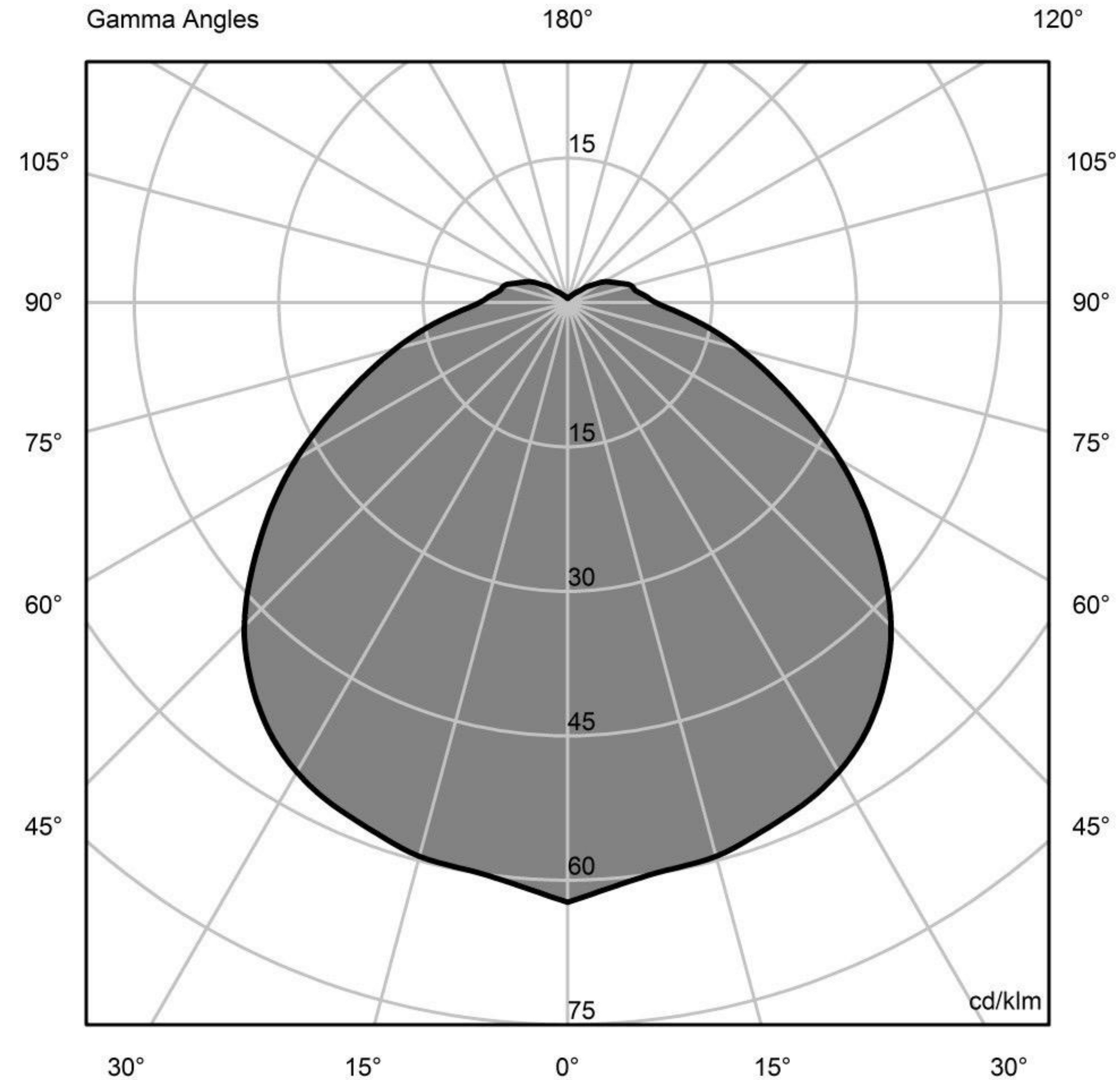


Figure credit: admesy ADVANCED MEASUREMENT SYSTEMS

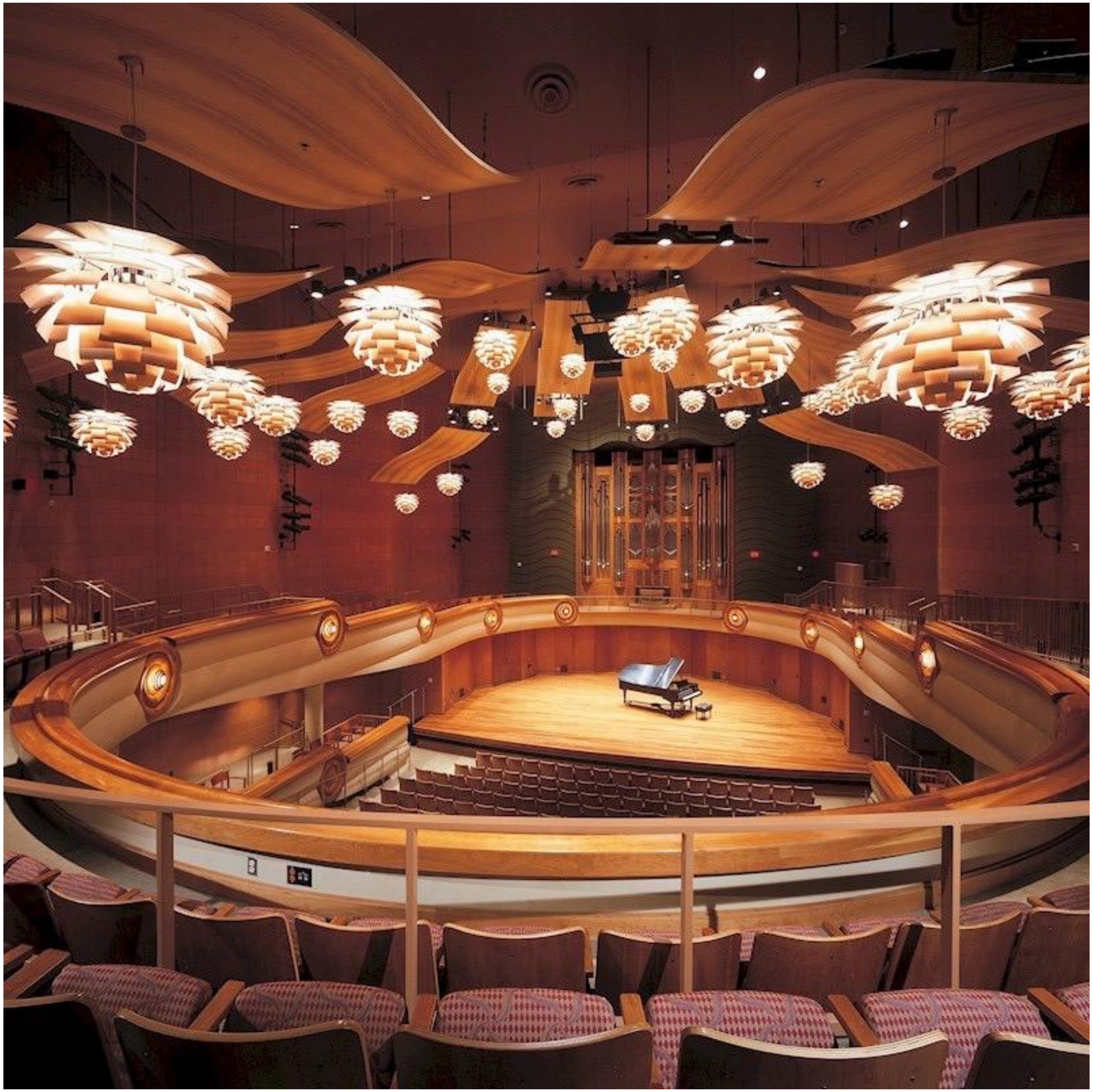
Light Fixture Measurements - Goniometric Diagram

