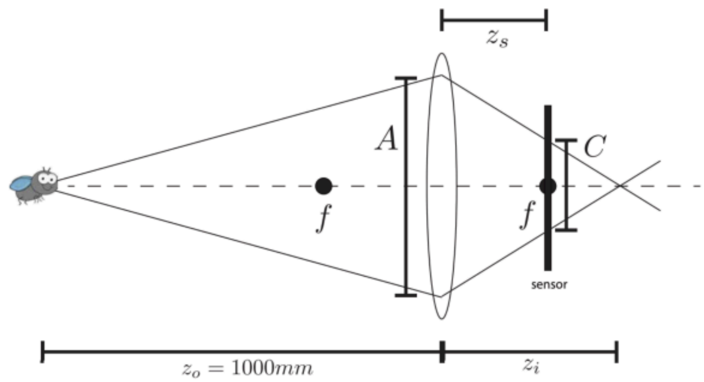
6. (Total : 12 points) Cameras, Lenses, Imaging

(6a) Consider the following diagram, which you may remember from discussion section. This diagram is not drawn to scale!



In this variant of that problem, assume that:

- $A = 25$mm
- $C = 2.5$mm

6a.i. [6 points] Calculate focal length f of this lens. Show your thinking and work for full or partial credit.

$f =$

Solution:
The image sensor is at the focal point, so $z_s = f$. Using similar triangles, notice that $\frac{A}{z_i} = \frac{C}{z_i - z_s} = \frac{C}{z_i - f}$. Also recall that using the Thin Lens Equation, $z_i = \frac{z_o f}{z_o - f}$.

$$z_i - f = \frac{C z_i}{A} \quad (1)$$

$$f = z_i \left(1 - \frac{C}{A}\right) \quad (2)$$

$$f = \frac{z_o f}{z_o - f} \left(1 - \frac{C}{A}\right) \quad (3)$$

$$z_o f - f^2 = z_o f \left(1 - \frac{C}{A}\right) \quad (4)$$

$$f \left(f + z_o \left(1 - \frac{C}{A}\right) - z_o\right) = 0 \quad (5)$$

Because f is not zero, we solve for the non-zero f value as $f = z_o \frac{C}{A} = 1000 \left(\frac{2.5}{25}\right) =$ 100mm. Equivalently, and maybe even more efficiently, you could plug in the calculation of $z_i = \frac{f}{1 - \frac{C}{A}}$ into the Thin Lens Equation, then solve for f.

6a.ii. [3 points] There is another sensor plane, call it z_s', where the circle of confusion would be exactly the same size as shown here for z_s. What is that depth z_s'? You can assume f is given. Show your work and you can round your answer to the nearest tenth of a millimeter.

$z_s' =$

Solution:

$$z_s' = z_i + (z_i - z_s) = 2\frac{z_o f}{z_o - f} - f$$

Plugging in the value of f as found in the previous part, $\frac{2 \cdot 1000 \cdot 100}{1000 - 100} - 100 = \frac{2000}{10 - 1} - 100 \approx 122.22$mm.

6a.iii. [3 points] What are the main visual differences between the photographs obtained at the two depths?

Solution: The image in the original camera setting will have smaller maginfication than the image in the second camera setting. So the fly will look larger at the second depth, as well as more blurry.
To be specific, the magnification in the first image is $M_1 = \frac{f}{z_o} = 0.1$. The magnification in the second image is $M_2 = \frac{z'_s}{z_o} \approx 0.122$, so the second image is 82%.

(3c) Focus and Field of View.
In this problem, assume a thin lens model.

3c.i. [2 points] You want to take a photo of your friend. You intend to compose the photo as a portrait that shows exactly 1 meter of your friend's height (from waist to top of the head). You take the picture with a lens that has a focal length of 100mm on a camera with a 25x25mm image sensor. Rounded to the nearest meter, how far away from your friend should you stand to take the photo? Show your work. Hint: for this part of the question you can safely assume that the camera is focused at infinity.

Answer = ____ (meters)

Show your work:

Solution: 4 meters. This is by similar triangles: need $\frac{25}{100} = \frac{0.5 \times 2\text{m}}{z}$, where z is the distance from your friend to the lens in meters.

3c.ii. [2 points] Unfortunately your friend is hit by a shrinking ray gun, and ends up exactly 25 mm tall! You resolve to take a photo of your friend to document this tragedy, but now want your friend's full height to exactly fill the photo. How far away from the lens should you place your friend to take the photo? Show your work. Hint: for this part of the question you cannot assume the camera will be focused at infinity.

Answer = [ ] (millimeters)

Show your work:

Solution: 200mm. Magnification desired is unity, since we want the friend to appear 25mm tall on the 25mm sensor. To achieve unit magnification, one can solve the thin lens equation, or recall that this occurs with both the subject and sensor at 2x the focal length.