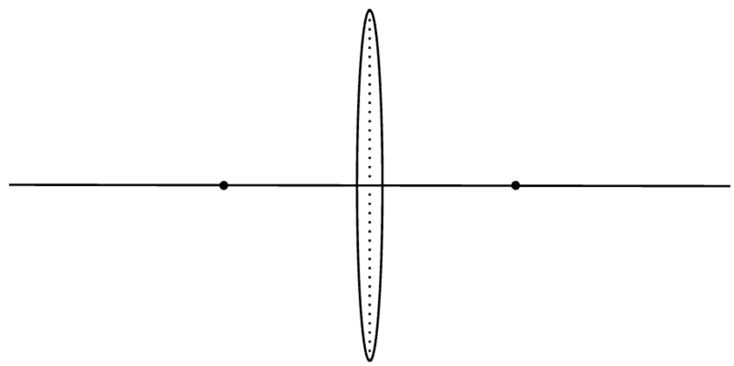
Cameras and Lenses 9

CS 184: Foundations of Computer Graphics

1 Blood Moon

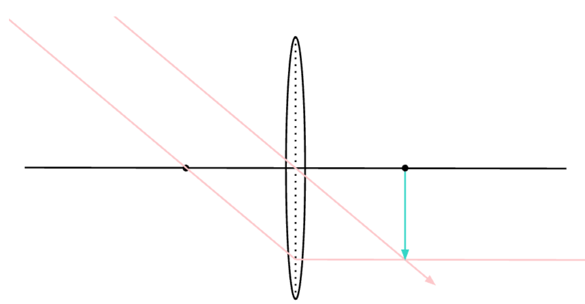
The following diagram shows the optical axis, focal points, and thin lens of a camera that was used to photograph the recent total lunar eclipse. Assume the moon is a distance of infinity away from the camera.

1. Draw the *focal ray*, which is the ray extending from the moon through the focal point of the thin lens. Recall that the focal ray exits the thin lens parallel to the optical axis.
2. Draw the *chief ray*, which is the ray extending from the moon through the center of the thin lens. Recall that all rays extending from an object at infinity are parallel to one another.



Solution:

Observe the image in turquoise.



3. The focal length of this thin lens is f_1. Calculate the distance between the lens and the image of the moon that is formed by the lens.

Solution: Use the Thin Lens Equation.

$$\frac{1}{f_1} = \frac{1}{z_i} + \frac{1}{z_o} = \frac{1}{z_i} + \frac{1}{\infty} = \frac{1}{z_i}$$

Therefore, the image forms at $z_i = f_1$. The image sensor is placed a distance of f_1 behind the lens (on the focal plane of the lens). This result could have also been found graphically from parts 1 and 2.

4. To achieve a clear photo of the moon, where should the image sensor be placed? What might occur if the distance between the lens and the image sensor is any smaller? What about any larger?

Solution: The distance between the lens and the image sensor should be the same as the distance between the image formed by the lens and the lens, which is f_1. Any smaller or any larger of a distance, the rays will not be converged, and the image will be blurry.

5. Let's now repurpose this lens as the *objective lens* of a telescope. In addition to the objective lens, a telescope has an *eyepiece lens*. The job of the eyepiece lens is to make the intermediate image formed by the objective lens appear infinitely far away to the viewer.

   The eyepiece lens has a focal length of f_2. Where should it be placed, relative to the objective lens?

Solution:

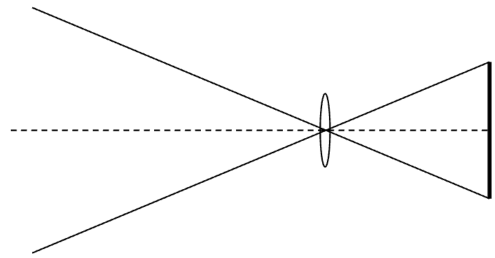
$$\frac{1}{f_2} = \frac{1}{z_i'} + \frac{1}{z_o'} = \frac{1}{\infty} + \frac{1}{z_o'} = \frac{1}{z_o'}$$

Therefore, the distance between the intermediate image and the eyepiece lens, z_o', is equal to f_2. So the eyepiece lens should be $f_1 + f_2$ behind the objective lens.

2 Raine's Field of View

Raine is doing a photoshoot. His camera has a sensor height of 36 millimeters and a focal length of 50 millimeters. It is focused at infinity. A bunny sits a little ways away.