Poul Henningsen's Artichoke Lamp



Polar AKA Goniometric Diagram

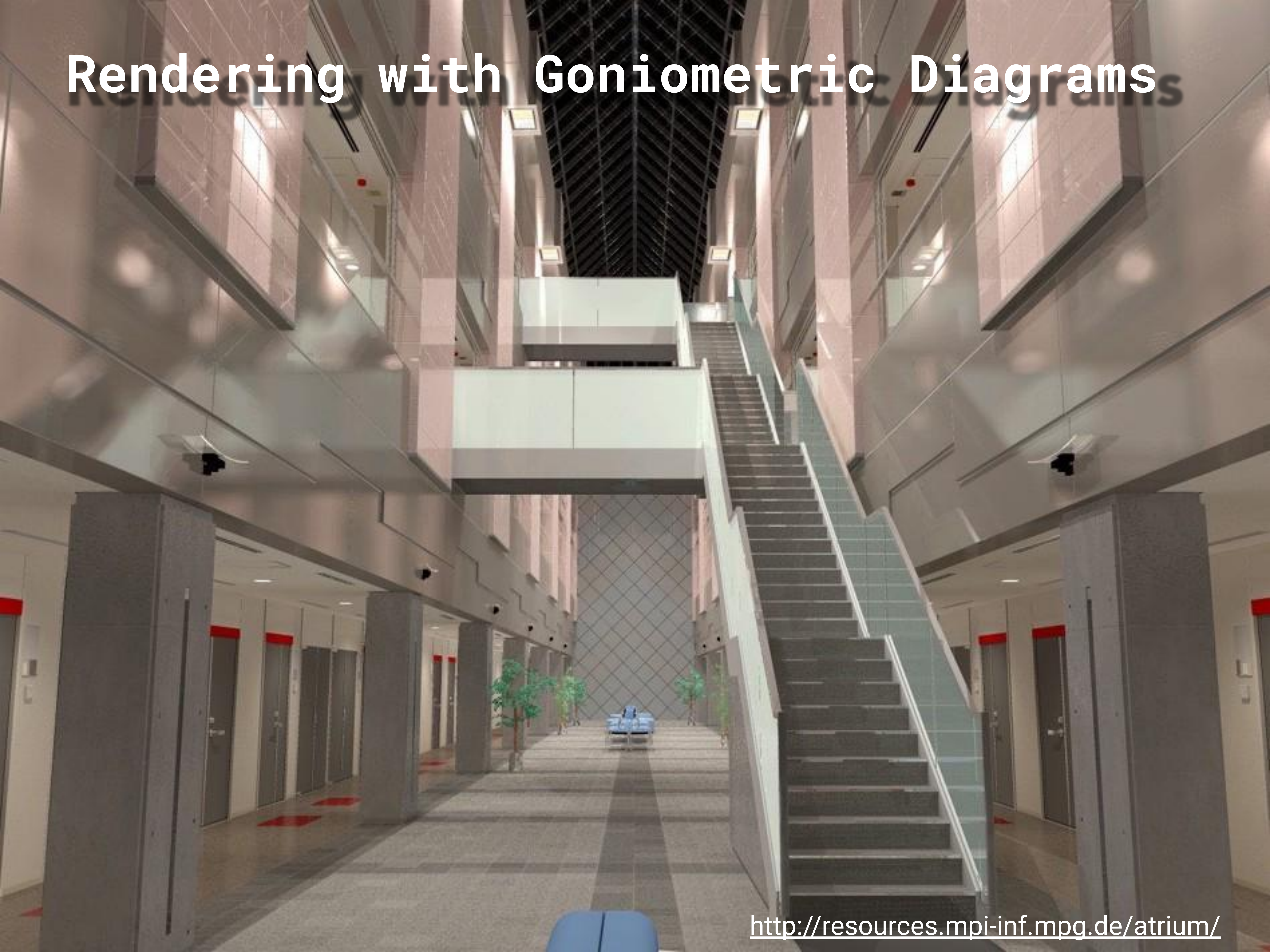
http://www.louispoulsen.com/



http://www.louispoulsen.com/

PH Artichoke Lamps in Rivercenter for the Performing Arts, Georgia

Rendering with Goniometric Diagrams



http://resources.mpi-inf.mpg.de/atrium/

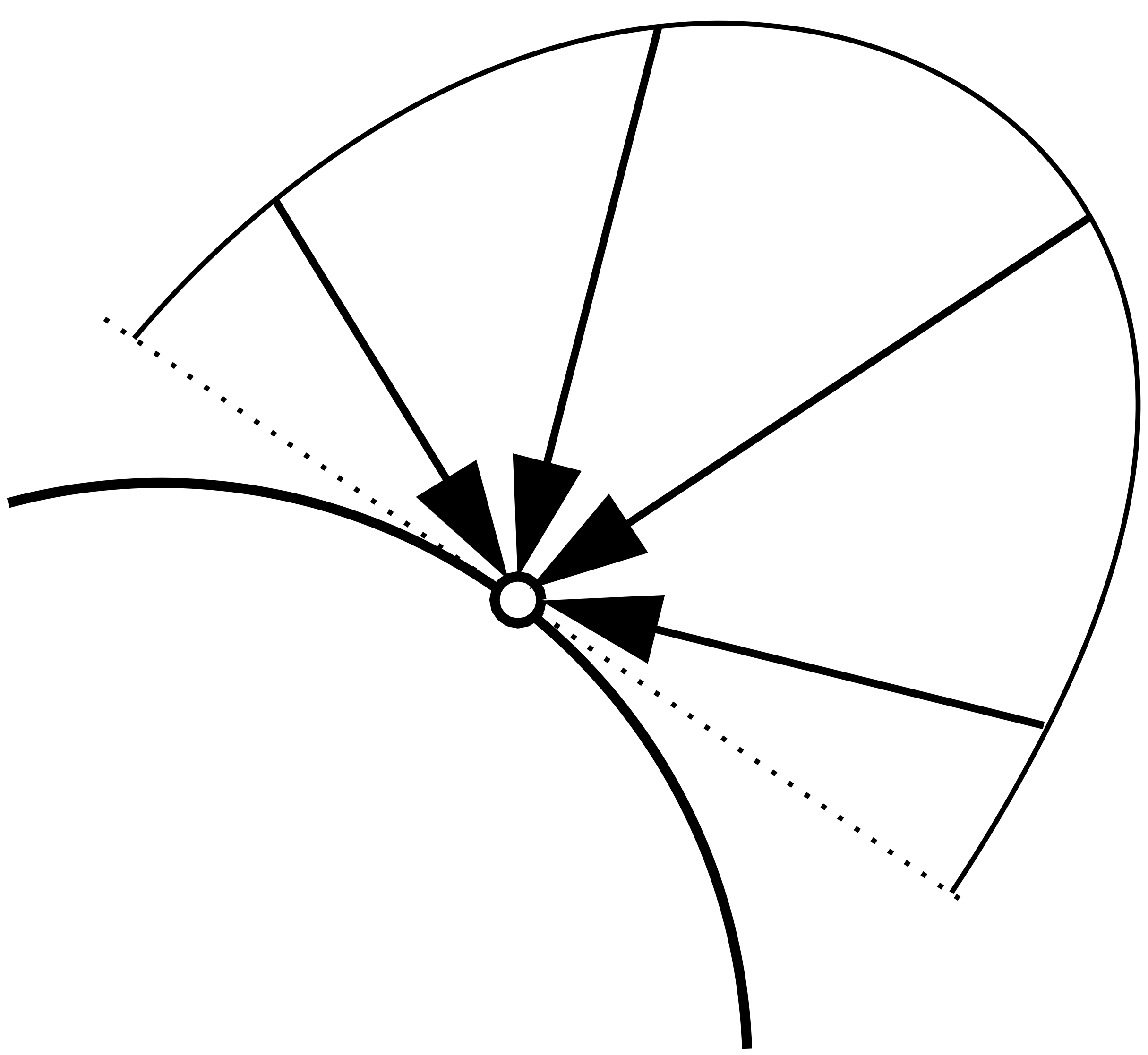
Irradiance

Irradiance

Definition: The irradiance *(illuminance)* is the power per unit area incident on a surface point.

$$E(\mathbf{x}) \equiv \frac{\mathrm{d}\Phi(\mathbf{x})}{\mathrm{d}A}$$

$$\left[\frac{\mathrm{W}}{\mathrm{m}^2}\right] \quad \left[\frac{\mathrm{lm}}{\mathrm{m}^2} = \mathrm{lux}\right]$$

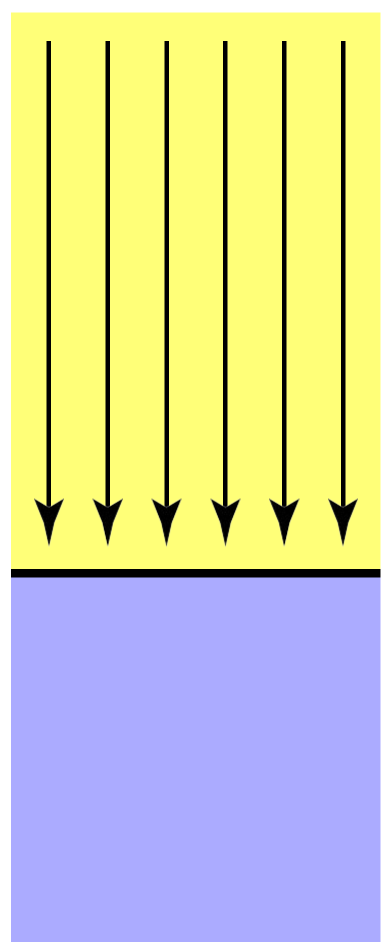


Typical Values of Illuminance [lm/m^2]

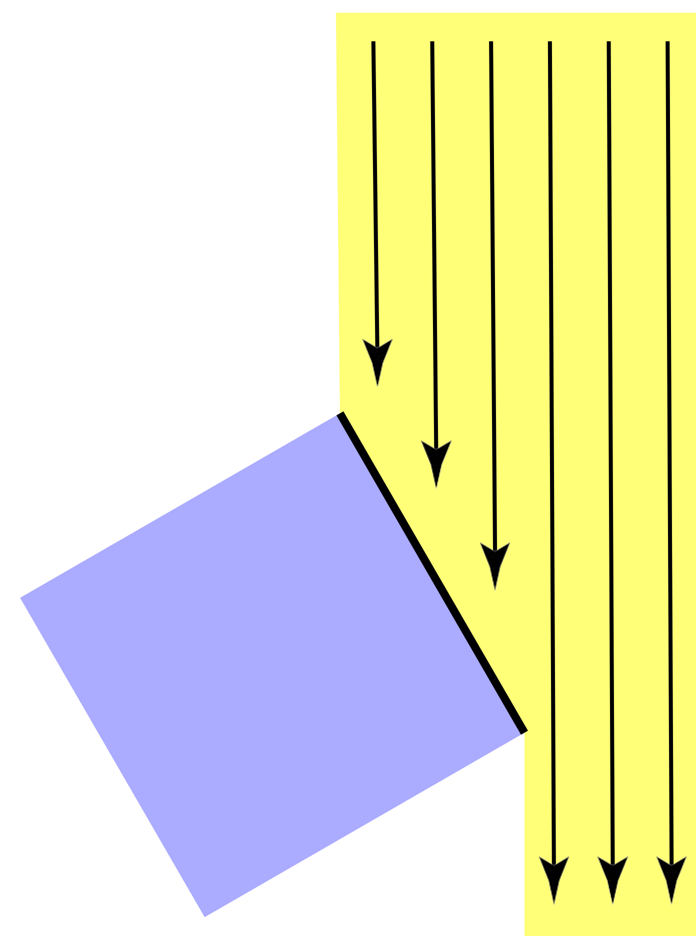
Brightest sunlight	120,000 lux
Overcast day (midday)	15,000
Interior near window (daylight)	1,000
Residential artificial lighting	300
Sunrise / sunset	40
Illuminated city street	10
Moonlight (full)	0.02
Starlight	0.0003

Light meter

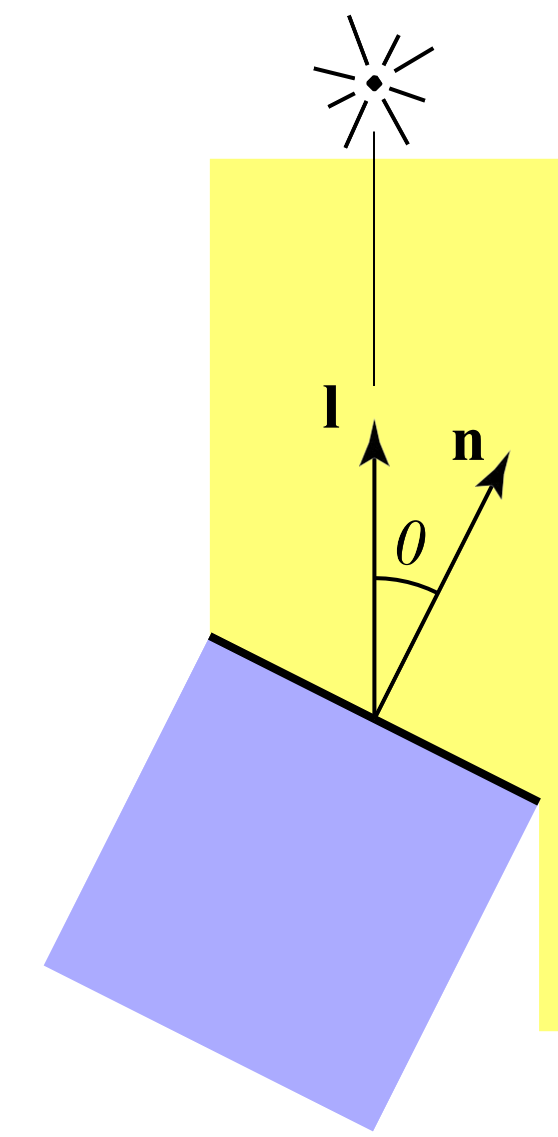
Lambert's Cosine Law



Top face of cube receives a certain amount of power



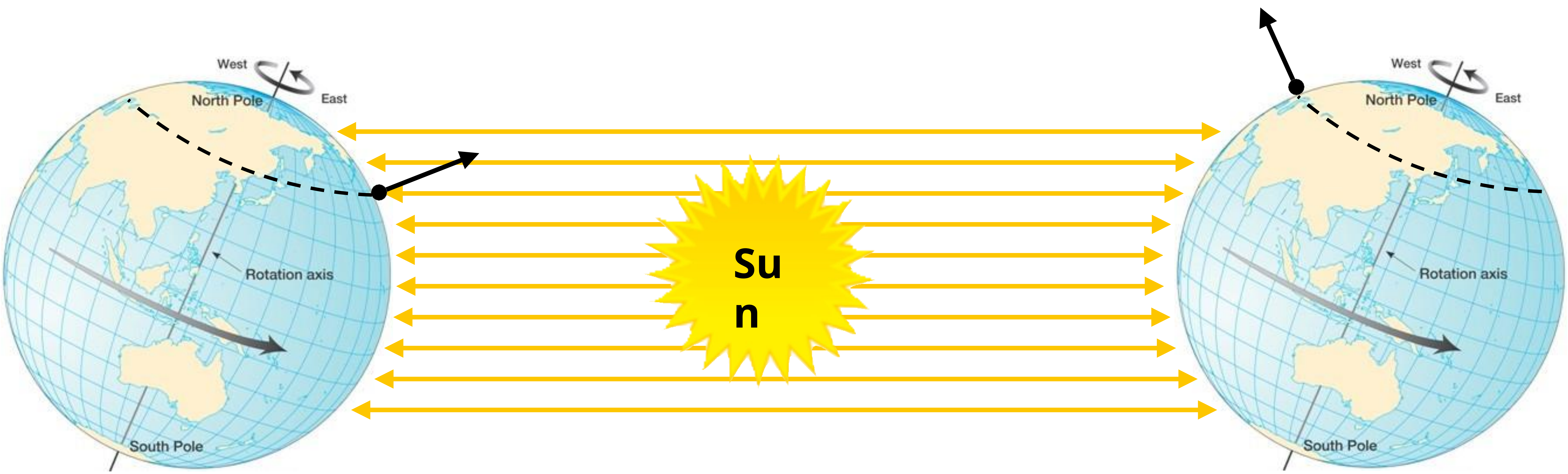
Top face of 60º rotated cube receives power



In general, power per unit area is proportional to

Irradiance at a surface is proportional to cosine of angle between light direction and surface normal.

Why Do We Have Seasons?



