Discussion 06

Ray Tracing, Radiometry, & Photometry

Computer Graphics and Imaging
UC Berkeley CS 184/284A

Week 6 Announcements

- Homework 2 is due on Tuesday 3/4.
    - As a whole, you have 8 slip days that are automatically applied.
    - Attend Homework 2 Parties! Please 🥺
- Updated discussion times.
- Participation tracker released on Gradescope.

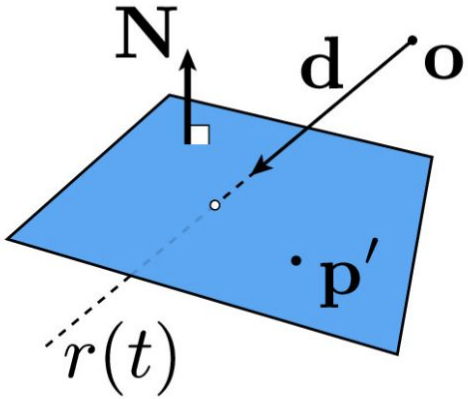
Date	Time	Location
Wed 2/26	5PM - 7PM	Soda 326
Fri 2/28	10AM - 12PM	Soda 310
Mon 3/3	11AM - 1PM	Cory 299
Mon 3/3	5PM - 7PM	Soda 380
Tue 3/4	5PM - 7PM	Berkeley Way West 1104

Ray Triangle Intersection

Last week: Ray-Plane Intersection

Ray equation:

$$\mathbf{r}(t) = \mathbf{o} + t\,\mathbf{d},\ \ 0 \leq t < \infty$$



Plane equation:

$$\mathbf{p} : (\mathbf{p} - \mathbf{p}') \cdot \mathbf{N} = 0$$

Solve for intersection

Set $\mathbf{p} = \mathbf{r}(t)$ and solve for t

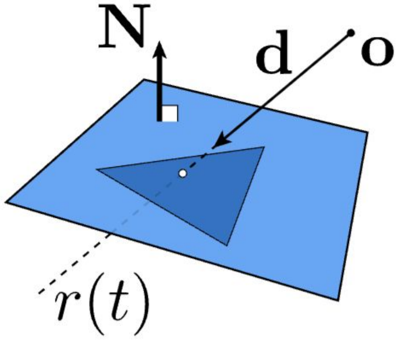
$$(\mathbf{p} - \mathbf{p}') \cdot \mathbf{N} = (\mathbf{o} + t\,\mathbf{d} - \mathbf{p}') \cdot \mathbf{N} = 0$$

$$t = \frac{(\mathbf{p}' - \mathbf{o}) \cdot \mathbf{N}}{\mathbf{d} \cdot \mathbf{N}}$$

Check: $0 \leq t < \infty$

Ray Triangle Intersection

But meshes are made of triangles, so we need ray-triangle intersection!



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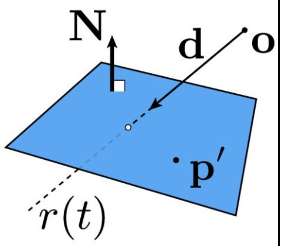

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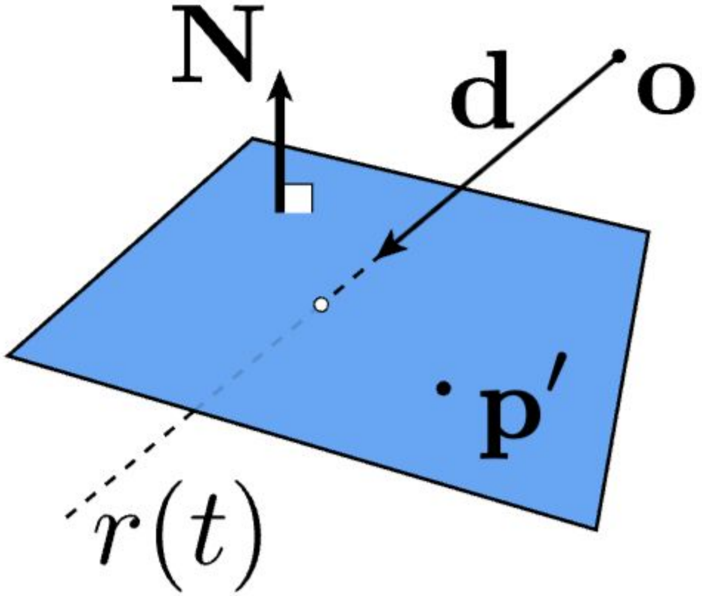
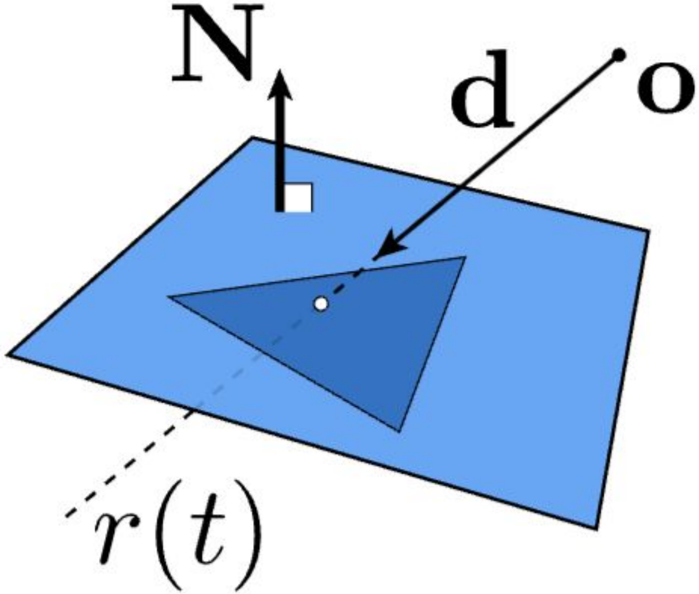
Ray Triangle Intersection





Ray Triangle Intersection Derivation

Given a mesh representation of an object, we would like to render it onto a display. To do so, we need to know which parts of the object are visible, where to put shadows, how to apply the scene's lighting, and more. The simplest idea to handle these problems is to take a ray and intersect it with each triangle in the mesh.

Recall that a ray is defined by its origin $\mathbf{O}$ and a direction vector $\mathbf{D}$ and varies with "time" t for $0 \leq t < \infty$.

$$\mathbf{r}(t) = \mathbf{O} + t\mathbf{D}. \tag{1}$$

Recall that a point within a triangle $\mathbf{P_0 P_1 P_2}$ can be represented as

$$\mathbf{P} = \alpha \mathbf{P}_0 + \beta \mathbf{P}_1 + \gamma \mathbf{P}_2, \tag{2}$$

where $\alpha + \beta + \gamma = 1$. Defining $b_1 = \beta$ and $b_2 = \gamma$, we obtain $\alpha = 1 - b_1 - b_2$. Thus, we can rewrite the point $\mathbf{P}$ in barycentric coordinates as:

$$\mathbf{P} = (1 - b_1 - b_2)\mathbf{P}_0 + b_1 \mathbf{P}_1 + b_2 \mathbf{P}_2. \tag{3}$$

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1. Let's solve for the intersection of a ray and a triangle. Specifically, if we arrange the unknowns t, b_1 and b_2 into a column vector $\mathbf{x} = [t, b_1, b_2]^T$, can you get a matrix $\mathbf{M}$ and a column vector $\mathbf{b}$ so that $\mathbf{M}\mathbf{x} = \mathbf{b}$?

$$\mathbf{P} = (1 - b_1 - b_2)\mathbf{P}_0 + b_1\mathbf{P}_1 + b_2\mathbf{P}_2. \tag{3}$$

1. Let's solve for the intersection of a ray and a triangle. Specifically, if we arrange the unknowns t, b_1 and b_2 into a column vector $\mathbf{x} = [t, b_1, b_2]^T$, can you get a matrix $\mathbf{M}$ and a column vector $\mathbf{b}$ so that $\mathbf{Mx} = \mathbf{b}$?

Solution:

Since the intersection is both along the ray and on the triangle, we have

$$\begin{aligned}
\mathbf{P} &= (1 - b_1 - b_2)\mathbf{P}_0 + b_1\mathbf{P}_1 + b_2\mathbf{P}_2 \\
&= \mathbf{P}_0 - b_1\mathbf{P}_0 - b_2\mathbf{P}_0 + b_1\mathbf{P}_1 + b_2\mathbf{P}_2 \qquad \text{(Expanding terms)} \\
\mathbf{O} + t\mathbf{D} &= \mathbf{P}_0 + b_1(\mathbf{P}_1 - \mathbf{P}_0) + b_2(\mathbf{P}_2 - \mathbf{P}_0) \qquad \text{(Plugging in } \mathbf{P} = \mathbf{O} + t\mathbf{D}\text{)}
\end{aligned} \tag{4}$$

Thus,

$$\mathbf{O} - \mathbf{P_0} = -t\mathbf{D} + b_1(\mathbf{P_1} - \mathbf{P_0}) + b_2(\mathbf{P_2} - \mathbf{P_0}). \tag{5}$$

Writing it in matrix form, we have

$$\begin{bmatrix} -\mathbf{D} & \mathbf{P_1} - \mathbf{P_0} & \mathbf{P_2} - \mathbf{P_0} \end{bmatrix} \begin{bmatrix} t \\ b_1 \\ b_2 \end{bmatrix} = \mathbf{O} - \mathbf{P_0} \tag{6}$$

So, we set $\mathbf{M} = [-\mathbf{D}, \mathbf{P_1} - \mathbf{P_0}, \mathbf{P_2} - \mathbf{P_0}]$, and $\mathbf{b} = \mathbf{O} - \mathbf{P_0}$.

2. Now let's derive the **Möller-Trumbore algorithm**!

$$\begin{bmatrix} t \\ b_1 \\ b_2 \end{bmatrix} = \frac{1}{\mathbf{S_1} \cdot \mathbf{E_1}} \begin{bmatrix} \mathbf{S_2} \cdot \mathbf{E_2} \\ \mathbf{S_1} \cdot \mathbf{S} \\ \mathbf{S_2} \cdot \mathbf{D} \end{bmatrix} \tag{7}$$

where $\mathbf{E_1} = \mathbf{P_1} - \mathbf{P_0}, \mathbf{E_2} = \mathbf{P_2} - \mathbf{P_0}, \mathbf{S} = \mathbf{O} - \mathbf{P_0}, \mathbf{S_1} = \mathbf{D} \times \mathbf{E_2}, \mathbf{S_2} = \mathbf{S} \times \mathbf{E_1}$.

Hint 1: (Cramer's rule) Linear equations $\mathbf{Mx} = \mathbf{b}$ can be simply solved using determinants of matrices as:

$$\mathbf{x} = \frac{1}{|\mathbf{M}|} \begin{bmatrix} |\mathbf{M_1}| \\ |\mathbf{M_2}| \\ |\mathbf{M_3}| \end{bmatrix}, \tag{8}$$

where $\mathbf{M_i}$ is the matrix $\mathbf{M}$ with its i-th column replaced by $\mathbf{b}$.

Hint 2: Suppose $\mathbf{A}$, $\mathbf{B}$, $\mathbf{C}$ are column vectors, the determinant of the 3×3 matrix $[\mathbf{A}, \mathbf{B}, \mathbf{C}]$ satisfy:

$$|\mathbf{A}, \mathbf{B}, \mathbf{C}| = -(\mathbf{A} \times \mathbf{C}) \cdot \mathbf{B} = -(\mathbf{C} \times \mathbf{B}) \cdot \mathbf{A} = -(\mathbf{B} \times \mathbf{A}) \cdot \mathbf{C}. \tag{9}$$

2. Now let's derive the **Möller-Trumbore algorithm**!

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where $\mathbf{E_1} = \mathbf{P_1} - \mathbf{P_0}, \mathbf{E_2} = \mathbf{P_2} - \mathbf{P_0}, \mathbf{S} = \mathbf{O} - \mathbf{P_0}, \mathbf{S_1} = \mathbf{D} \times \mathbf{E_2}, \mathbf{S_2} = \mathbf{S} \times \mathbf{E_1}$.

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Solution: Applying Cramer's rule, we immediately have

$$\begin{bmatrix} t \\ b_1 \\ b_2 \end{bmatrix} = \frac{1}{|\mathbf{M}|} \begin{bmatrix} |\mathbf{M_1}| \\ |\mathbf{M_2}| \\ |\mathbf{M_3}| \end{bmatrix} \tag{10}$$

$$= \frac{1}{|\ -\mathbf{D} \quad \mathbf{P_1} - \mathbf{P_0} \quad \mathbf{P_2} - \mathbf{P_0}\ |} \begin{bmatrix} |\ \mathbf{O} - \mathbf{P_0} \quad \mathbf{P_1} - \mathbf{P_0} \quad \mathbf{P_2} - \mathbf{P_0}\ | \\ |\ -\mathbf{D} \quad \mathbf{O} - \mathbf{P_0} \quad \mathbf{P_2} - \mathbf{P_0}\ | \\ |\ -\mathbf{D} \quad \mathbf{P_1} - \mathbf{P_0} \quad \mathbf{O} - \mathbf{P_0}\ | \end{bmatrix} \tag{11}$$

$$= \frac{1}{|\ -\mathbf{D} \quad \mathbf{E_1} \quad \mathbf{E_2}\ |} \begin{bmatrix} |\ \mathbf{S} \quad \mathbf{E_1} \quad \mathbf{E_2}\ | \\ |\ -\mathbf{D} \quad \mathbf{S} \quad \mathbf{E_2}\ | \\ |\ -\mathbf{D} \quad \mathbf{E_1} \quad \mathbf{S}\ | \end{bmatrix} \tag{12}$$

Now let's take a look at these determinants, we have

$$|\ -\mathbf{D} \quad \mathbf{E_1} \quad \mathbf{E_2}\ | = -(-\mathbf{D} \times \mathbf{E_2}) \cdot \mathbf{E_1} = \mathbf{S_1} \cdot \mathbf{E_1}, \tag{13}$$

$$|\ \mathbf{S} \quad \mathbf{E_1} \quad \mathbf{E_2}\ | = -(\mathbf{E_1} \times \mathbf{S}) \cdot \mathbf{E_2} = \mathbf{S_2} \cdot \mathbf{E_2}, \tag{14}$$

$$|\ -\mathbf{D} \quad \mathbf{S} \quad \mathbf{E_2}\ | = -(-\mathbf{D} \times \mathbf{E_2}) \cdot \mathbf{S} = \mathbf{S_1} \cdot \mathbf{S}, \tag{15}$$

$$|\ -\mathbf{D} \quad \mathbf{E_1} \quad \mathbf{S}\ | = -(\mathbf{S} \times \mathbf{E_1}) \cdot -\mathbf{D} = \mathbf{S_2} \cdot \mathbf{D}. \tag{16}$$

Plugging these back in, we have the Möller-Trumbore algorithm!

$$\begin{bmatrix} t \\ b_1 \\ b_2 \end{bmatrix} = \frac{1}{\mathbf{S_1} \cdot \mathbf{E_1}} \begin{bmatrix} \mathbf{S_2} \cdot \mathbf{E_2} \\ \mathbf{S_1} \cdot \mathbf{S} \\ \mathbf{S_2} \cdot \mathbf{D} \end{bmatrix} \tag{17}$$

$$\mathbf{P} = (1 - b_1 - b_2)\mathbf{P}_0 + b_1\mathbf{P}_1 + b_2\mathbf{P}_2$$

$$\begin{bmatrix} t \\ b_1 \\ b_2 \end{bmatrix} = \frac{1}{\mathbf{S_1} \cdot \mathbf{E_1}} \begin{bmatrix} \mathbf{S_2} \cdot \mathbf{E_2} \\ \mathbf{S_1} \cdot \mathbf{S} \\ \mathbf{S_2} \cdot \mathbf{D} \end{bmatrix}$$

3. Once you've solved for t, b_1 and b_2, what conditions must be satisfied so that you have a valid ray-triangle intersection?