1. Label the sensor height, focal length, and field of view (FOV) of Raine's camera.



2. Write an expression for the FOV of Raine's camera in the vertical direction.

Solution:

$$2\arctan\left(\frac{36\text{ mm}/2}{50\text{ mm}}\right) = 2\arctan(0.36)$$

3. Raine snaps a picture of the bunny, who is 20 centimeters tall. In the image, the bunny's height is one-ninth of the height of the image. Calculate the horizontal distance from the bunny to Raine's camera.

Solution: We are given that $h_o = 20$ cm, $h_i = \frac{h}{9} = 4$ mm, and $f = 50$ mm. Furthermore, since the camera is focused at infinity, we have

$$\frac{1}{f} = \frac{1}{z_i} + \frac{1}{\infty} \implies z_i = f.$$

By similar triangles using the chief ray0,

$$\frac{z_o}{h_o} = \frac{f}{h_i}.$$

Therefore,

$$z_o = f \cdot \frac{h_o}{h_i} = 50\text{ mm} \cdot \frac{20\text{ cm}}{4\text{ mm}} = 2.5\text{ m}.$$

Raine's camera is 2.5 meters away from the bunny.

4. Raine wants the bunny to take up a larger fraction of the vertical space of the image. Assume his camera's sensor height is fixed, and he doesn't want to move for fear of disturbing the bunny. How can he adjust his camera's *focal length* to achieve the desired effect? How is focal length related to FOV?

Solution: Raine can increase the focal length, which narrows his camera's FOV, thus resulting in the bunny taking up a larger fraction of the vertical space of the image. Focal length and FOV are inversely related.

5. How is sensor size related to FOV?

Solution: Raine can choose a camera with a smaller sensor height, which narrows FOV in the vertical direction, thus resulting in the bunny taking up a larger fraction of the vertical space of the image. Sensor size and FOV are directly related.

6. Finally, assume Raine's position is fixed. Which of the following configurations for his camera results in the bunny taking up the largest fraction of vertical space?

   (a) 36 millimeters tall sensor and 50 millimeters focal length lens.

   (b) 12 millimeters tall sensor and 18 millimeters focal length lens.

   (c) 24 millimeters tall sensor and 8 millimeters focal length lens.

Solution: B. Recall that the field of view can be computed as an angle:

$$\text{FOV} = 2\arctan\left(\frac{h}{2f}\right)$$

where h is the sensor size and f is the focal length.

Using this expression, we see that (a) has FOV $2\arctan(\frac{36}{100})$, (b) has FOV $2\arctan(\frac{12}{36})$, and (c) has FOV $2\arctan(\frac{24}{16})$. (c) has the largest FOV at approximately 56.31 degrees, whereas (a) has FOV 19.8 degrees and (b) has FOV 18.43 degrees.

3 Exposé of Exposure

James's camera supports several aperture, shutter duration, and ISO gain settings:

- Aperture: f/1.4, f/2, f/2.8, f/4, f/5.6, f/8, f/11, f/16.
- Shutter duration: Anything longer than 1/4000 seconds.
- ISO gain: 100, 200, 400, 800, 1600.

1. James's camera is currently on the following settings: aperture f/5.6, shutter duration 1/100 seconds, and ISO 200. In each part, leaving all other factors equal, select the new setting that allows James to double the exposure of images he captures.

   (a) Aperture (b) Shutter duration (c) ISO gain

   Solution:

   (a) Each $\sqrt{2}$ increase in aperture doubles the amount of light. Thus, James can step up the aperture to f/4.0.

   (b) Doubling the shutter duration doubles the exposure. Thus, James can double the duration to $1/50$ seconds.

   (c) Doubling the ISO gain doubles the exposure. Thus, James can double the ISO gain to 400.

2. James is now taking a shot in extremely bright light, where the correct exposure is equivalent to an aperture of f/8 with a shutter duration of 1/1000 seconds at ISO 100. He is taking a portrait of a person, and aesthetically would like to blur the background as much as possible without changing the exposure. Help James select his camera settings!

   (a) Aperture (b) Shutter duration (c) ISO gain

   Solution: Blurring the background requires a shallow depth of field, which is achieved by using the largest possible aperture. To allow this while maintaining correct exposure, we adjust the following settings:

   (b) Minimize the shutter duration: We shorten the exposure time to 1/4000 seconds, reducing the amount of light reaching the sensor by $4\times$.

   (c) Minimize the ISO gain: 100 is already the minimum, so no change is required.

   (a) To maintain the same exposure level, we must increase light intake by 2 stops. This means opening the aperture from f/8 to f/4, which is supported by James's camera. (Each step up in f-stop doubles the light, so f/8 $\rightarrow$ f/5.6 $\rightarrow$ f/4.)