[Image credit: Pearson Prentice Hall]

Summer
(Northern hemisphere)

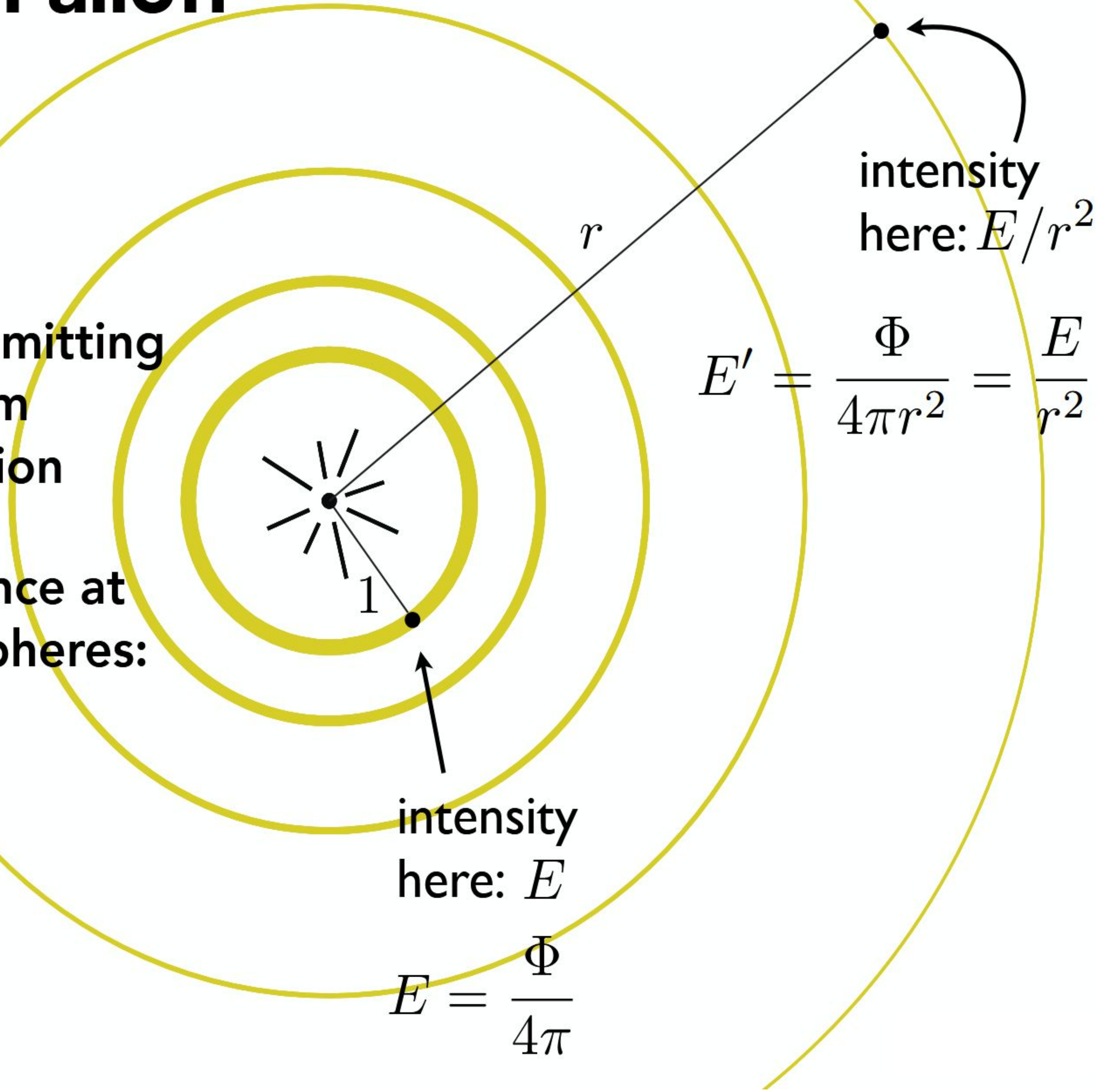
Winter
(Northern hemisphere)

Earth’s axis of rotation: ~23.5° off axis

Irradiance Falloff

Assume light is emitting flux Φ in a uniform angular distribution

Compare irradiance at surface of two spheres:

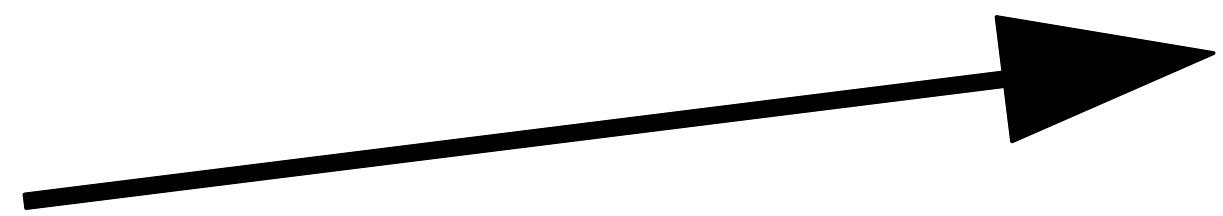


$$E' = \frac{\Phi}{4\pi r^2} = \frac{E}{r^2}$$

$$E = \frac{\Phi}{4\pi}$$

Radiance

Radiance



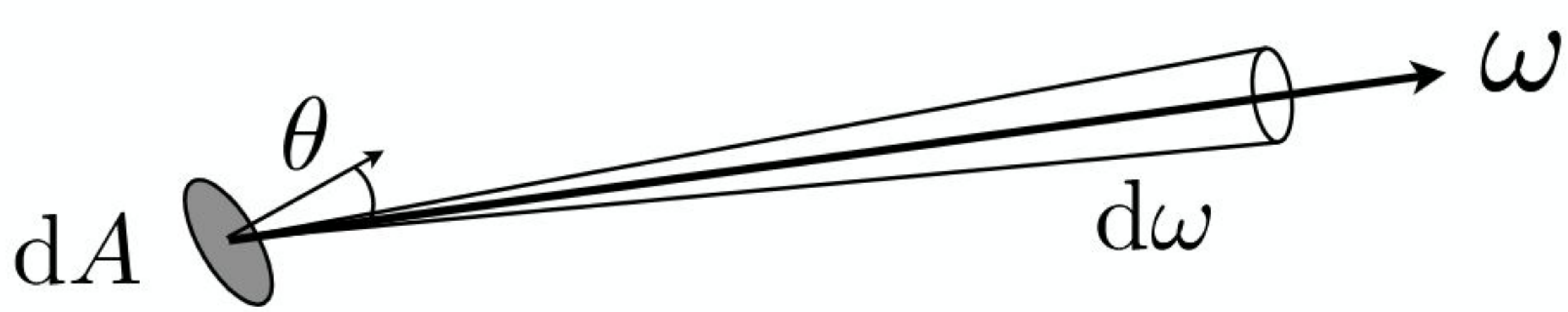
Light Traveling Along A Ray

Radiance is the fundamental field quantity that describes the distribution of light in an environment

- Radiance is the quantity associated with a ray
- Rendering is all about computing radiance
- Radiance is invariant along a ray in a vacuum

Surface Radiance

Definition: The radiance (luminance) is the power emitted, reflected, transmitted or received by a surface, per unit solid angle, per unit projected area.



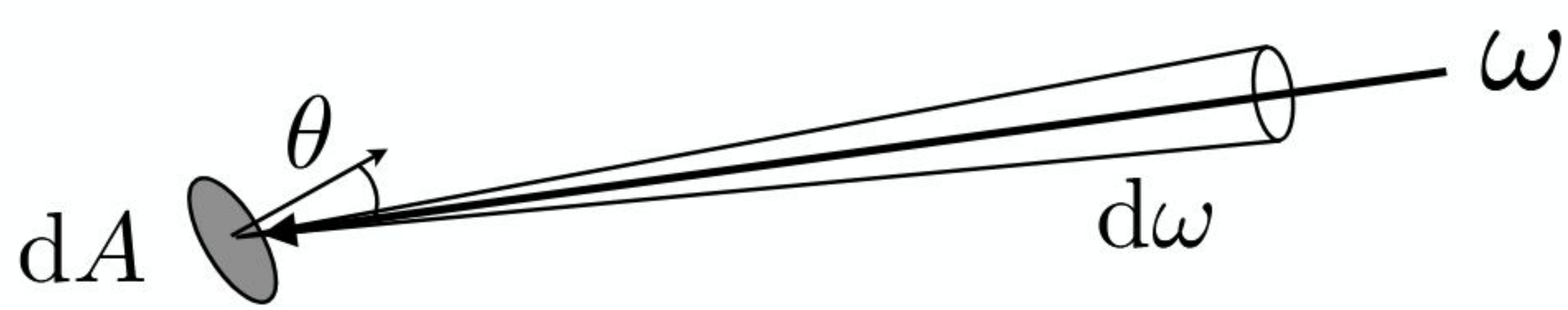
$$L(\mathrm{p},\omega) \equiv \frac{\mathrm{d}^2\Phi(\mathrm{p},\omega)}{\mathrm{d}\omega\,\mathrm{d}A\cos\theta}$$

$\cos\theta$ accounts for projected surface area

$$\left[\frac{\mathrm{W}}{\mathrm{sr\,m}^2}\right]\ \left[\frac{\mathrm{cd}}{\mathrm{m}^2} = \frac{\mathrm{lm}}{\mathrm{sr\,m}^2} = \mathrm{nit}\right]$$

Incident Surface Radiance

Equivalent: Incident surface radiance (luminance) is the irradiance per unit solid angle arriving at the surface.



recall:

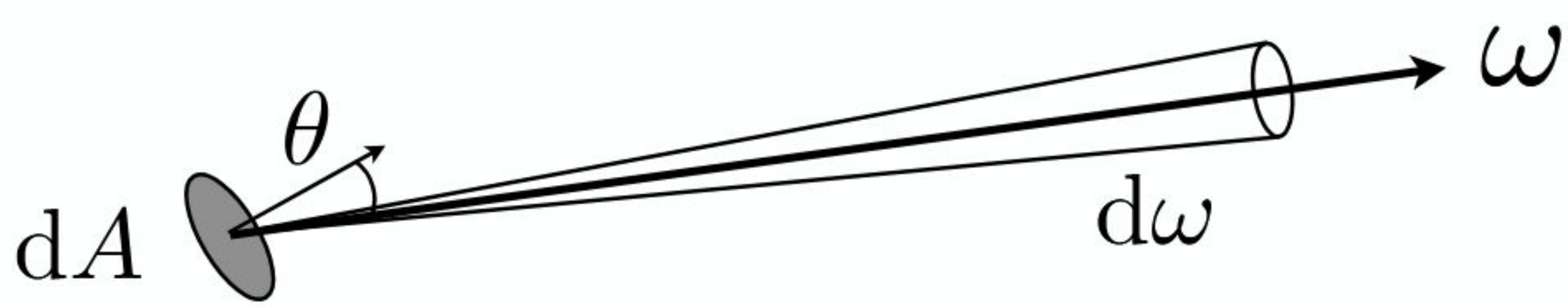
$$E(\mathbf{x}) \equiv \frac{\mathrm{d}\Phi(\mathbf{x})}{\mathrm{d}A}$$

$$L(\mathrm{p}, \omega) = \frac{\mathrm{d}E(\mathrm{p})}{\mathrm{d}\omega \cos\theta}$$

i.e. it is the light arriving at the surface along a given ray (point on surface and incident direction).

Exiting Surface Radiance

Equivalent: Exiting surface radiance (luminance) is the intensity per unit projected area leaving the surface.



recall:

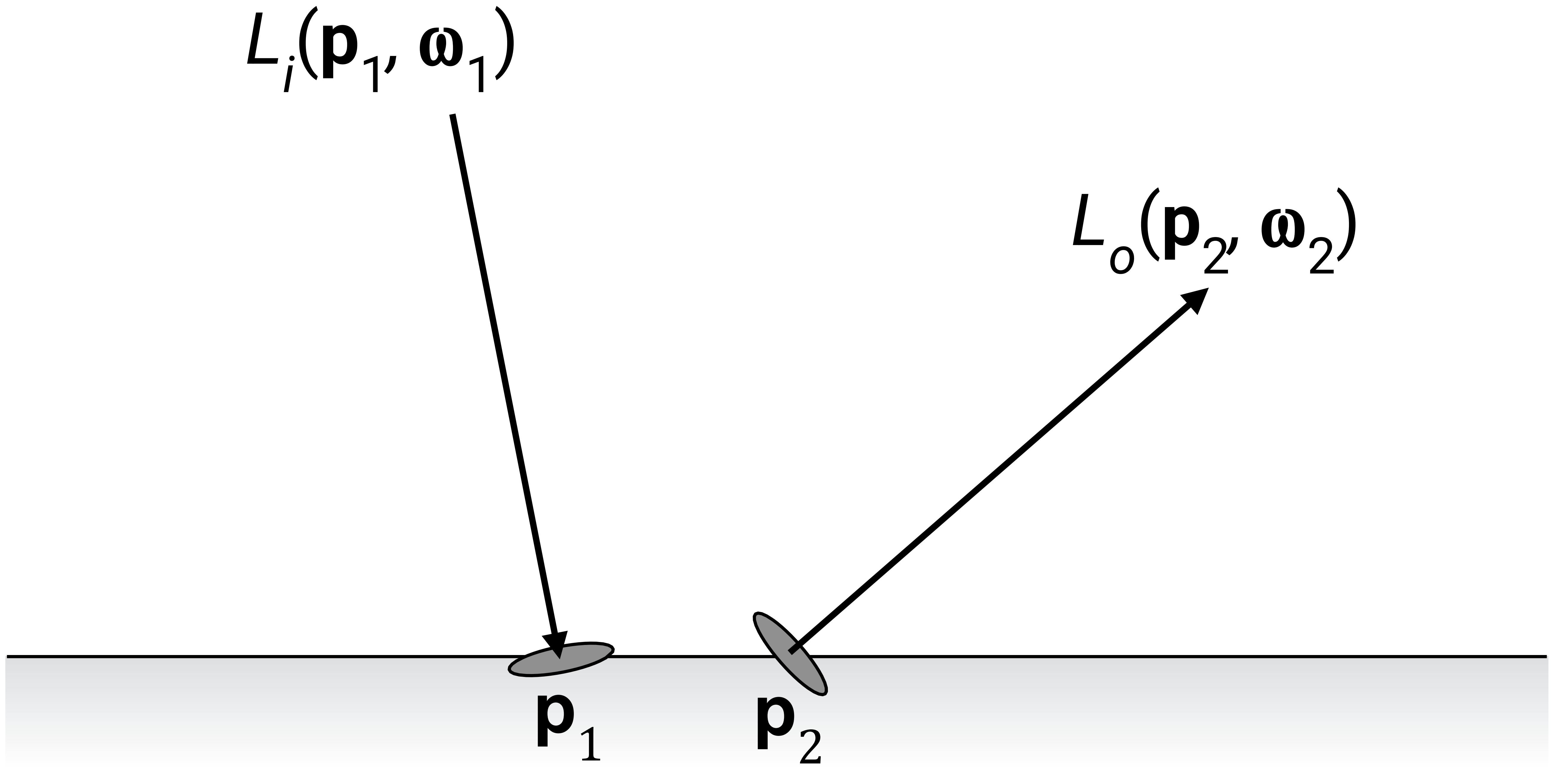
$$I(\omega) \equiv \frac{\mathrm{d}\Phi}{\mathrm{d}\omega}$$

$$L(\mathrm{p}, \omega) = \frac{\mathrm{d}I(\mathrm{p}, \omega)}{\mathrm{d}A \cos\theta}$$

e.g. for an area light it is the light emitted along a given ray (point on surface and exit direction).

Incident & Exiting Surface Radiance Differ!

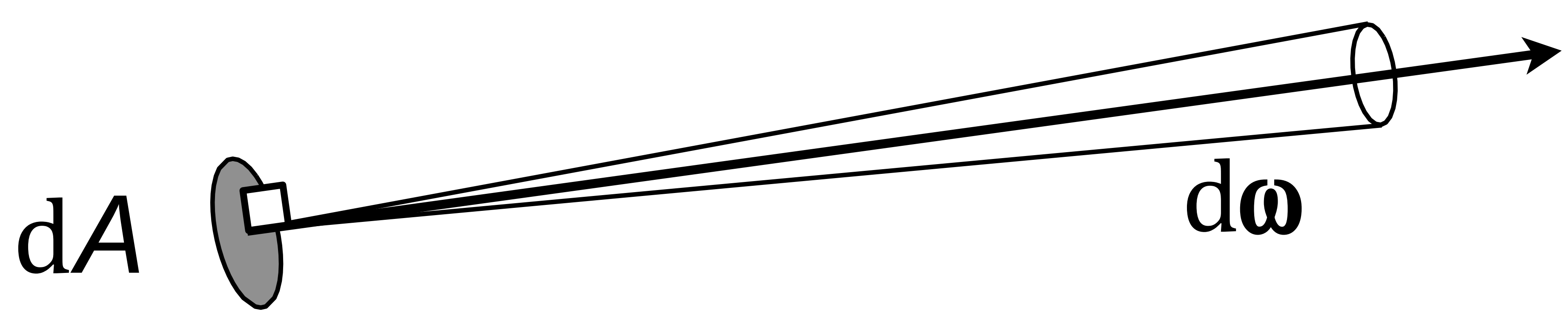
Need to distinguish between incident radiance and exitant radiance functions at a point on a surface



In general: $L_i(\mathbf{p}, \boldsymbol{\omega}) \neq L_o(\mathbf{p}, \boldsymbol{\omega})$

Field Radiance or Light Field

Definition: The field radiance (luminance) at a point in space in a given direction is the power per unit solid angle per unit area perpendicular to the direction.



Typical Values of Luminance [cd/m^2]

Surface of the sun	2,000,000,000 nits
Sunlight clouds	30,000
Clear sky	3,000
Cell Phone display	500
Overcast sky	300
Scene at sunrise	30
Scene lit by moon	0.001
Threshold of vision	0.000001

Calculating with Radiance

Irradiance from the Environment

Computing flux per unit area on surface, due to incoming light from all directions.

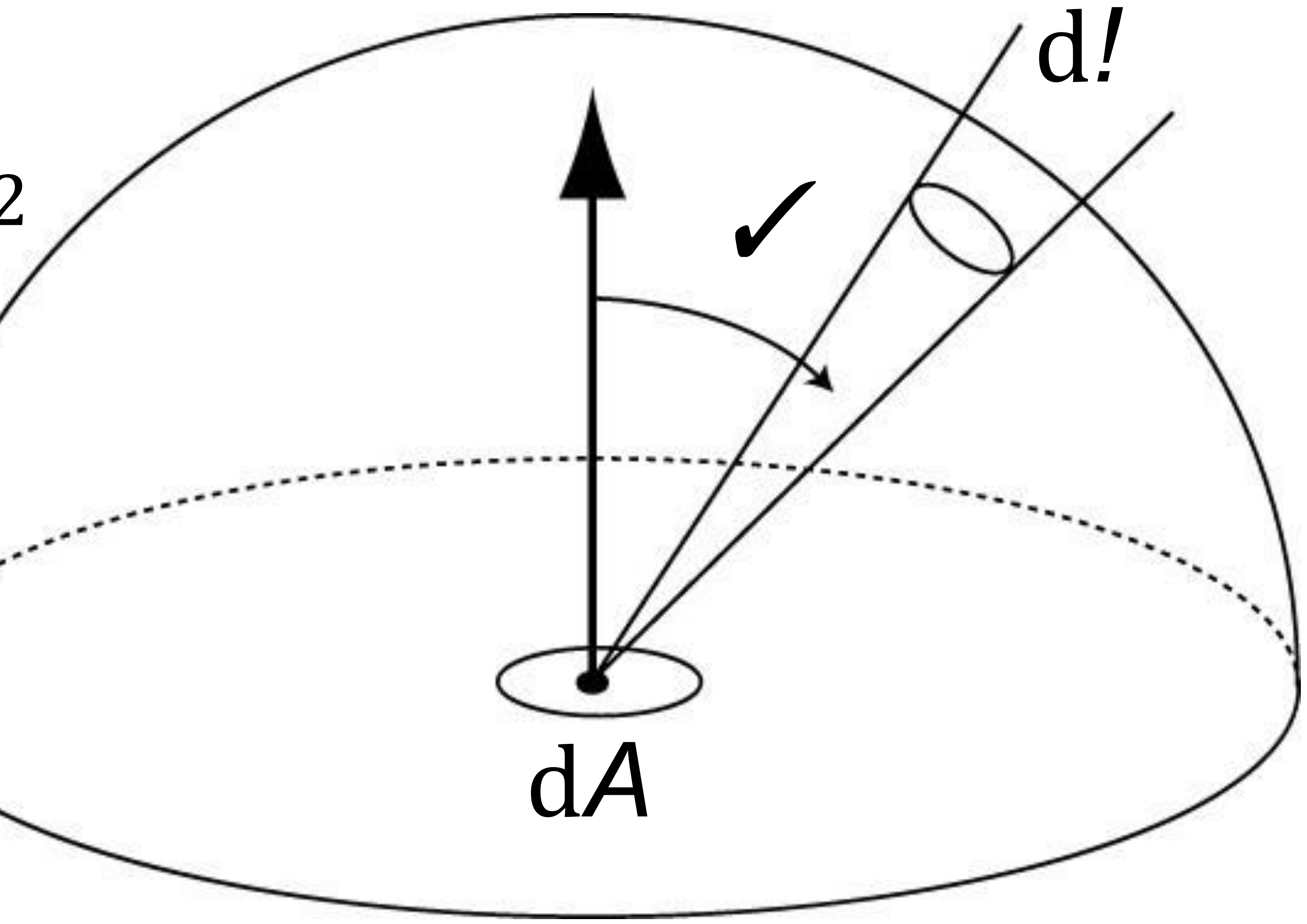
$$dE(\mathrm{p}, \omega) = L_i(\mathrm{p}, \omega) \cos\theta \, \mathrm{d}\omega$$

Contribution to irradiance from light arriving from direction

$$E(\mathrm{p}) = \int_{H^2} L_i(\mathrm{p}, \omega) \cos\theta \, \mathrm{d}\omega$$

Light meter

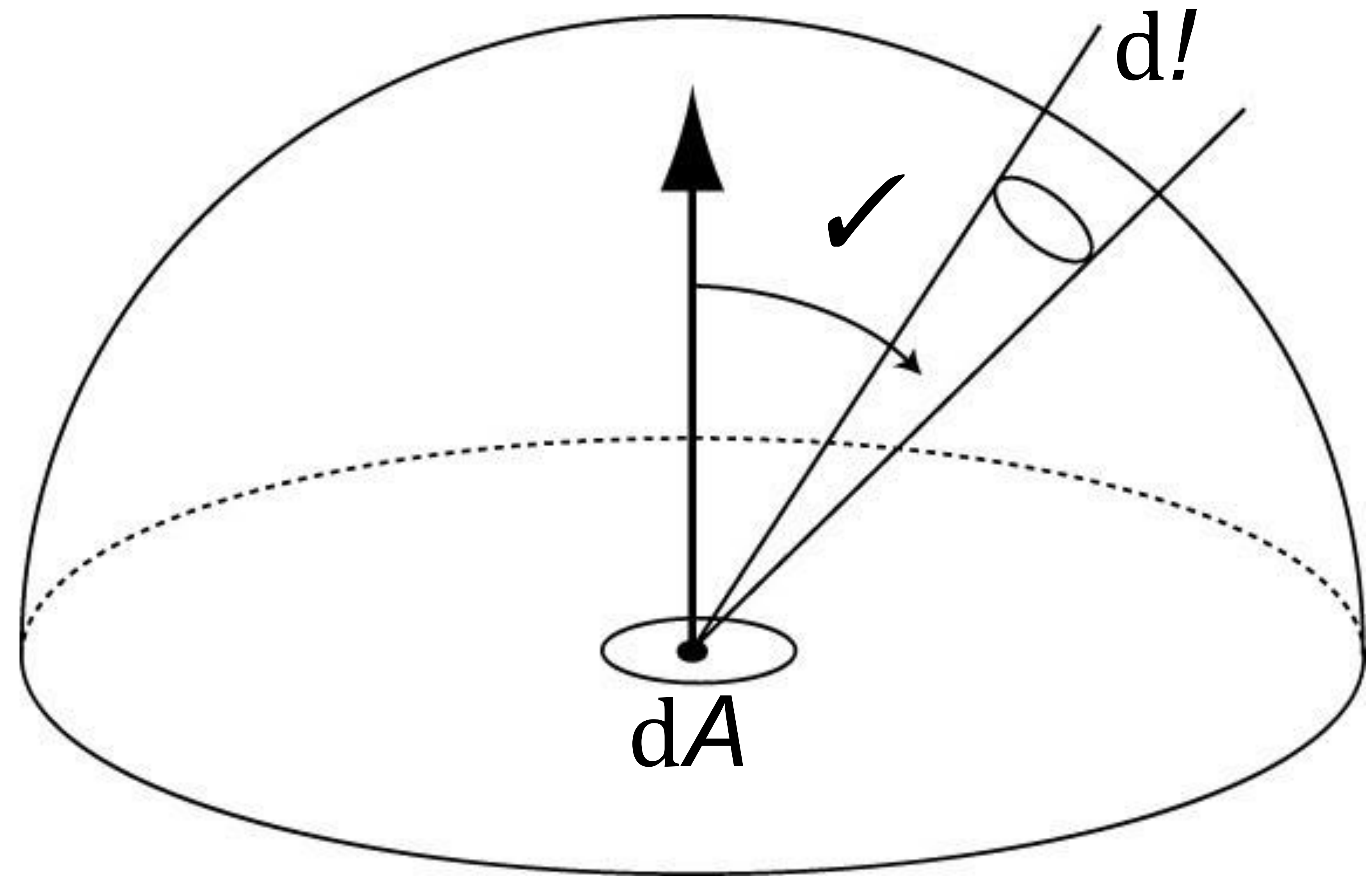
Hemisphere: H^2



Irradiance from Uniform Hemispherical Light

$$
\begin{aligned}
E(\mathrm{p}) &= \int_{H^2} L \cos\theta \, \mathrm{d}\omega \\
&= L \int_0^{2\pi} \int_0^{\frac{\pi}{2}} \cos\theta \sin\theta \, \mathrm{d}\theta \, \mathrm{d}\phi \\
&= L\pi
\end{aligned}
$$

Note: integral of cosine over hemisphere is only 1/2 the area of the hemisphere.