4. What does it mean when $\mathbf{S}_1 \cdot \mathbf{E}_1 = 0$ in the context of the Möller–Trumbore algorithm?

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3. Once you've solved for t, b_1 and b_2, what conditions must be satisfied so that you have a valid ray-triangle intersection?

Solution: $t \geq 0, 0 \leq b_1 \leq 1, 0 \leq b_2 \leq 1, 0 \leq 1 - b_1 - b_2 \leq 1.$

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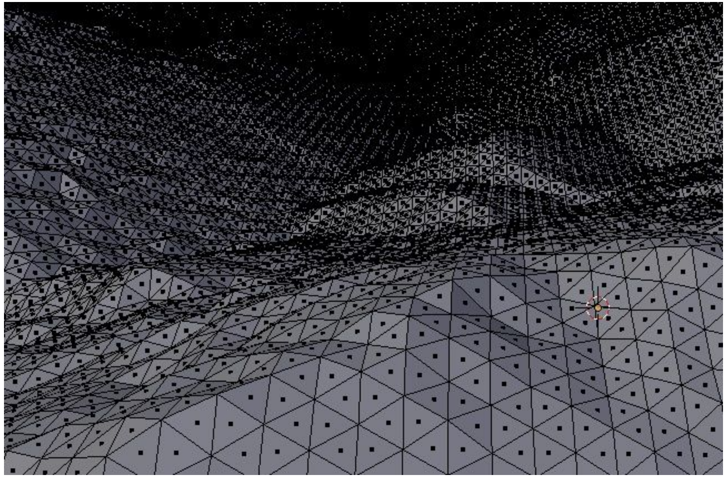
4. What does it mean when $\mathbf{S}_1 \cdot \mathbf{E}_1 = 0$ in the context of the Möller–Trumbore algorithm?

> **Solution:** If $\mathbf{S}_1 \cdot \mathbf{E}_1 = 0$, it means $(\mathbf{D} \times \mathbf{E}_2)$ is perpendicular to $(\mathbf{P}_1 - \mathbf{P}_0)$, which geometrically indicates that the ray $\mathbf{D}$ is parallel to the plane of the triangle (or that the triangle is degenerate). Hence, there is no unique intersection solution in this case.

Acceleration Structures

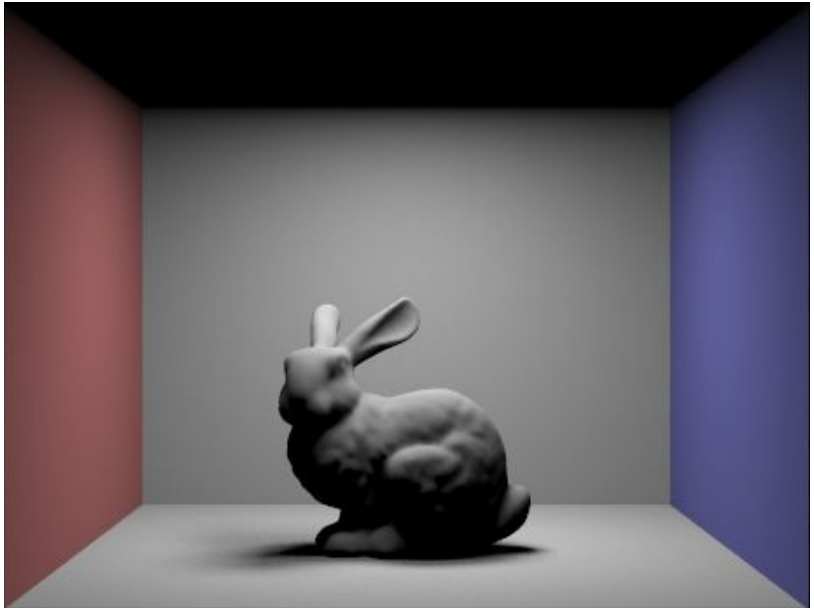
Acceleration Structures Motivation

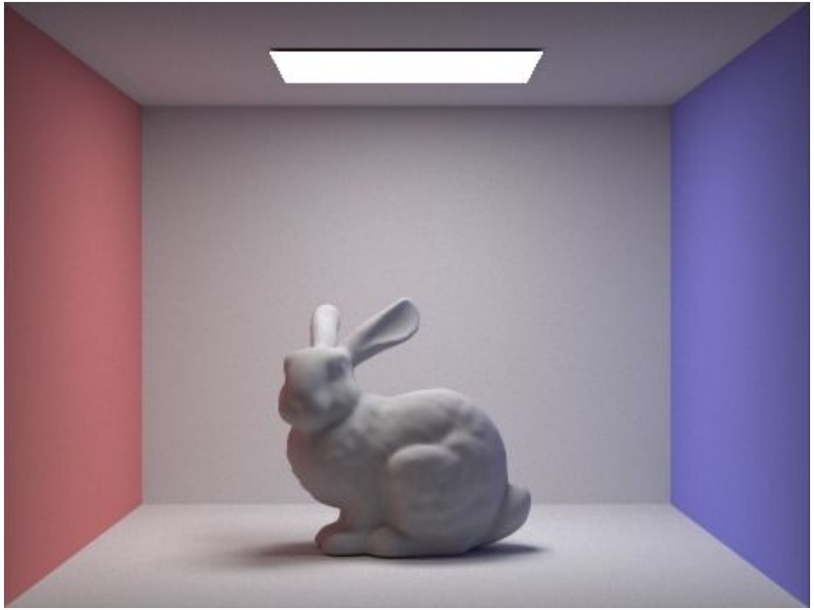
- Meshes can be made of hundreds of thousands of triangles.
- We sample at least one ray per pixel. (Imagine a 4K = 3840x2160 TV.)
- Calculating intersections can quickly get expensive!



Sneak Peek

In the next assignment, you will be able to render images like these!

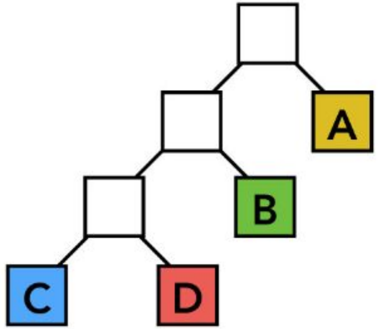


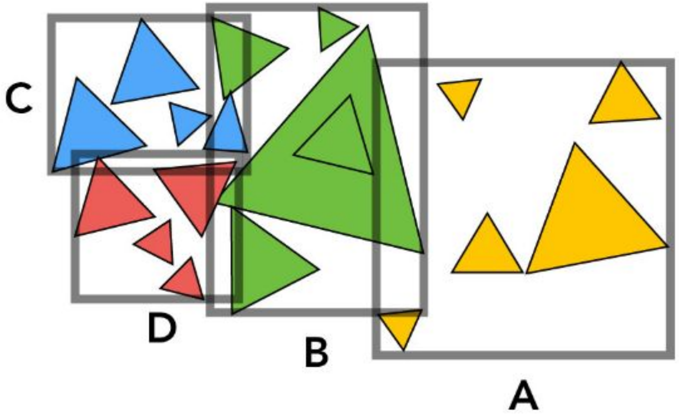


Idea: Bounding Volume Hierarchy

Internal nodes:

1. Bounding box.
2. Reference to children.





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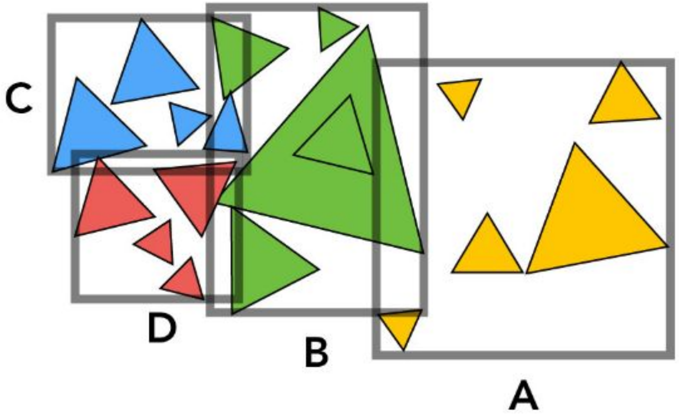
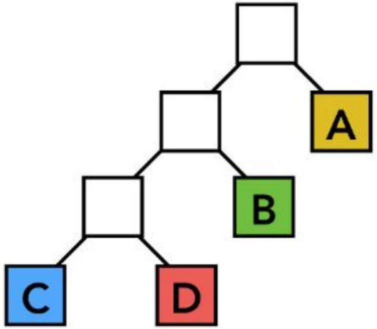


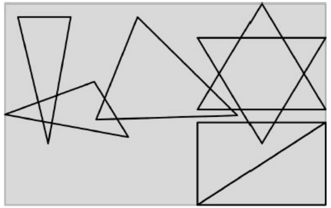


Leaf nodes:

1. Bounding box.
2. List of primitives in the box.

Worksheet Question 2

- Always pick the longest axis to divide.
- Use center of mass of triangles to decide their relative positions.
- Keep the BVH balanced. Try to ensure the same number of triangles for children nodes.



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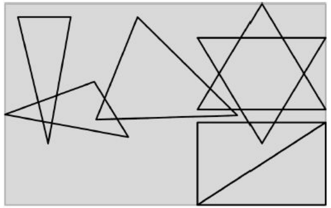

split this side

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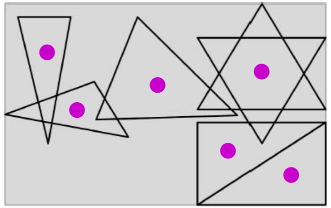

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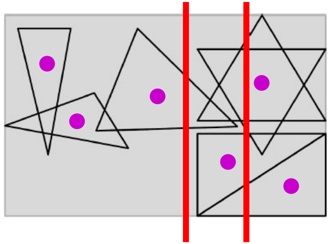

There are 2 valid splits!

- Always pick the longest axis to divide.
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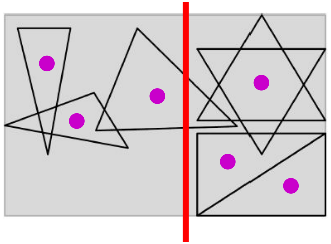

We'll just consider this one.

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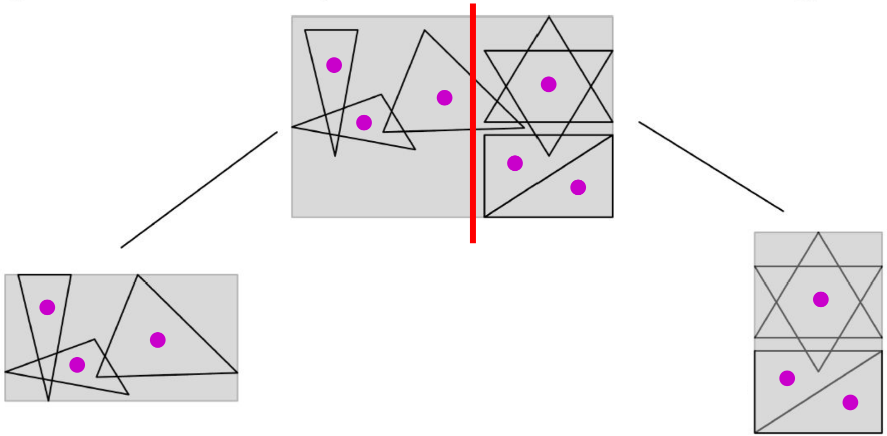

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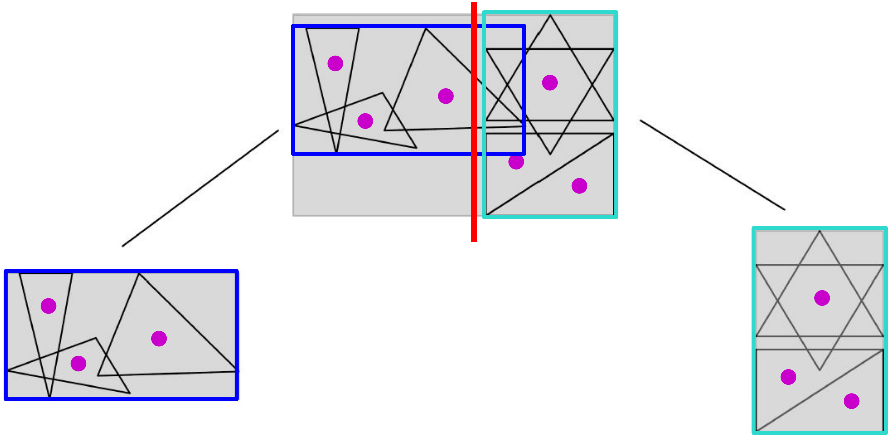

- It’s fine if these bounding boxes intersect!
- **Key idea:** we have partitioned objects into disjoint sets.

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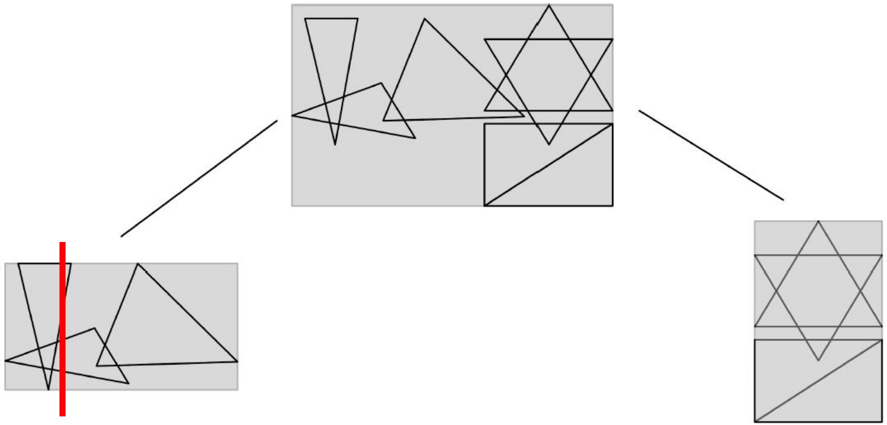

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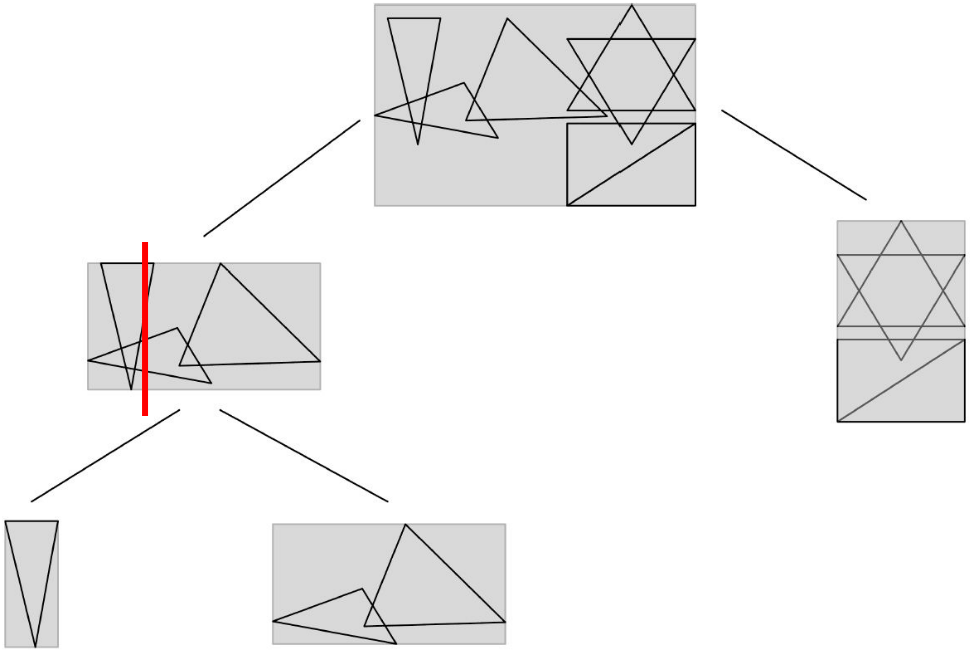