3. James has had a long day of photography and takes his last shot in the evening. The correct exposure is equivalent to an aperture of f/8 with a shutter duration of 1/8 second at ISO 100. This shot is a landscape, and aesthetically James would like to maximize the depth of field without changing the exposure.

A rule of thumb in photography is that the shutter duration should be at least as fast as the inverse of the focal length of the lens to avoid motion blur due to hand-shake. James's camera has a 32 mm focal length lens, so make sure to choose a shutter duration at most 1/32 seconds.

What camera settings should he choose?

(a) Aperture

(b) Shutter duration

(c) ISO gain

Solution: Maximizing the depth of field is achieved by using the smallest possible aperture. To allow this while maintaining correct exposure, we adjust the following settings:

(b) Maximize the shutter duration: 1/32 seconds (decrease by 2 stops from 1/8 seconds).

(c) Maximize the ISO gain: 1600 (increase by 4 stops from ISO 100).

(a) To maintain the same exposure, we decrease the aperture by 2 stops from f/8 to f/16. (Each step down in f-stop doubles the light, so f/8 $\rightarrow$ f/11 $\rightarrow$ f/16.)

4 You Might Not Believe Your Eyes...

A camera has an ideal thin lens with a 50 millimeter focal length and f/4.0 F-number. It is focused at optical infinity, so the image sensor is placed at the focal plane. A firefly floats 1 meter in front of the camera lens.

1. Calculate z_i, the horizontal distance between the lens and the image formed of the firefly. Does the image form in front, on, or behind the image sensor?

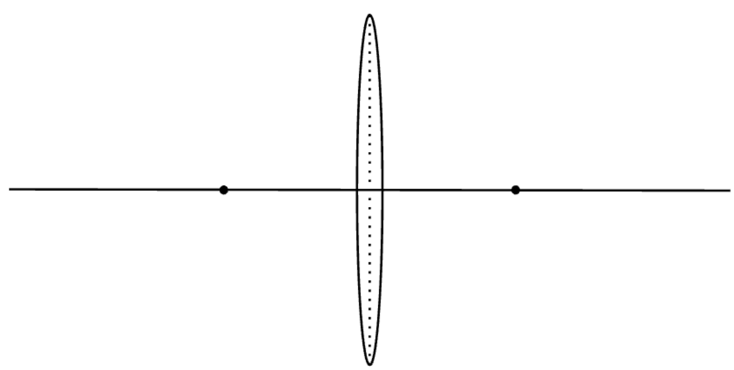
Solution:

$$\begin{aligned}\frac{1}{f} &= \frac{1}{z_o} + \frac{1}{z_i}\\ \frac{1}{50\text{ mm}} &= \frac{1}{1000\text{ mm}} + \frac{1}{z_i}\\ \frac{19}{1000\text{ mm}} &= \frac{1}{z_i}\\ \frac{1000}{19}\text{mm} &= z_i\end{aligned}$$

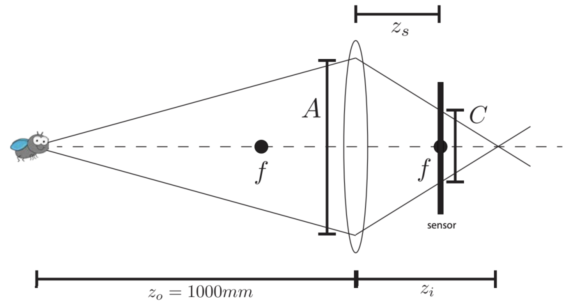
Image forms behind the image sensor.

2. Draw a ray diagram representing this problem. In your diagram, include the firefly, lens, image sensor, and rays from the firefly passing through the lens. Label the focal lengths (f), the distance from firefly to lens (z_o), and the distance from the lens to where the image of the firefly converges (z_i).

(Your drawing does not need to be to scale.)



Solution:



3. Recall that F-number is the ratio of focal length to aperture diameter. What is the aperture diameter of the lens, A, in millimeters?

Solution: $A = f/4.0 = \frac{50 \text{ mm}}{4.0} = 12.5 \text{ mm}.$

4. C is the diameter of the circle of confusion formed on the image sensor. Calculate C using A, z_i, z_o, and f. (Hint: can you spot any similar triangles?)

Solution: By similar triangles,

$$\frac{C}{z_i - f} = \frac{A}{z_i}.$$

Therefore,

$$C = A \cdot \frac{z_i - f}{z_i} = A\left(1 - \frac{f}{z_i}\right) = 12.5 \text{ mm}\left(1 - \frac{50 \text{ mm}}{\frac{1000}{19} \text{ mm}}\right) = 0.625 \text{ mm}.$$