Irradiance from a Uniform Area Source

$$E(\mathrm{p}) = \int_{H^2} L(\mathrm{p}, \omega) \cos\theta \, \mathrm{d}\omega$$

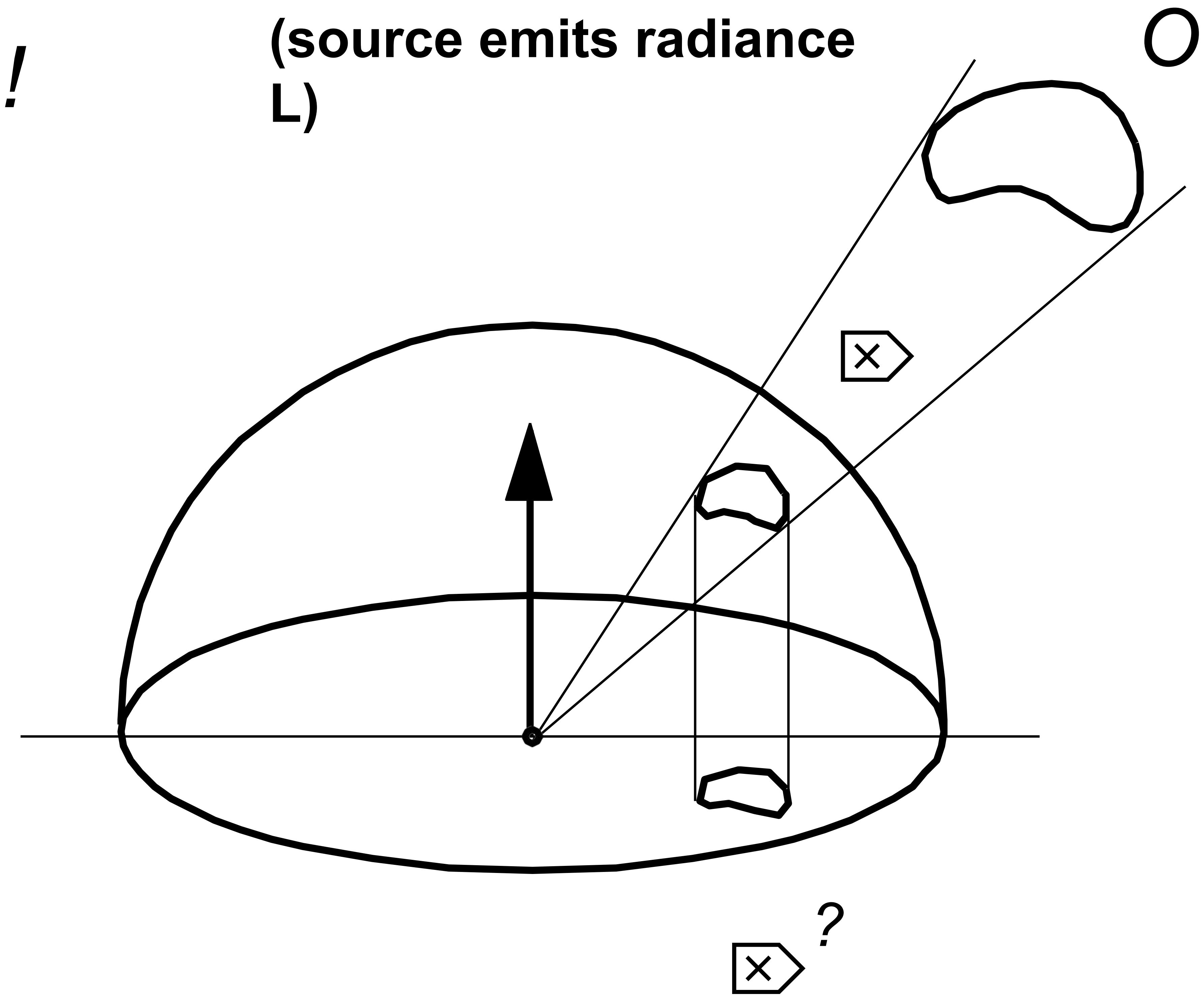
$$= L \int_{\Omega} \cos\theta \, \mathrm{d}\omega$$

$$= L\,\Omega^{\perp}$$

Projected solid angle:

- **Cosine-weighted solid angle**
- **Area of object O projected onto unit sphere, then projected onto plane**

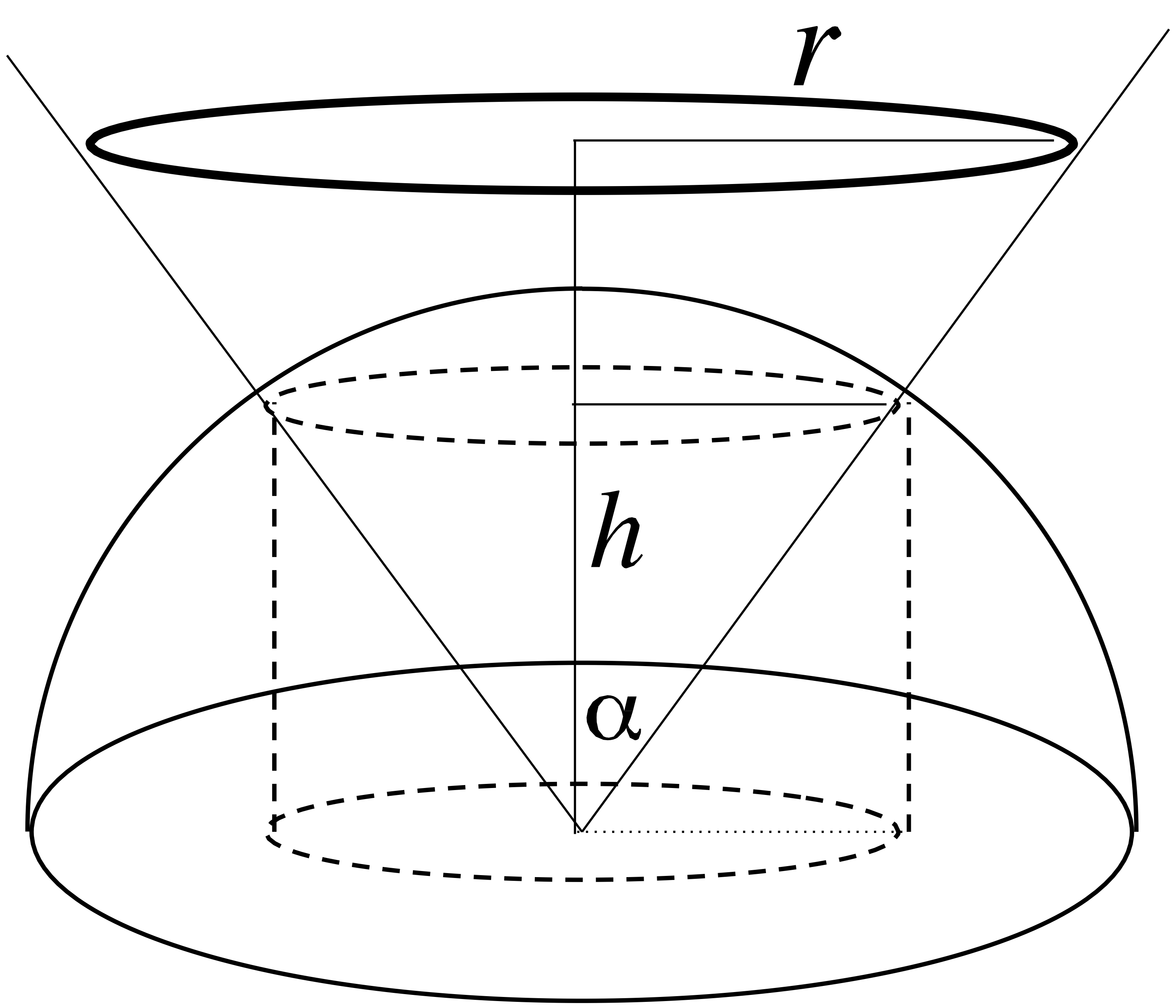
(source emits radiance L)



$$\mathrm{d}\omega^{\perp} = |\cos\theta| \, \mathrm{d}\omega$$

Uniform Disk Source Overhead

Geometric Derivation

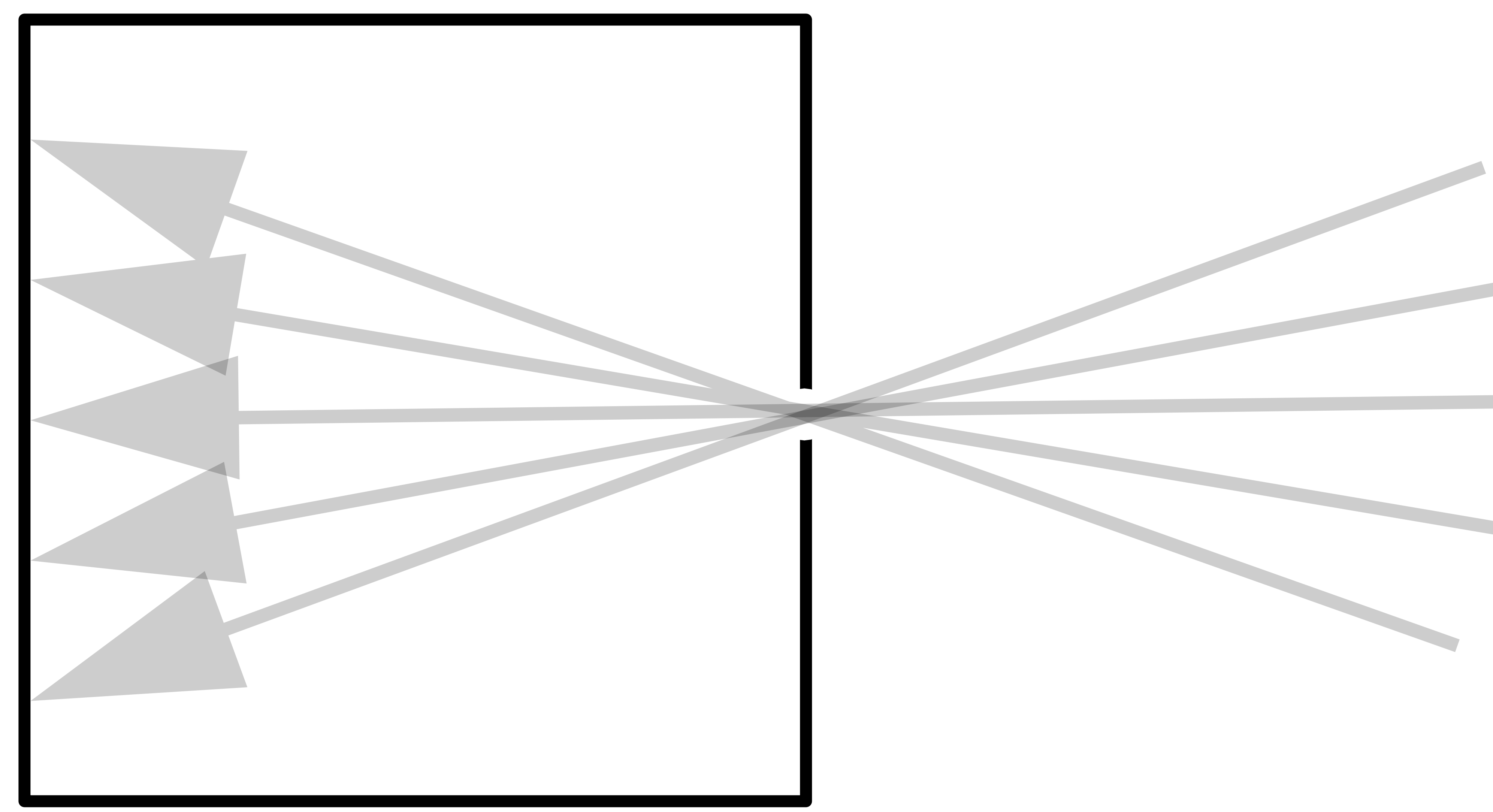


Algebraic Derivation

Measuring Radiance

A Pinhole Camera Samples Radiance

Photograph pixels measure radiance for rays passing through pinhole in different directions

