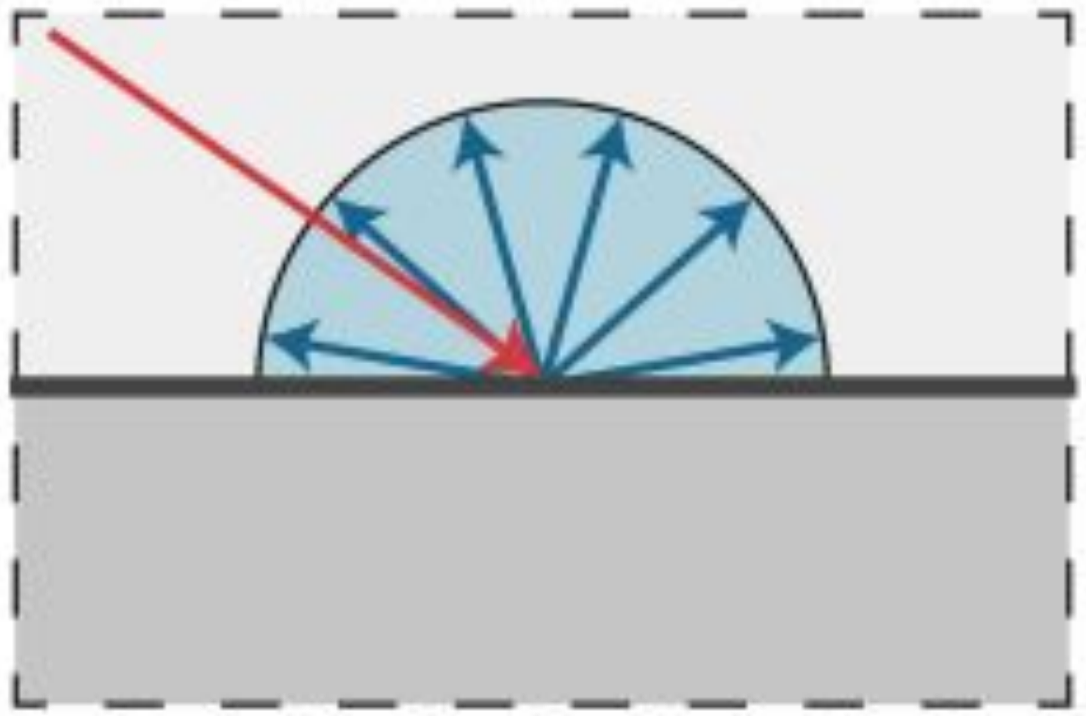
Discussion 08

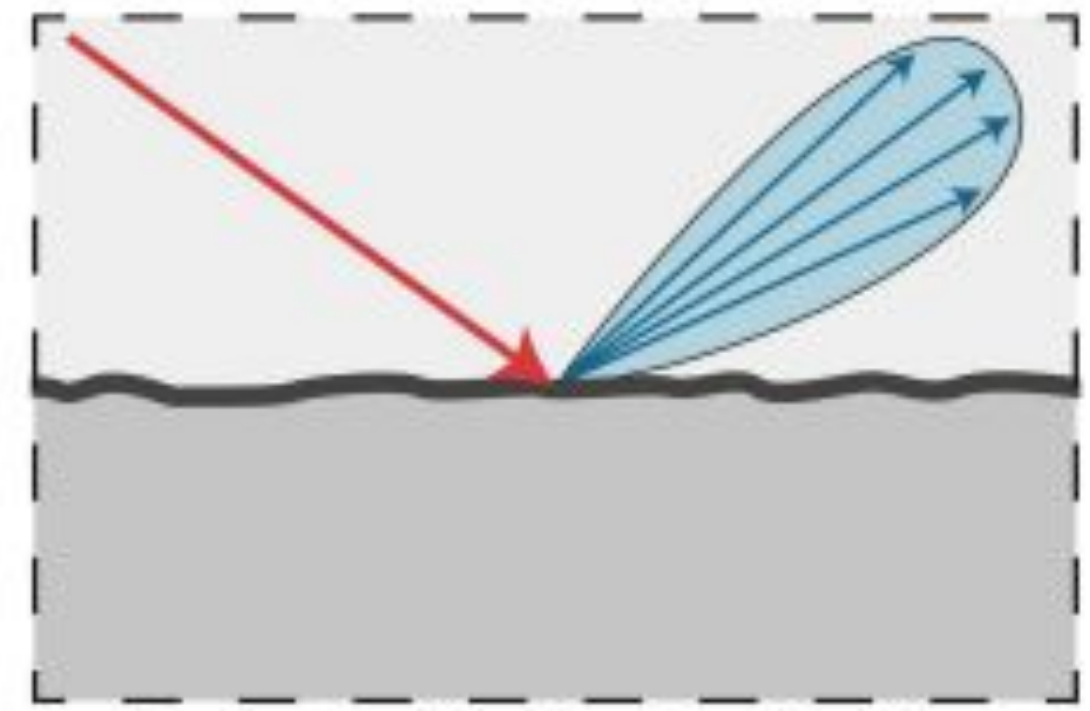
Material Modeling

Computer Graphics and Imaging
UC Berkeley CS 184/284A

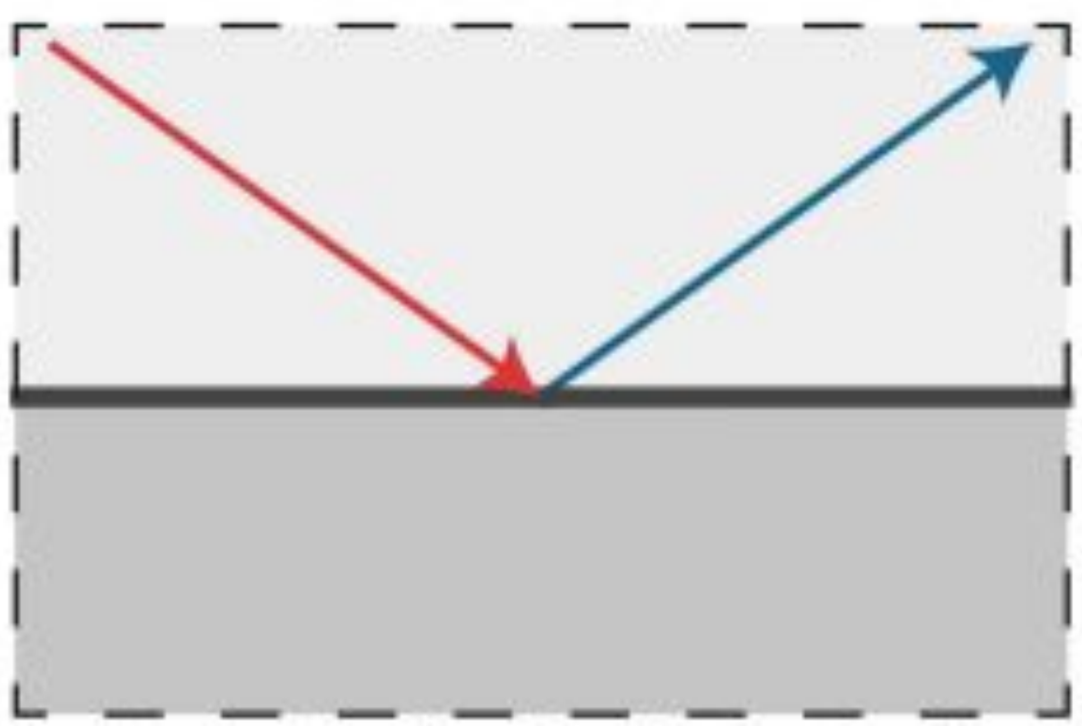
Optics for Ideal Surfaces



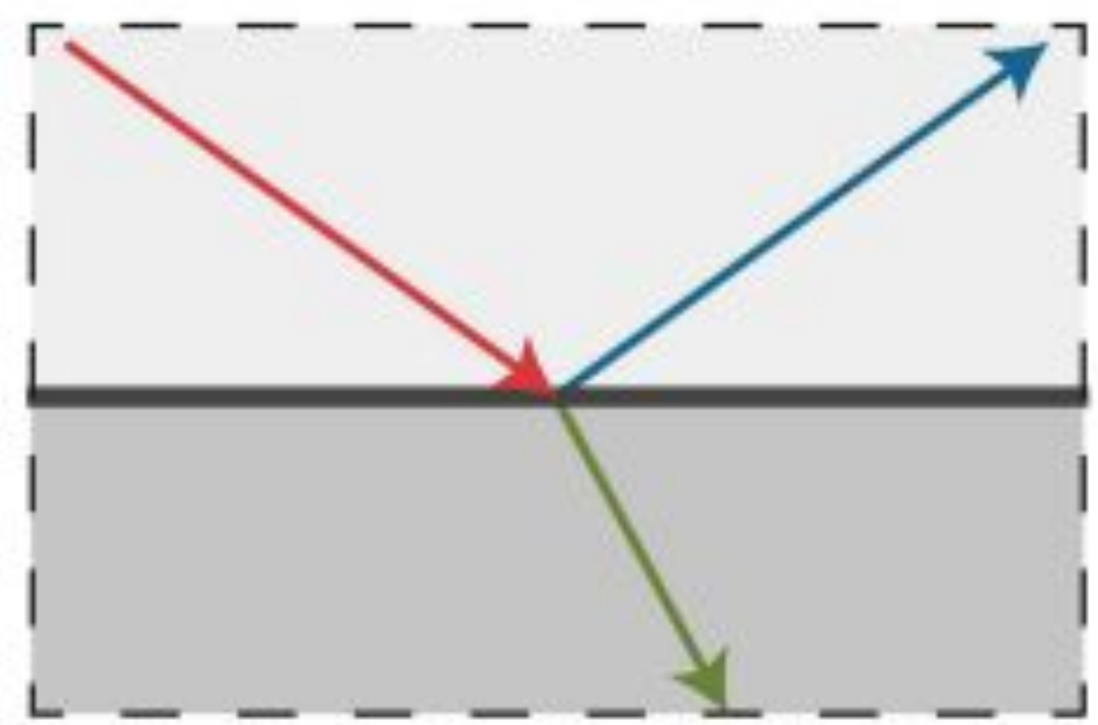
Diffuse / Lambertian
(Scatters light equally in all directions)



Glossy / Specular
(Scatters light preferentially around a reflection direction)



Ideal Mirror (Reflects light into a single direction)



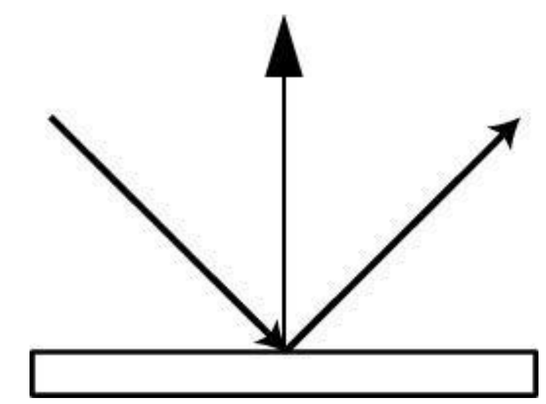
Ideal Refractive (Glass)
(Transmits and reflects light)

Categories of Reflection Functions

Ideal specular

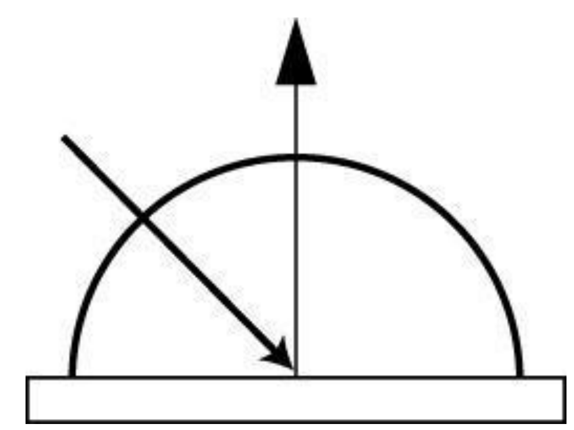
Ideal specular

- **Perfect mirror reflection**



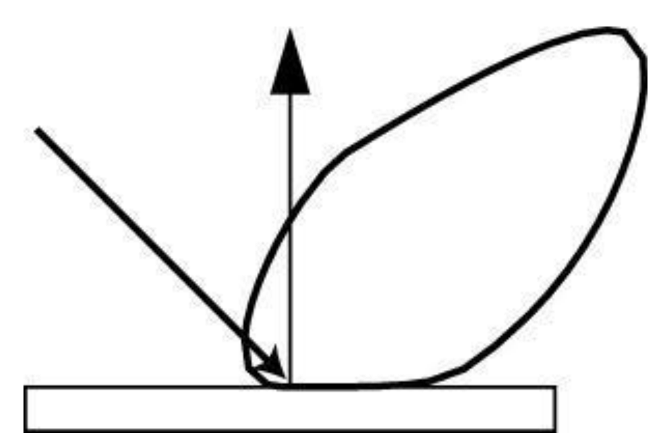
Ideal diffuse

- **Equal reflection in all directions**



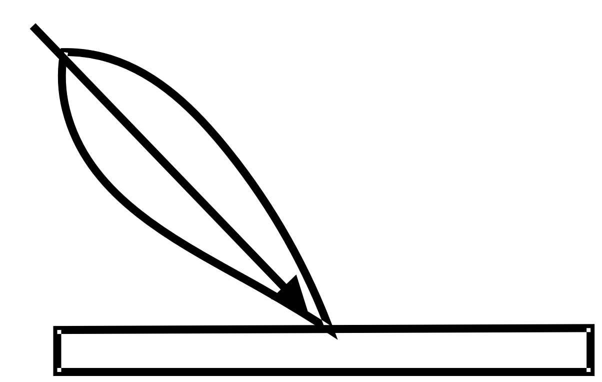

Glossy specular

- **Majority of light reflected near mirror direction**



Retro-reflective

- **Light reflected back towards light source**



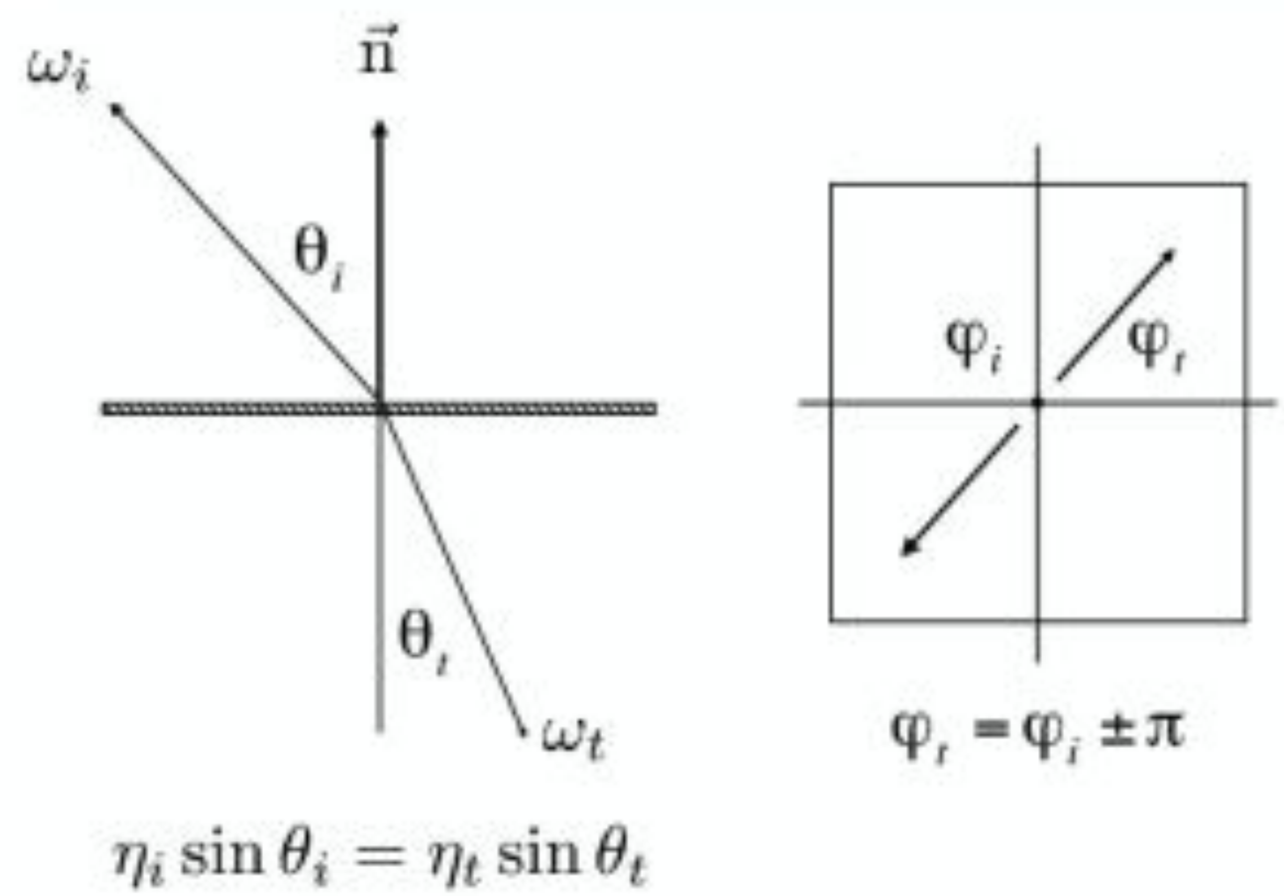

Diagrams illustrate how light from incoming direction is reflected in various outgoing directions.