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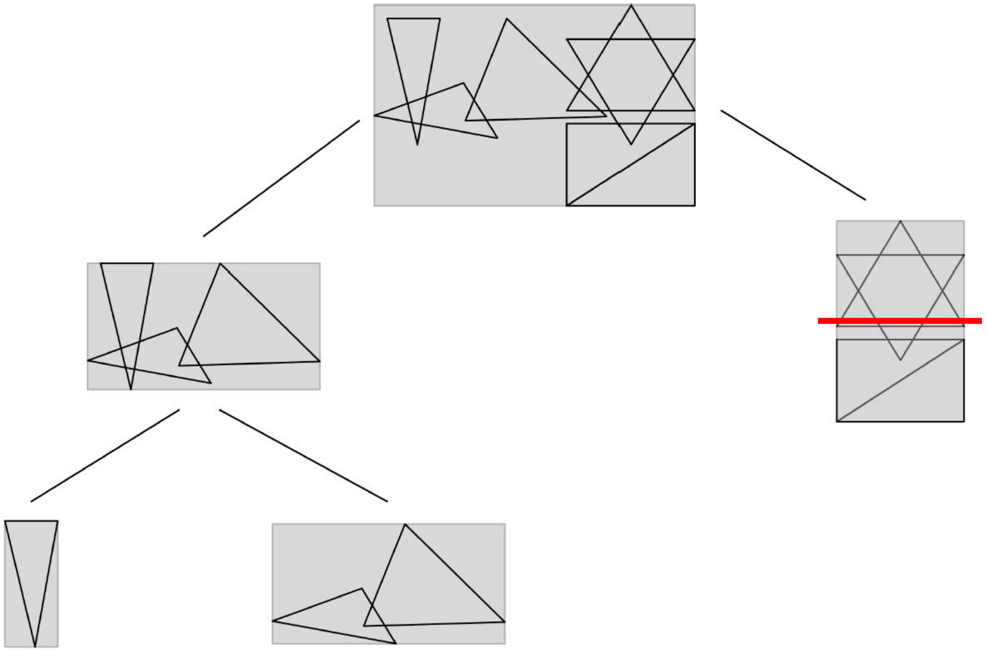

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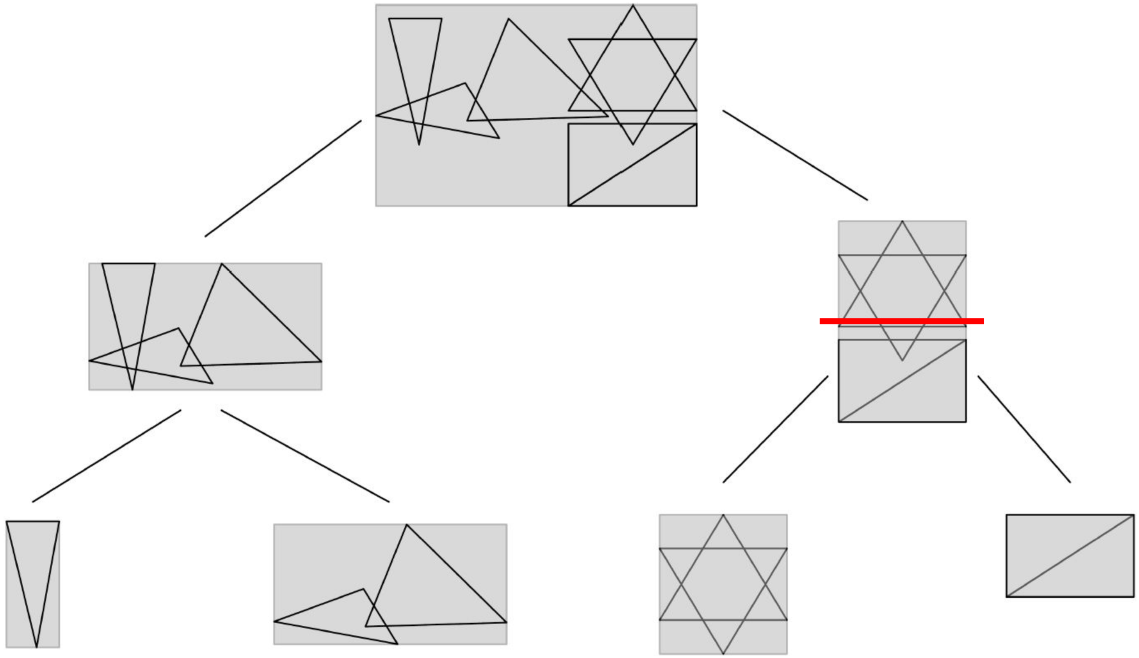

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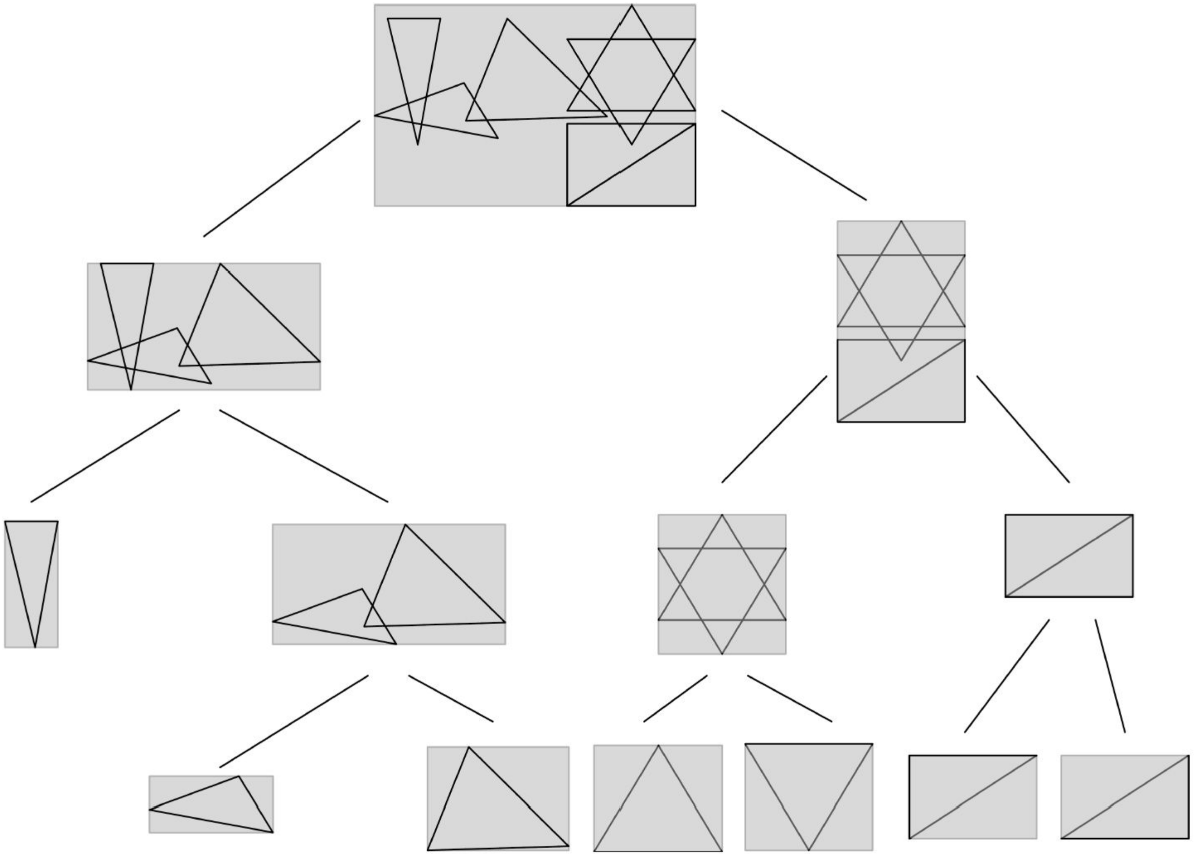
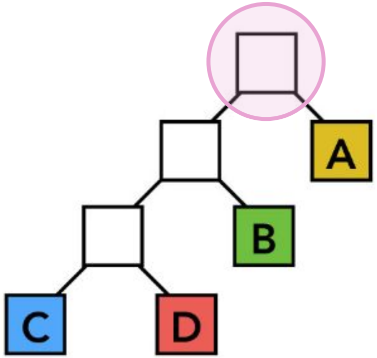
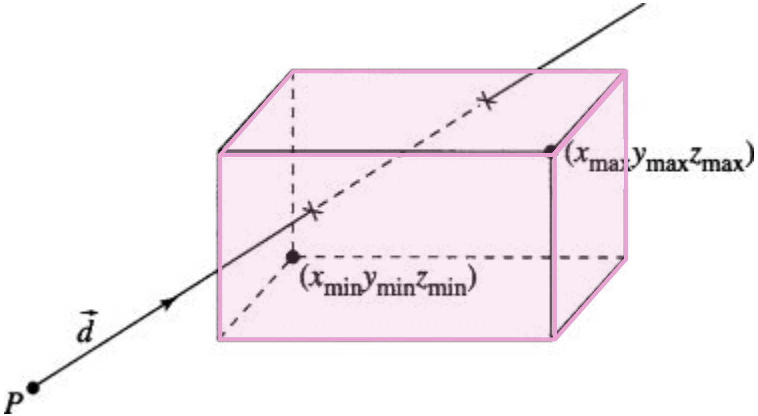


Using a Bounding Volume Hierarchy

```
Intersect (Ray ray, BVH node)
  if (ray misses node.bbox) return;
  if (node is a leaf node)
     test intersection with all objs;
     return closest intersection;
  hit1 = Intersect (ray, node.child1);
  hit2 = Intersect (ray, node.child2);
  return closer of hit1, hit2;
```





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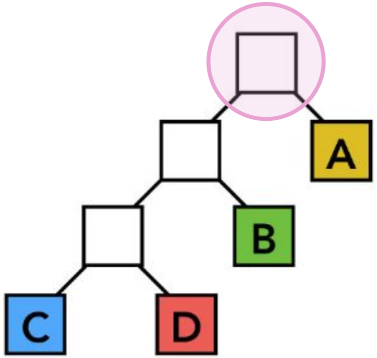

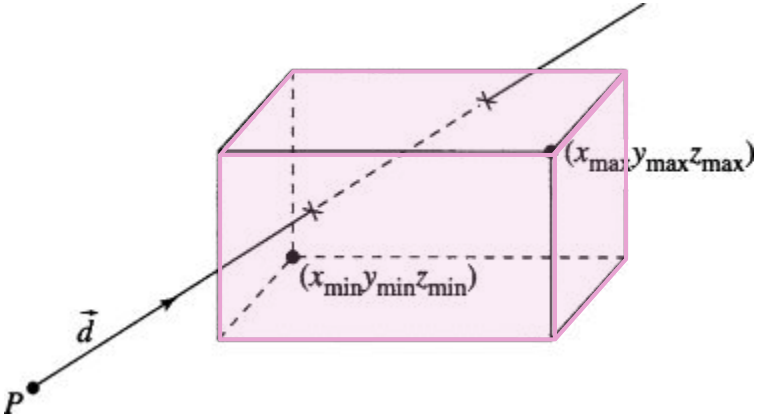

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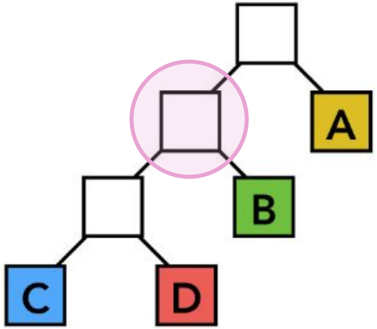

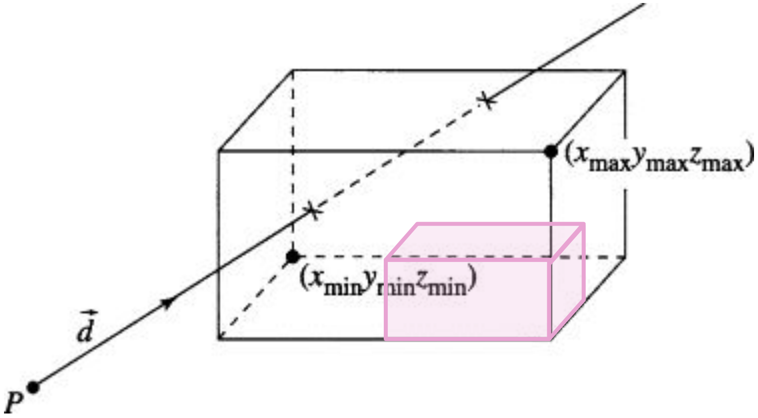

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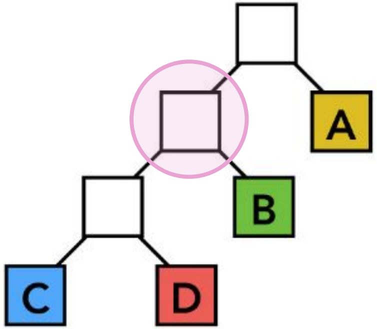

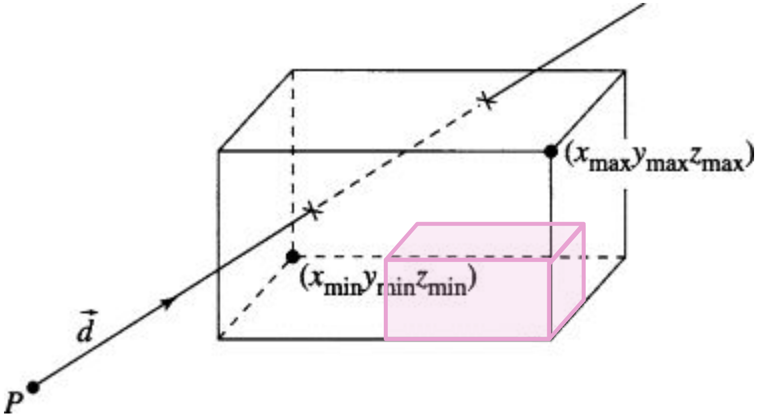

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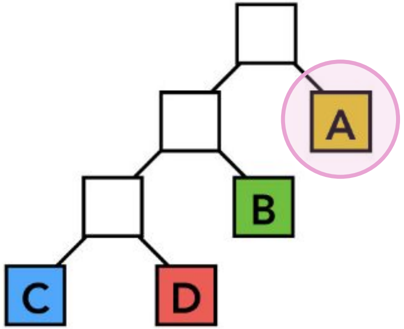

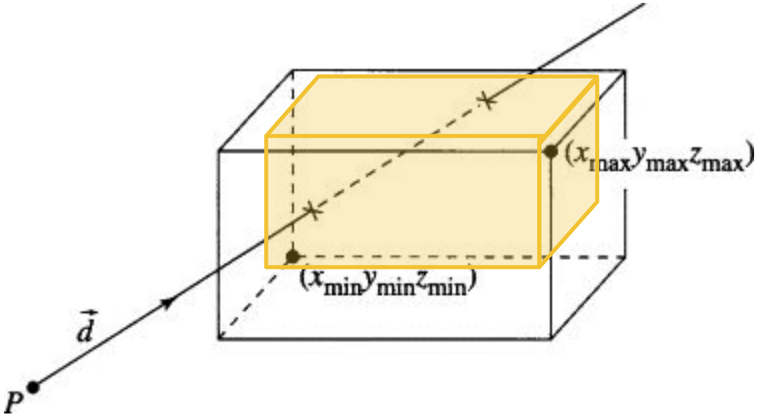

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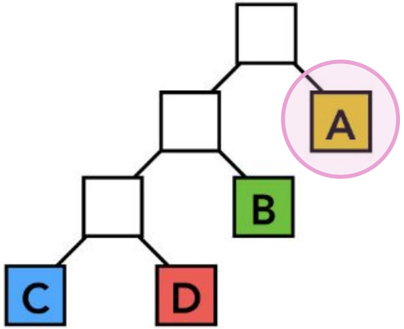