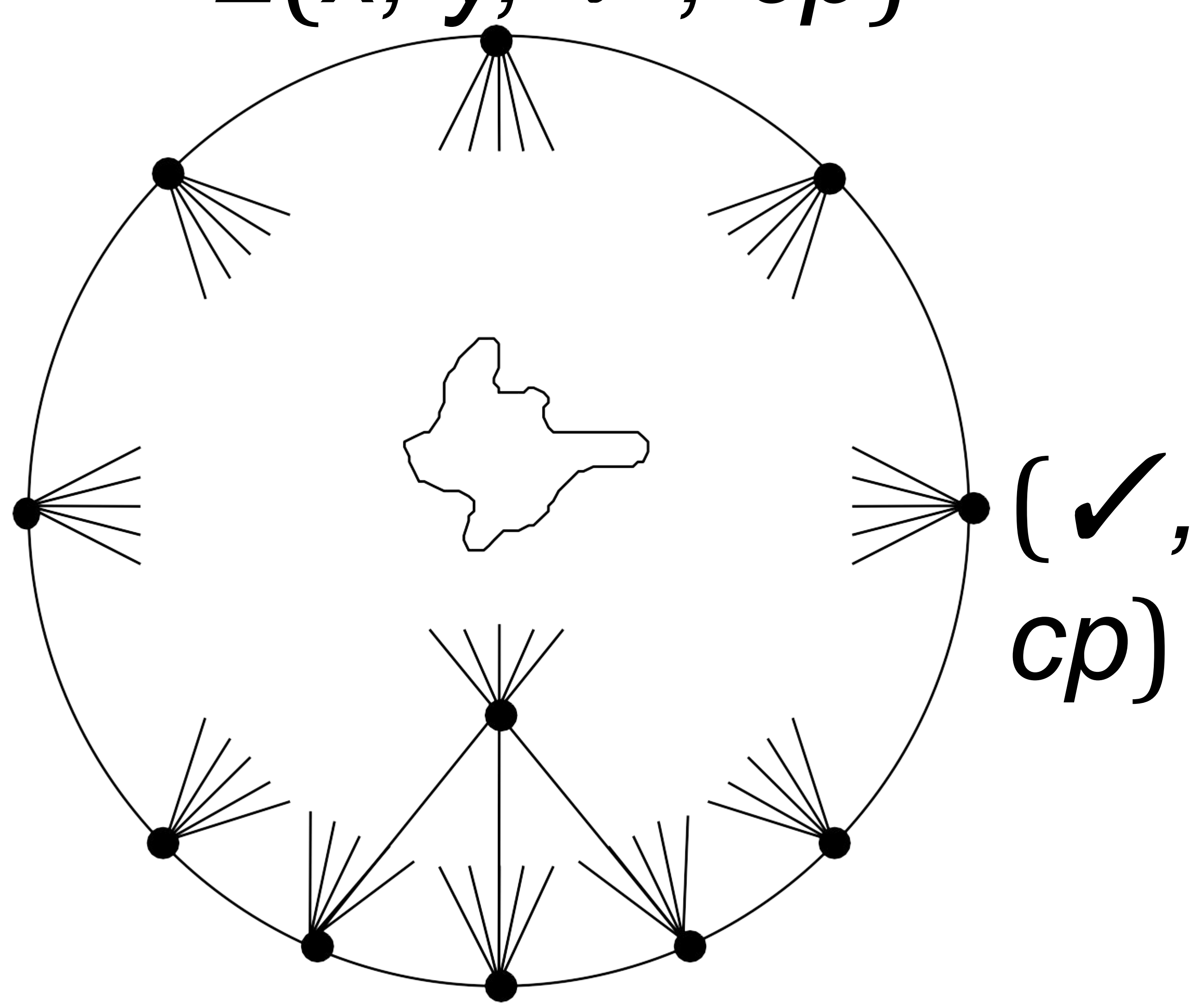


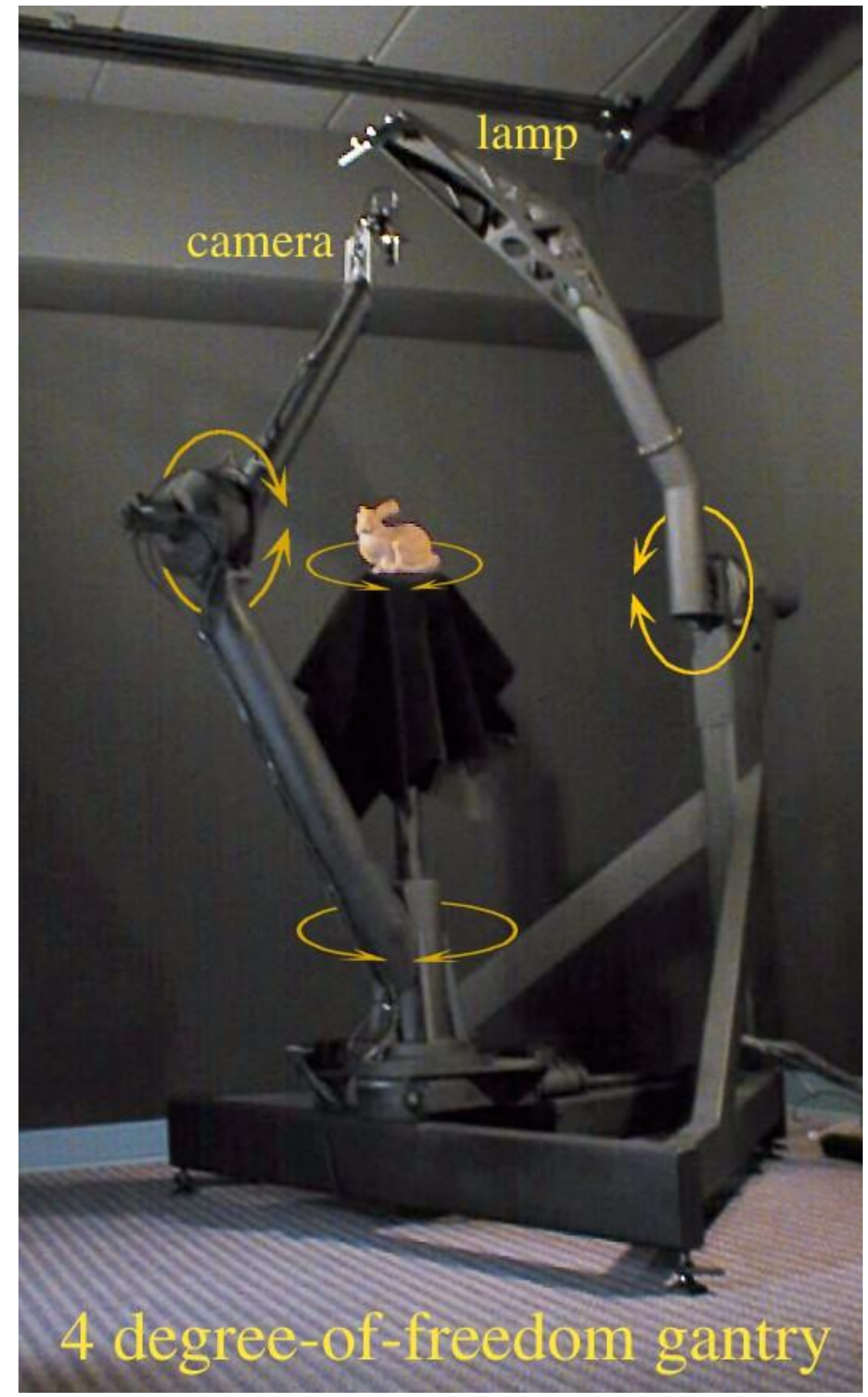
Spherical Gantry ⇒ 4D Light Field

Take photographs of an object from all points on an enclosing sphere

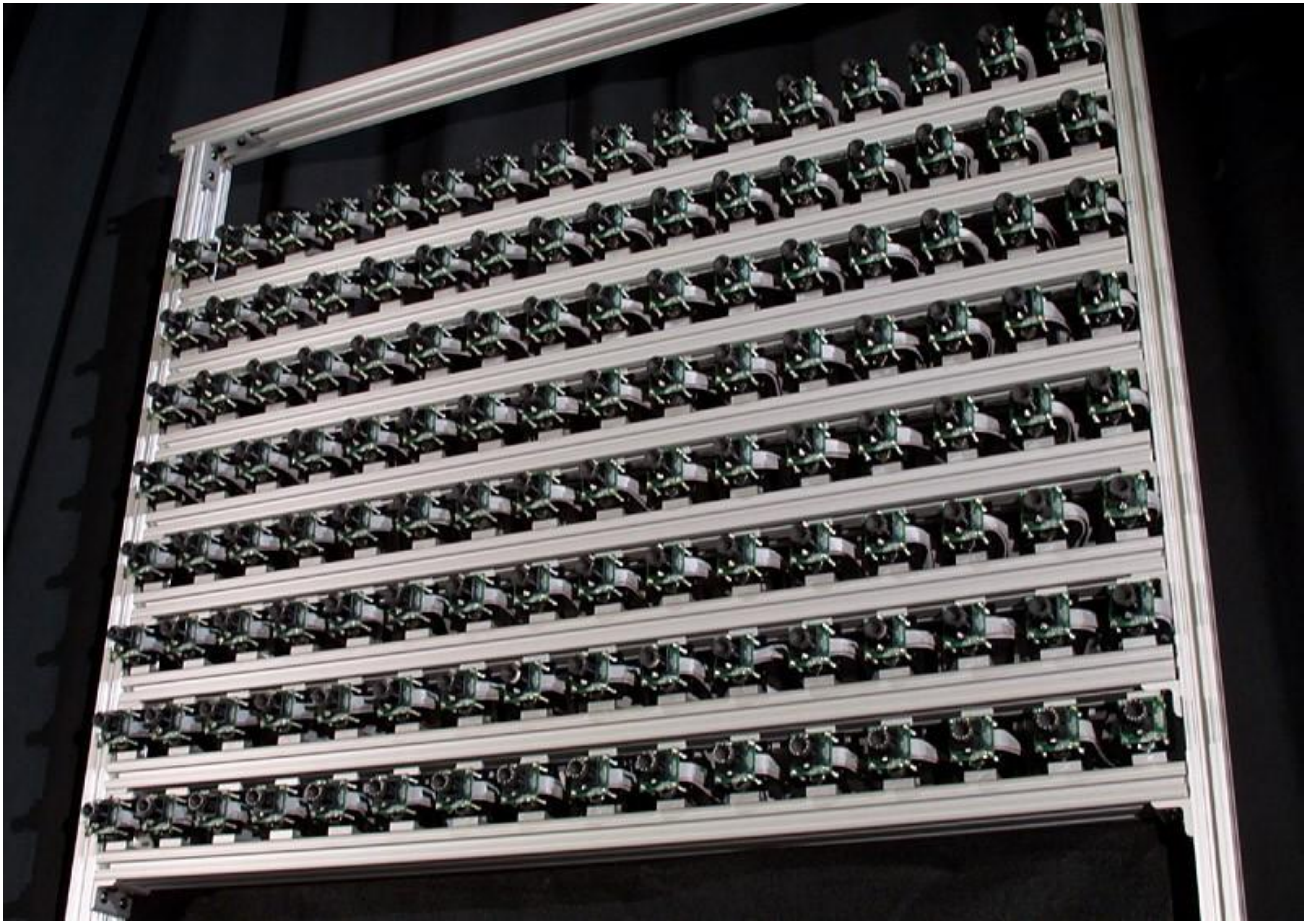
Captures all light leaving an object – like a hologram