Snell’s Law of Refraction

Light bends when it crosses a boundary between two media with different indices of refraction (IOR).

$$\eta_i \sin(\theta_i) = \eta_t \sin(\theta_t)$$

- η_i, η_t: IOR of incident, transmitted media
- θ_i, θ_t: Angles to the surface normal

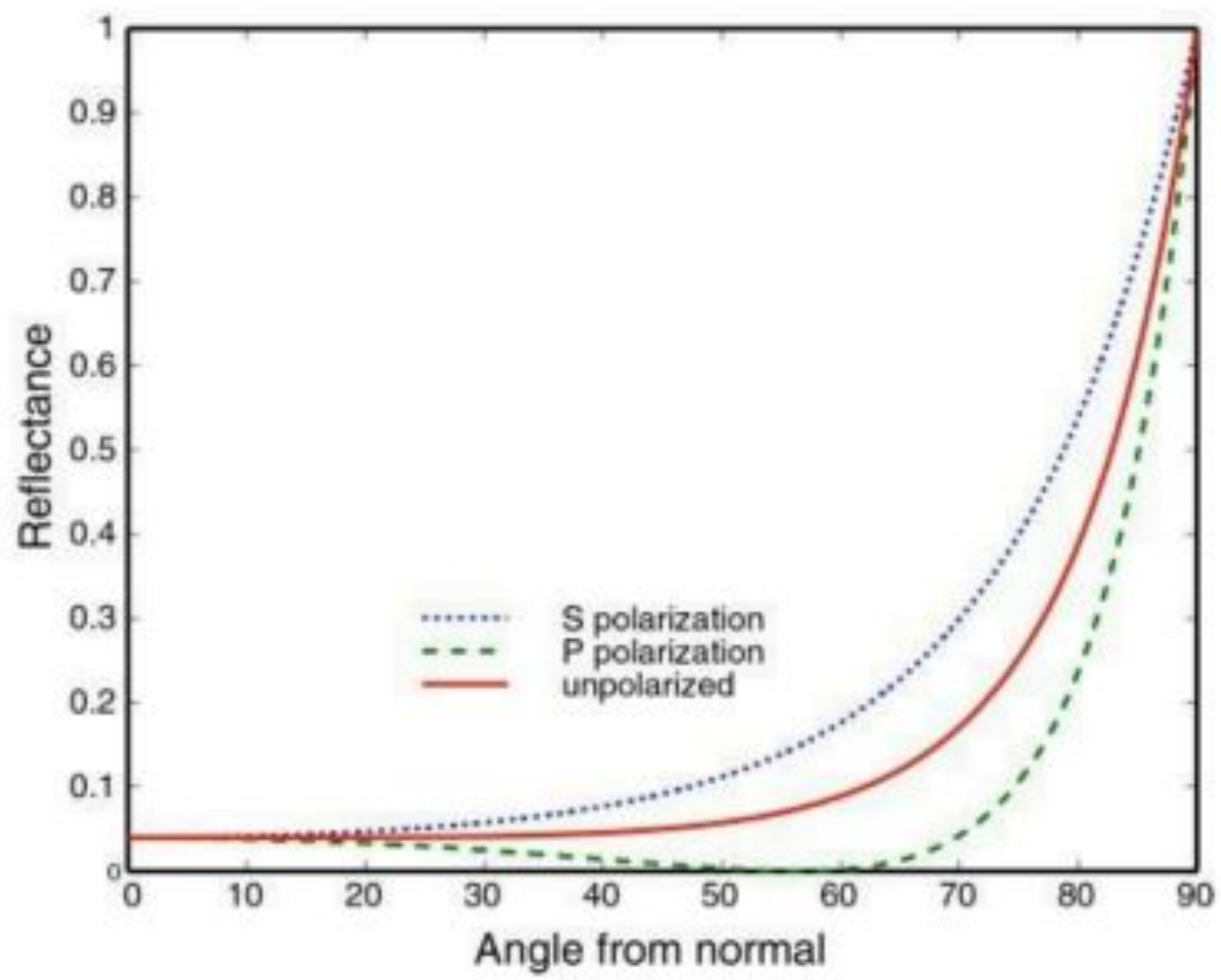


Medium	η*
Vacuum	1.0
Air (sea level)	1.00029
Water (20°C)	1.333
Glass	1.5-1.6
Diamond	2.42

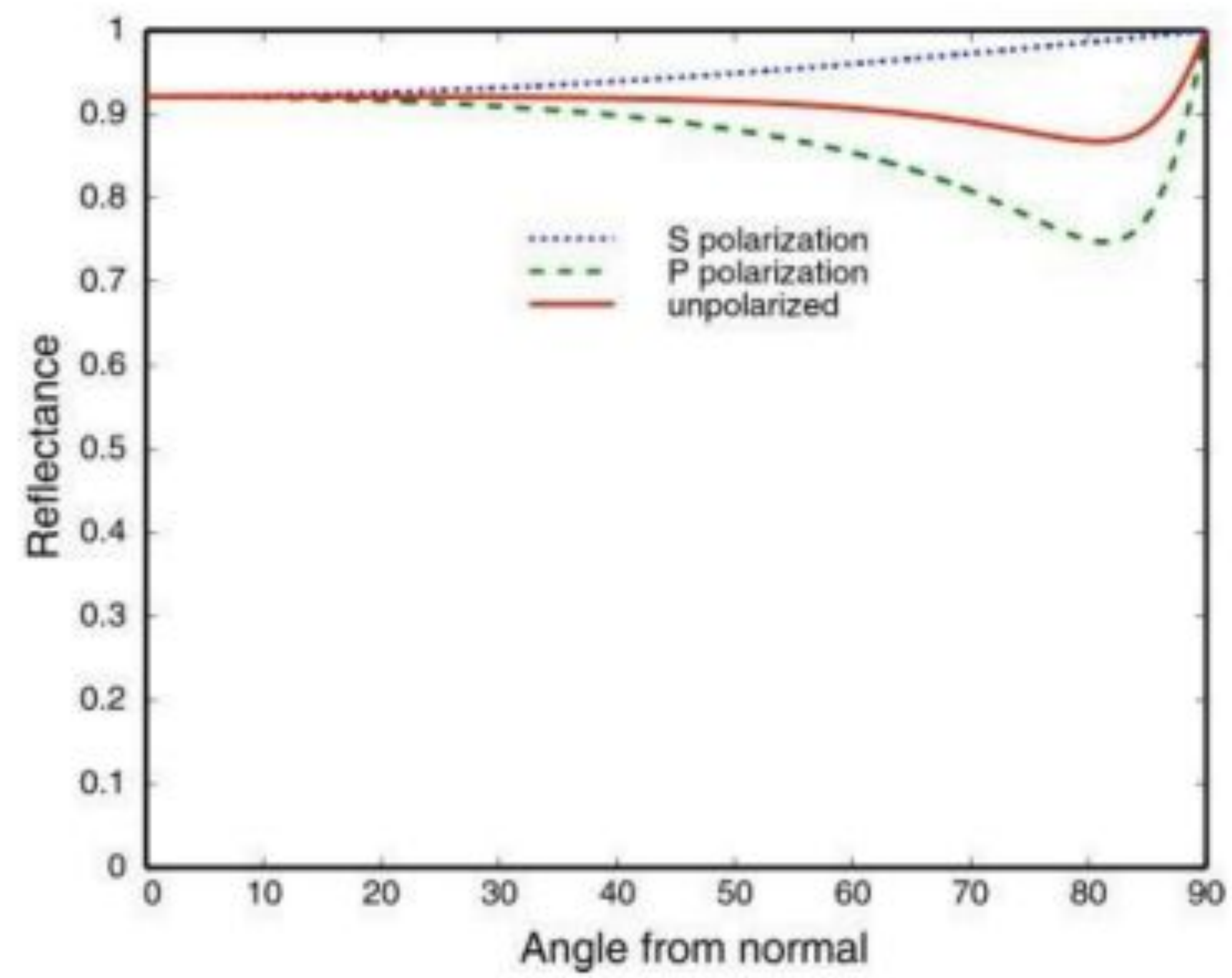
* index of refraction is wavelength dependent (these are averages)

The Fresnel Term (n*)

The Fresnel equations describe how a surface's reflectivity changes with viewing angle and material type. They determine the ratio of reflected to transmitted light.



Dielectrics (e.g., plastic, water) reflect very little head-on, but much more at grazing angles.



Conductors (e.g., metals) are highly reflective at all angles and have a colored reflectance.

Example Problem: Snell's Law

Problem

A ray of light enters a flat water surface ($\eta_t = 1.33$) from air ($\eta_i \approx 1.0$) at an incident angle of $\theta_i = 45^\circ$. What is the angle of refraction θ_t?

Example Problem: Snell's Law

Problem

A ray of light enters a flat water surface ($\eta_t = 1.33$) from air ($\eta_i \approx 1.0$) at an incident angle of $\theta_i = 45^\circ$. What is the angle of refraction θ_t?

Derivation

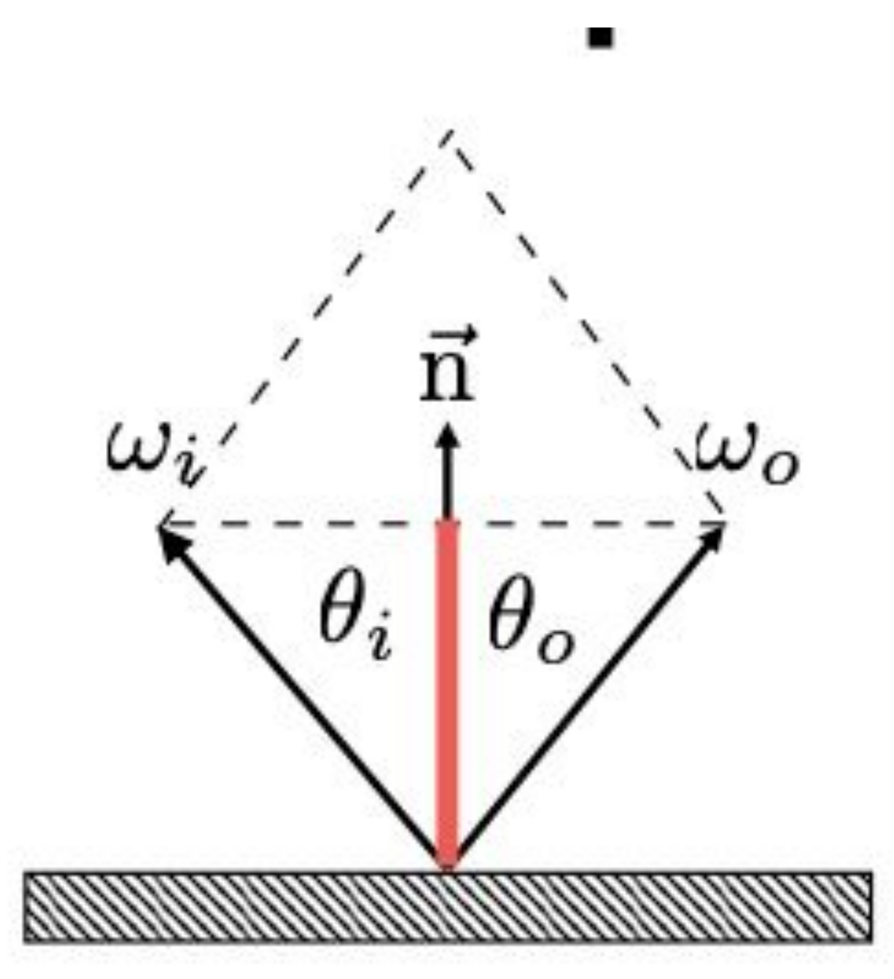
1. Start with Snell's Law: $\eta_i \sin(\theta_i) = \eta_t \sin(\theta_t)$
2. Isolate $\sin(\theta_t)$: $\sin(\theta_t) = \frac{\eta_i}{\eta_t} \sin(\theta_i)$
3. Solve for θ_t: $\theta_t = \arcsin\left(\frac{\eta_i}{\eta_t} \sin(\theta_i)\right)$

Solution

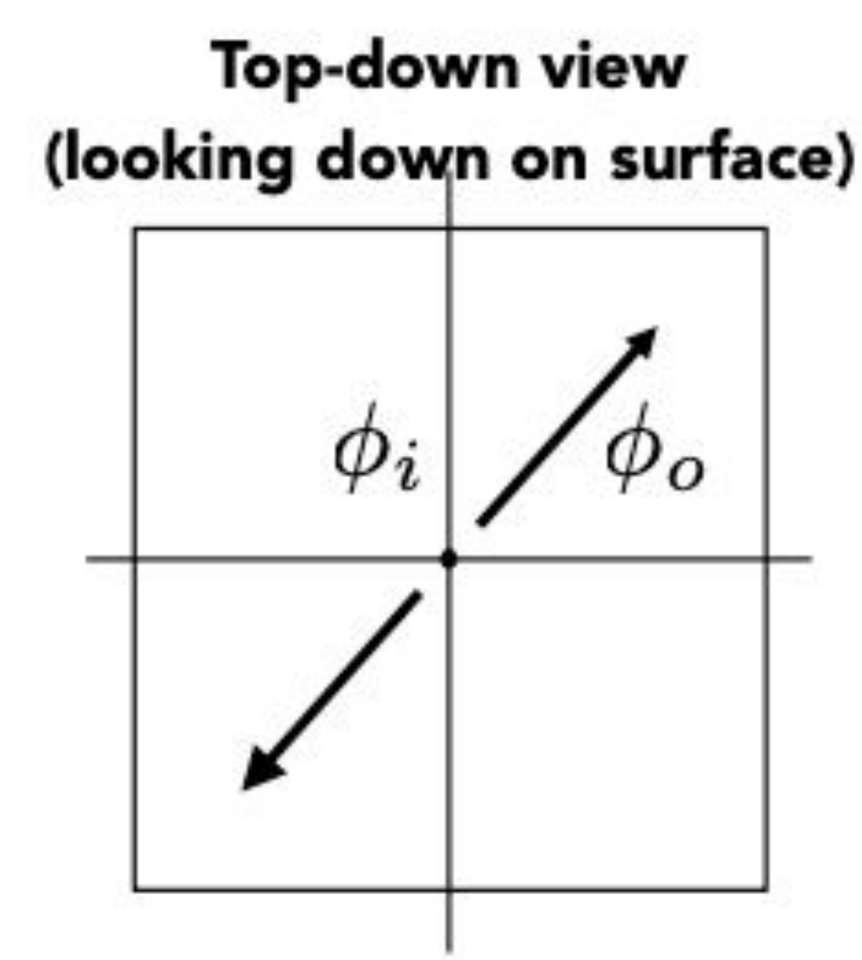
$$\begin{aligned}
\theta_t &= \arcsin\left(\frac{1.0}{1.33} \sin(45^\circ)\right) \\
&= \arcsin\left(\frac{1.0}{1.33} \cdot 0.707\right) \\
&= \arcsin(0.531) \\
&\approx 32.1^\circ
\end{aligned}$$

Perfect Specular Reflection:

How to compute bounce



$$\theta = \theta_o = \theta_i$$



$$\phi_o = (\phi_i + \pi) \bmod 2\pi$$

$$\omega_o + \omega_i = 2\cos\theta\,\vec{n} = 2(\omega_i \cdot \vec{n})\vec{n}$$

$$\omega_o = -\omega_i + 2(\omega_i \cdot \vec{n})\vec{n}$$

Example Problem: Specular Reflection

Problem

A light ray comes from the direction $\boldsymbol{v} = [-1, -1, 0]$ and hits a surface with normal $\boldsymbol{n} = [0, 1, 0]$. Note that $\boldsymbol{v}$ points *towards* the surface. The vector $\boldsymbol{\omega_i}$ used in the formula points *away*, so $\boldsymbol{\omega_i} = -\boldsymbol{v} = [1, 1, 0]$. Calculate the outgoing reflection direction $\boldsymbol{\omega_o}$.

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Derivation

1. Calculate the dot product: $(\boldsymbol{\omega_i} \cdot \boldsymbol{n})$
2. Scale the normal by $2\times$ the dot product: $2(\boldsymbol{\omega_i} \cdot \boldsymbol{n})\boldsymbol{n}$
3. Use the reflection formula: $\boldsymbol{\omega_o} = -\boldsymbol{\omega_i} + 2(\boldsymbol{\omega_i} \cdot \boldsymbol{n})\boldsymbol{n}$

Solution

$$
\begin{aligned}
\boldsymbol{\omega_i} \cdot \boldsymbol{n} &= (1)(0) + (1)(1) + (0)(0) = 1 \\
\boldsymbol{\omega_o} &= -[1, 1, 0] + 2(1)[0, 1, 0] \\
&= [-1, -1, 0] + [0, 2, 0] \\
&= [-1, 1, 0]
\end{aligned}
$$

The light ray leaves in the direction $[-1, 1, 0]$.

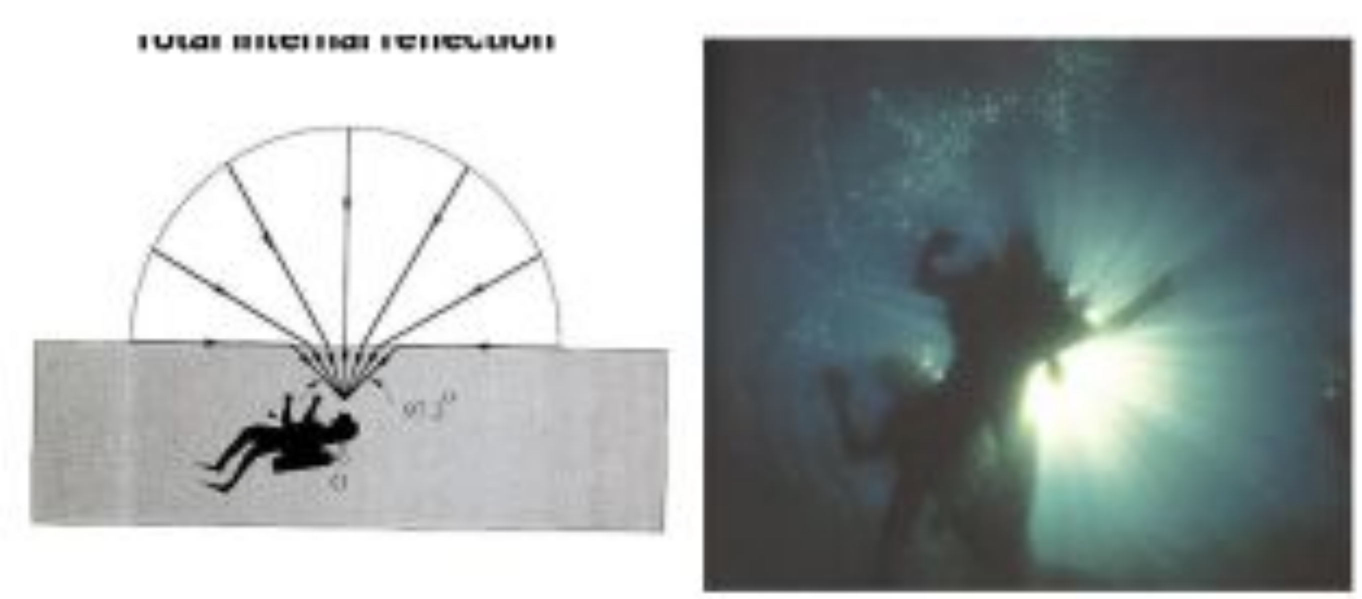
Total Internal Reflection (TIR)

When light goes from a dense to a less dense medium ($\eta_i > \eta_t$), if θ_i is greater than the **critical angle** θ_c, all light is reflected, and none is refracted.

TIR occurs when Snell's law would require $\sin(\theta_t) > 1$, which is impossible. The boundary case $\sin(\theta_t) = 1$ (i.e., $\theta_t = 90^\circ$) gives the critical angle:

$$\eta_i \sin(\theta_c) = \eta_t \sin(90^\circ) = \eta_t$$

$$\theta_c = \arcsin\left(\frac{\eta_t}{\eta_i}\right)$$



Snell's Window is a result of TIR.

Example Problem: Total Internal Reflection

Problem

A diver is underwater ($\eta_i = 1.33$) and shines a flashlight up towards the surface (air, $\eta_t = 1.0$).

1. What is the critical angle θ_c for the water-air interface?
2. What happens if the light hits the surface at 40°? At 60°?

Problem

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1. What is the critical angle θ_c for the water-air interface?
2. What happens if the light hits the surface at 40°? At 60°?