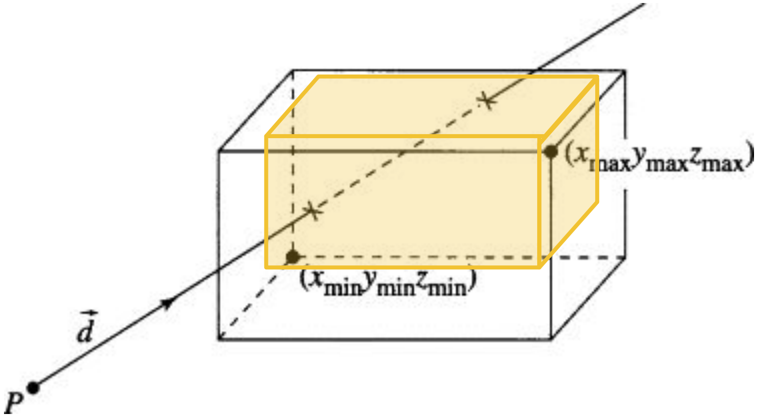

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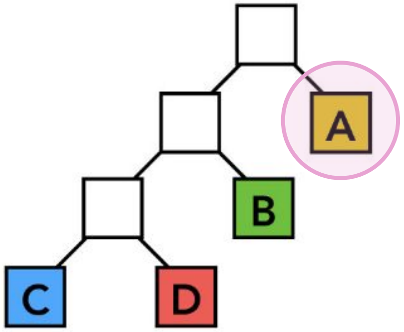

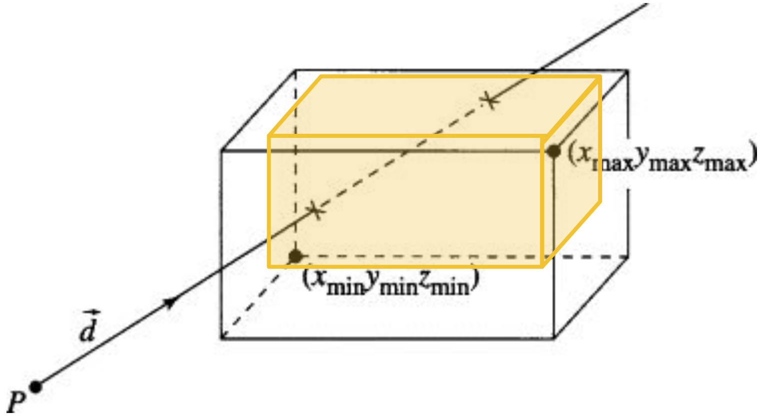

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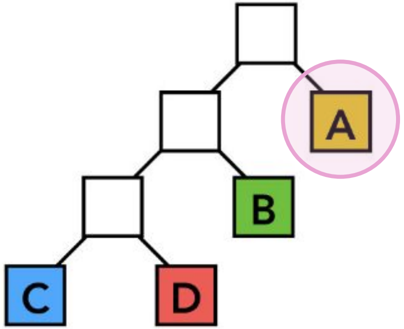

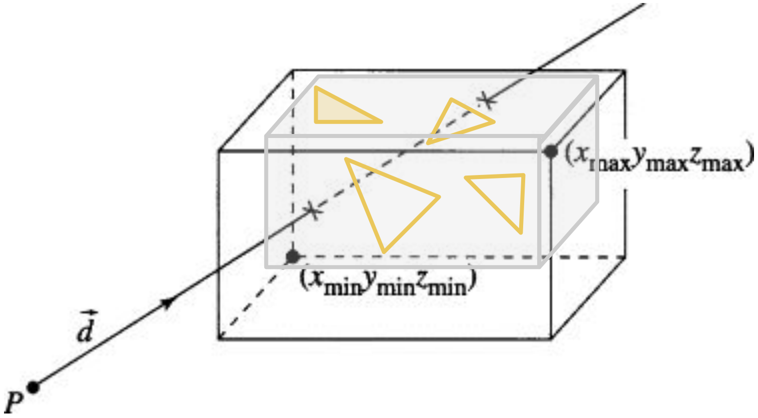

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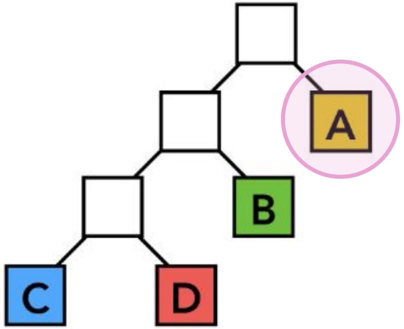

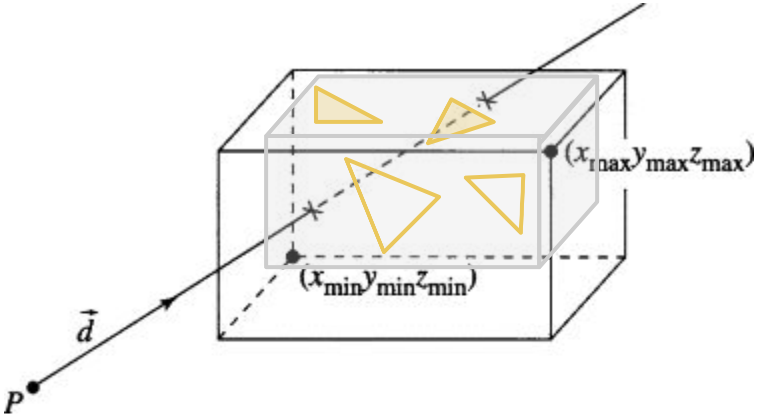

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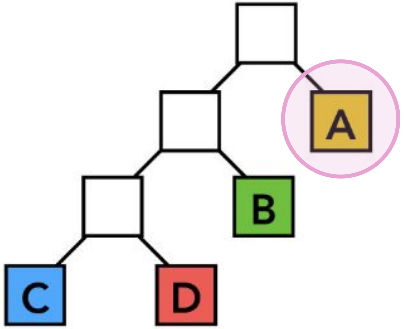

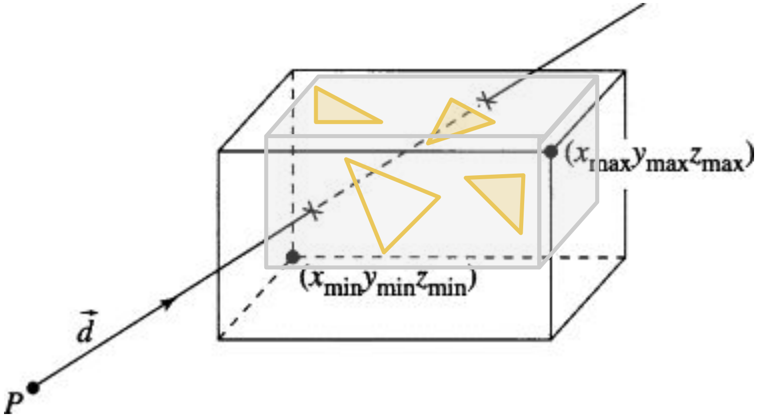

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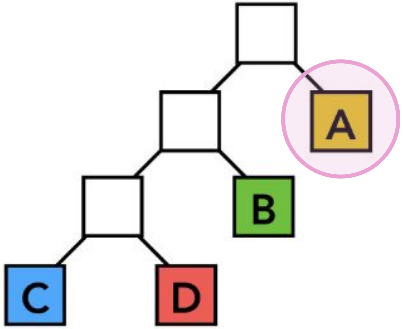

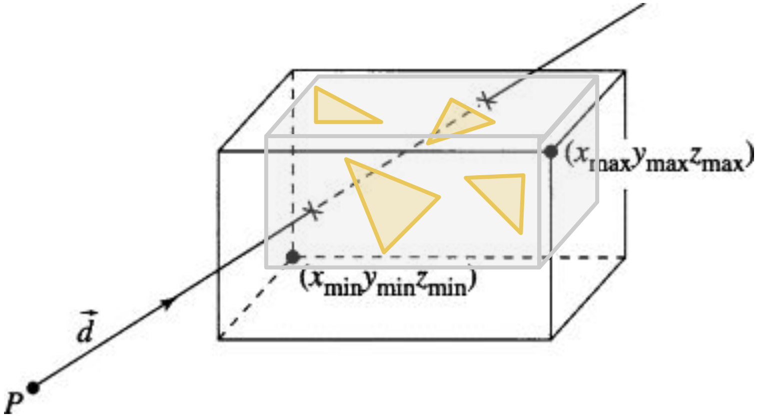

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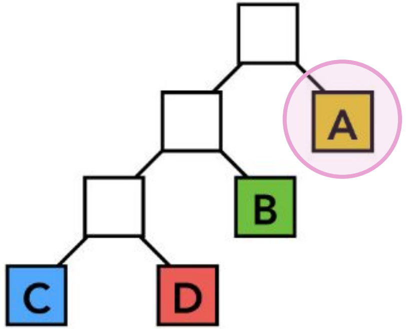

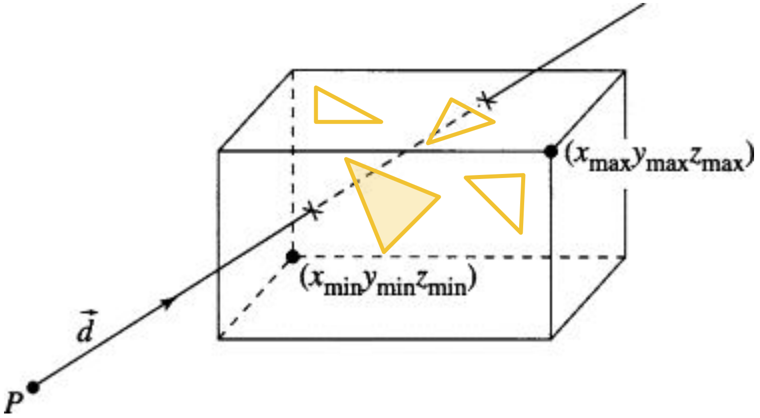

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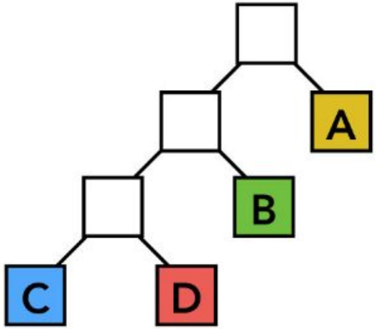
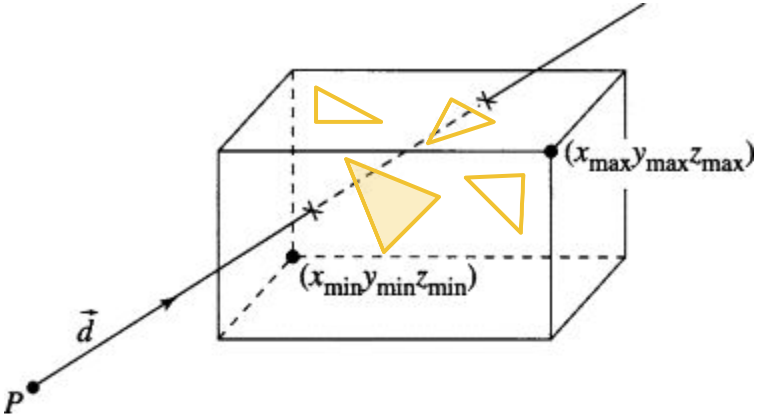
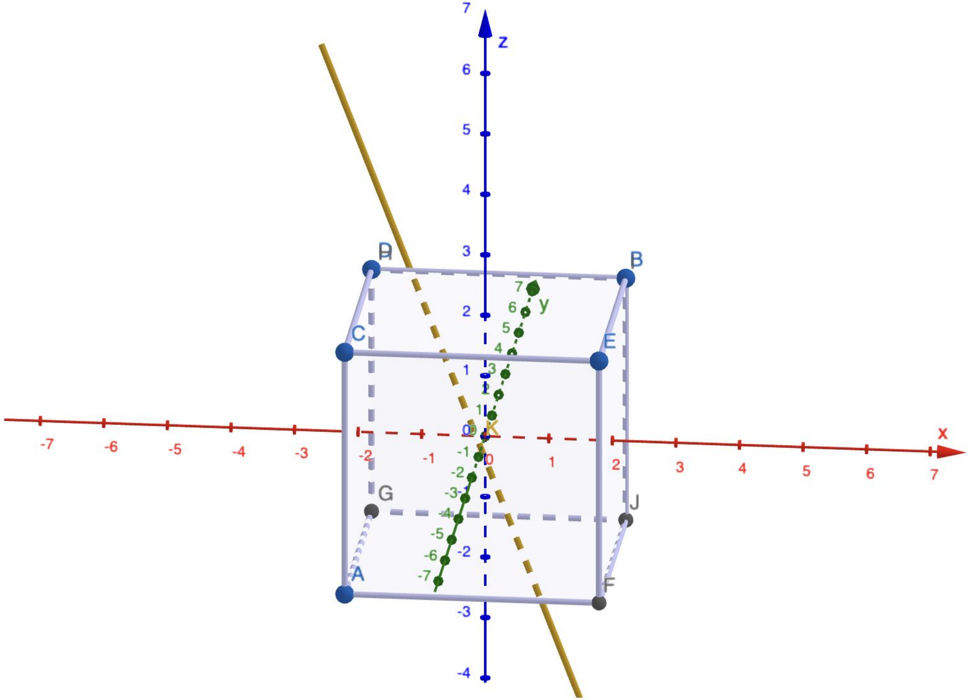




https://www.geogebra.org/3d/umn78ybr

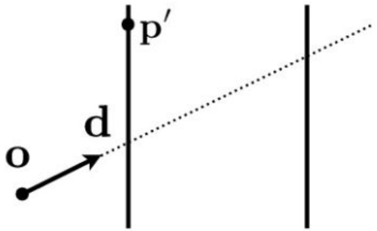


2.2. Using a Bounding Volume Hierarchy

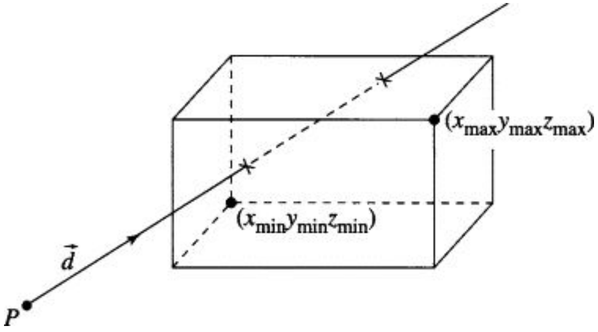
2. Given a box with corners $(-2, -2, -2)$ and $(2, 2, 2)$. Compute the entry and exit point of this box for a ray that has origin $(-3, 4, 5)$ and direction $(1, -1, -2)$.

Hint: Axis-aligned ray-plane intersection equation

Perpendicular to x-axis



$$t = \frac{\mathbf{p'}_x - \mathbf{o}_x}{\mathbf{d}_x}$$



2.2. Using a Bounding Volume Hierarchy

2. Given a box with corners $(-2,-2,-2)$ and $(2,2,2)$. Compute the entry and exit point of this box for a ray that has origin $(-3,4,5)$ and direction $(1,-1,-2)$.

Solution: Intersecting the yz-slabs, we have

$$t_{x,1} = (-2-(-3))/1 = 1,$$
$$t_{x,2} = (2-(-3))/1 = 5.$$

Intersecting the xz-slabs, we have

$$t_{y,1} = (-2-4)/(-1) = 6,$$
$$t_{y,2} = (2-4)/(-1) = 2.$$

Intersecting the xy-slabs, we have

$$t_{z,1} = (-2-5)/(-2) = 3.5,$$
$$t_{z,2} = (2-5/(-2) = 1.5.$$

2.2. Using a Bounding Volume Hierarchy

2. Given a box with corners $(-2, -2, -2)$ and $(2, 2, 2)$. Compute the entry and exit point of this box for a ray that has origin $(-3, 4, 5)$ and direction $(1, -1, -2)$.