$L(x, y, ✓, cp)$

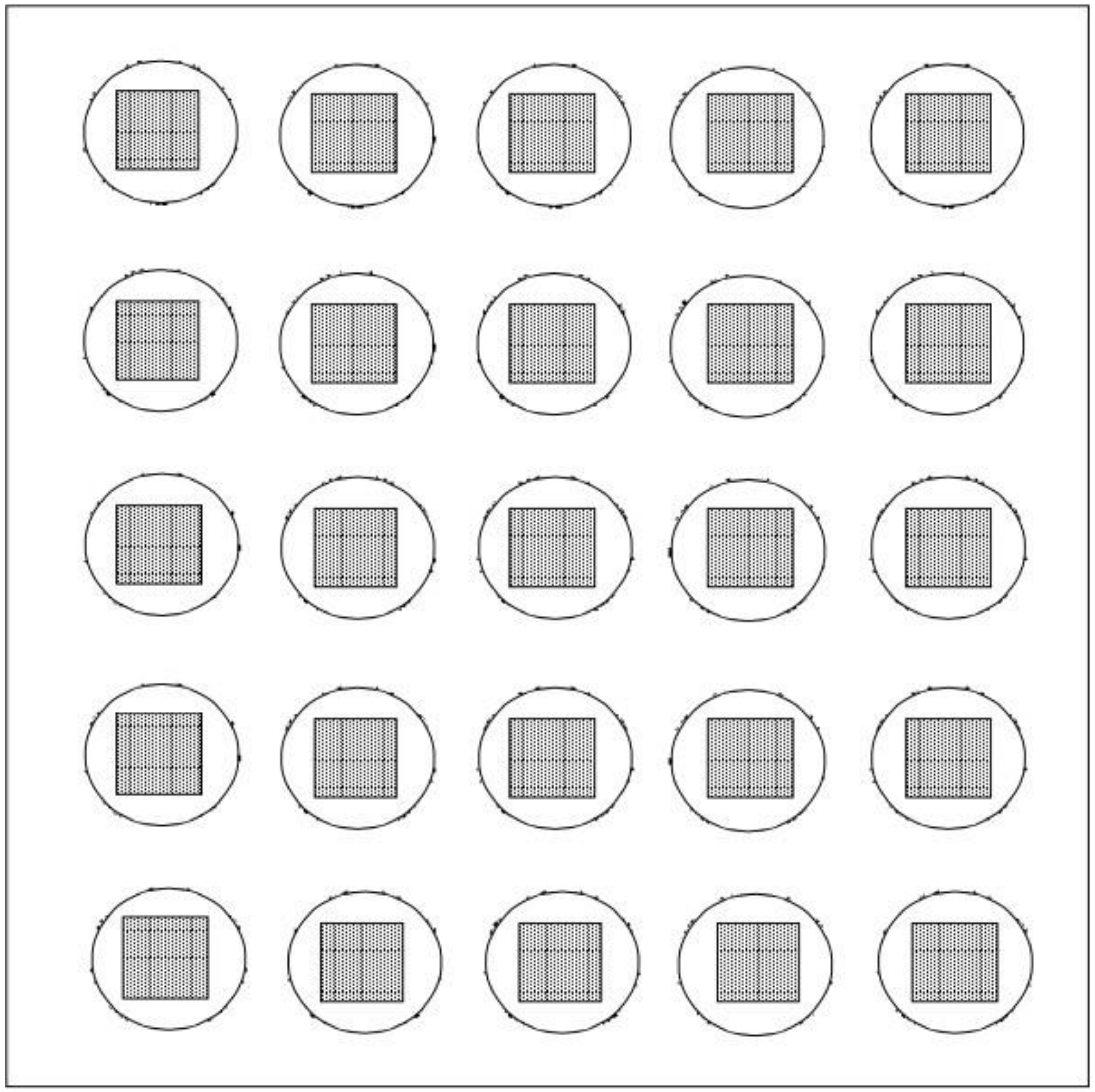




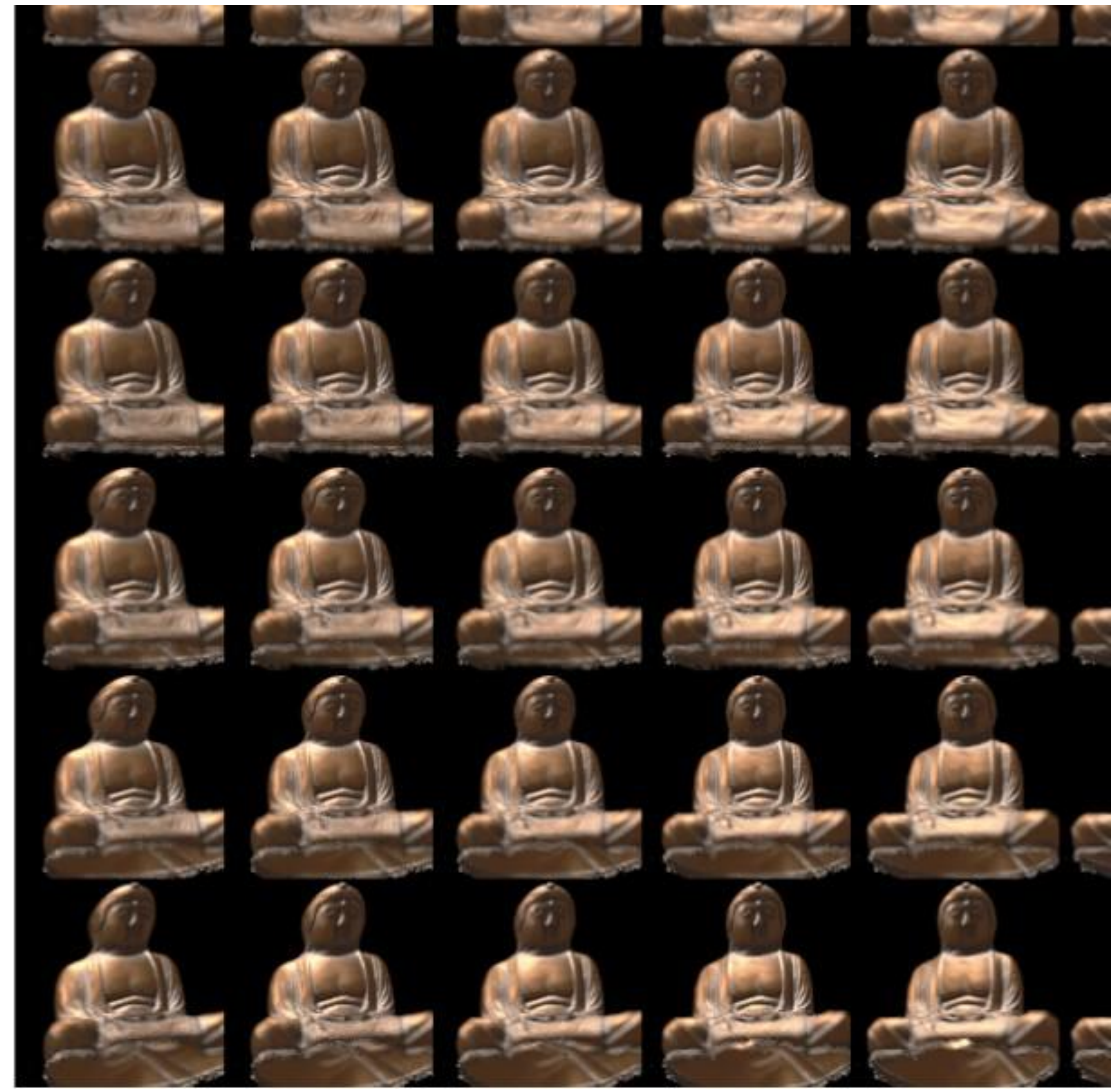
Multi-Camera Array ⇒ 4D Light Field



Two-Plane Light Field



2D Array of Cameras



2D Array of Images

L(x,y,u,v)

Radiometry & Photometry
Terms & Units

Radiometric & Photometric Terms & Units

Physics		Radiometry	Units	Photometry	Units
Energy	Q	Radiant Energy	Joules (W·sec)	Luminous Energy	Lumen·sec
Flux (Power)		Radiant Power	W	Luminous Power	Lumen (Candela sr)
Angular Flux Density		Radiant Intensity	W/sr	Luminous Intensity	Candela (Lumen/sr)
Spatial Flux Density	E	Irradiance (in) Radiosity (out)	W/m^2	Illuminance (in) Luminosity (out)	Lux (Lumen/m^2)
Spatio-Angular Flux Density	L	Radiance	W/m^2/sr	Luminance	Nit (Candela/m^2)

"Thus one nit is one lux per steradian is one candela per square meter is one lumen per square meter per steradian. Got it?" — *James Kajiya*

Things to Remember

- **Radiometry vs photometry: physics vs human**
- **response Spatial measures of light:**
  - Flux, intensity, irradiance, radiance
- **Pinhole cameras and light field cameras**
- **Integration on sphere / hemisphere**
- **Cosine weight: project from hemisphere onto disk**
- **Photon counting**

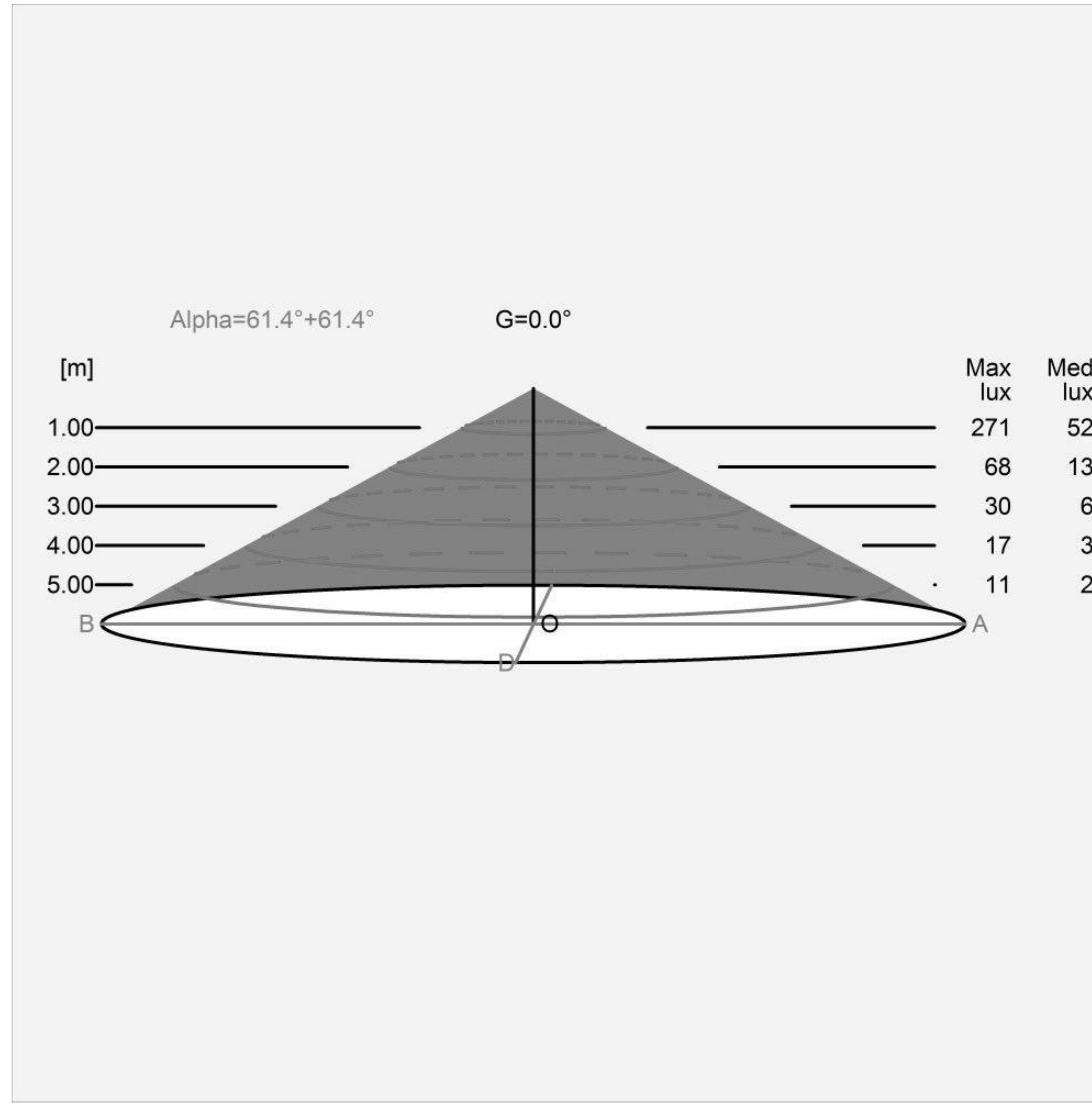
Acknowledgments

Many thanks to Ren Ng, Kayvon Fatahalian, Matt Pharr, Pat Hanrahan, and Steve Marschner for presentation resources.