Solution (Part 1)

The critical angle only exists when going from a more dense to a less dense medium ($\eta_i > \eta_t$), which is the case here.

$$\theta_c = \arcsin\left(\frac{\eta_t}{\eta_i}\right) = \arcsin\left(\frac{1.0}{1.33}\right)$$
$$= \arcsin(0.752) \approx 48.8^\circ$$

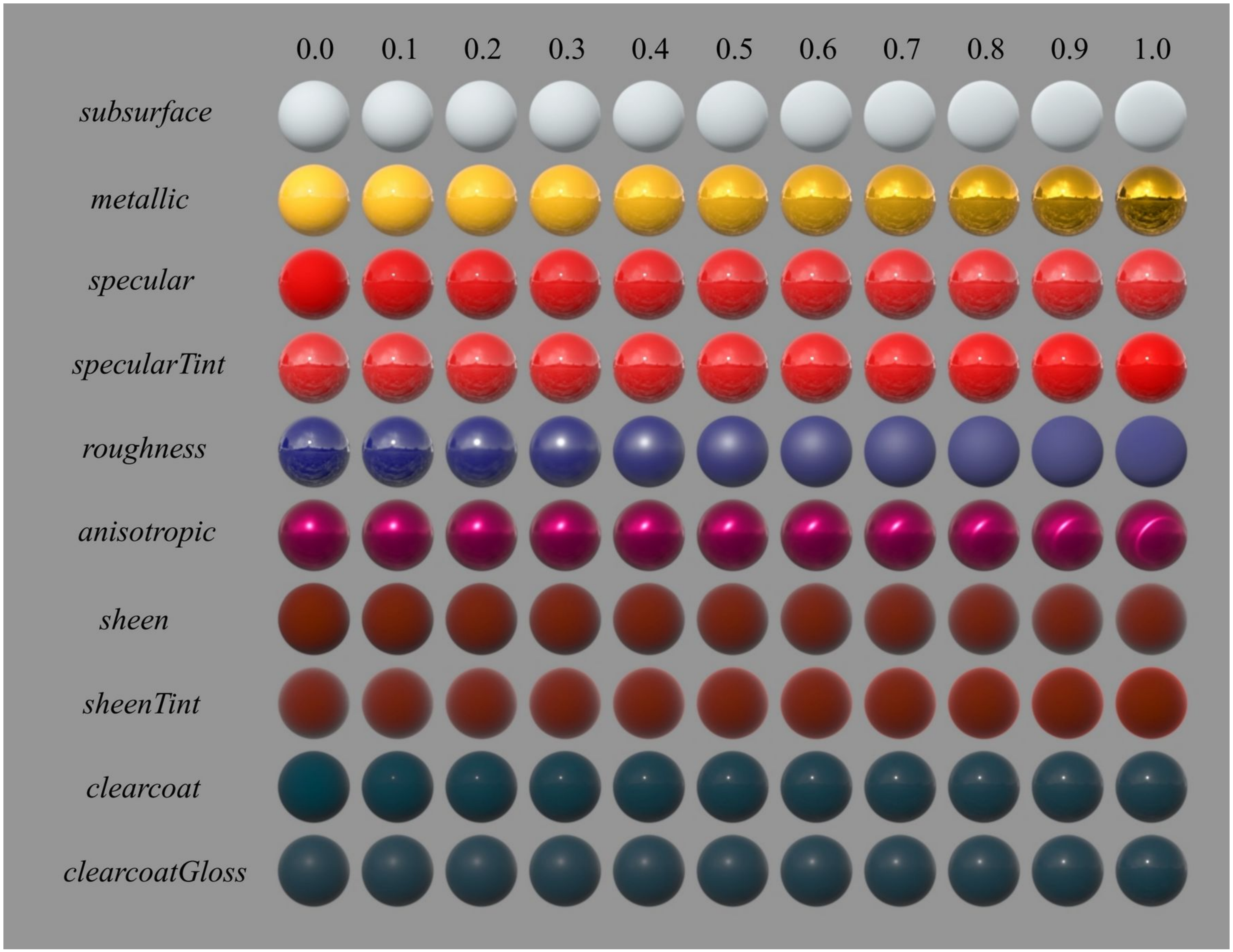
Solution (Part 2)

- **At 40**$^\circ$: Since $40^\circ < \theta_c$, the light ray is both refracted out into the air and partially reflected back into the water.
- **At 60**$^\circ$: Since $60^\circ > \theta_c$, the ray undergoes **total internal reflection**. No light escapes into the air; it all reflects back into the water as if from a perfect mirror.

BRDF

The bidirectional reflectance distribution function

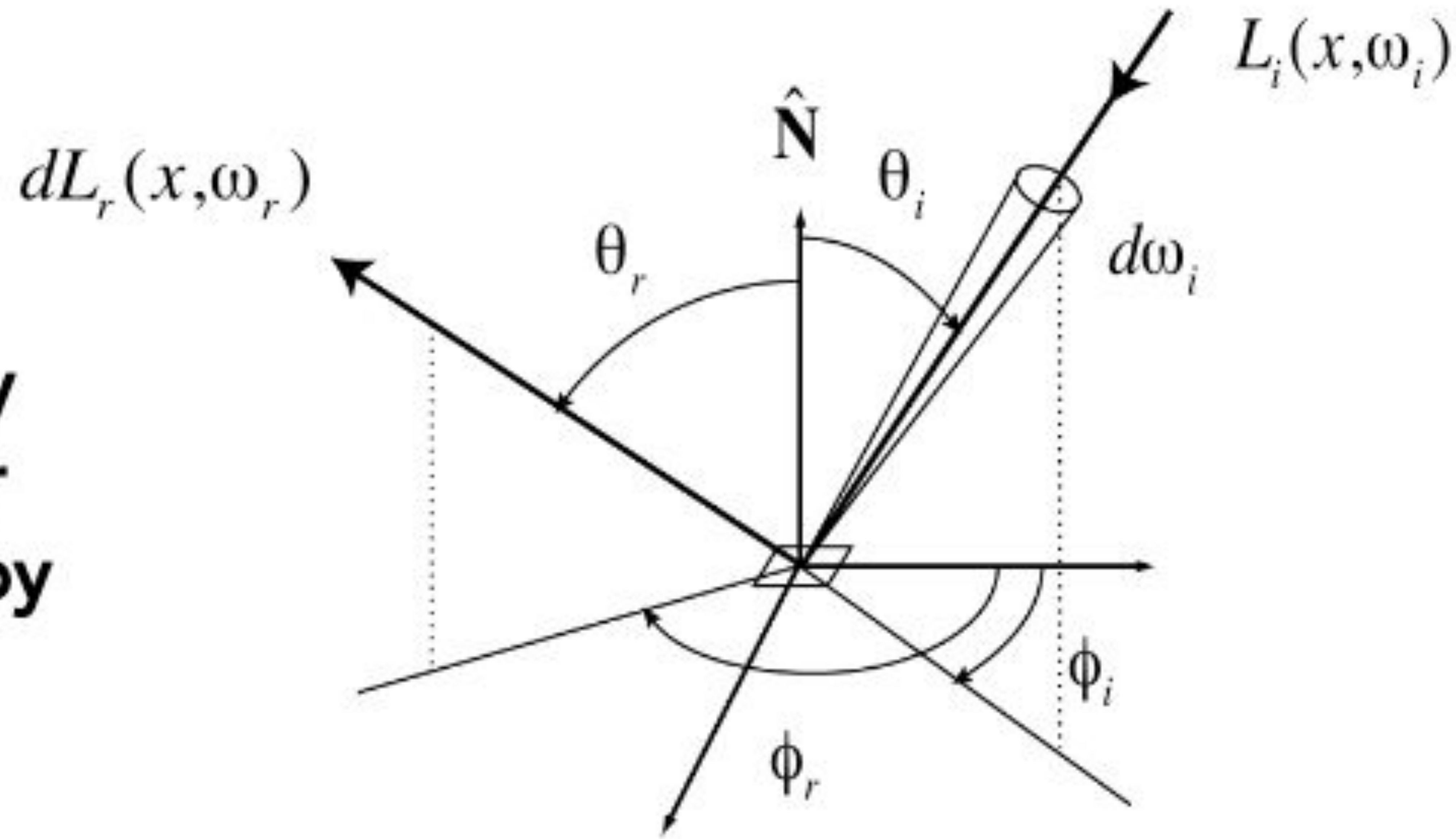
Disney BRDF: flexible model used in production



Physically Based Shading at Disney, 2012

BRDF

Definition: The bidirectional reflectance distribution function (BRDF) represents how much light is reflected into each outgoing direction ω_r from each incoming direction

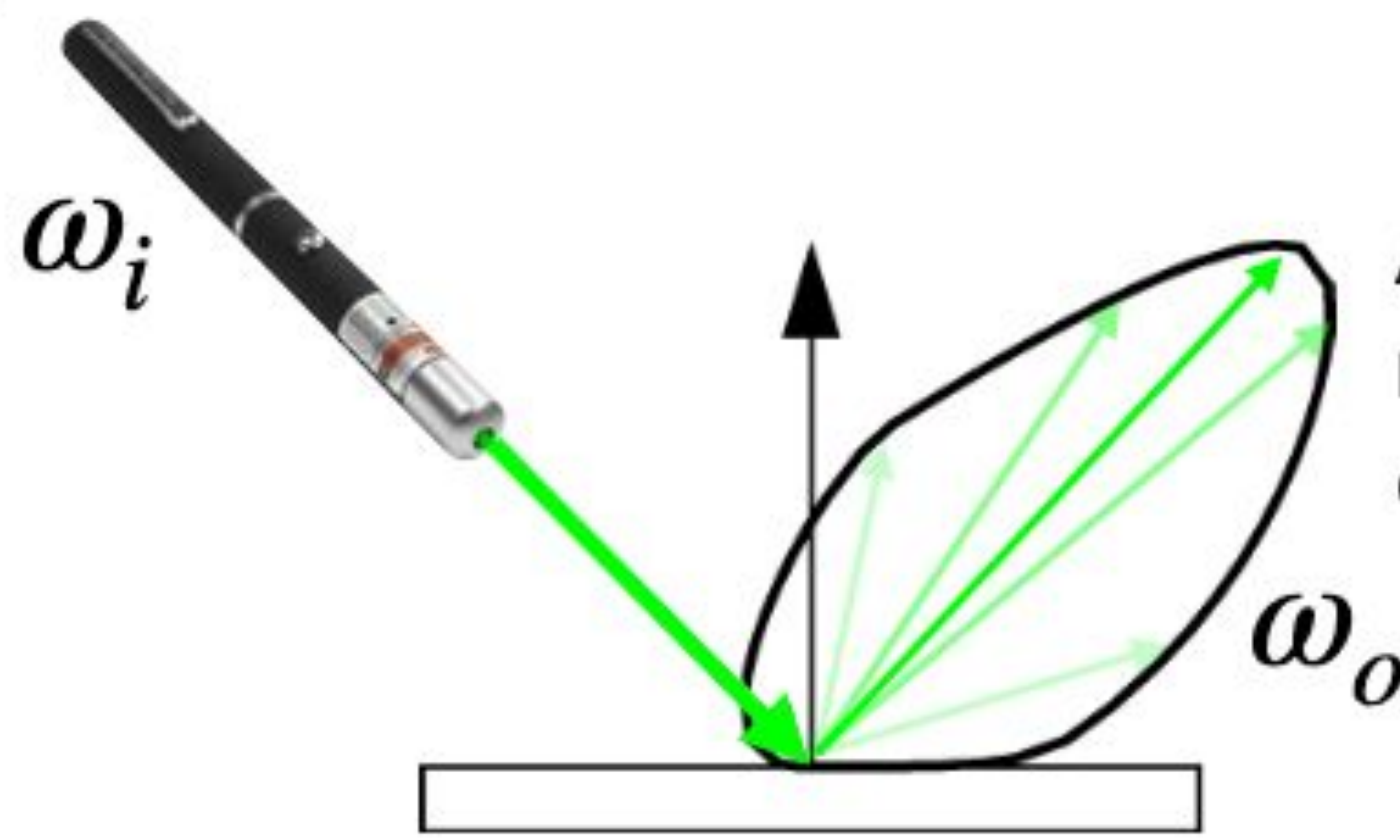


NB: ω_i points away from surface rather than into surface, by convention.

$$f_r(\omega_i \to \omega_r) = \frac{\mathrm{d}L_r(\omega_r)}{\mathrm{d}E_i(\omega_i)} = \frac{\mathrm{d}L_r(\omega_r)}{L_i(\omega_i)\cos\theta_i\,\mathrm{d}\omega_i} \quad \left[\frac{1}{\mathrm{sr}}\right]$$

What is a BRDF?

Imagine shining a laser pointer at a specific point on a surface

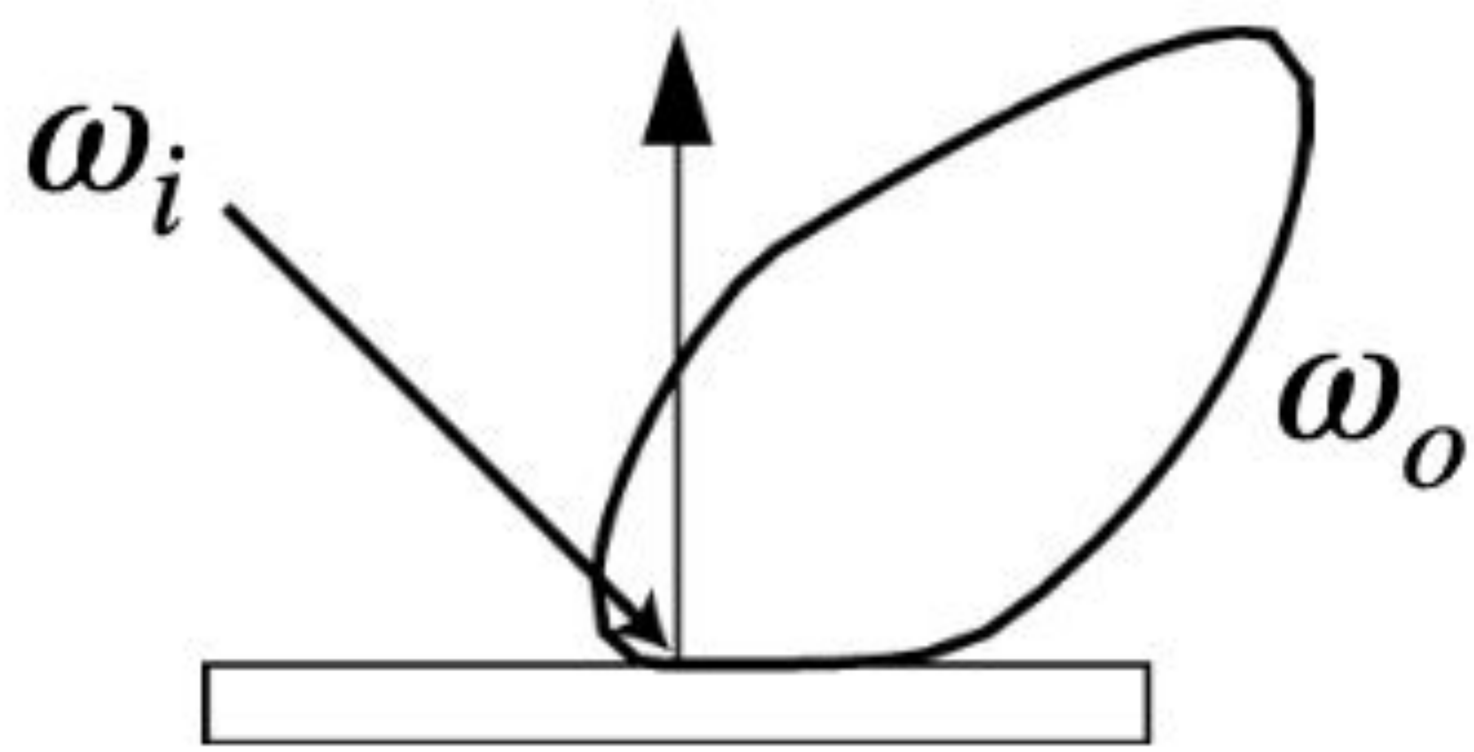


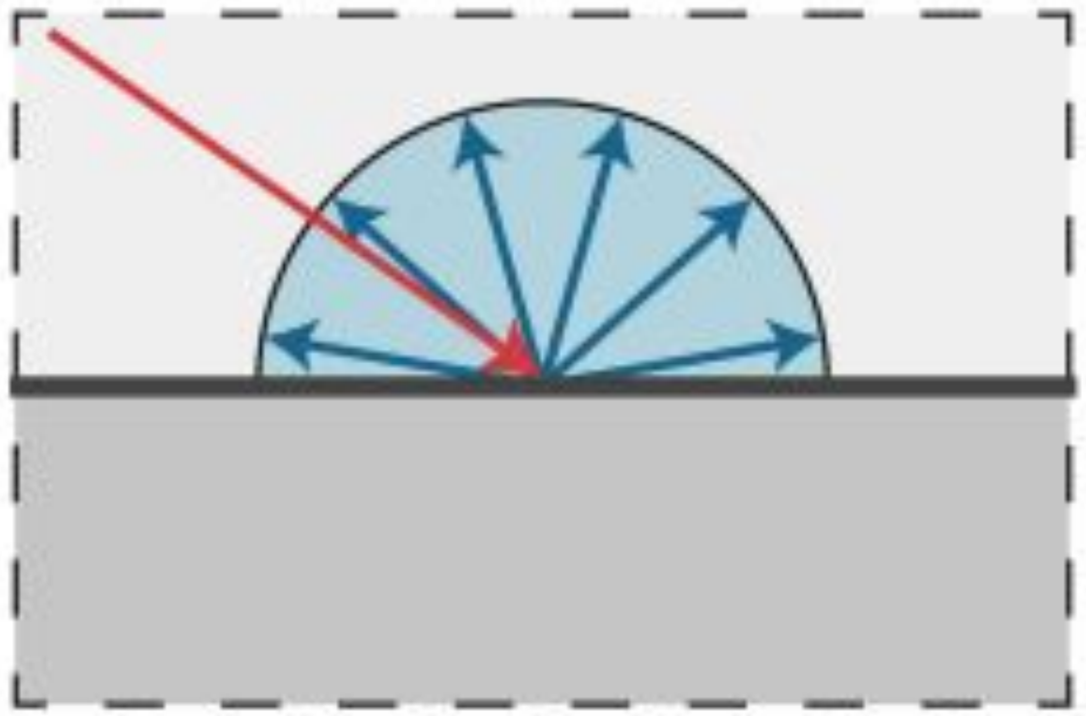
And observing the reflected light in all directions

What is a BRDF?