So we have

$$t_{x,\min} = 1,\ t_{x,max} = 5,$$
$$t_{y,\min} = 2,\ t_{y,max} = 6,$$
$$t_{z,\min} = 1.5,\ t_{z,max} = 3.5.$$

Then

$$t_{\min} = \max\{t_{x,\min}, t_{y,\min}, t_{z,\min}\} = 2,$$
$$t_{\max} = \min\{t_{x,\max}, t_{y,\max}, t_{z,\max}\} = 3.5.$$

2.2. Using a Bounding Volume Hierarchy

2. Given a box with corners $(-2,-2,-2)$ and $(2,2,2)$. Compute the entry and exit point of this box for a ray that has origin $(-3,4,5)$ and direction $(1,-1,-2)$.

Since $t_{\min} <= t_{\max}$ and $t_{\min} > 0$ and $t_{\max} > 0$, we have two intersections. The entry and exit points are at

$$(-3,4,5) + t_{\min}(1,-1,-2) = (-1,2,1) \tag{28}$$

and

$$(-3,4,5) + t_{\max}(1,-1,-2) = (0.5,0.5,-2). \tag{29}$$

Radiometry & Photometry

Radiometry

Flux (Power) is measured in Watts.

Radiometry

Flux (Power) is measured in Watts.

Radiant intensity is flux per solid angle.

Radiometry

Flux (Power) is measured in Watts.

Radiance is flux per area per solid angle.

AKA brightness!

Radiant intensity is flux per solid angle.

Radiometry

Flux (Power) is measured in Watts.

Irradiance is flux per area.

Radiance is flux per area per solid angle.

AKA brightness!

Radiant intensity is flux per solid angle.

Radiant flux is energy received per time. Watts.

$$\Phi \equiv \frac{\mathrm{d}Q}{\mathrm{d}t}$$

Photometry mirrors radiometry but adjusts for human vision.

Radiant (luminous) flux is energy received per time. Watts (lumens).

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Radiant (luminous) intensity is flux per solid angle. W/sr (candela).

$$I(\omega) \equiv \frac{\mathrm{d}\Phi}{\mathrm{d}\omega}$$

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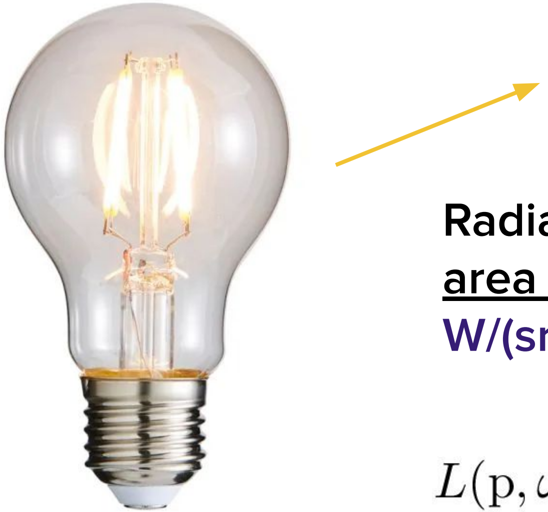


Radiance is flux per area per solid angle. W/(sr m^2) (nit).

$$L(\mathrm{p}, \omega) \equiv \frac{\mathrm{d}^2\Phi(\mathrm{p}, \omega)}{\mathrm{d}\omega\, \mathrm{d}A \cos\theta}$$

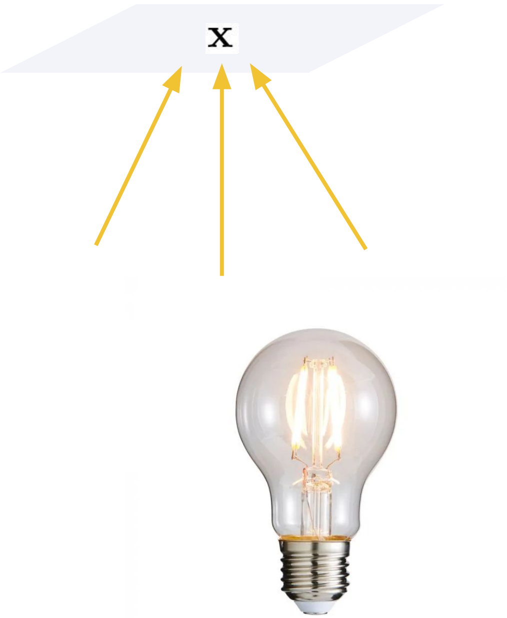
Photometry mirrors radiometry but adjusts for human vision.

Irradiance is flux per area. W/m^2 (lux).

$$E(\mathbf{x}) \equiv \frac{\mathrm{d}\Phi(\mathbf{x})}{\mathrm{d}A}$$

Radiant (luminous) flux is energy received per time. Watts (lumens).

$$\Phi \equiv \frac{\mathrm{d}Q}{\mathrm{d}t}$$



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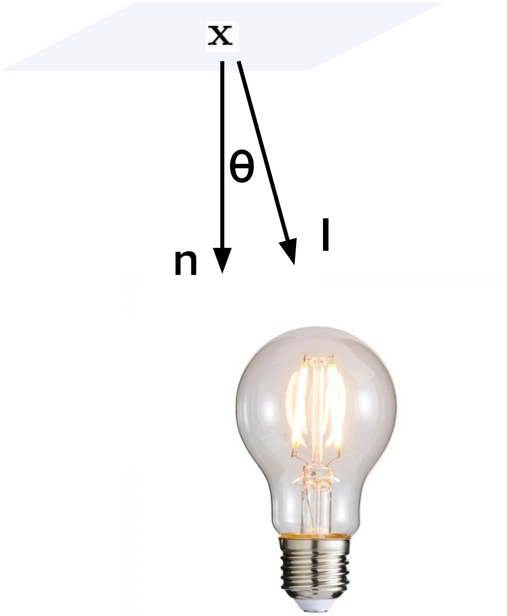


Irradiance is flux per area. W/m² (lux).

$$E(\mathbf{x}) \equiv \frac{\mathrm{d}\Phi(\mathbf{x})}{\mathrm{d}A} = \frac{\Phi}{A}\cos\theta$$

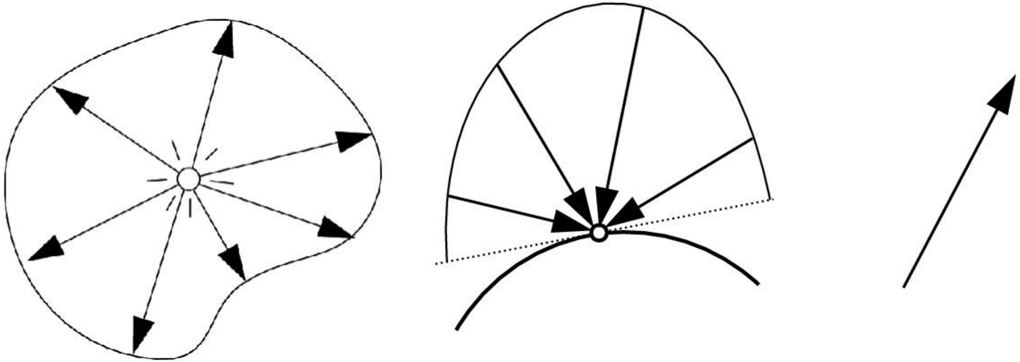
Lambert's cosine law

Radiance is flux per area per solid angle. W/(sr m²) (nit).

$$L(\mathrm{p},\omega) \equiv \frac{\mathrm{d}^2\Phi(\mathrm{p},\omega)}{\mathrm{d}\omega\,\mathrm{d}A\cos\theta}$$

3.1. Radiometry Definitions and Relations

1. What's the difference between **radiant flux / power** (Φ), **radiant intensity** (I), **irradiance** (E) and **radiance** (L)? How does increasing the distance from the light source affect these values?



Light Emitted From A Source	Light Falling On A Surface	Light Traveling Along A Ray
"Radiant Intensity"	"Irradiance"	"Radiance"

3.1. Radiometry Definitions and Relations

Physics Symbol/-Name	Radiometry Unit/Name	Photometry Unit/Name	Definition
Q Energy	Radiant Energy **Joules (W·s)**	Luminous Energy **Lumen·sec**	$Q = \int_{t_0}^{t_1} \Phi dt$
Φ Flux(Power)	Radiant Power **W**	Luminous Power **Lumen (Candela·sr)**	$\Phi = \frac{dQ}{dt}$
I Angular Flux Density	Radiant Intensity **W/sr**	Luminous Intensity **Candela (Lumen/sr)**	$I(\omega) = \frac{d\Phi}{d\omega}$
E Spatial Flux Density	Irradiance (in), Radiosity (out) **W/m²**	Illuminance (in), Luminosity (out) **Lux (Lumen/m²)**	$E(p) = \frac{d\Phi(p)}{dA}$
L Spatio-Angular Flux Density	Radiance **W/m²/sr**	Luminance **Nit (Candela/m²)**	$L(p,\omega) = \frac{d^2\Phi(p,\omega)}{d\omega dA \cos\theta} = \frac{dE(p)}{d\omega \cos\theta} = \frac{dI(p,\omega)}{dA \cos\theta}$

3.1. Radiometry Definitions and Relations

1. What's the difference between **radiant flux / power** (Φ), **radiant intensity** (I), **irradiance** (E) and **radiance** (L)?

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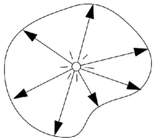
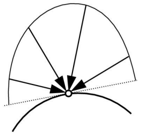
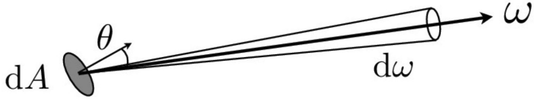
$I =$ 

$E =$ 

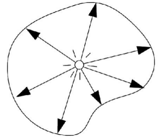
$L =$ 

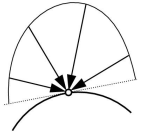
3.1. Radiometry Definitions and Relations

1. What's the difference between **radiant flux / power** (Φ), **radiant intensity** (I), **irradiance** (E) and **radiance** (L)?

Solution: The radiant flux (power) Φ is the energy emitted, reflected, transmitted or received, per unit time.

$$\Phi \equiv \frac{dQ}{dt}$$

$I =$ 

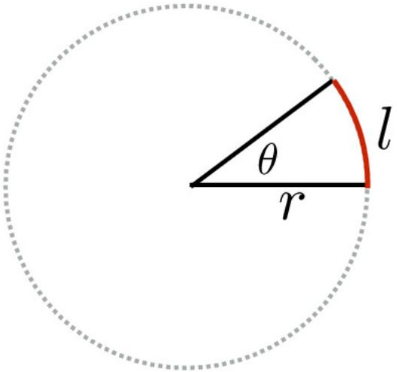
$E =$ 

$L =$ 

Angles and Solid Angles

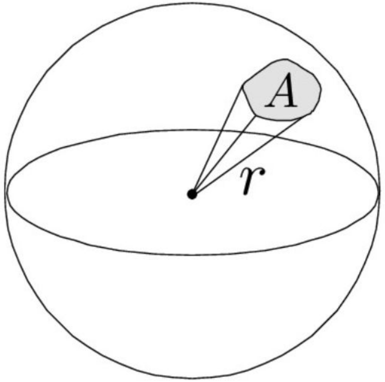
Angle: ratio of subtended arc length on circle to radius

- $\theta = \frac{l}{r}$
- Circle has 2π radians

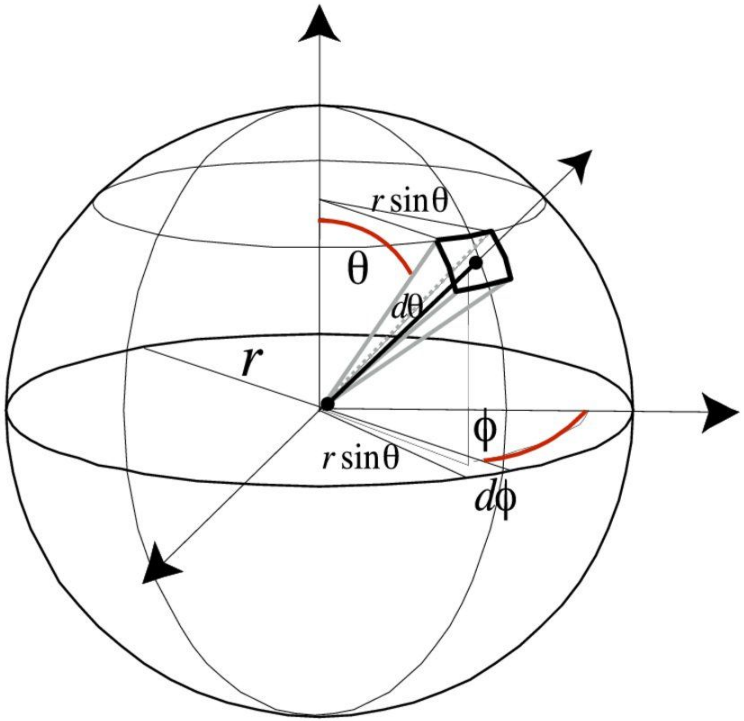


Solid angle: ratio of subtended area on sphere to radius squared

- $\Omega = \frac{A}{r^2}$
- Sphere has 4π steradians



Solid Angle

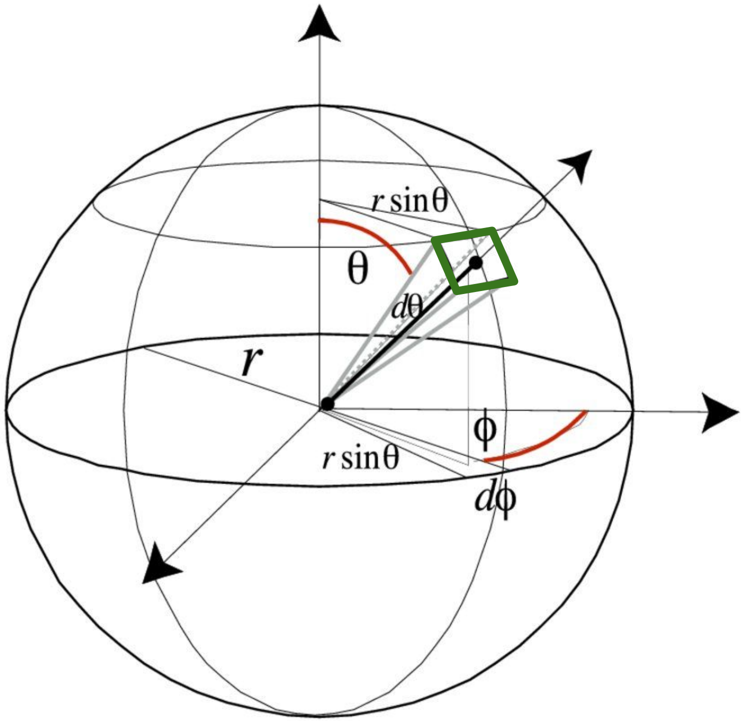


$\mathrm{d}A =$

$=$

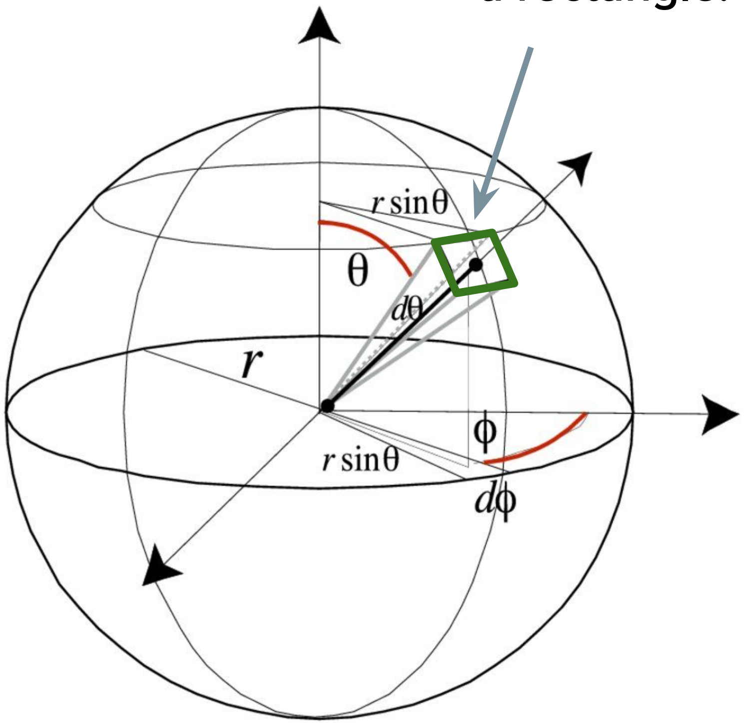
$\mathrm{d}\omega =$

Solid Angle



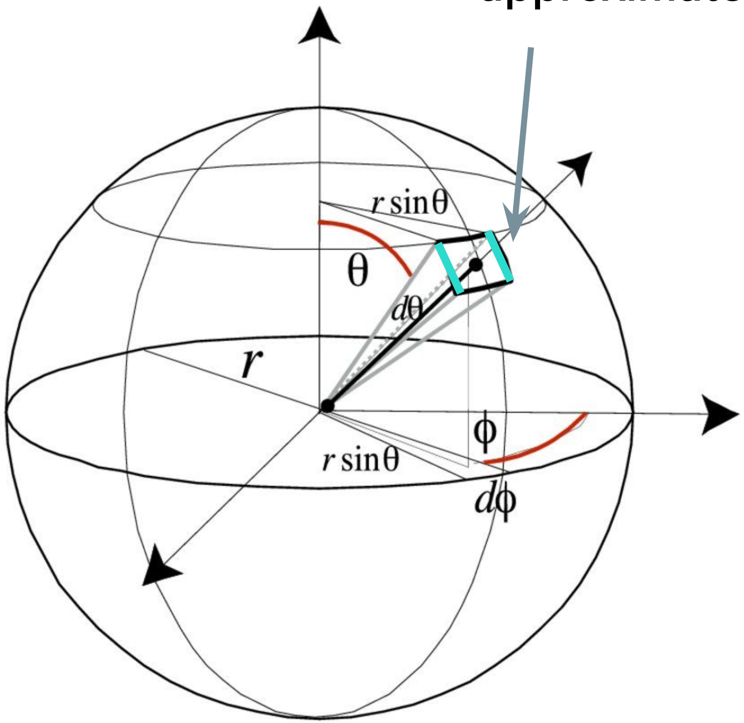
Solid Angle

Approximate dA as a rectangle!



