Triangles and Transforms 2

CS 184: Foundations of Computer Graphics

1 Rasterization Potpourri

1. What is the connection between rasterizing a polygon and discretizing a continuous function?

2. Why does aliasing occur? Describe or draw an example of aliasing of any kind.

3. What is the connection between applying a box blur to an image and supersampling each pixel within the image?

2 Tying Shoelaces

The cross product between two vectors, **a** and **b**, in three-dimensional space is given by:

$$\mathbf{a} \times \mathbf{b} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

1. Given two vectors, $\mathbf{a} = \begin{pmatrix} 10 \\ 7 \\ 0 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} -2 \\ -11 \\ 0 \end{pmatrix}$, calculate $\mathbf{a} \times \mathbf{b}$. Next, calculate $\mathbf{b} \times \mathbf{a}$.

2. Draw the triangle given by points $(2, -1)$, $(12, 6)$, $(10, -5)$. What is the *winding order* of the triangle? In other words, are the points given in clockwise or counter-clockwise order?

3. What are the three vectors defined by the edges of this triangle? Assume the triangle is lying in the xy-plane. Select two of the three vectors. Calculate their cross product.

4. In general, given triangle vertices $\mathbf{p}_0$, $\mathbf{p}_1$, $\mathbf{p}_2$, how can winding order be determined?

3 Basic Transforms

1. Isometric transformations preserve the distances between every pair of points on an object. Which transformations are isometric? Which transformations aren't?

2. Using homogeneous coordinates, define a point, $\mathbf{p} = \begin{pmatrix} 2 \\ 5 \\ 1 \end{pmatrix}$, a vector, $\mathbf{v} = \begin{pmatrix} 2 \\ 5 \\ 0 \end{pmatrix}$, and a translation transformation, $\mathbf{T} = \begin{pmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{pmatrix}$. What is the result of applying $\mathbf{T}$ to $\mathbf{p}$?

3. What is the result of applying $\mathbf{T}$ to $\mathbf{v}$? Justify this result.

4. Write the transformation matrix for a 2D object that is reflected across the y-axis, translated up by 1 unit and right by 3 units, then rotated around the origin by $90°$ counterclockwise.

4 Rebecca Takes a Look

In world space coordinates, a teapot sits at $(0, 0, 0)$. Rebecca holds her camera at position $(1, 0, 2)$ — the eye point, **e**, of the camera.

1. Rebecca points her camera at the teapot, intending to take a picture. What is the view direction, **v**, of her camera?

2. In world space coordinates, in what direction does the positive z-axis of Rebecca's "standard" camera space point?

3. The up vector, **u**, points in the direction of the positive y-axis of "standard" camera space. In world space coordinates, $\mathbf{u} = \begin{pmatrix}0 & 1 & 0\end{pmatrix}^\intercal$. Calculate the right vector, **r**, which points in the direction of the positive x-axis of "standard" camera space.

4. Write the matrix that transforms coordinates from "standard" camera space to world space. The inverse of this matrix is the *Look-At* matrix. What coordinate system transformation does the *Look-At* matrix perform?