That's how you read "lobe" diagrams:

Length of lobe = amount of incoming light reflected in that direction





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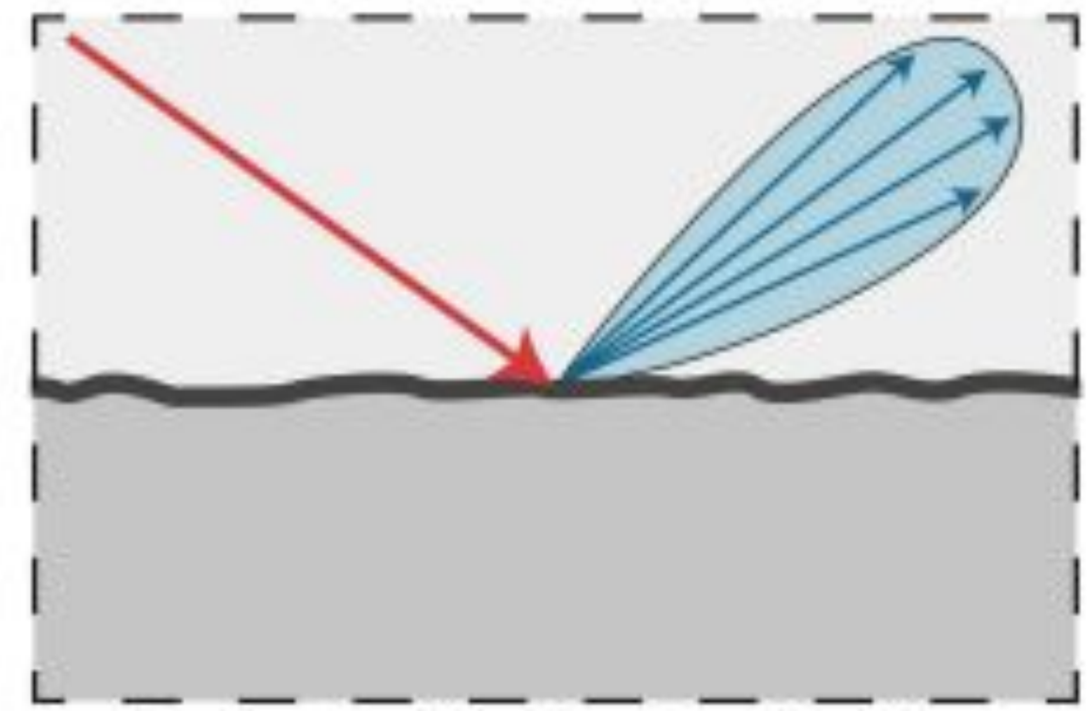

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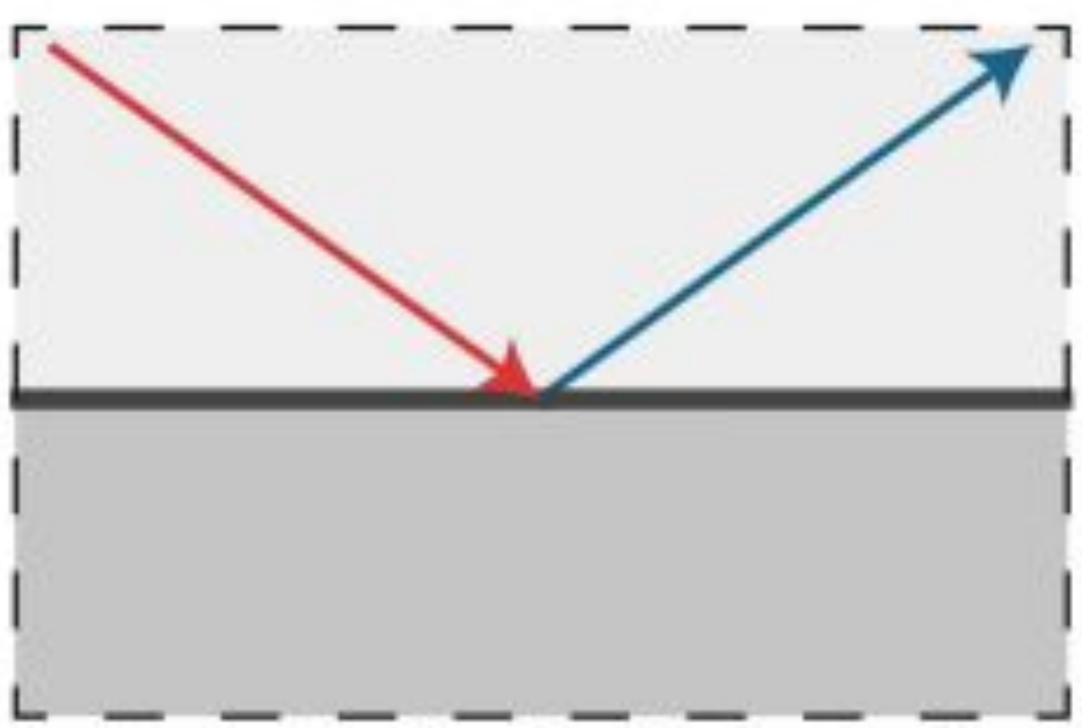
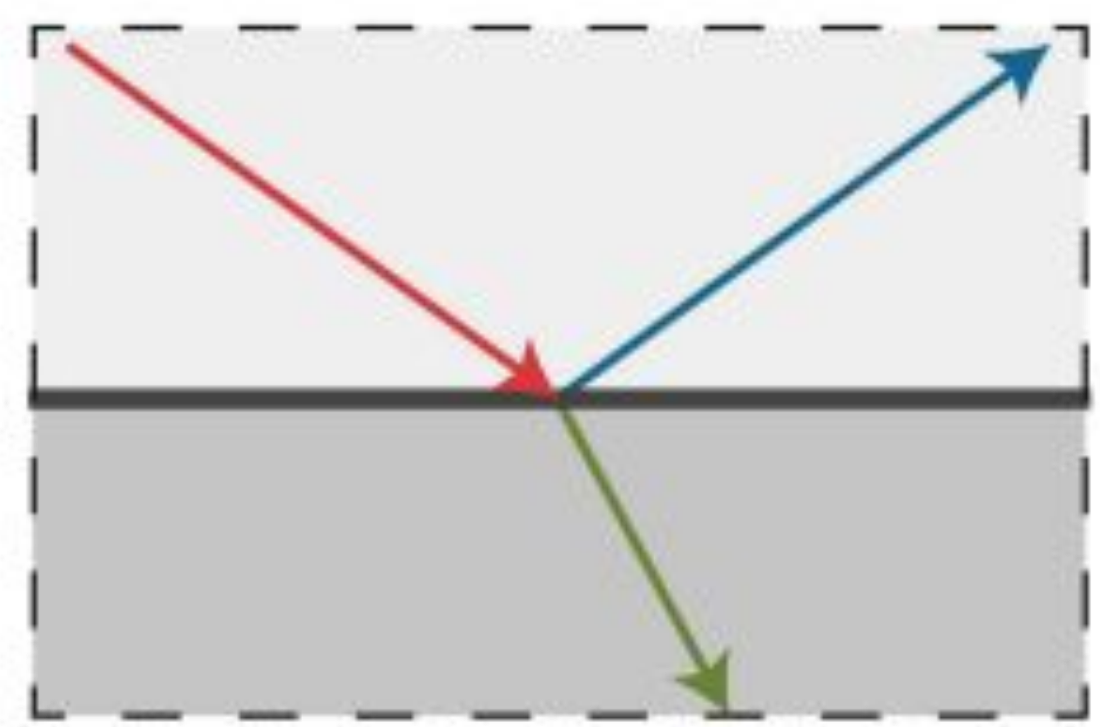


Ideal Mirror (Reflects light into a single direction)



Ideal Refractive (Glass)
(Transmits and reflects light)

Important properties

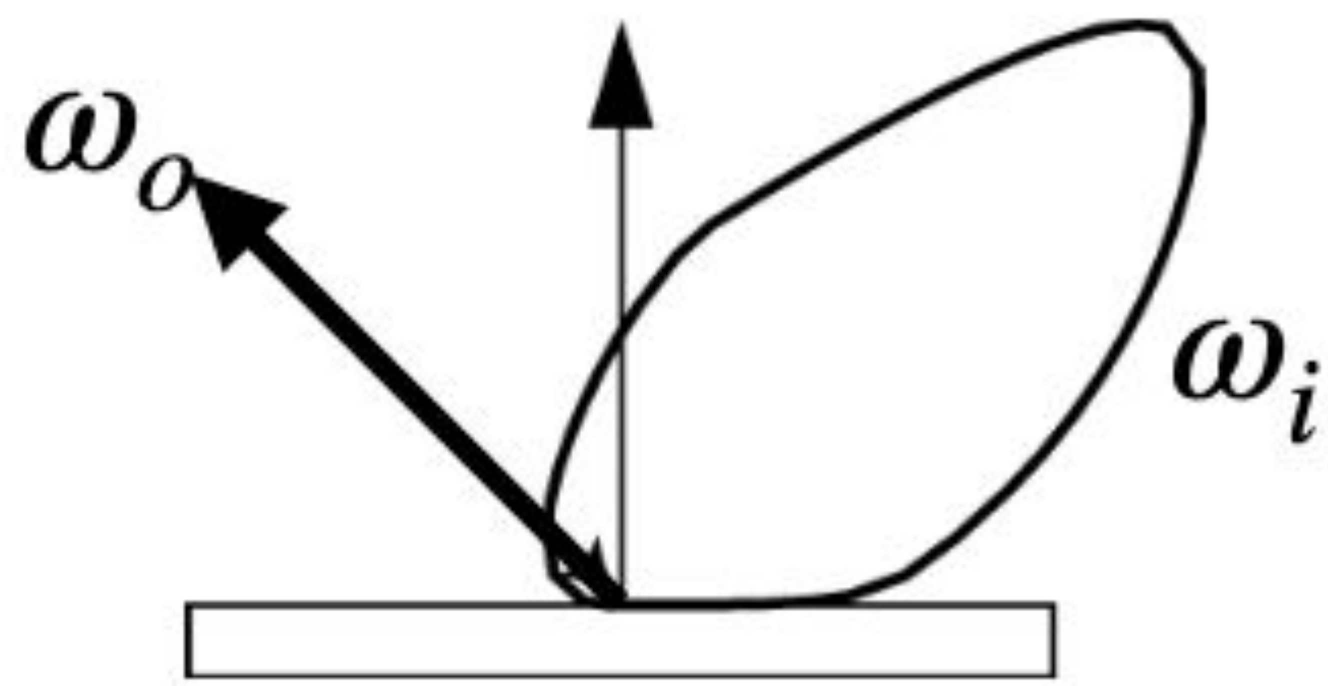
Positivity (self explanatory)

$$f(\omega_i, \omega_o) \geq 0$$

Important properties

- **Reciprocity (can trace light paths either direction)**

$$f(\omega_i, \omega_o) = f(\omega_o, \omega_i)$$

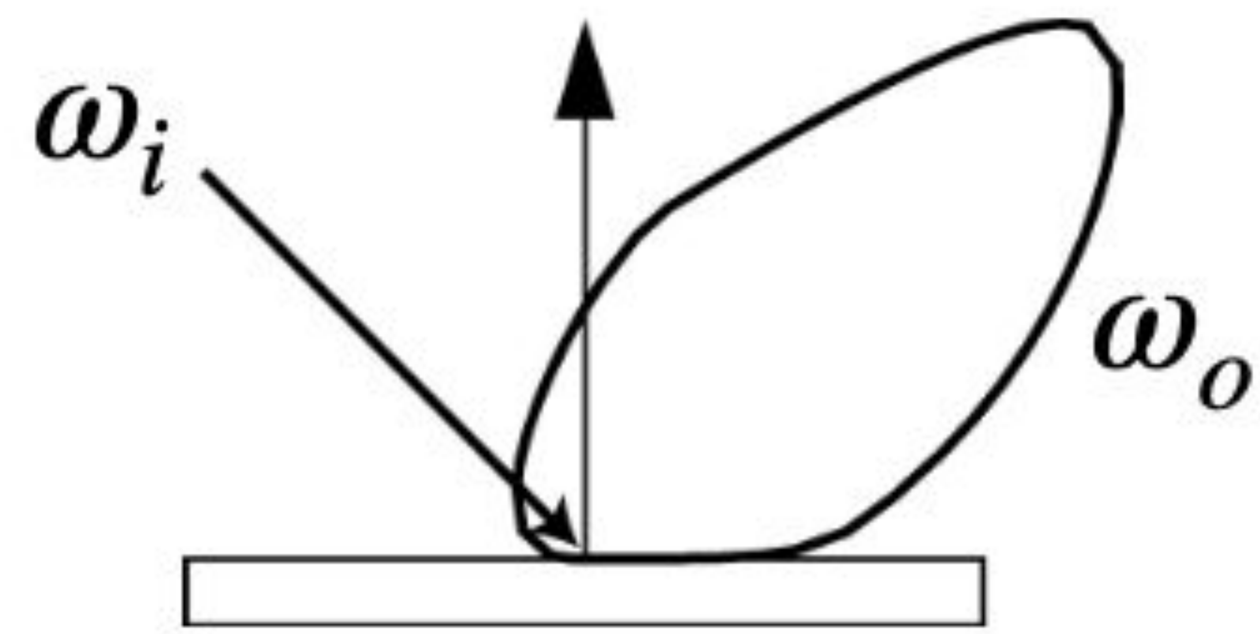


Means you can swap the labels in this diagram

Important properties

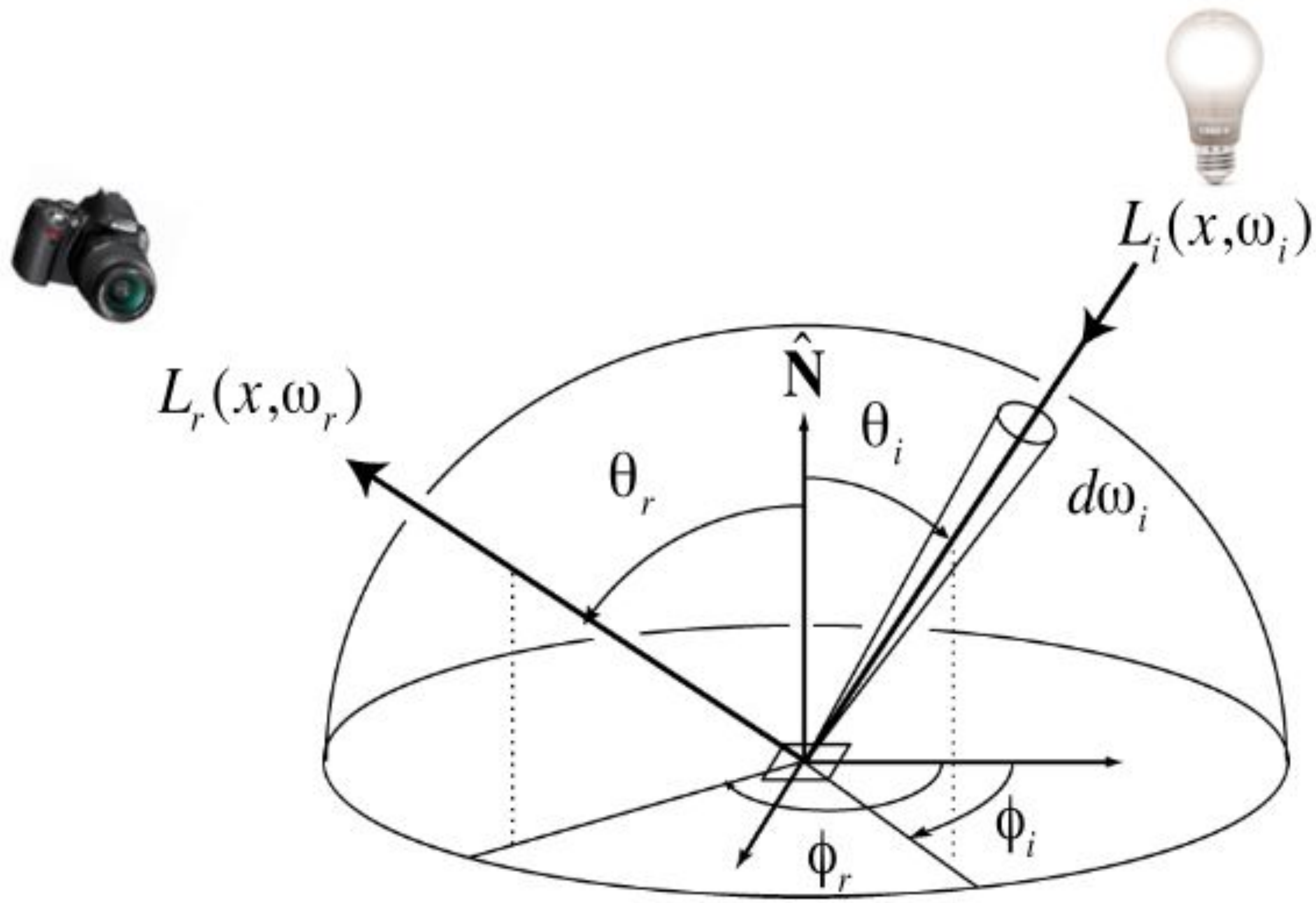
- Conservation of energy (can't reflect > 100%)

$$\int_{\Omega} f(\omega_i, \omega_o)\cos\theta_o d\omega_o \leq 1$$



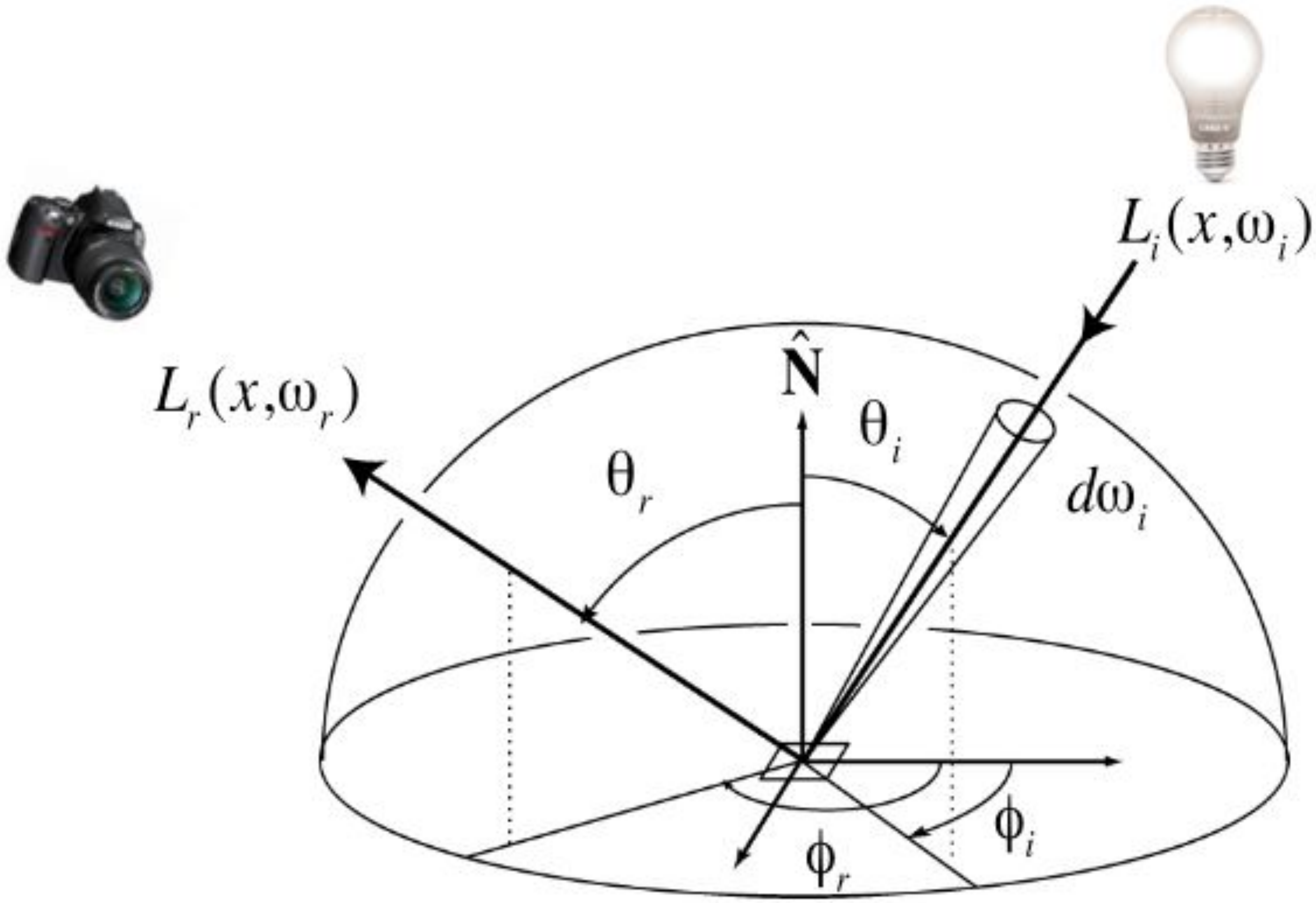
If you add up all the radiance in the "lobe" it can't exceed 1, given 1 unit of input radiance

How you use a BRDF: reflection equation



$$L_r(\mathrm{p},\omega_r) = \int_{H^2} f_r(\mathrm{p},\omega_i \rightarrow \omega_r)\, L_i(\mathrm{p},\omega_i)\, \cos\theta_i \,\mathrm{d}\omega_i$$