Solid Angle

How do we approximate?



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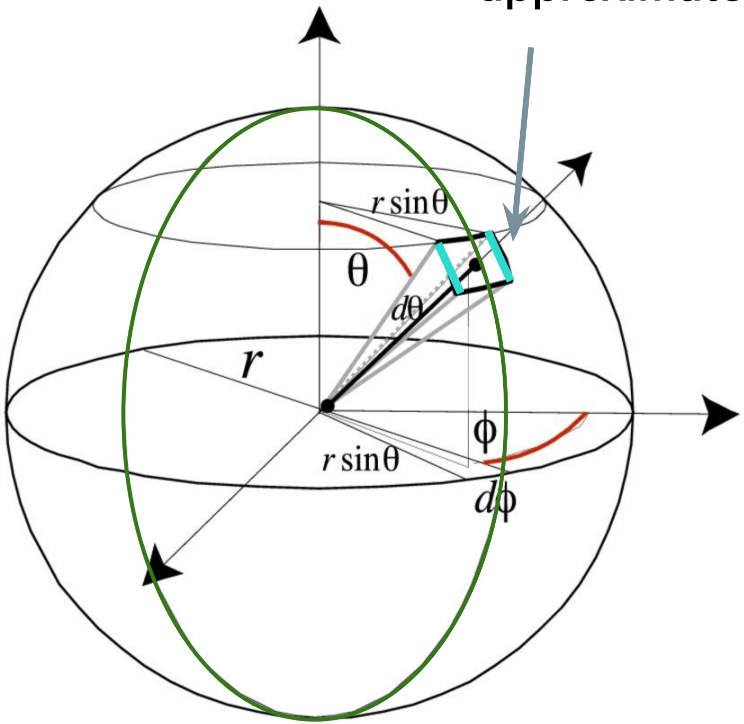

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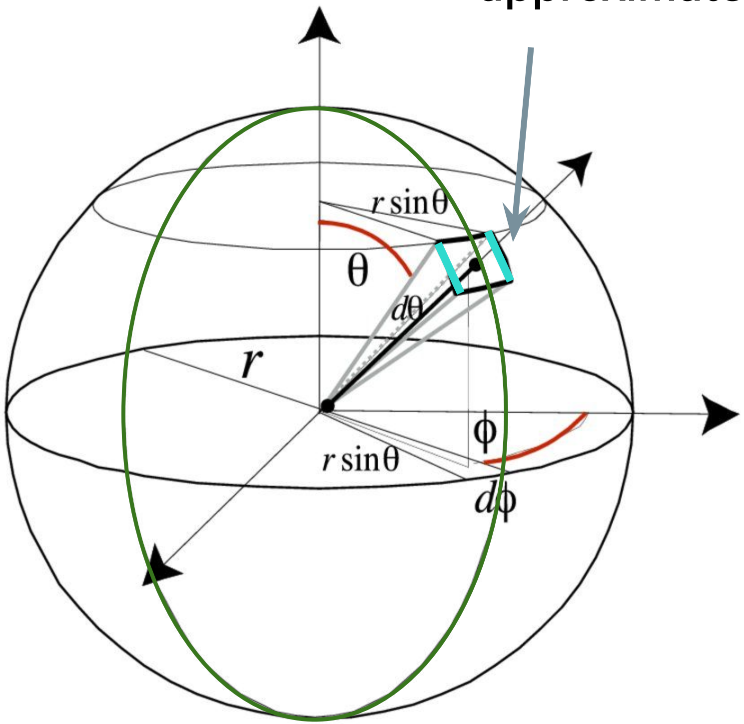

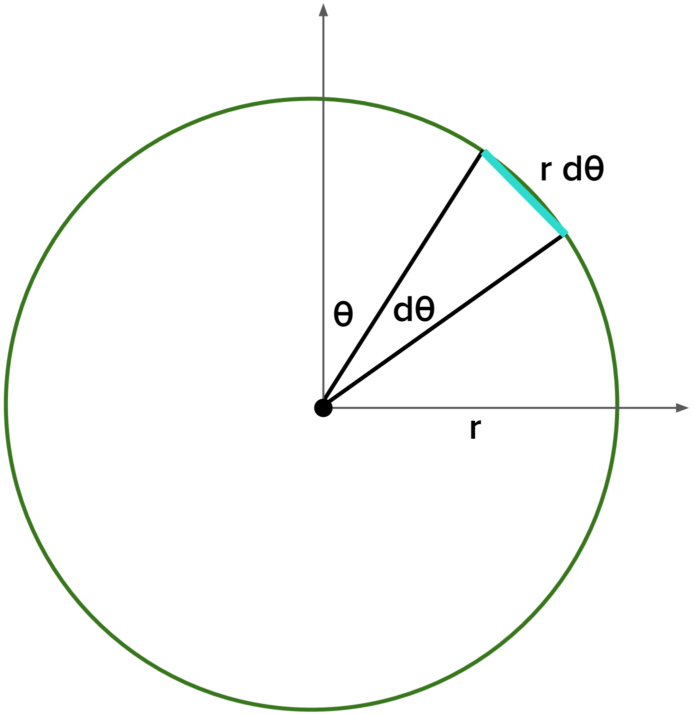

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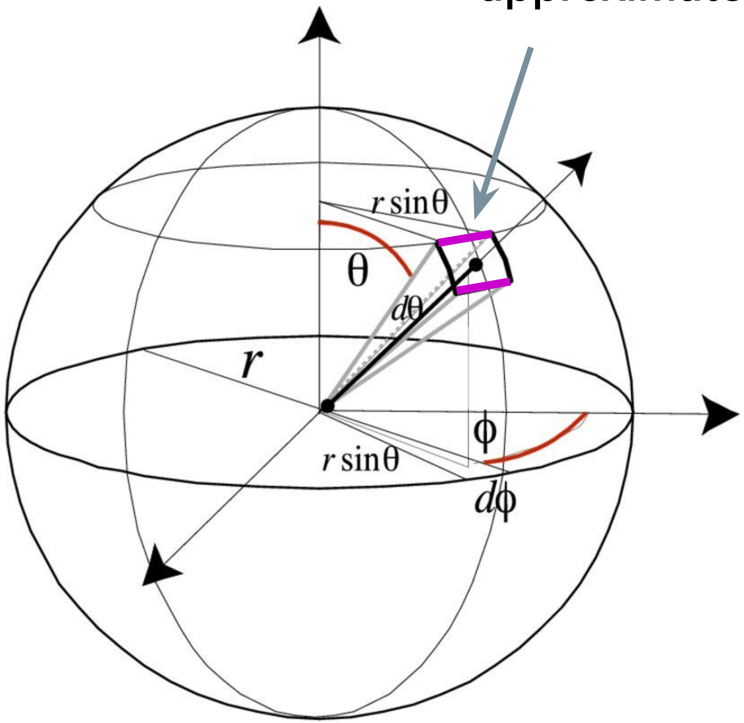

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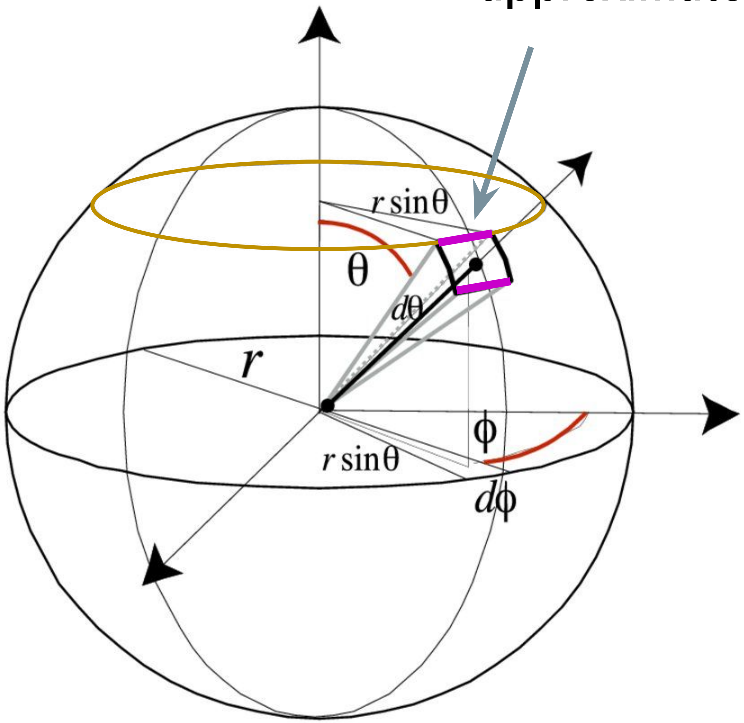

Solid Angle

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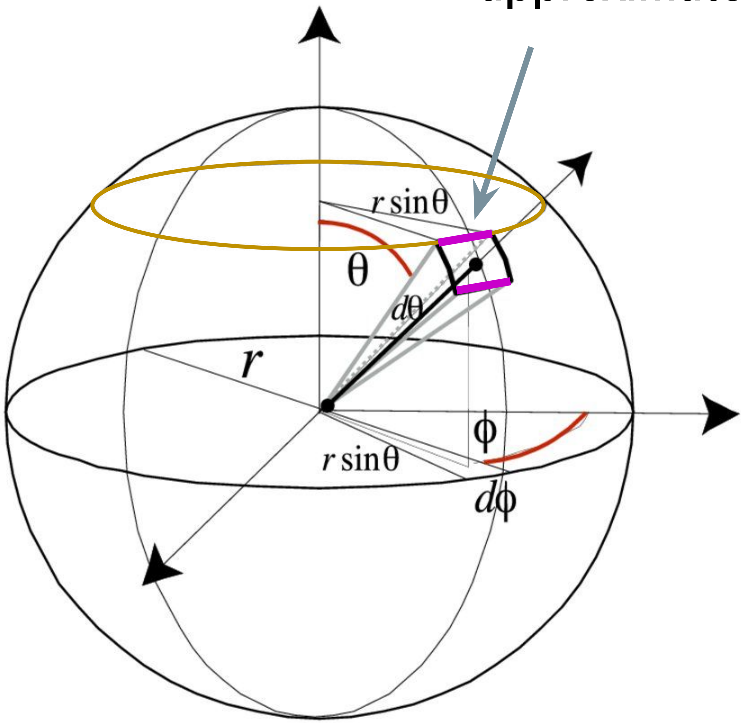
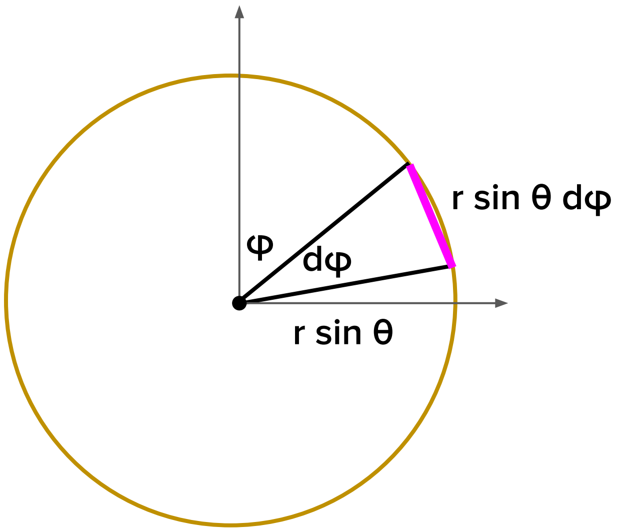
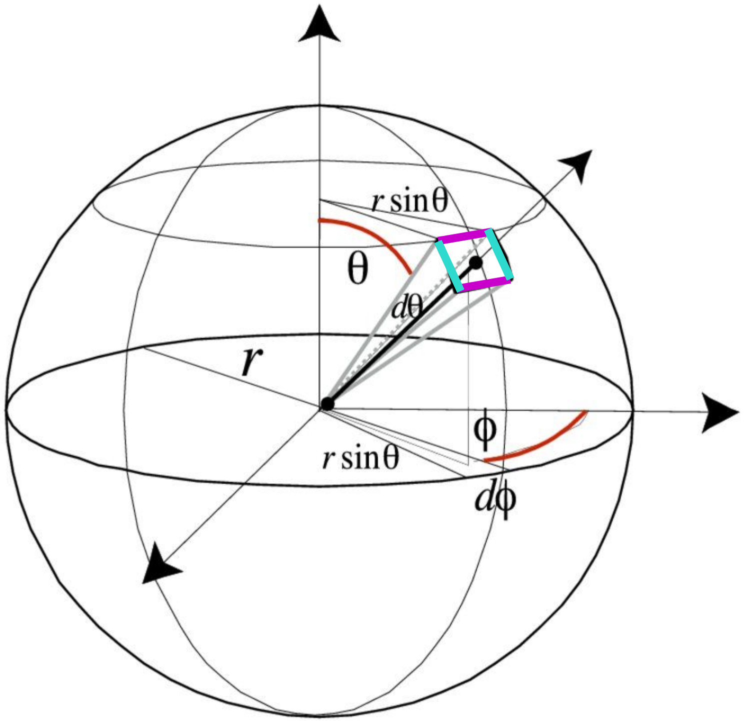




Solid Angle



$$\mathrm{d}A = (r\,\mathrm{d}\theta)(r\sin\theta\,\mathrm{d}\phi)$$
$$= r^2 \sin\theta\,\mathrm{d}\theta\,\mathrm{d}\phi$$

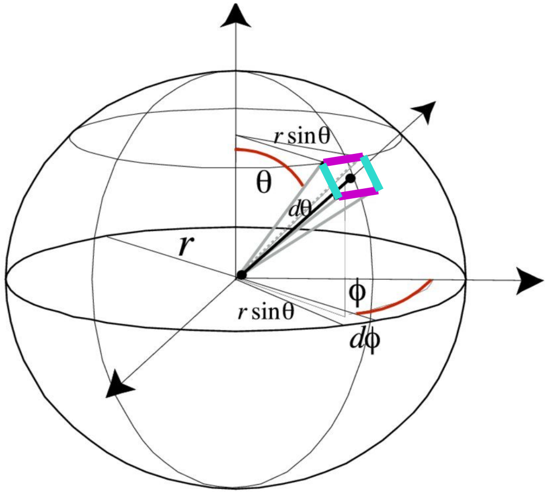
$$\mathrm{d}\omega = \frac{\mathrm{d}A}{r^2} = \sin\theta\,\mathrm{d}\theta\,\mathrm{d}\phi$$

Worksheet Question 4.1

4 Shedding Some Light

1. Suppose we use (θ, ϕ)-parameterization of directions. Recall that the solid angle represents the ratio of the subtended area on a sphere to the radius squared, $\Omega = \frac{A}{r^2}$. Estimate the solid angle subtended by a patch that covers $\theta \in [\pi/6 - \pi/12, \pi/6 + \pi/12]$ and $\phi \in [\pi/5 - \pi/24, \pi/5 + \pi/24]$?