Extra

Light Fixture Measurements

Poul Henningsen's Artichoke Lamp

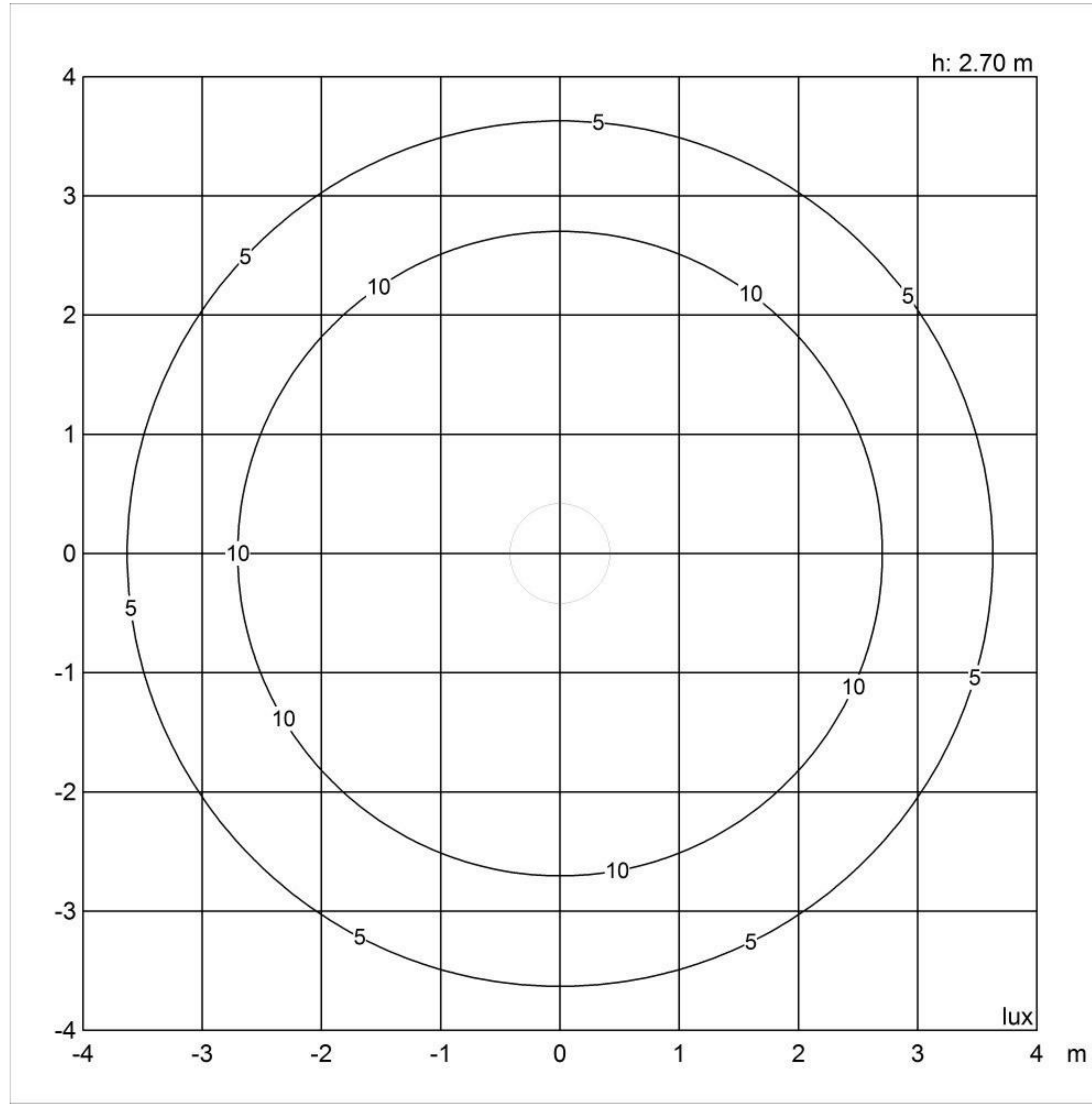


Cartesian Diagram

http://www.louispoulsen.com/

Light Fixture Measurements

Poul Henningsen's Artichoke Lamp



Isolux Diagram

http://www.louispoulsen.com/

Quantitative Photometry

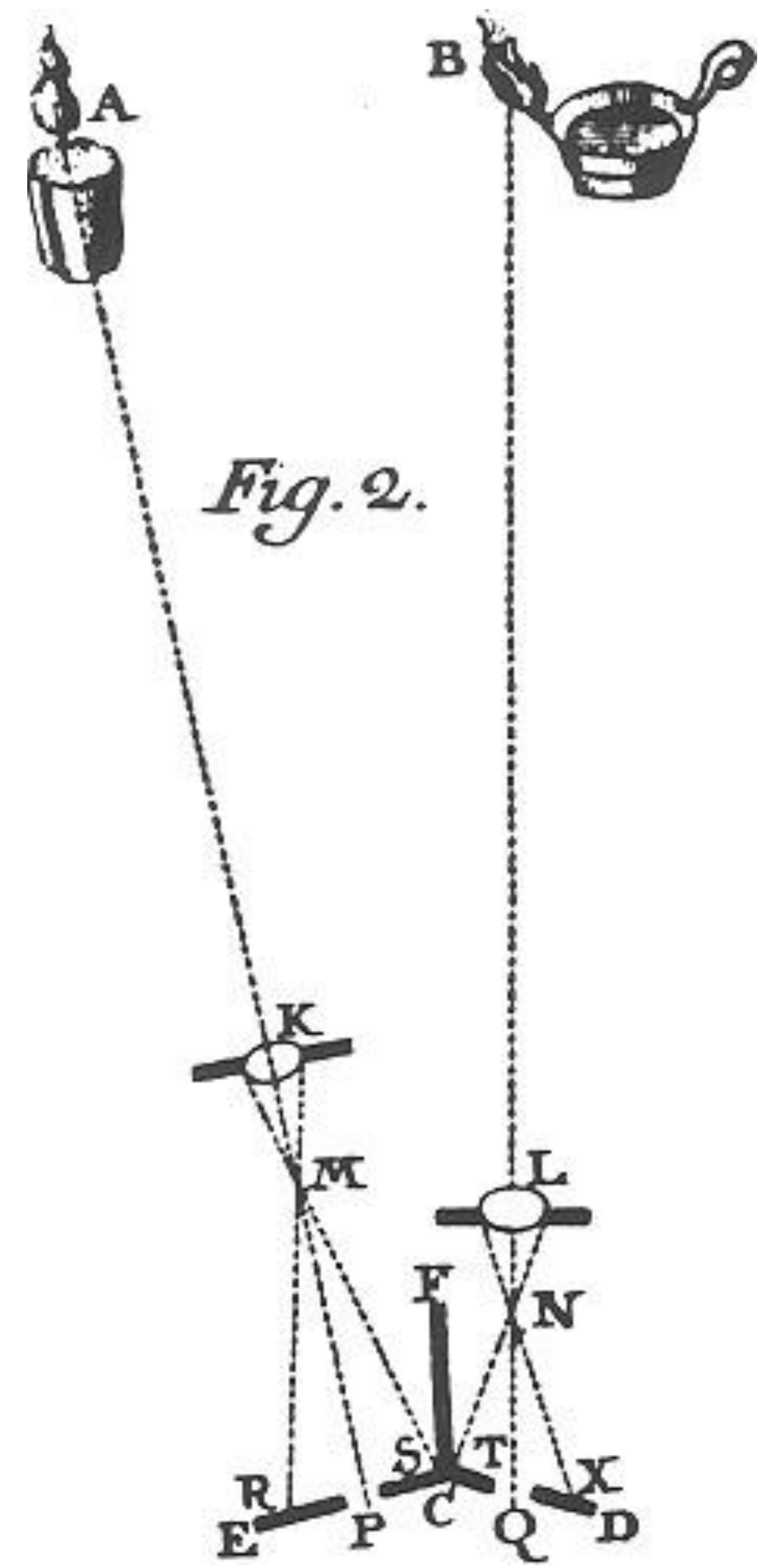
The Invention of Photometry

Bouguer’s classic experiment

- Compare a light source and a candle
- Move until appear equally bright
- Intensity is proportional to ratio of distances squared

Definition of a candela

- Originally a “*standard*” candle
- Currently 555 nm laser with power 1/683 W/sr



Counting Photons

Given a sensor/light, we can count how many photons it receives/emits

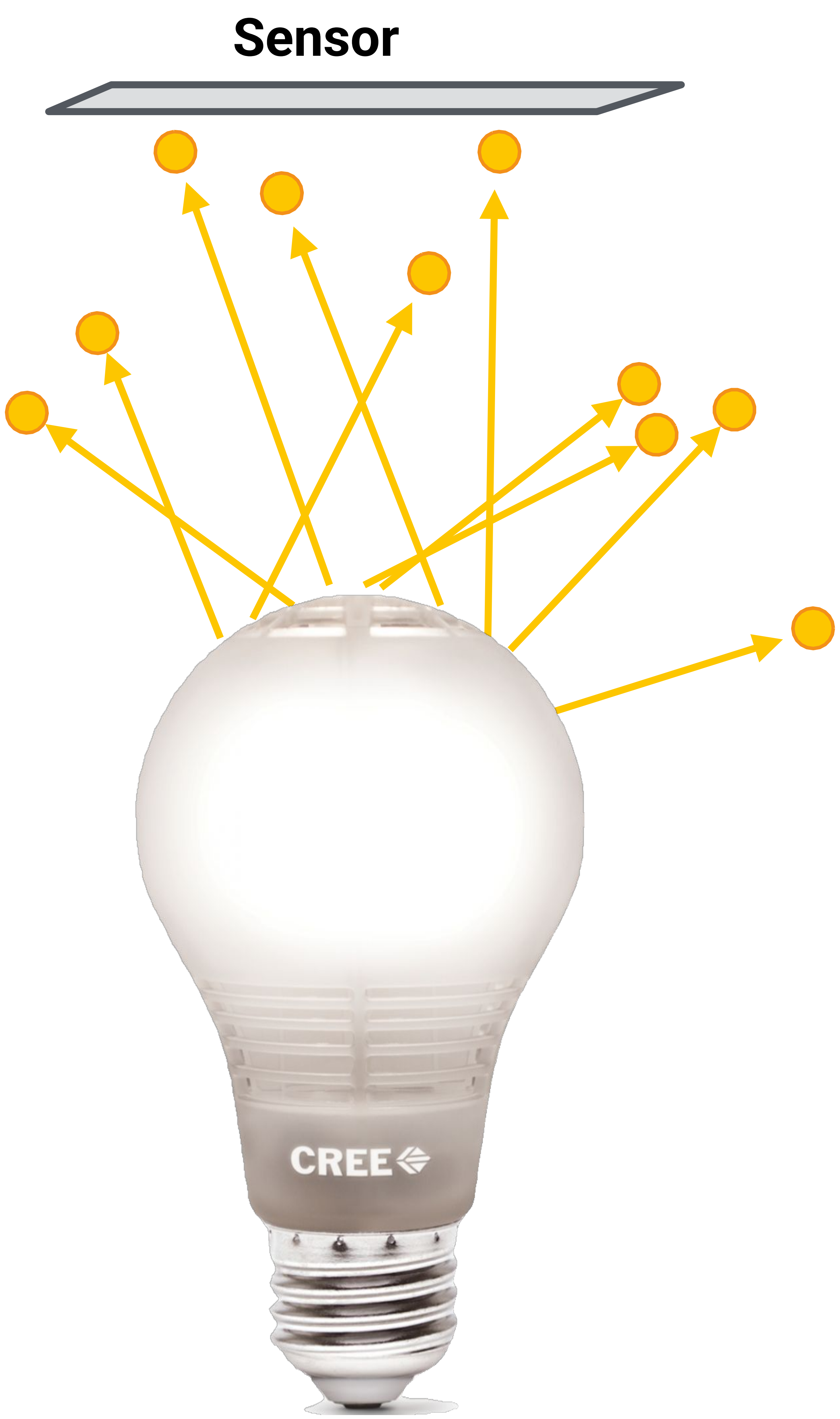
- **Over a period of time, gives the energy Q and flux (power) Φ received/ emitted by the sensor/light**
- **Energy carried by a photon:**

$$Q = \frac{hc}{\lambda}, \text{where} \quad h \approx 6.626 \times 10^{-34} \mathrm{m^2\,kg/s}$$

$$c = 299,792,458 \ \mathrm{m/s}$$

λ = wavelength of photon

- **~ 3.6 E-19J for a 555nm green photon**
- **~ 2.8 E18 green photons for 1W of radiant energy**



Modern LED Light: Estimate Efficiency?

Input power: 11 W

Output: 815 lumens
(~80 lumens / Watt)

Incandescent bulb?
Input power: 60W
Output: ~700 lumens
(~12 lumens / Watt)

Modern LED Light: Estimate Efficiency?

Input power: 11 W
If all power into light with 555nm average wavelength, get 3.1E19 photons/s

Intensity rating is 815 lumens, equivalent to 555nm laser at 815/683W. If average wavelength is 555nm, get 3.3E18 photons/s.

Efficiency*:
3.3E18/3.1E19 = 11%