How you use a BRDF: reflection equation



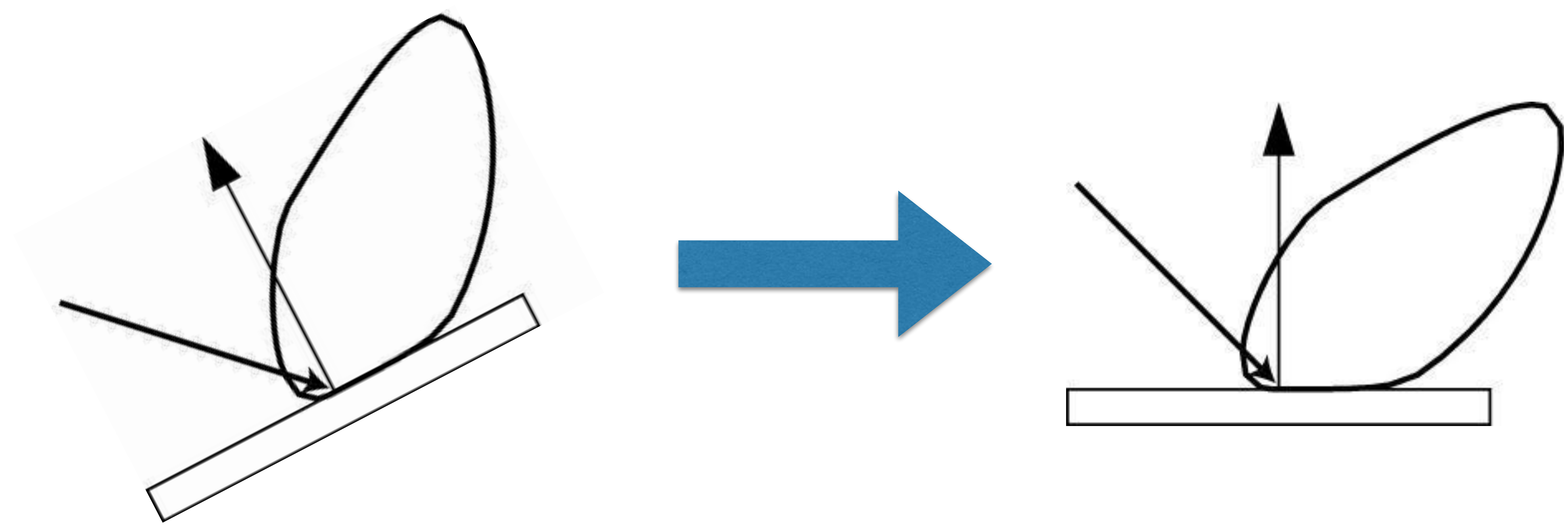
For a given fixed output direction ω_r , add up all the radiance reflected from incoming light over all directions of the hemisphere

Points of clarification: terminology

- BRDF, BTDF, BSDF, BxDF, B*DF
  - R = reflection, T = transmission, S = scattering
  - x, * = catch-all
  - "BSDF" used in project 3 since it covers refraction
- Direction vectors: ω_r and ω_o are the same, subscript is short for "reflected" or "outgoing"
- BRDF sometimes written as $f(\theta_i, \phi_i, \theta_o, \phi_o)$, in terms of spherical coordinates for the two input directions

BRDF "coordinate system" in assignment

- Align the normal with the z axis (0,0,1)
- This simplifies BRDF evaluation math
- ONLY valid for a single point on an object!
- Not the same as "object space"



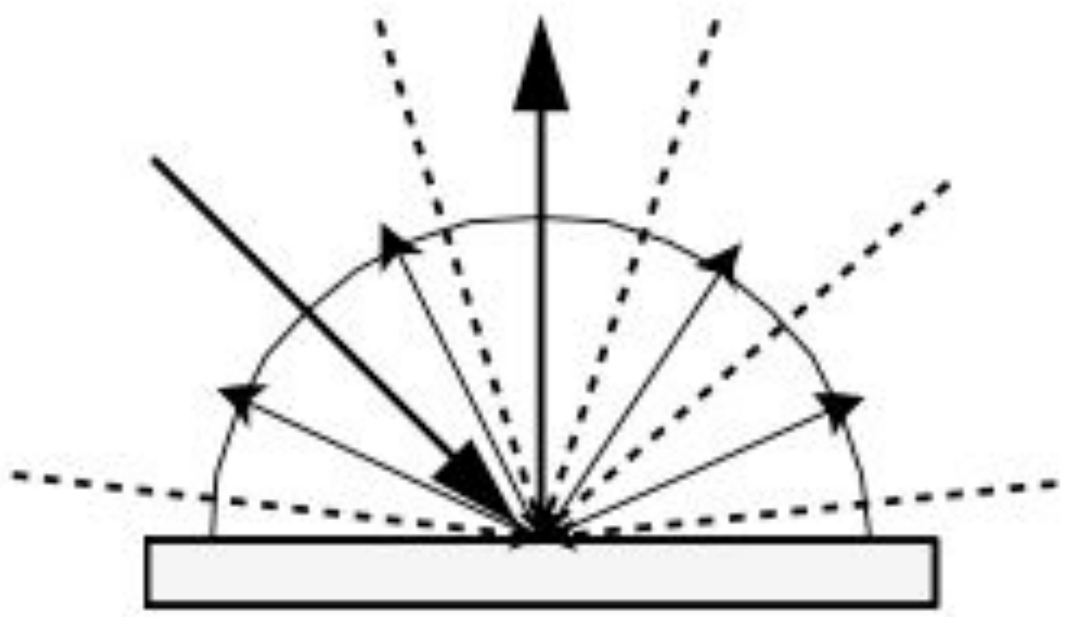
World-space orientation of surface

Local BRDF-space defined by normal vector

Simple BRDFs

Diffuse / Lambertian Material

Light is equally reflected in each output direction



$$f_r = c$$

Suppose the incident lighting is uniform:

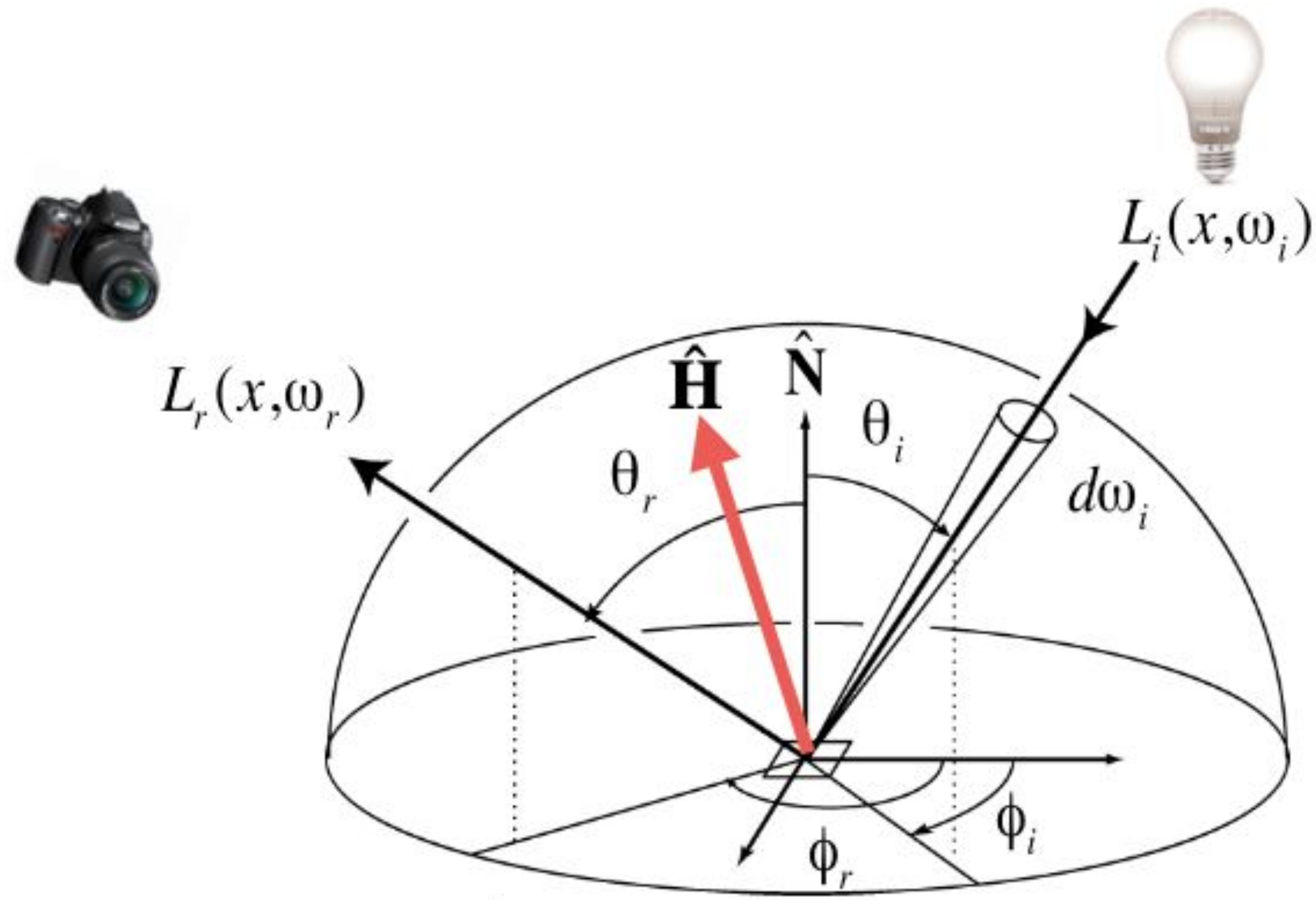
$$L_o(\omega_o) = \int_{H^2} f_r L_i(\omega_i) \cos\theta_i \, \mathrm{d}\omega_i$$

$$= f_r L_i \int_{H^2} \cancel{(\omega_i)} \cos\theta_i \, \mathrm{d}\omega_i$$

$$= \pi f_r L_i$$

$$f_r = \frac{\rho}{\pi}$$ — albedo (color)

Important concept: the "half angle"

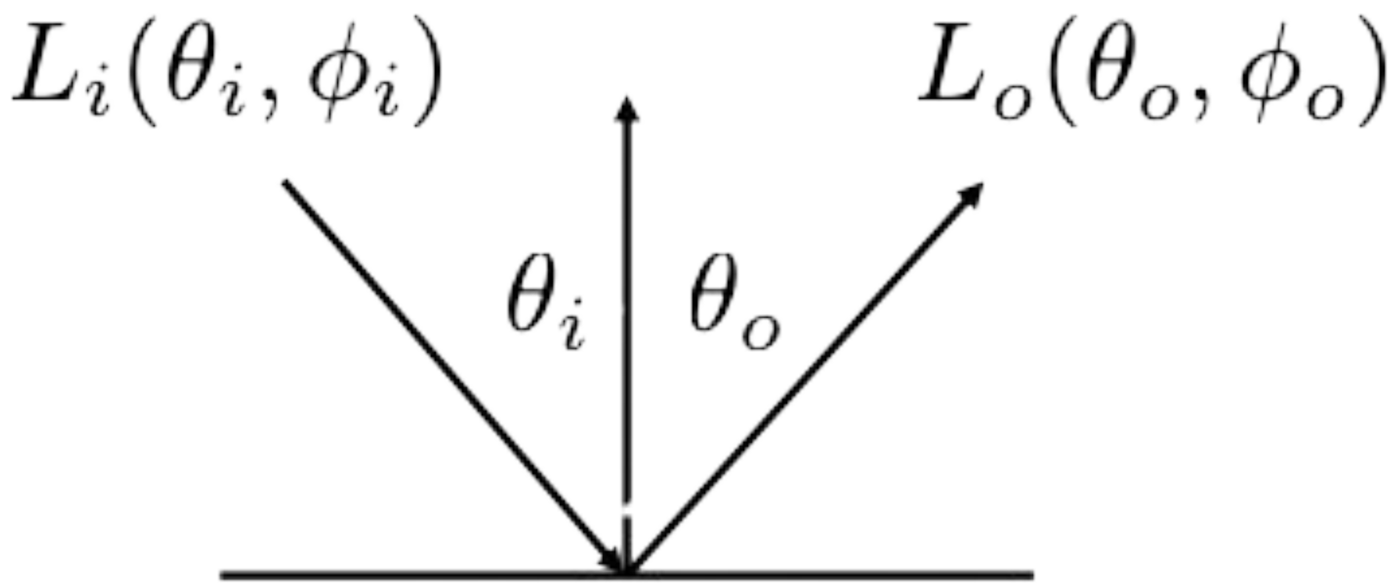


$$\hat{\mathbf{H}} = \text{normalize}(\omega_r + \omega_i)$$

Perfect specular reflection occurs when $\hat{\mathbf{H}} = \hat{\mathbf{N}}$

Glossy BRDFs often written as a function of $\hat{\mathbf{H}} \cdot \hat{\mathbf{N}}$

How does this work with reflection integral?



$$L_o(\theta_o, \phi_o) = L_i(\theta_i, \phi_i \pm \pi)$$

$$f_r(\theta_i, \phi_i; \theta_o, \phi_o) = \frac{\delta(\cos\theta_i - \cos\theta_o)}{\boxed{\cos\theta_i}} \delta(\phi_i - \phi_o \pm \pi)$$

- **Why $\cos\theta_i$?**

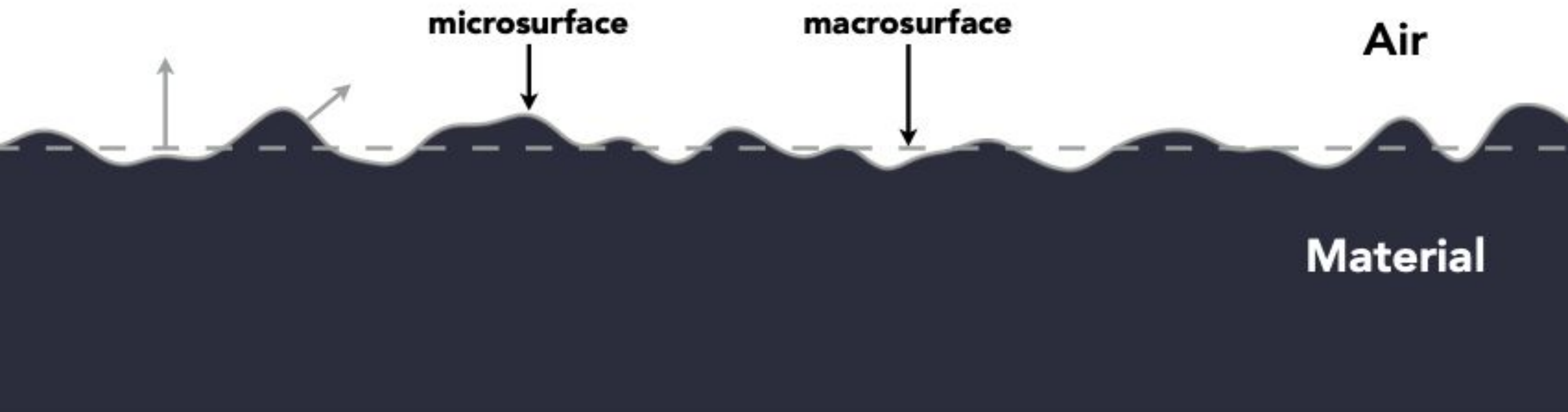
$$\begin{aligned} L_o(\theta_o, \phi_o) &= \int f_r(\theta_i, \phi_i; \theta_o, \phi_o)\, L_i(\theta_i, \phi_i) \boxed{\cos\theta_i} \mathrm{d}\cos\theta_i \, \mathrm{d}\phi_i \\ &= \int \frac{\delta(\cos\theta_i - \cos\theta_o)}{\cos\theta_i} \delta(\phi_i - \phi_o \pm \pi)\, L_i(\theta_i, \phi_i)\, \cos\theta_i \, \mathrm{d}\cos\theta_i \, \mathrm{d}\phi_i \\ &= L_i(\theta_r, \phi_r \pm \pi) \end{aligned}$$

Microfacet BRDFs

Microfacet Theory

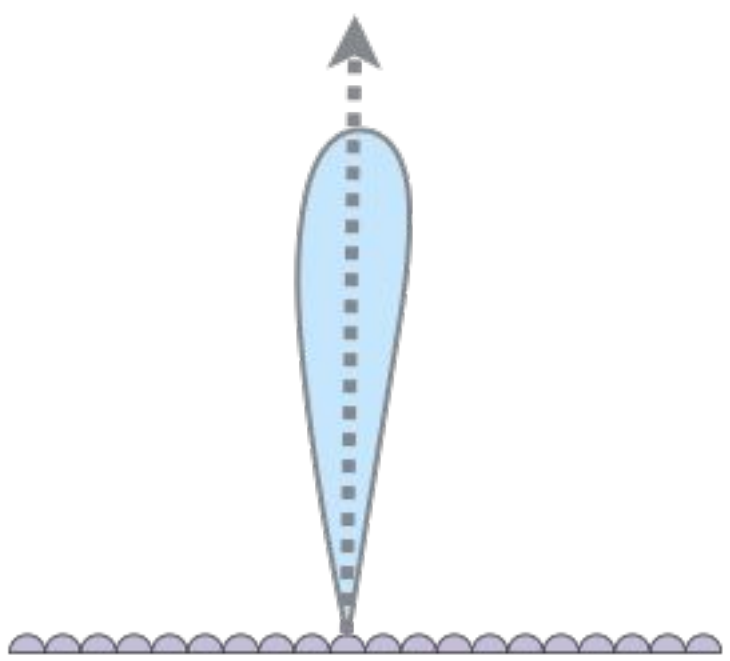
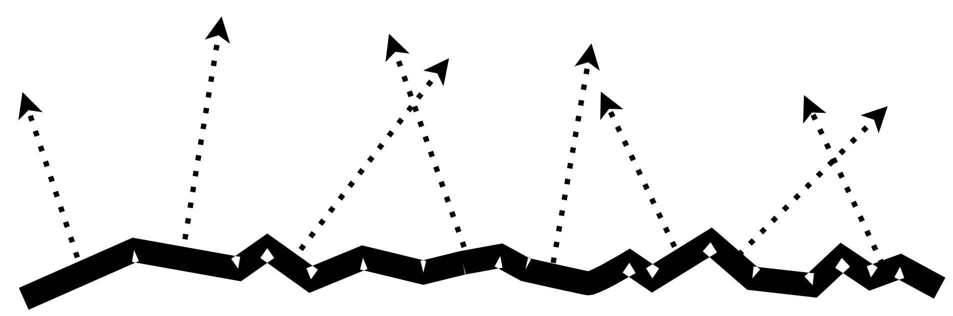
Rough surface
- **Macroscale: flat & rough**
- **Microscale: bumpy & specular**

Individual elements of surface act like mirrors
- **Known as Microfacets**
- **Each microfacet has its own normal**