   (Hint: you may assume that the patch is small enough. Recall or derive the differential solid angle $d\omega$, then use the values given.)



$$\mathrm{d}A = (r\,\mathrm{d}\theta)(r \sin\theta\,\mathrm{d}\phi)$$
$$= r^2 \sin\theta\,\mathrm{d}\theta\,\mathrm{d}\phi$$

$$\mathrm{d}\omega = \frac{\mathrm{d}A}{r^2} = \sin\theta\,\mathrm{d}\theta\,\mathrm{d}\phi$$

4 Shedding Some Light

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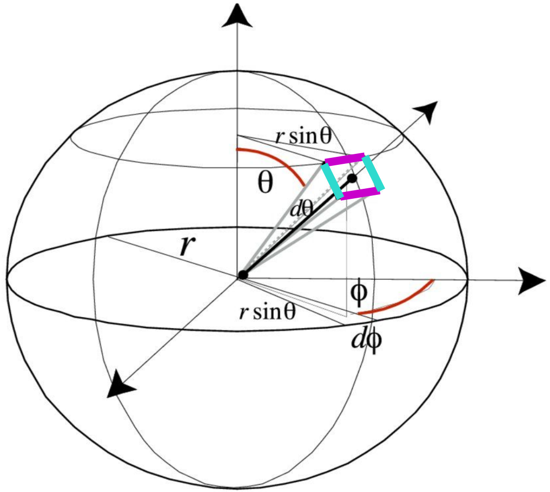


$$
\begin{aligned}
\mathrm{d}A &= (r\,\mathrm{d}\theta)(r\sin\theta\,\mathrm{d}\phi) \\
&= r^2 \sin\theta\,\mathrm{d}\theta\,\mathrm{d}\phi
\end{aligned}
$$

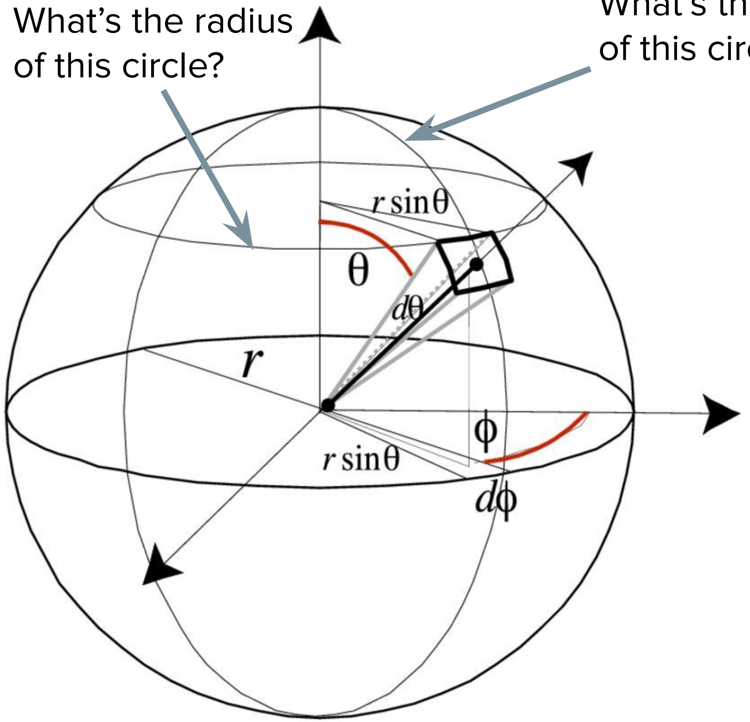
$$
\mathrm{d}\omega = \frac{\mathrm{d}A}{r^2} = \sin\theta\,\mathrm{d}\theta\,\mathrm{d}\phi
$$

$$
\begin{aligned}
\Delta\omega &\approx \sin\frac{\pi}{6} \cdot \left[\frac{\pi}{12} - \left(-\frac{\pi}{12}\right)\right] \cdot \left[\frac{\pi}{24} - \left(-\frac{\pi}{24}\right)\right] \\
&= \frac{\pi^2}{144}.
\end{aligned}
$$

3.2. Solid Angle

What's the radius of this circle?

What's the radius of this circle?



$\mathrm{d}A =$

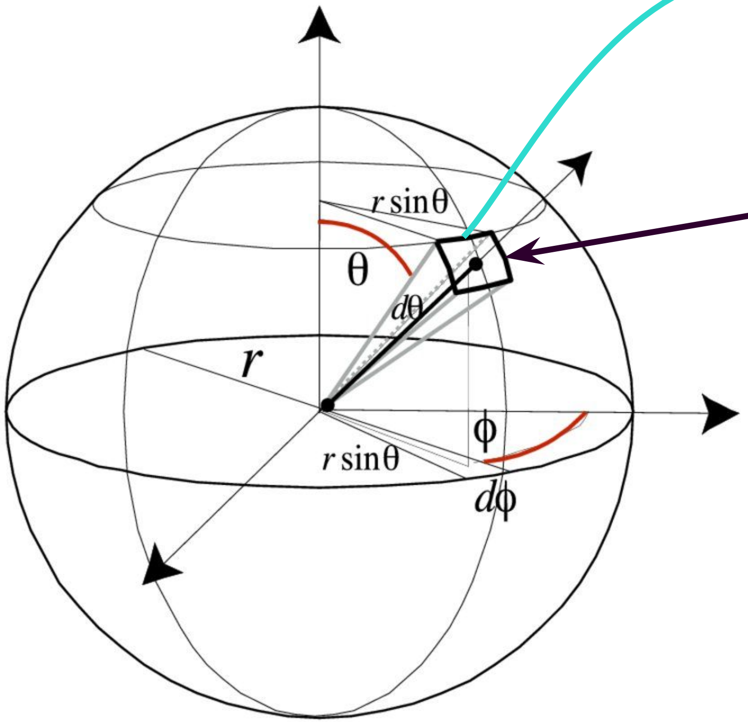
$=$

$\mathrm{d}\omega =$

Approximate this value as a rectangle.

Recall the arclength of a circle: $L = r \cdot \theta$

3.2. Solid Angle



$$\mathrm{d}A = (r\,\mathrm{d}\theta)(r\sin\theta\,\mathrm{d}\phi)$$
$$= r^2\sin\theta\,\mathrm{d}\theta\,\mathrm{d}\phi$$

$$\mathrm{d}\omega = \frac{\mathrm{d}A}{r^2} = \sin\theta\,\mathrm{d}\theta\,\mathrm{d}\phi$$

In our specific case, the solid angle subtended by the patch is now

$$\Delta\omega \approx \sin\frac{\pi}{6}\cdot\left[\frac{\pi}{12}-\left(-\frac{\pi}{12}\right)\right]\cdot\left[\frac{\pi}{24}-\left(-\frac{\pi}{24}\right)\right]$$
$$= \frac{\pi^2}{144}.$$

Radiant Energy and Flux (Power)

Definition: Radiant (luminous*) energy is the energy of electromagnetic radiation. It is measured in units of joules, and denoted by the symbol:

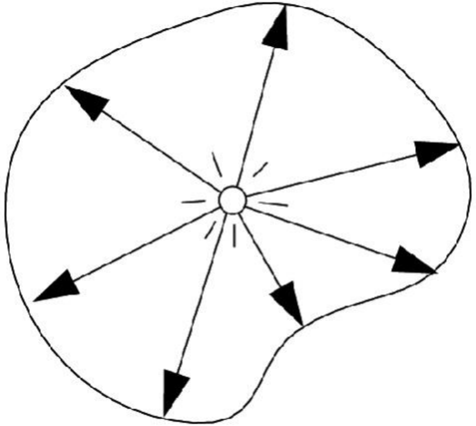
$$Q \ [\mathrm{J} = \mathrm{Joule}]$$

Definition: Radiant (luminous*) flux is the energy emitted, reflected, transmitted or received, per unit time.

$$\Phi \equiv \frac{\mathrm{d}Q}{\mathrm{d}t} \ [\mathrm{W} = \mathrm{Watt}] \ [\mathrm{lm} = \mathrm{lumen}]^*$$

Radiant Intensity

Definition: The radiant (luminous) intensity is the power per unit solid angle emitted by a point light source.



$$I(\omega) \equiv \frac{\mathrm{d}\Phi}{\mathrm{d}\omega}$$

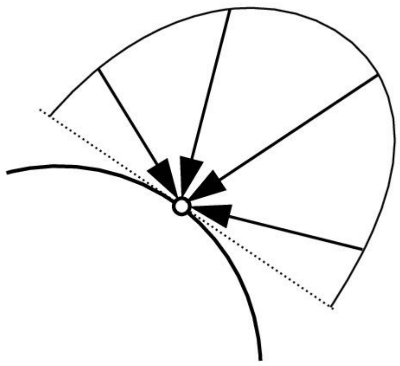
$$\left[\frac{\mathrm{W}}{\mathrm{sr}}\right] \left[\frac{\mathrm{lm}}{\mathrm{sr}} = \mathrm{cd} = \mathrm{candela}\right]$$

Irradiance

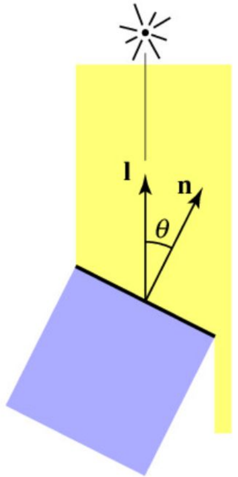
Definition: The irradiance (illuminance) is the power per unit area incident on a surface point.

$$E(\mathbf{x}) \equiv \frac{\mathrm{d}\Phi(\mathbf{x})}{\mathrm{d}A}$$

$$\left[\frac{\mathrm{W}}{\mathrm{m}^2}\right] \left[\frac{\mathrm{lm}}{\mathrm{m}^2} = \mathrm{lux}\right]$$



Lambert’s Cosine Law



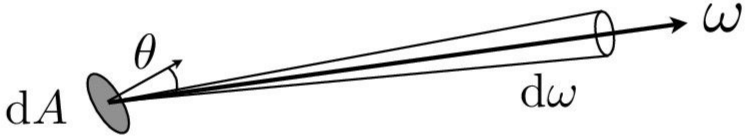
In general, power per unit area is proportional to

$$\cos\theta = l \cdot n$$

$$E = \frac{\Phi}{A}\cos\theta$$

Surface Radiance

Definition: The radiance (luminance) is the power emitted by a surface, per unit solid angle, per unit projected area.



$$L(\mathrm{p}, \omega) \equiv \frac{\mathrm{d}^2\Phi(\mathrm{p}, \omega)}{\mathrm{d}\omega\, \mathrm{d}A \cos\theta}$$

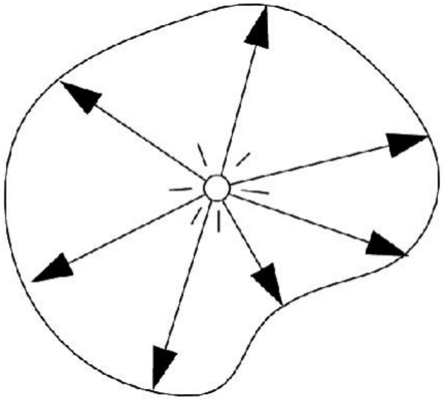
$\cos\theta$ accounts for projected surface area

$$\left[\frac{\mathrm{W}}{\mathrm{sr}\,\mathrm{m}^2}\right] \left[\frac{\mathrm{cd}}{\mathrm{m}^2} = \frac{\mathrm{lm}}{\mathrm{sr}\,\mathrm{m}^2} = \mathrm{nit}\right]$$

Radiance

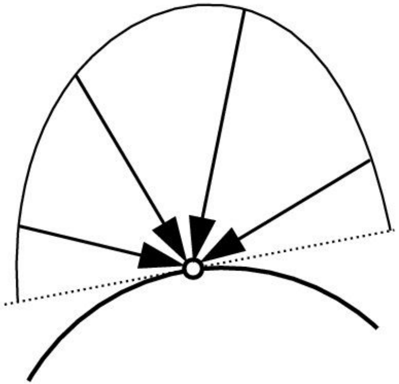
Light Traveling Along A Ray

Radiance is invariant along a ray in a vacuum



Light Emitted
From A Source

"Radiant Intensity"



Light Falling
On A Surface

"Irradiance"

Light Traveling
Along A Ray

"Radiance"

Units Summary

Symbol/Name	**Radiometry Unit/Name**	**Photometry Unit/Name**	**Effect of Increased R**
Q: Energy	Radiant Energy Joules (W·s)	Luminous Energy Lumen·sec	$=$
Φ: Flux (Power)	Radiant Power W	Luminous Power Candela·sr	
I: Angular Flux Density	Radiant Intensity W/sr	Luminous Intensity Candela = Lumen/sr	
E: Spatial Flux Density	Irradiance (in), Radiosity (out) W/m^2	Illuminance (in), Luminosity (out) Lux = Lumen/m^2	
L: Spatio-Angular Flux Density	Radiance $W/m^2/sr$	Luminance Nit = Candela/m^2	

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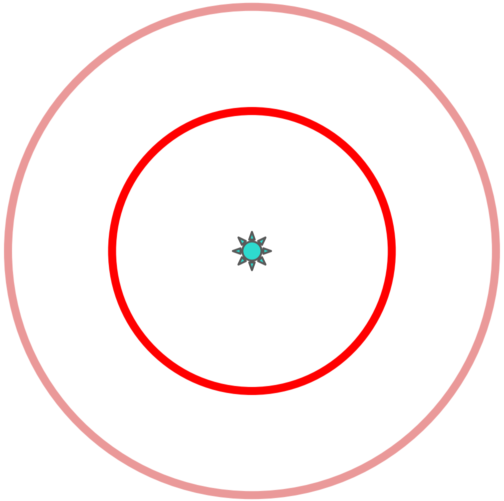
$$E(\mathbf{x}) \equiv \frac{\mathrm{d}\Phi(\mathbf{x})}{\mathrm{d}A}$$

A scales as R^2, so E(x) decreases as R increases.



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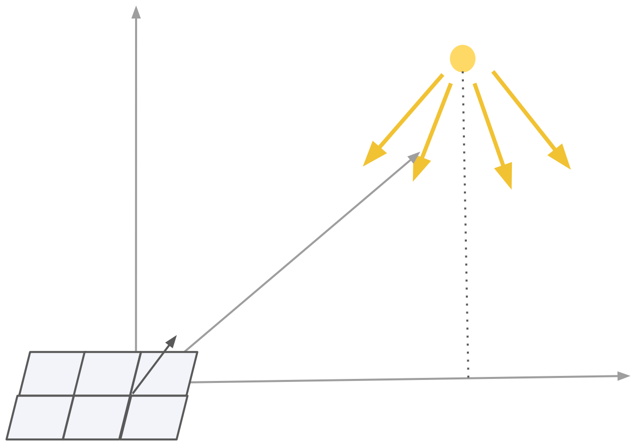
Worksheet Question 4.2

4.2 Irradiance Calculation

2. A point light at position $(6, 0, 8)$ (in meters) with radiant flux (power) of 100 watts directs all its light uniformly into the hemisphere directly below it. Some of this light falls on a flat, tilted surface passing through the origin, with surface normal vector $(1, 1, 1)$.

   What is the irradiance at the origin? Show your work and use the correct units.