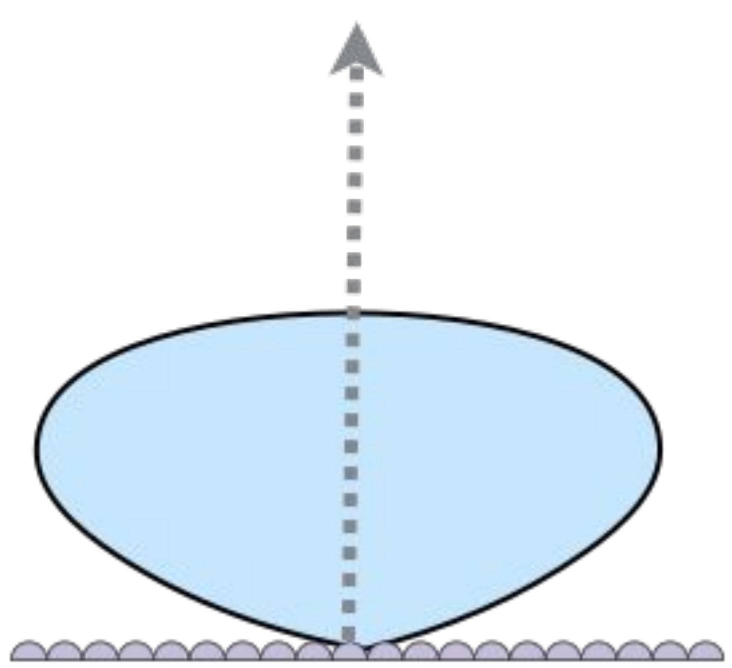
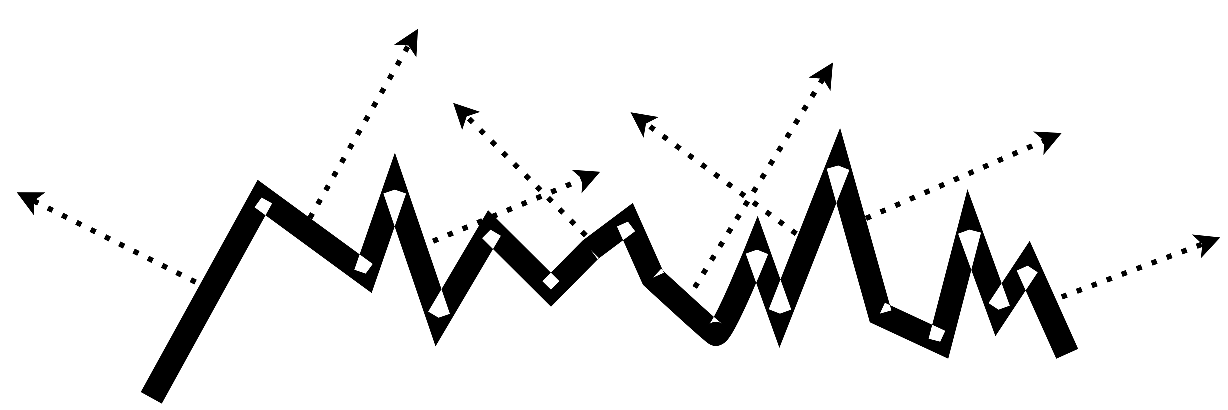
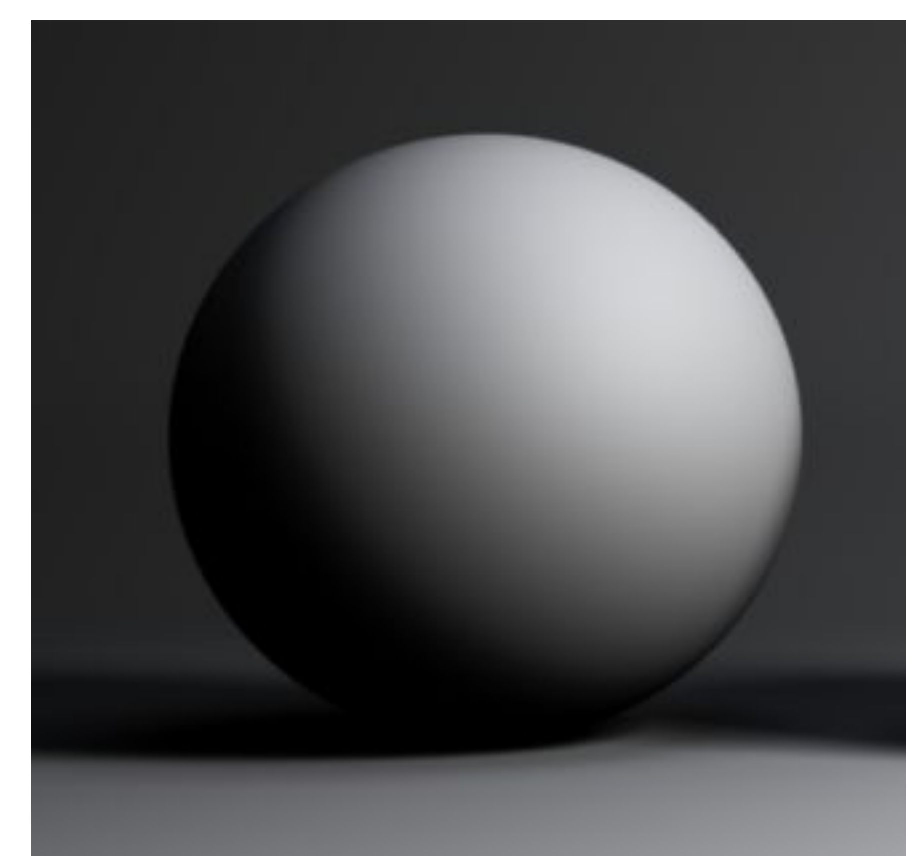


Microfacet BRDF

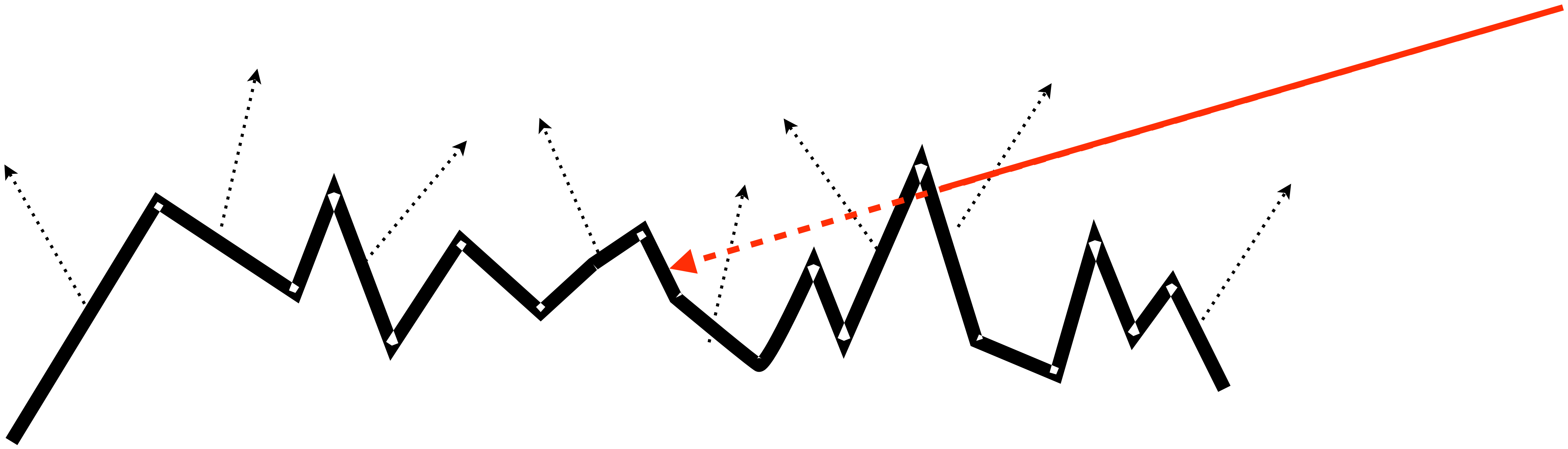
- Key: the distribution of microfacets' normals
  - Concentrated <==> glossy


  - Spread <==> diffuse

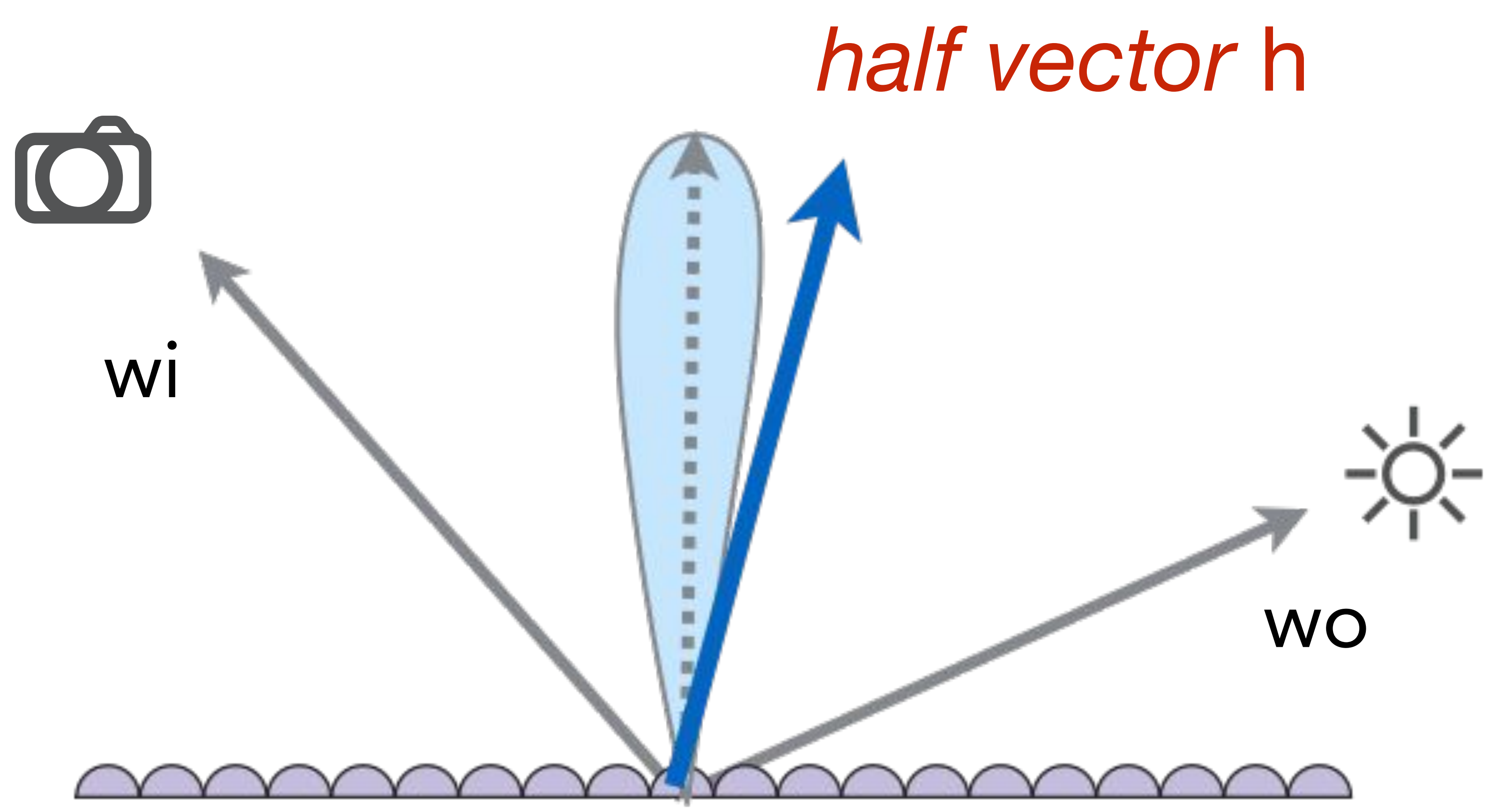
Shadowing/masking term

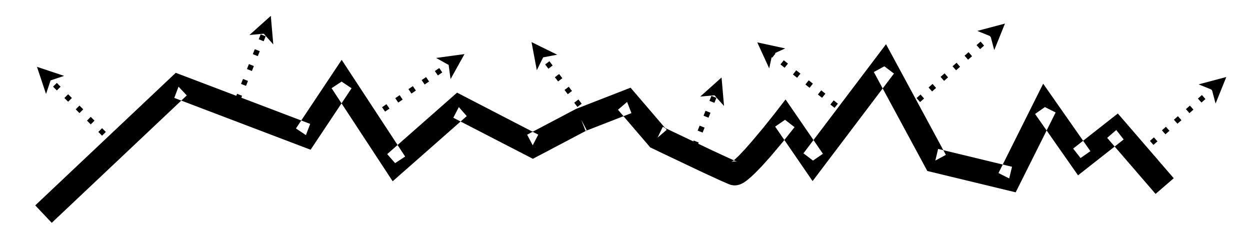


At grazing incoming light angles, some microfacets will block light from reaching other parts of surface

Microfacet BRDF

- What kind of microfacets reflect wi to wo?
  (hint: microfacets are mirrors)





Fresnel term

shadowing-masking term

distribution of normals

$$f(\mathbf{i},\mathbf{o}) = \frac{\mathbf{F}(\mathbf{i},\mathbf{h})\,\mathbf{G}(\mathbf{i},\mathbf{o},\mathbf{h})\,\mathbf{D}(\mathbf{h})}{4(\mathbf{n},\mathbf{i})(\mathbf{n},\mathbf{o})}$$

This is the standard microfacet model used in modern rendering:

$$f_r(\omega_i, \omega_o) = \frac{D(\boldsymbol{h})G(\omega_i, \omega_o, \boldsymbol{h})F(\omega_i, \boldsymbol{h})}{4(\boldsymbol{n} \cdot \omega_i)(\boldsymbol{n} \cdot \omega_o)}$$

Where $\boldsymbol{h} = \frac{\omega_i + \omega_o}{||\omega_i + \omega_o||}$ is the halfway vector.

- **D(h): Normal Distribution Function** - How are the microfacet normals oriented?
- **G(i,o,h): Geometry Term** - How much are the microfacets shadowed or masked?
- **F(i,h): Fresnel Term** - How much light reflects from a single microfacet?

Problem

You are calculating the BRDF for a point on a surface with normal $\boldsymbol{n} = [0, 1, 0]$. The light comes from $\omega_i = [0.6, 0.8, 0]$ and the viewer is at $\omega_o = [-0.6, 0.8, 0]$.

Given pre-calculated values for this configuration: $D(\boldsymbol{h}) = 0.8$, $G(...) = 0.9$, and $F(...) = 0.5$. Calculate the final value of the Cook-Torrance BRDF, $f_r(\omega_i, \omega_o)$.

Problem

You are calculating the BRDF for a point on a surface with normal $\boldsymbol{n} = [0, 1, 0]$. The light comes from $\boldsymbol{\omega_i} = [0.6, 0.8, 0]$ and the viewer is at $\boldsymbol{\omega_o} = [-0.6, 0.8, 0]$.

Given pre-calculated values for this configuration: $D(\boldsymbol{h}) = 0.8$, $G(...) = 0.9$, and $F(...) = 0.5$. Calculate the final value of the Cook-Torrance BRDF, $f_r(\boldsymbol{\omega_i}, \boldsymbol{\omega_o})$.

Solution

First, calculate the denominator dot products:

$$\boldsymbol{n} \cdot \boldsymbol{\omega_i} = [0, 1, 0] \cdot [0.6, 0.8, 0] = 0.8$$
$$\boldsymbol{n} \cdot \boldsymbol{\omega_o} = [0, 1, 0] \cdot [-0.6, 0.8, 0] = 0.8$$

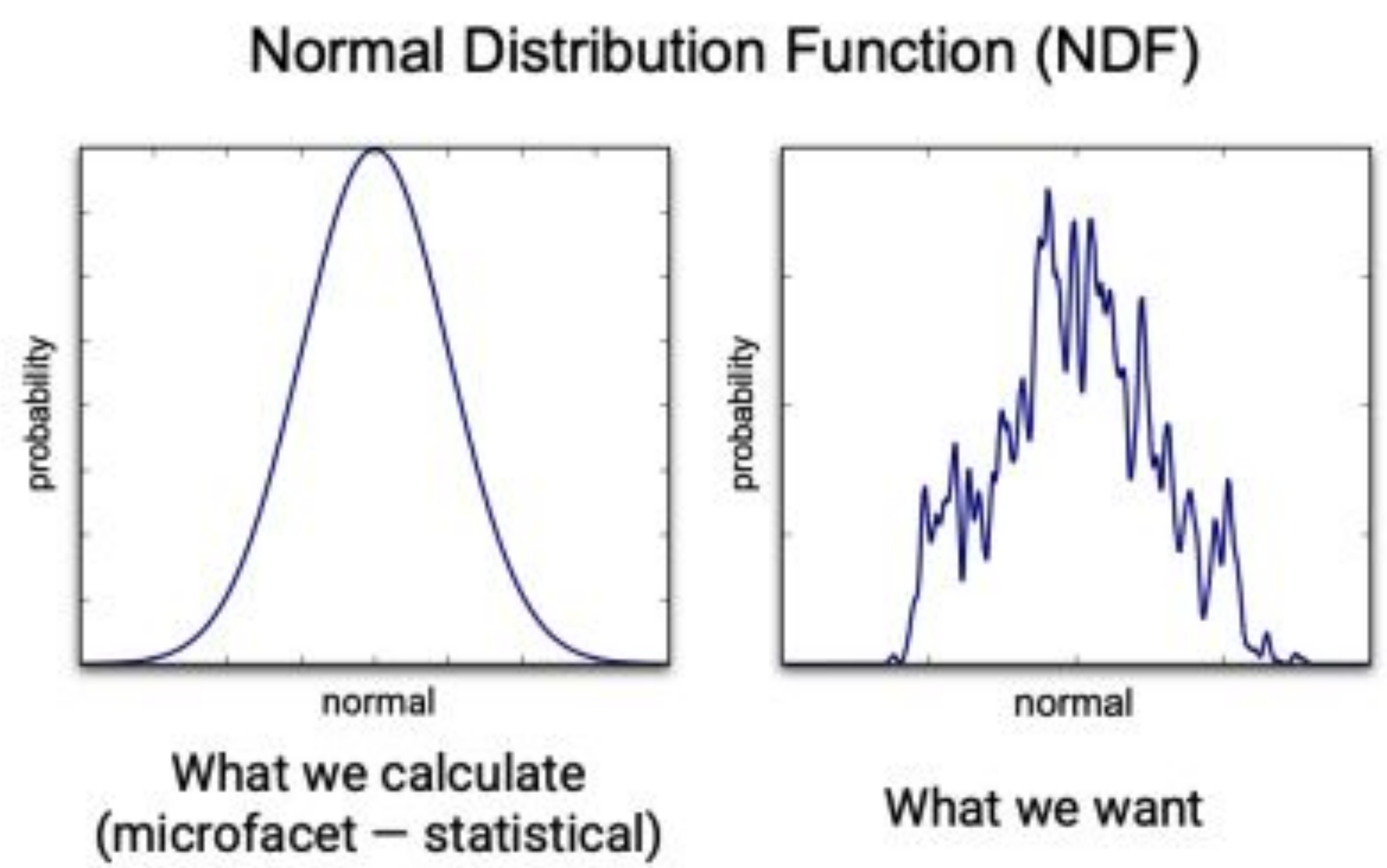
Now, plug everything into the Cook-Torrance equation:

$$\begin{aligned} f_r &= \frac{D(\boldsymbol{h})G(...)F(...)}{4(\boldsymbol{n} \cdot \boldsymbol{\omega_i})(\boldsymbol{n} \cdot \boldsymbol{\omega_o})} \\ &= \frac{(0.8) \cdot (0.9) \cdot (0.5)}{4 \cdot (0.8) \cdot (0.8)} \\ &= \frac{0.36}{2.56} \approx 0.14 \end{aligned}$$

Term 1: Normal Distribution Function (NDF)

The NDF, $D(\boldsymbol{h})$, models the statistical distribution of microfacet normals. A key idea is that only facets whose normal $\boldsymbol{m}$ is equal to the halfway vector $\boldsymbol{h}$ will reflect light from $\boldsymbol{\omega_i}$ to $\boldsymbol{\omega_o}$.

- A **smooth** surface has a very narrow NDF (normals are all aligned).
- A **rough** surface has a wide NDF (normals are scattered).



Example: Beckmann NDF

Problem

The Beckmann distribution is a common NDF defined by a roughness parameter α:

$$D(\boldsymbol{h}) = \frac{1}{\pi\alpha^2 \cos^4\theta_h} \exp\left(-\frac{\tan^2\theta_h}{\alpha^2}\right)$$

Calculate $D(\boldsymbol{h})$ for a rough material ($\alpha = 0.5$) where the halfway vector makes an angle $\theta_h = 30^\circ$ with the macro-surface normal.

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Calculate $D(\boldsymbol{h})$ for a rough material ($\alpha = 0.5$) where the halfway vector makes an angle $\theta_h = 30^\circ$ with the macro-surface normal.

Solution

- $\cos(30^\circ) \approx 0.866 \implies \cos^4(30^\circ) \approx 0.563$
- $\tan(30^\circ) \approx 0.577 \implies \tan^2(30^\circ) \approx 0.333$
- $\alpha^2 = 0.5^2 = 0.25$

$$\begin{aligned} D(\boldsymbol{h}) &= \frac{1}{\pi(0.25)(0.563)} \exp\left(-\frac{0.333}{0.25}\right) \\ &= \frac{1}{0.442} \exp(-1.332) \\ &= 2.262 \times 0.264 \approx 0.597 \end{aligned}$$

Isotropic / Anisotropic materials

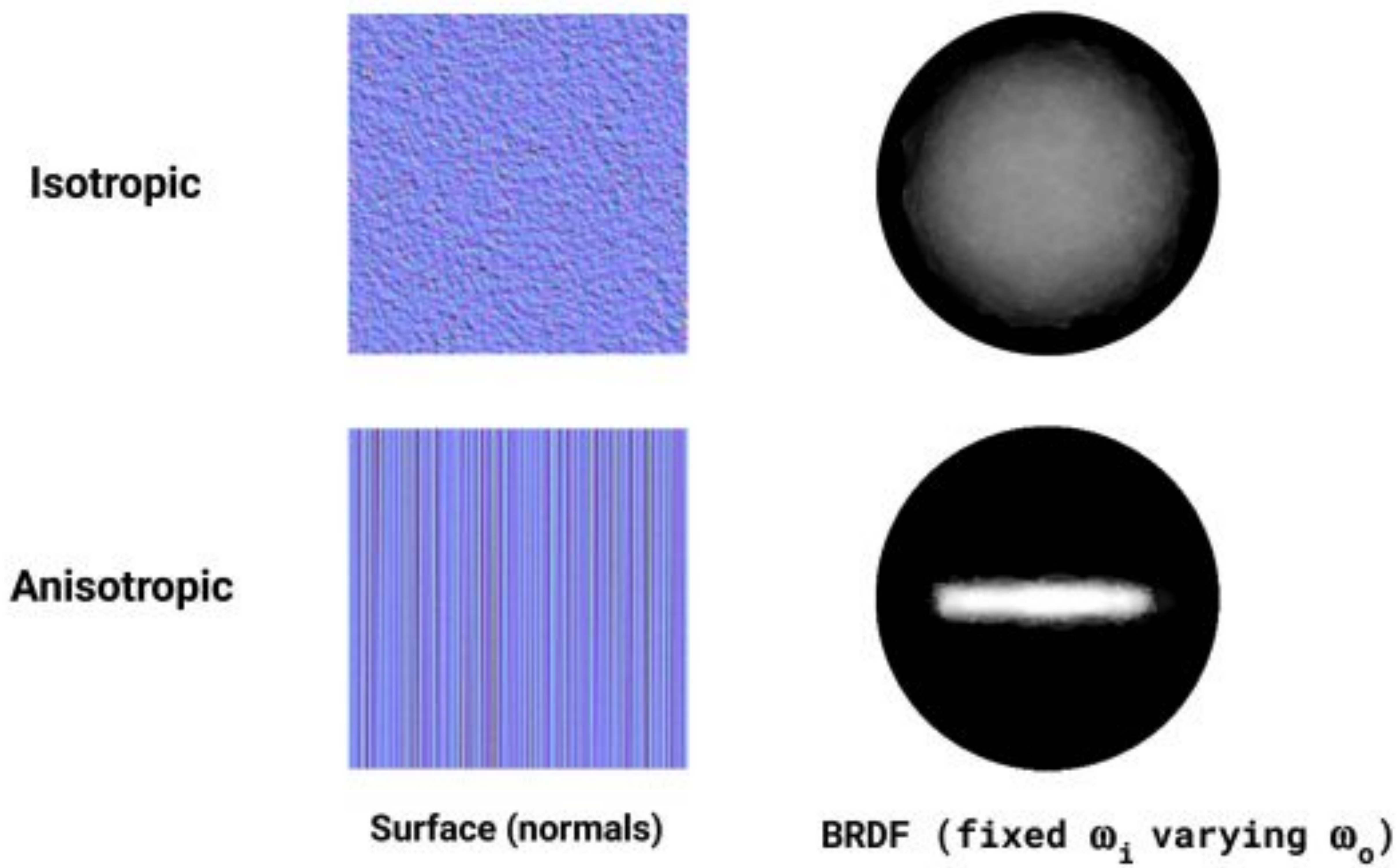
Isotropic / Anisotropic Materials (BRDFs)

Isotropic materials look the same regardless of how they are rotated around the surface normal. Their microstructure is random.

$$f_r(\theta_i, \phi_i; \theta_r, \phi_r) = f_r(\theta_i, \theta_r, \phi_r - \phi_i)$$

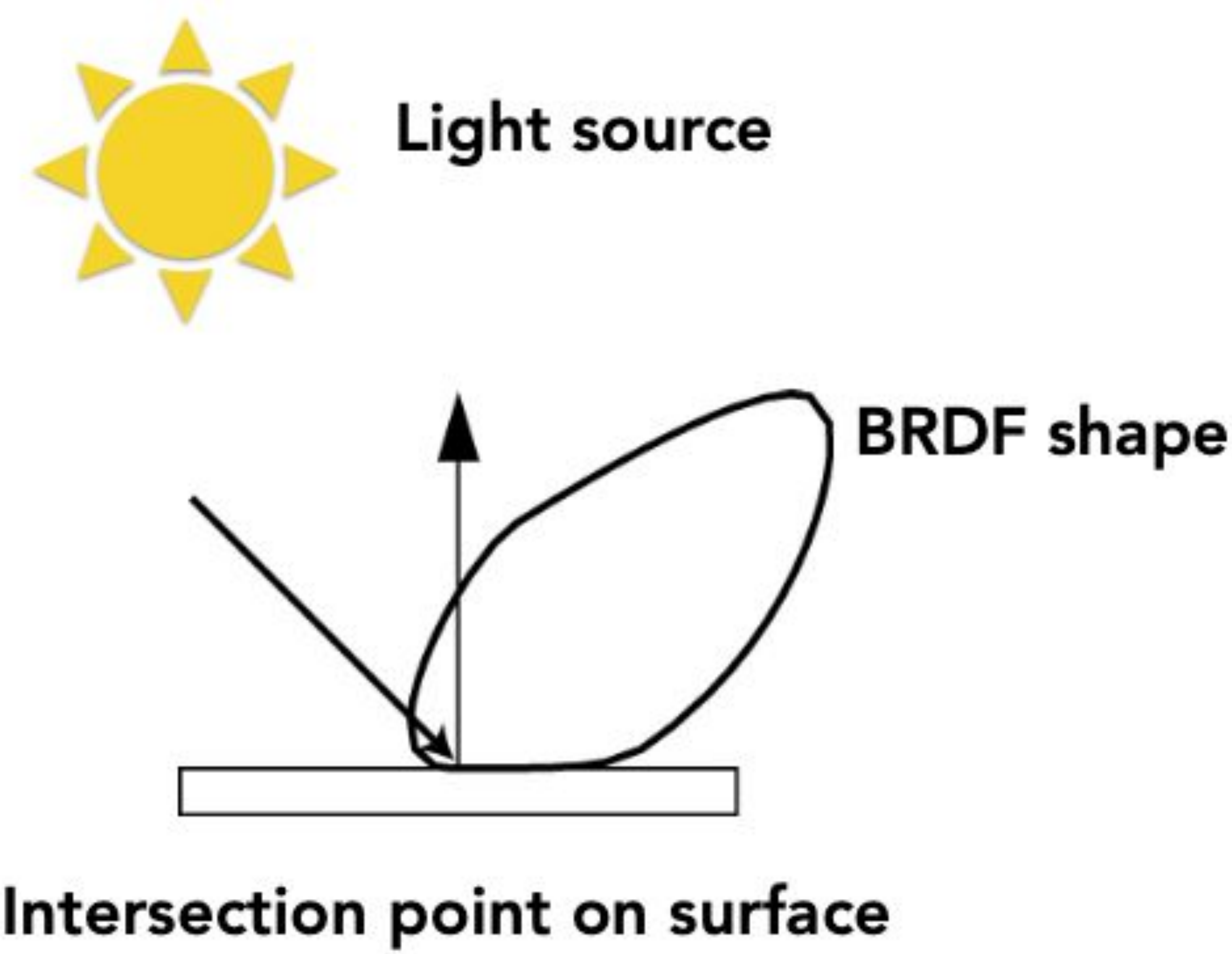
Anisotropic materials have an oriented microstructure, like brushed metal or wood grain. This causes reflections to stretch.

$$f_r(\theta_i, \phi_i; \theta_r, \phi_r) \neq f_r(\theta_i, \theta_r, \phi_r - \phi_i)$$



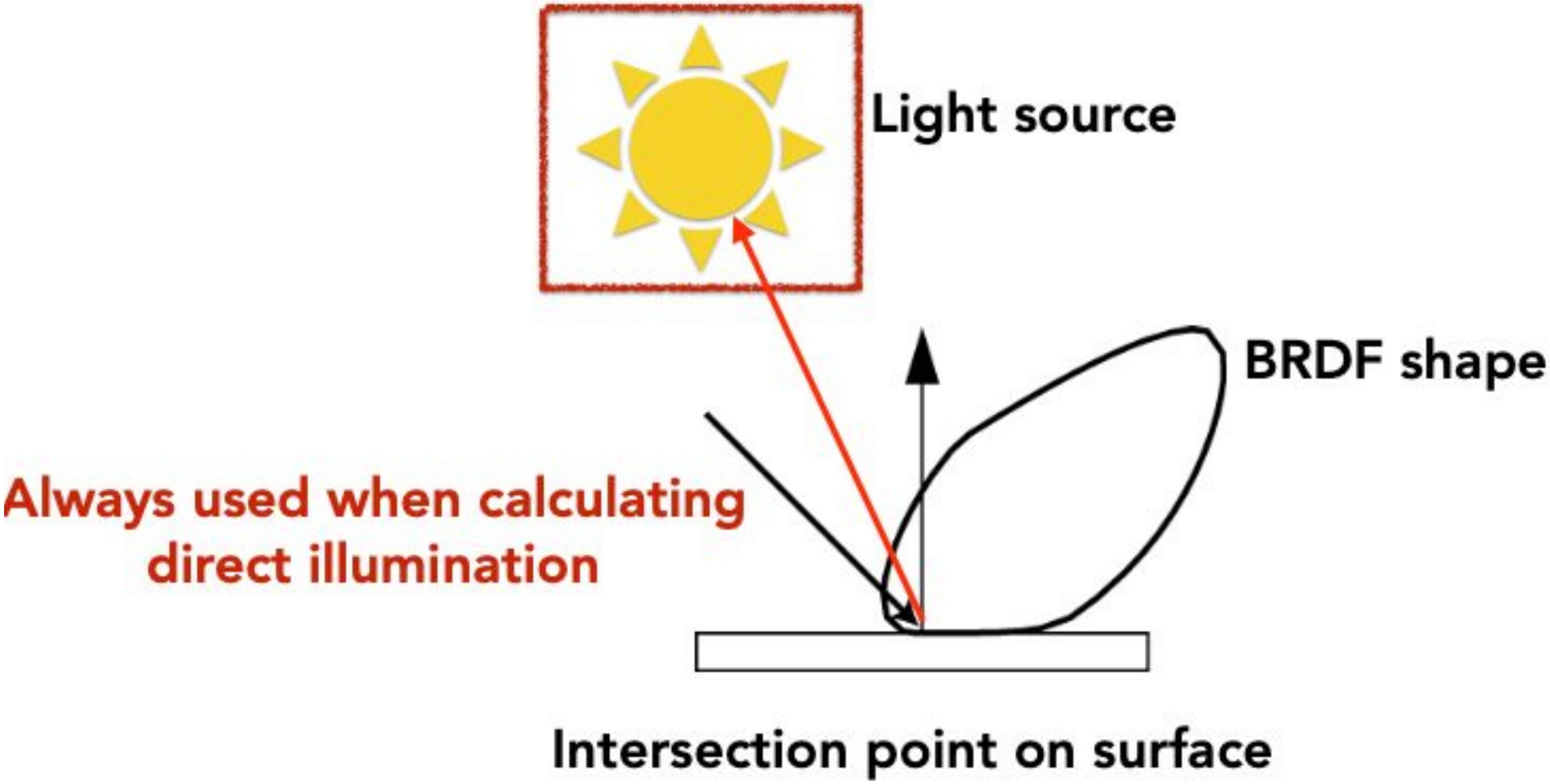
Importance Sampling

Importance sampling: lights and BRDF



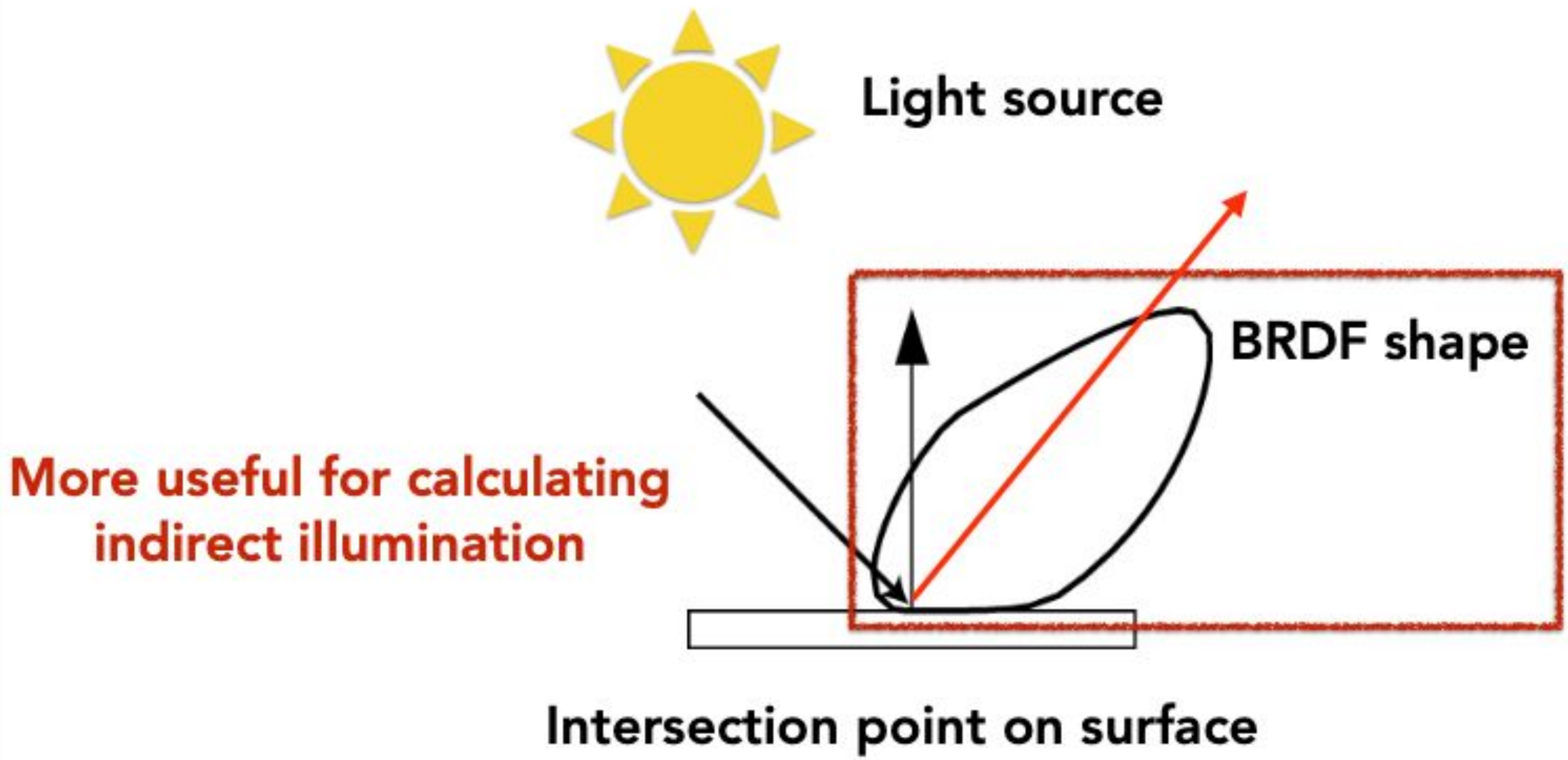
$$L_r(\mathrm{p}, \omega_r) = \int_{H^2} f_r(\mathrm{p}, \omega_i \rightarrow \omega_r)\, L_i(\mathrm{p}, \omega_i)\, \cos\theta_i \, \mathrm{d}\omega_i$$

Shoot rays toward random point on light surface



$$L_r(\mathrm{p}, \omega_r) = \int_{H^2} f_r(\mathrm{p}, \omega_i \rightarrow \omega_r) \boxed{L_i(\mathrm{p}, \omega_i)} \cos\theta_i \, \mathrm{d}\omega_i$$

Shoot rays in proportion to BRDF strength



$$L_r(\mathrm{p},\omega_r) = \int_{H^2} f_r(\mathrm{p},\omega_i \to \omega_r)\, L_i(\mathrm{p},\omega_i)\, \cos\theta_i \, \mathrm{d}\omega_i$$

Importance sampling diffuse BRDF

$$L_o(\omega_o) = \frac{\rho}{\pi} \int L_i(\omega_i) \cos\theta_i \mathrm{d}\omega_i$$

The BRDF factors out of the reflectance integral since it's constant

Can just use cosine-weighted random samples on hemisphere

Importance sampling microfacet BRDF

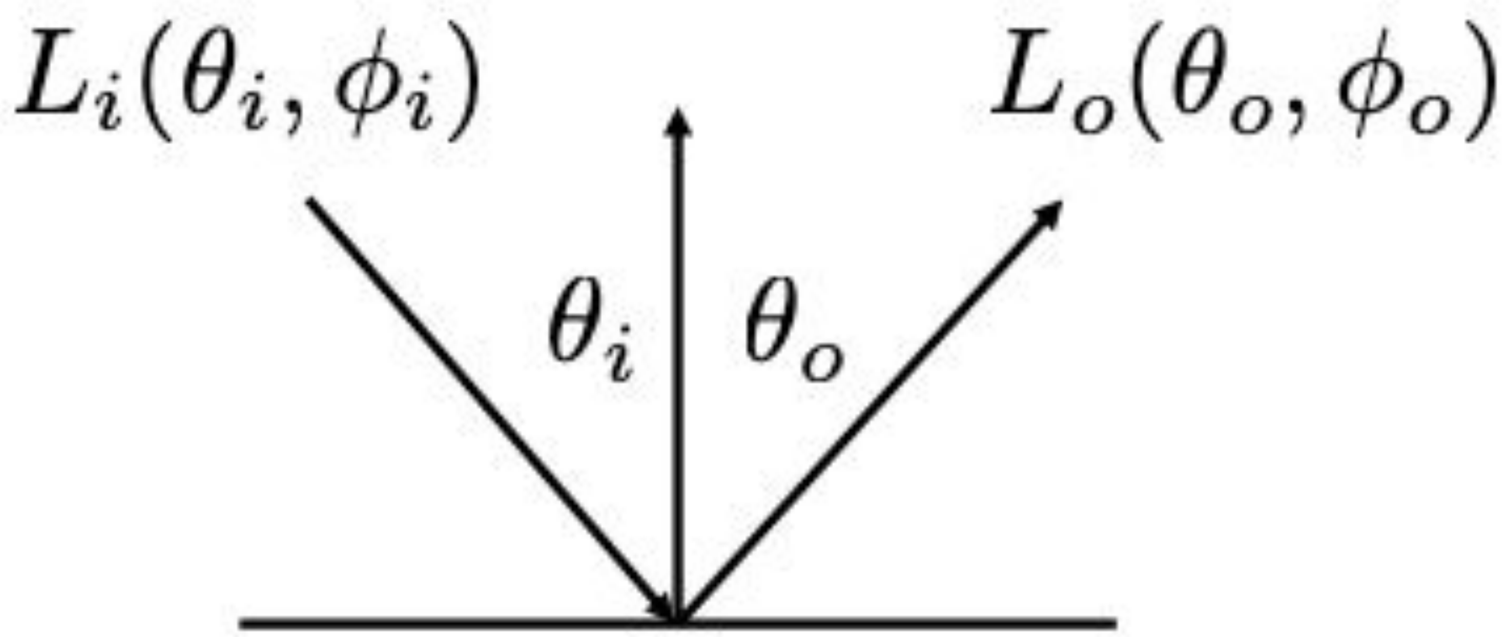
distribution of normals

$$f(\mathbf{i}, \mathbf{o}) = \frac{\mathbf{F}(\mathbf{i}, \mathbf{h})\mathbf{G}(\mathbf{i}, \mathbf{o}, \mathbf{h})\boxed{\mathbf{D}(\mathbf{h})}}{4(\mathbf{n}, \mathbf{i})(\mathbf{n}, \mathbf{o})}$$

They come with a probability distribution built right in!