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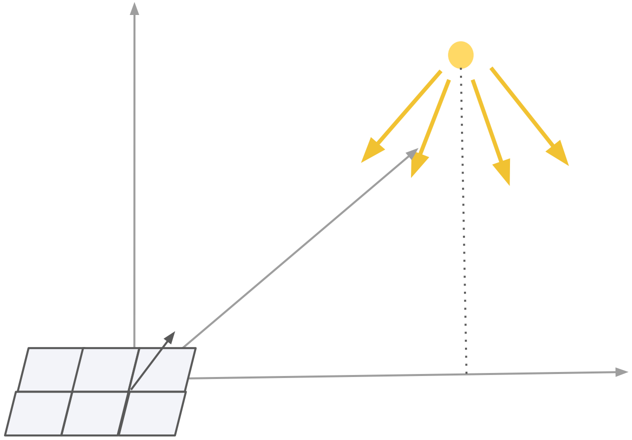

Irradiance is power/area: $\frac{\Phi}{2\pi r^2} \cos(\Theta)$

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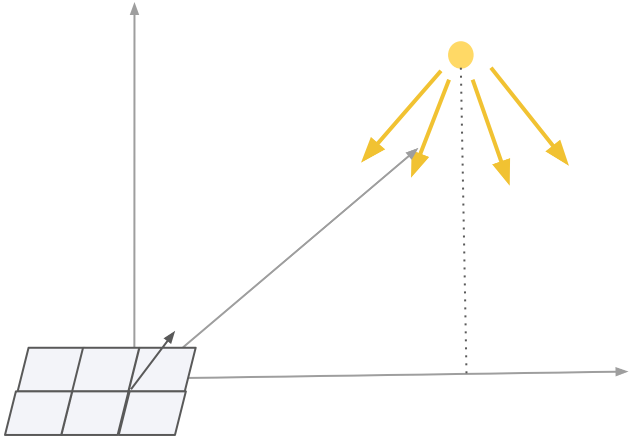

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What do we know?

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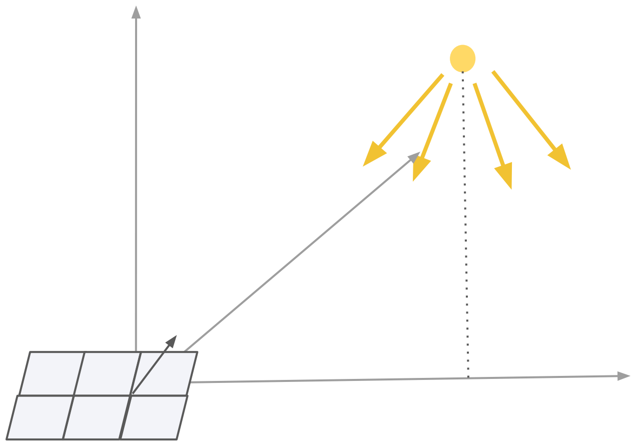

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What do we know?

$\Phi = 100$ W

What do we *not* know?

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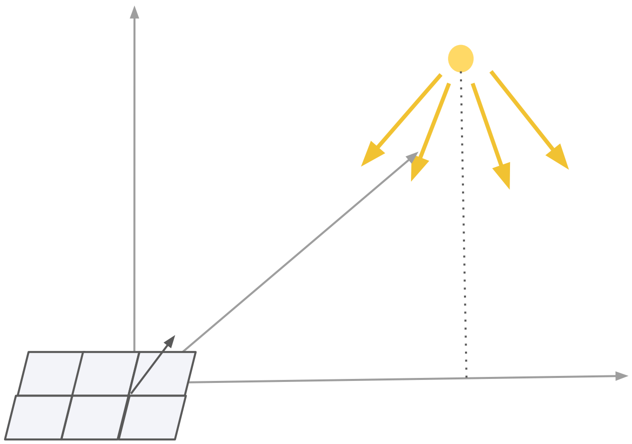

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What do we *not* know?

r, Θ

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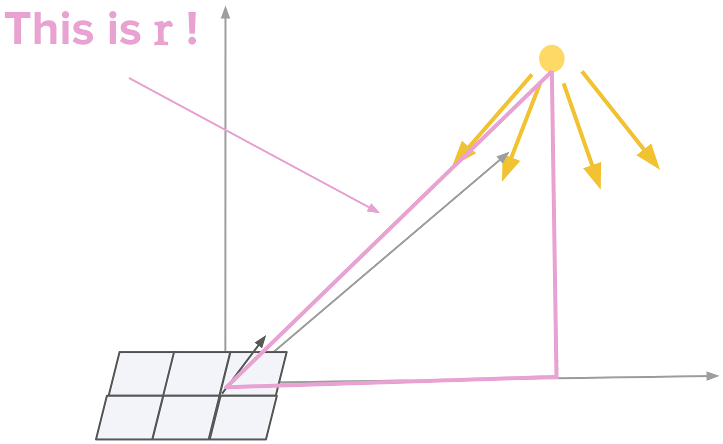

Irradiance is power/area: $\frac{\Phi}{2\pi r^2} \cos(\Theta)$

Use the Pythagorean Theorem

$r = (6^2 + 8^2)^{½} = 10$

4.2 Irradiance Calculation

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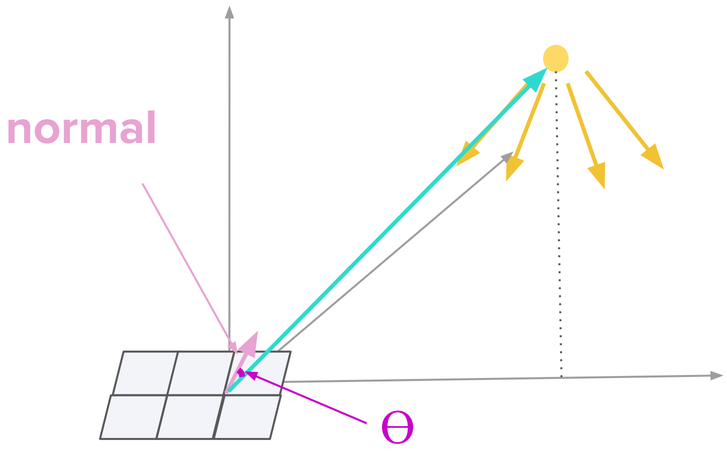

Irradiance is power/area: $\frac{\Phi}{2\pi r^2} \cos(\Theta)$

Cosine angle between surface normal and the light position is given by the dot product of normalized vectors:

$\cos(\Theta) = (6, 0, 8)/10 \cdot (1, 1, 1)/\sqrt{3}$

$= \frac{14}{10\sqrt{3}}$

4.2 Irradiance Calculation

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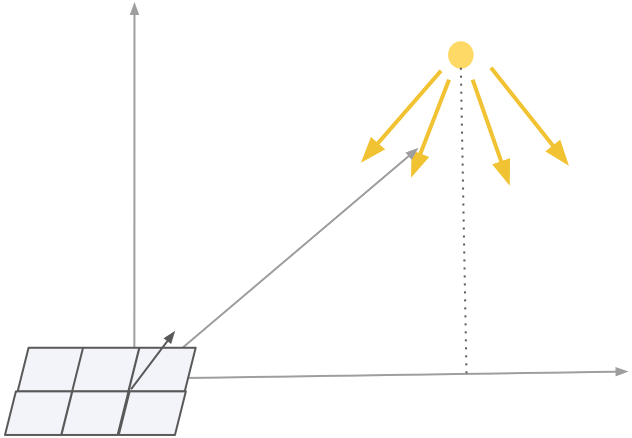
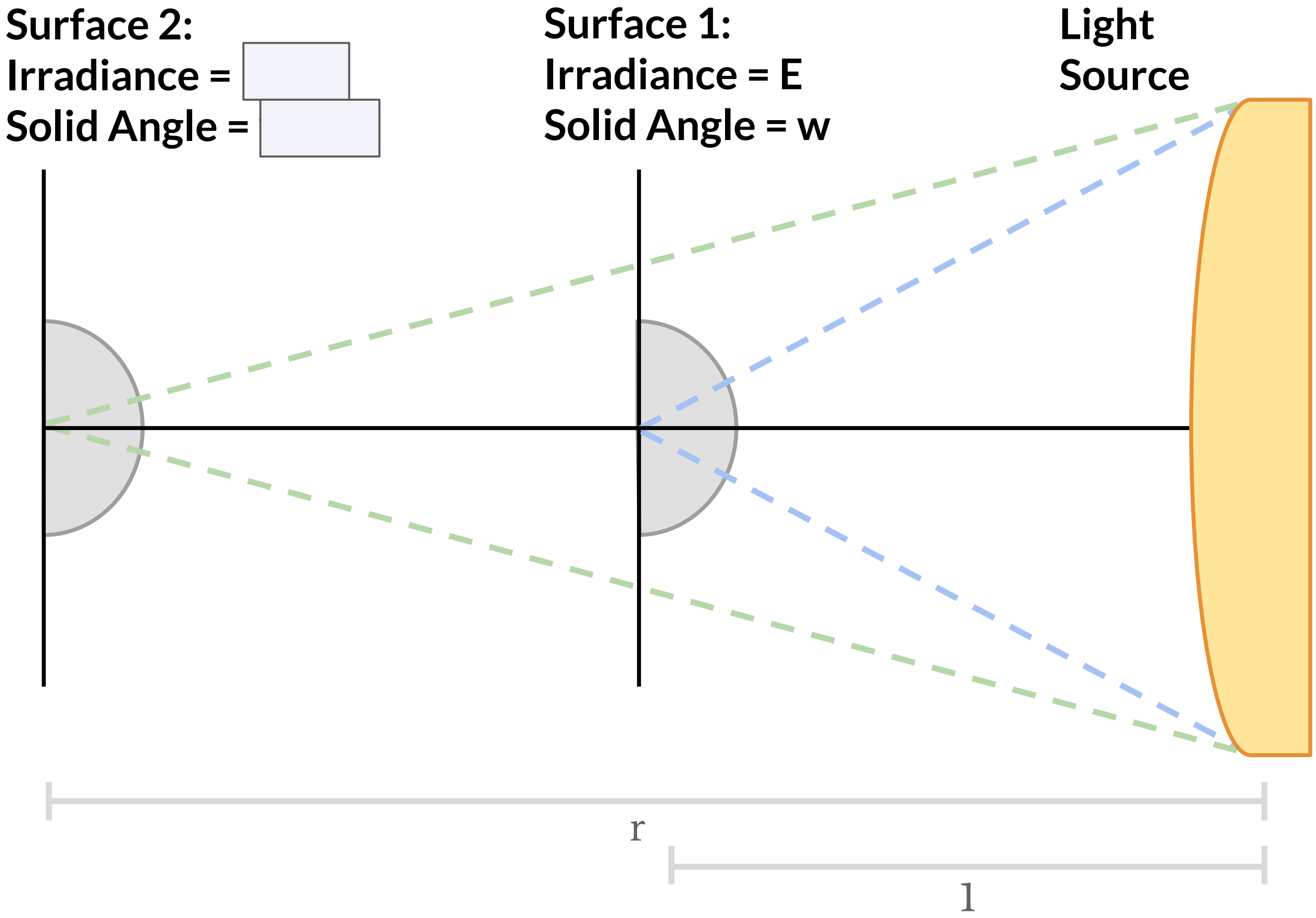


Irradiance is power/area: $\frac{\Phi}{2\pi r^2} \cos(\Theta)$

$$E = \frac{\Phi}{2\pi r^2} \cos(\Theta)$$

$$= \frac{100}{2\pi(10)^2} \frac{14}{10\sqrt{3}} \text{ W/m}^2$$

$$= \frac{7}{10\pi\sqrt{3}} \text{ W/m}^2$$

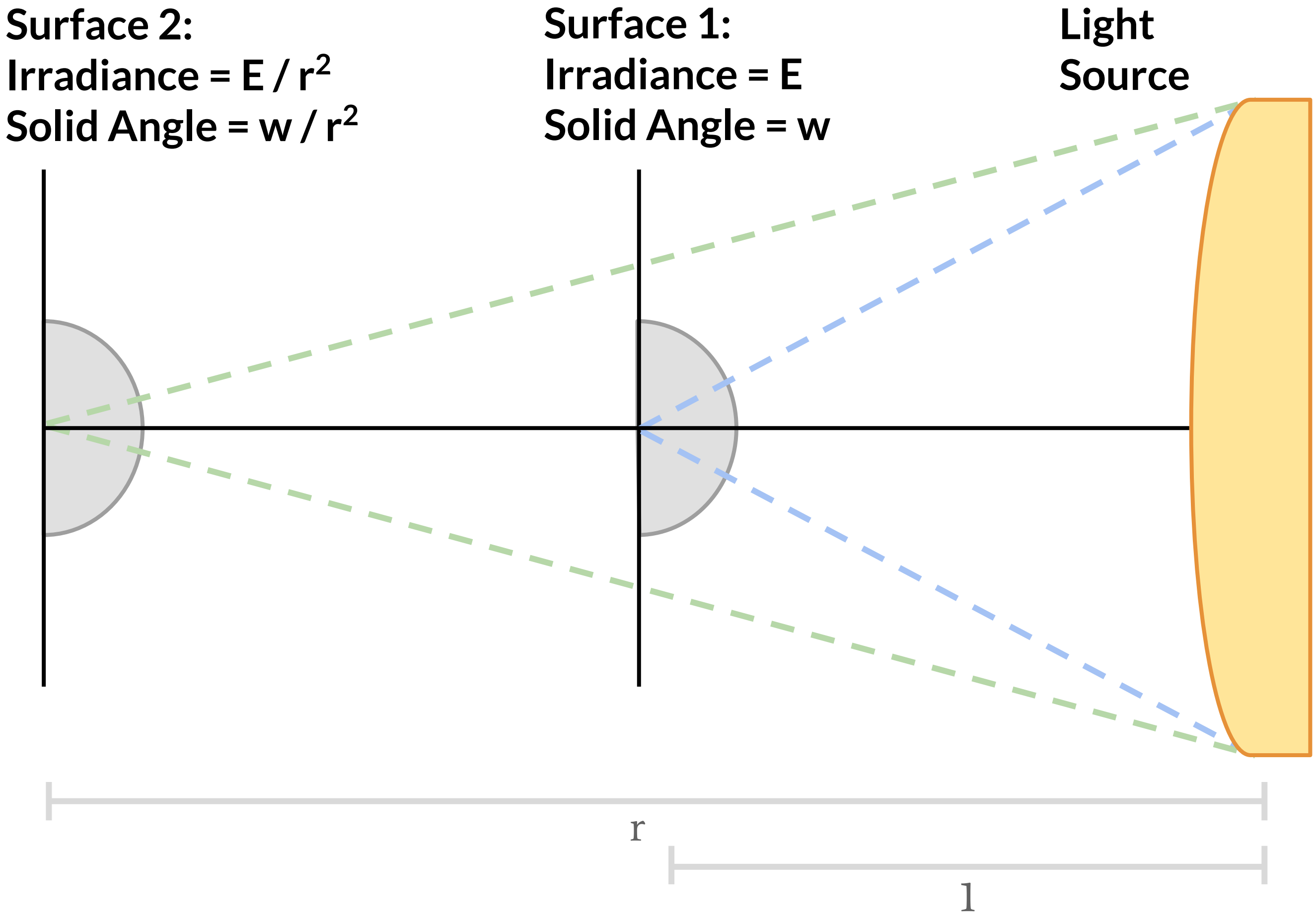


How does radiance change with distance?

Hint:

1. The light source emits uniform flux.

2. With irradiance E and solid angle w at surface 1, what are these values at the surface 2?



Since L = dE / dw (cos θ = 1 here), the radiance at the two surfaces is the same.

Radiance does not change with distance.

Radiance

Light Traveling Along A Ray

1. Radiance is the fundamental field quantity that describes the distribution of light in an environment
   - Radiance is the quantity associated with a ray
   - Rendering is all about computing radiance
2. Radiance is invariant along a ray in a vacuum

Let's Take Attendance.

- Be sure to select Week 6 and input your TA's secret word 😉
- Any feedback? Let us know!