Sampling a half-angle from $\mathbf{D}$ works well to match specular lobe

“Importance sampling” perfect specular BRDFs



$$L_o(\theta_o, \phi_o) = L_i(\theta_i, \phi_i \pm \pi)$$

$$f_r(\theta_i, \phi_i; \theta_o, \phi_o) = \frac{\delta(\cos\theta_i - \cos\theta_o)}{\cos\theta_i}\delta(\phi_i - \phi_o \pm \pi)$$

In the case of a perfect specularity, the BRDF is a “delta function”

No energy will bounce from ANY other direction

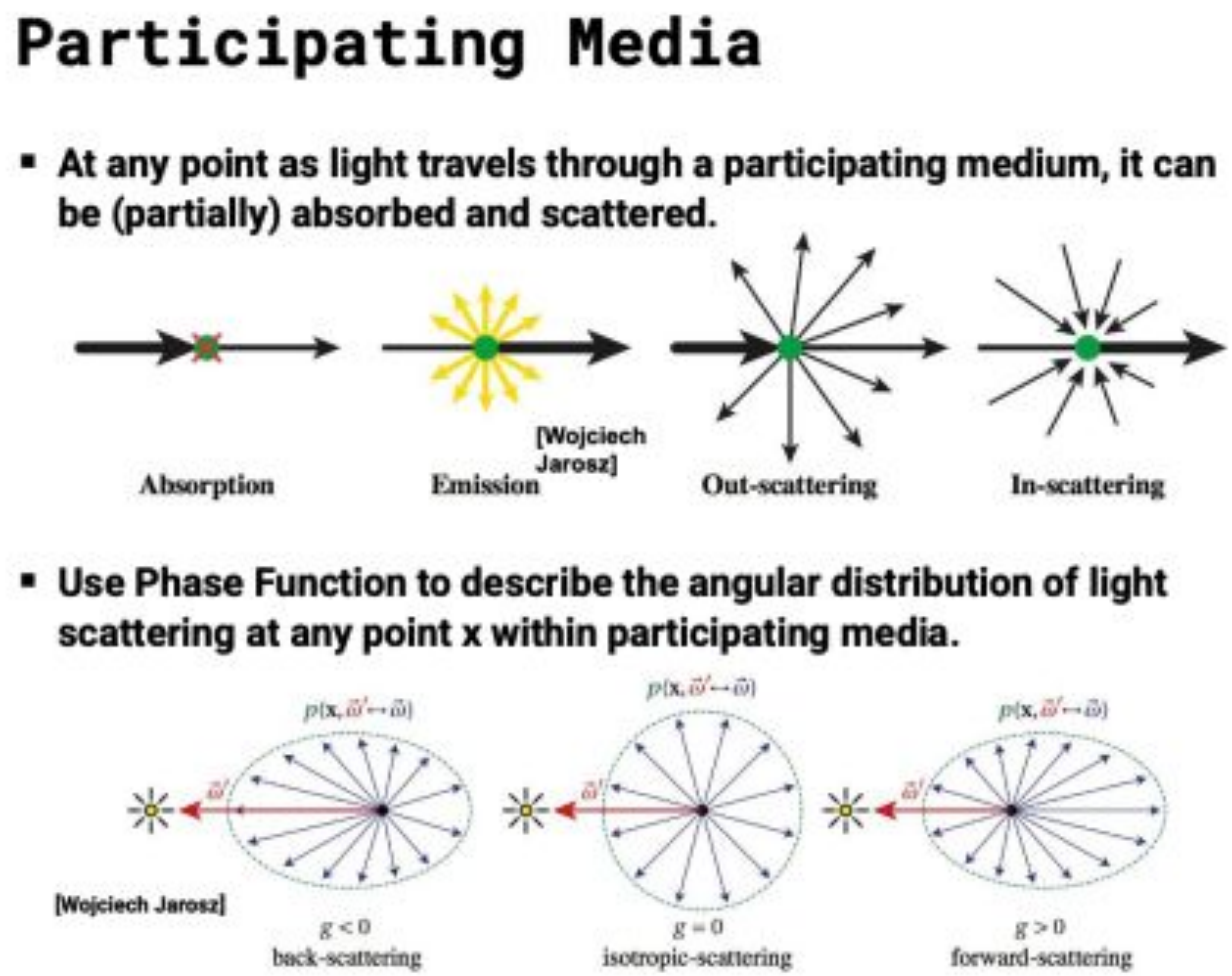
Importance sampling lights is useless here

Participating Media

Participating Media

Some materials aren't just surfaces. Light travels *through* them, scattering and being absorbed along the way.

- Examples: Fog, smoke, clouds, murky water.
- At any point, light can be:
  - **Absorbed** (energy is lost)
  - **Scattered** (changes direction)
- A **Phase Function** describes the angular distribution of scattering.

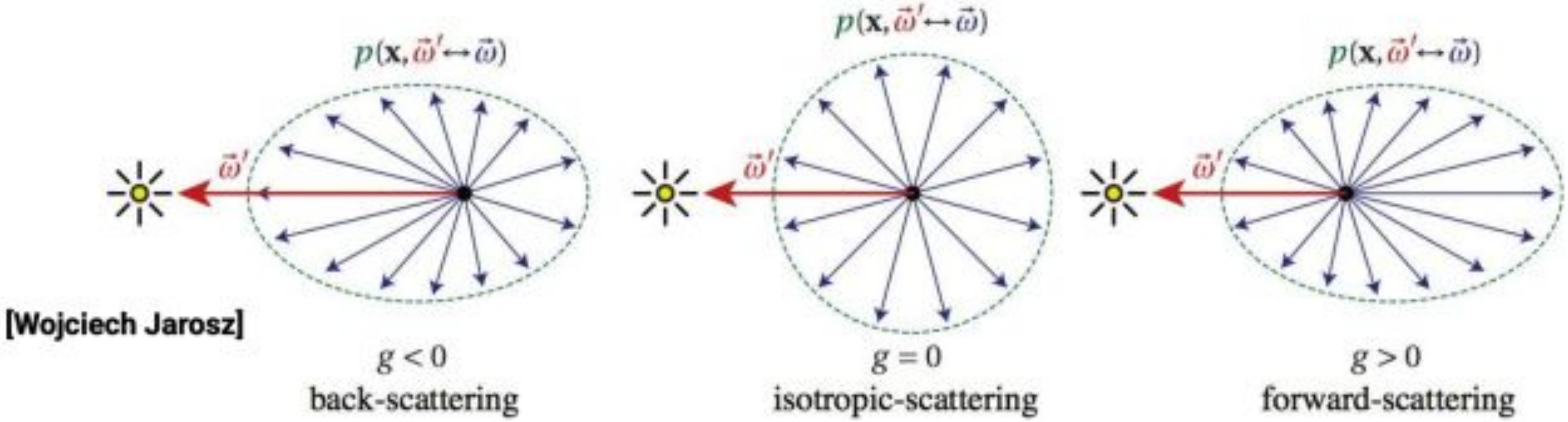


The Henyey-Greenstein Phase Function

A common model for describing scattering direction, controlled by an asymmetry parameter $g \in [-1, 1]$.

$$p(\cos\theta) = \frac{1}{4\pi} \frac{1 - g^2}{(1 + g^2 - 2g\cos\theta)^{3/2}}$$

Use Phase Function to describe the angular distribution of light scattering at any point x within participating media.



- $g > 0$: **Forward scattering** (fog, mist)
- $g = 0$: **Isotropic scattering** (random)
- $g < 0$: **Back scattering**

Example Problem: Phase Functions

Problem

Using the Henyey-Greenstein formula, calculate the value of $4\pi \cdot p(\cos\theta)$ for light scattering at 90° ($\cos\theta = 0$) in two media:

1. A forward-scattering medium (mist), $g = 0.7$.
2. A backward-scattering medium, $g = -0.7$.

Example Problem: Phase Functions

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Using the Henyey-Greenstein formula, calculate the value of $4\pi \cdot p(\cos\theta)$ for light scattering at 90° ($\cos\theta = 0$) in two media:

1. A forward-scattering medium (mist), $g = 0.7$.
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Solution

Let the core term be $P' = \frac{1-g^2}{(1+g^2-2g\cos\theta)^{3/2}}$. With $\cos\theta = 0$, this simplifies to $P' = \frac{1-g^2}{(1+g^2)^{3/2}}$.

1. **Forward-scattering ($g = 0.7$):**

$$P' = \frac{1-(0.7)^2}{(1+(0.7)^2)^{3/2}} = \frac{1-0.49}{(1+0.49)^{3/2}}$$
$$= \frac{0.51}{(1.49)^{1.5}} = \frac{0.51}{1.82} \approx 0.28$$

2. **Backward-scattering ($g = -0.7$):**

$$P' = \frac{1-(-0.7)^2}{(1+(-0.7)^2)^{3/2}} = \frac{1-0.49}{(1+0.49)^{3/2}}$$
$$= \frac{0.51}{(1.49)^{1.5}} = \frac{0.51}{1.82} \approx 0.28$$

At 90°, the scattering probability is the same for g and $-g$. The difference is only seen for forward/backward angles.

Subsurface Scattering (BSSRDF)

For translucent materials (skin, marble, milk), light enters the surface, scatters internally, and exits at a **different point**.

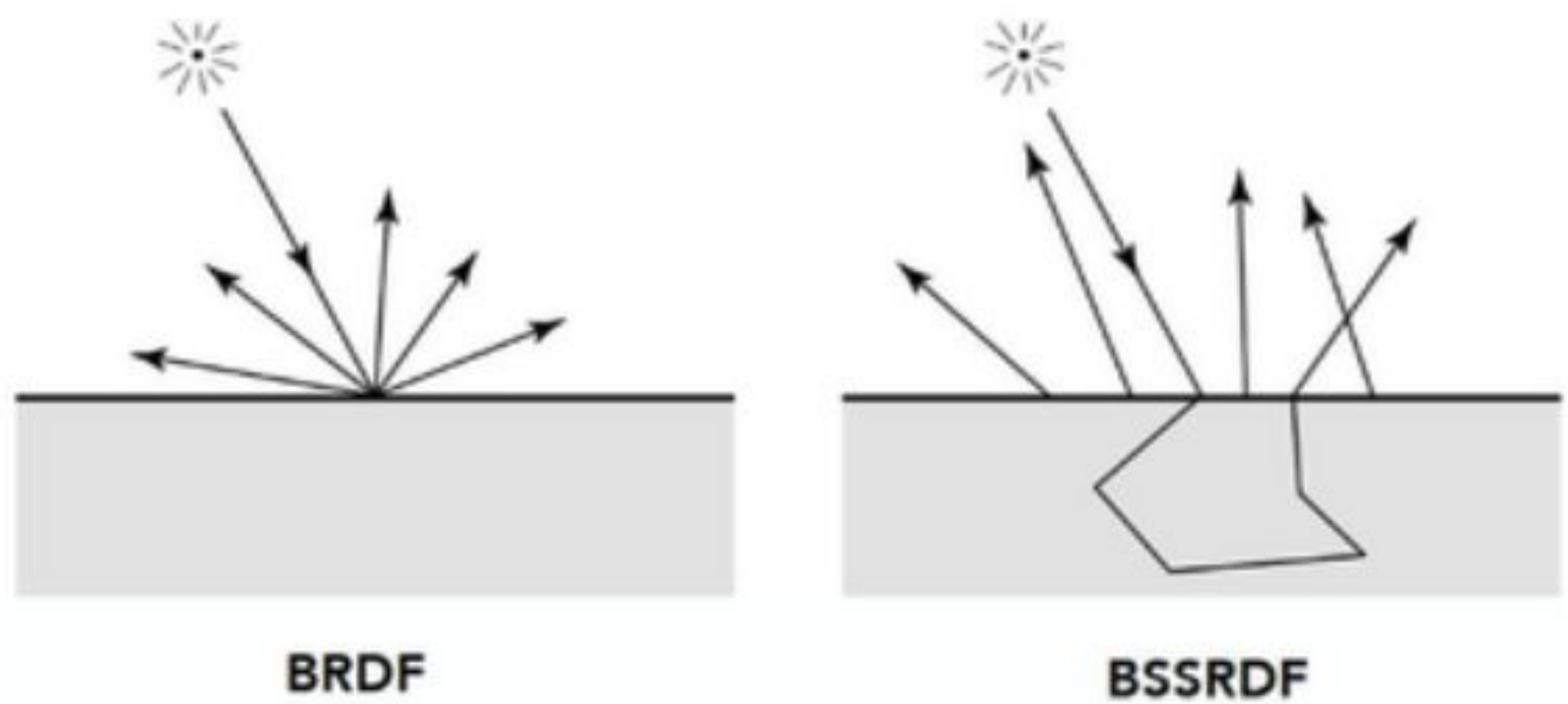

Figure: A BRDF assumes light enters and exits at the same point (left). A BSSRDF generalizes this to allow for different entry and exit points (right).

The Bidirectional Surface Scattering Reflectance Distribution Function (BSSRDF), S, is a generalization of the BRDF.

$$S(x_i, \boldsymbol{\omega_i}, x_o, \boldsymbol{\omega_o})$$

This leads to a more general rendering equation that integrates over surface area A as well as the hemisphere H^2:

$$L(x_o, \boldsymbol{\omega_o}) = \int_A \int_{H^2} S(\ldots) L_i(x_i, \boldsymbol{\omega_i}) \cos\theta_i \mathrm{d}\omega_i \mathrm{d}A(x_i)$$

- x_i: Entry point of light
- x_o: Exit point of light

This integral is much more expensive to solve than the standard rendering equation.

Inverse Rendering

What is Inverse Rendering?

The process of recovering scene properties (geometry, materials, lighting) from one or more captured images. It is the reverse of the standard graphics pipeline.

Forward Rendering:	**Inverse Rendering:**
(Scene Description) $\rightarrow$ Image	Image(s) $\rightarrow$ (Scene Description)

Key Idea

Optimization-Based Methods (Classic Approach): This is a "guess and check" strategy. The algorithm makes an initial guess for the scene properties, renders an image from that guess, and compares it to the real photo. It then calculates the "error" or difference between the two images and systematically adjusts its guess to minimize that error. This process is repeated until the rendered image closely matches the real one.
Machine Learning-Based Methods (Modern Approach): Modern methods use machine learning models trained on enormous datasets of images and 3D scenes. The model learns the incredibly complex relationship between 3D properties and 2D image features. This allows it to make a much more informed and plausible reconstruction of the scene, effectively cutting through the ambiguities that plague classic methods. Techniques like Neural Radiance Fields (NeRFs) and Gaussian Splatting are powerful examples of this approach

Application: Inverse Rendering for Hair

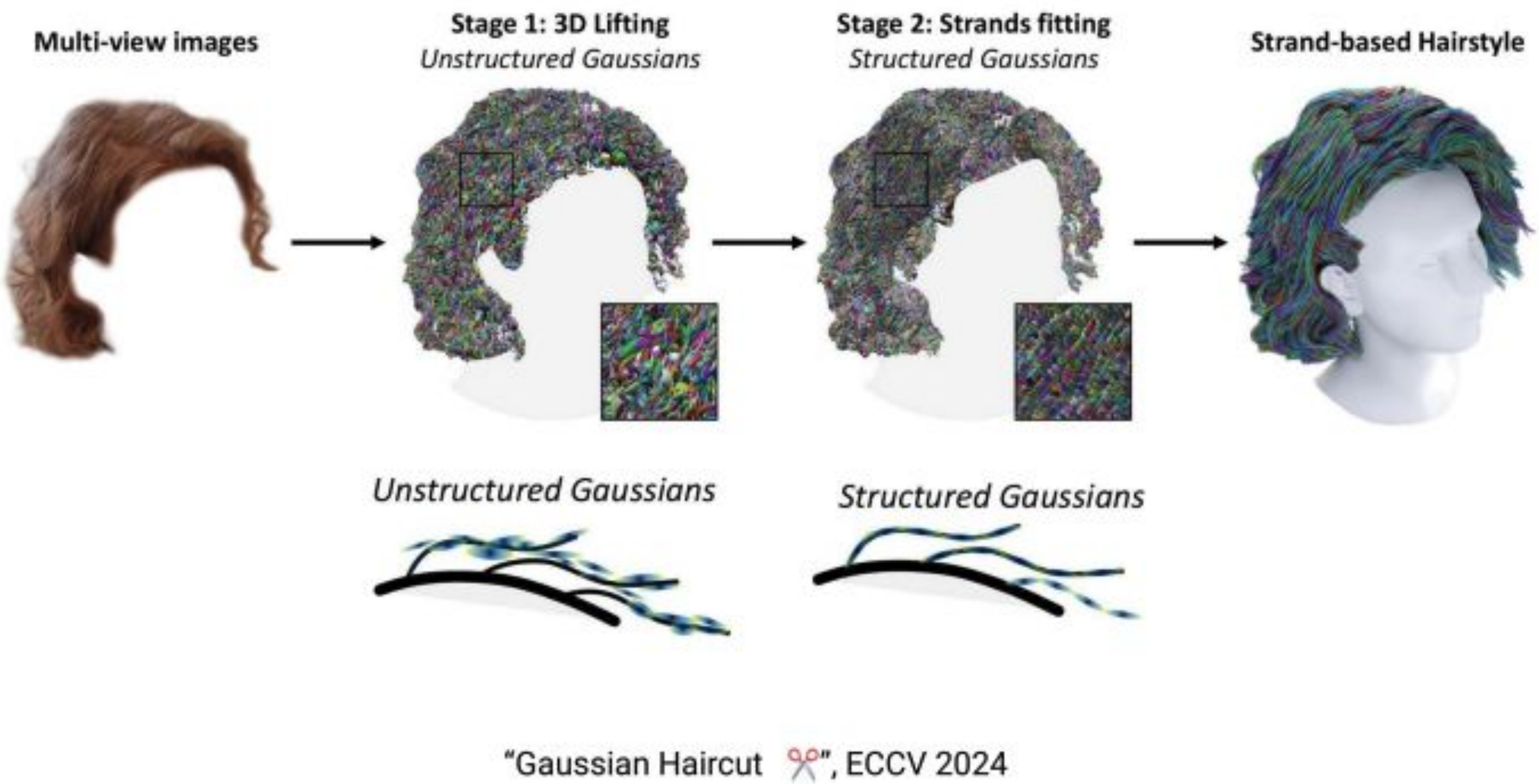

Figure: Reconstructing a 3D strand-based hairstyle from multi-view images. [Source: "Gaussian Haircut", ECCV 2024]