(3a) [4 points] Nami, a display engineer, is building a new kind of TV that has 4 color primaries (Red, Green, Blue, and X) instead of just the usual 3. Nami believes that the added primary X will expand the color gamut of the TV, and therefore allow it to show new colors it could not before. Zoro, the company boss, is unconvinced. After all, 3 primaries is apparently good enough for every other TV on the market.

You decide to resolve this dispute by running a formal color-matching test. Before you run the test, you think about how things could turn out. Which of the following hypothetical outcomes would show that Nami is right? **Select all that apply.**

- ○ 0% Green + 30% Blue + 5% X = 35% Red.
- ○ 30% Red + 60% Blue + 10% X = 10% Green.
- ○ 5% Red + 5% Green + 5% X = 15% Blue.
- ○ 10% Red + 5% Green + 20% Blue = 70% X.

Briefly explain your answer in the box below.

Solution: Only the first three are correct. First, re-arrange the equations to solve for X. In each case, at least one of the RGB primaries needs to be negative in order to make a match, which is impossible. These situations would show that X is outside the gamut of RGB, and thus is a new color to add to the TV. In contrast, the fourth option instead shows that X can already be made with RGB.

(3b) [4 points] You run the experiment and find that Nami is right. However, you discover later that your test subject, Chopper, is color-blind and has only two out of the three cone types. Chopper's result indicated he saw that X is outside of the gamut of RGB. Would this still be true for a color-normal observer, i.e. someone with all three cone types? Briefly explain why or why not.

Solution: Yes, X would still be outside of the RGB gamut. There was no setting of Red, Green, or Blue that could satisfy both of Chopper's cone types at once. Those two cones would still be unsatisfied regardless if the missing third one was. Therefore, X is still outside of the gamut of Red, Green, and Blue.

7. (Total : 25 points) Color

(7a) Color Reproduction for Aliens

An alien species magically transports to earth. They are almost exactly like us humans in every way, except that their color perception is different. It turns out that instead of S, M and L cone cells, they have S, P and L and cone cells. The spectral response of the P cone cell is approximately midway between that for S and M cones (similar to one of the cone cells found in pigeons).

One thing that is discovered is that our RGB smartphone displays don't produce convincing colors for aliens. Unlike for humans, for aliens the display colors don't appear the same as the corresponding colors in the real world.

Assume that r_S, r_P, r_M and r_L are vectors representing the spectral response of the S, P, M and L cone cells of the human and alien retinas as described in lecture. Also, assume that s_R, s_G and s_B are the spectral emission functions for the R, G and B pixels in the smartphone displays we are testing.

You are a color display engineer, and you take it upon yourself to reprogram some smartphones with modified color processing, with the goal of producing optimal color for our new alien friends. Given an SPD s of a color in the real world that we want to reproduce for an alien, in the next few subparts of this problem, you will derive new intensities R_A, G_A and B_A for the R, G, and B pixels in the smartphones, to accomplish the goal.

7a.i. (2 points) Spectral Power Distribution Emitted by Reprogrammed Smartphones
First, write down an expression for the SPD emitted by the reprogrammed smartphones, as a function of R_A, G_A and B_A. You may use any of the other variables defined above.

$s_{\text{display}} =$

Solution: $S_R R_A + S_G G_A + S_B B_A$

7a.ii. (4 points) Tristimulus Alien Response
Next, write down expressions for the tristimulus response of the alien visual system to the real SPD s and the displayed SPD s_{display}.

$$\begin{bmatrix} S \\ P \\ L \end{bmatrix}_{\text{real}} =$$

$$\begin{bmatrix} S \\ P \\ L \end{bmatrix}_{\text{display}} =$$

for the s_R, s_G and s_B pixels in the smartphone.

Solution: $\begin{bmatrix} S \\ P \\ L \end{bmatrix}_{\text{real}} = \begin{bmatrix} --r_S-- \\ --r_P-- \\ --r_L-- \end{bmatrix} \begin{bmatrix} | \\ s \\ | \end{bmatrix}$

Solution:
$$\begin{bmatrix} S \\ P \\ L \end{bmatrix}_{\text{display}} = \begin{bmatrix} --r_S-- \\ --r_P-- \\ --r_L-- \end{bmatrix} \begin{bmatrix} --S_R-- \\ --S_G-- \\ --S_B-- \end{bmatrix}^T \begin{bmatrix} R_A \\ B_A \\ B_A \end{bmatrix}$$

7a.iii. (6 points) Alien Color Reproduction Formula
Finally, for the display to reproduce the color of s for the aliens, the tristimulus values in the previous part must be equal. Set them equal and solve for the intensities R_A, G_A and B_A as a function of the input s.

$$\begin{bmatrix} R_A \\ G_A \\ B_A \end{bmatrix} =$$

Solution:

$$\begin{bmatrix} --r_S-- \\ --r_P-- \\ --r_L-- \end{bmatrix} \begin{bmatrix} | \\ s \\ | \end{bmatrix} = \begin{bmatrix} --r_S-- \\ --r_P-- \\ --r_L-- \end{bmatrix} \begin{bmatrix} | & | & | \\ S_R & S_G & S_B \\ | & | & | \end{bmatrix} \begin{bmatrix} R_A \\ G_A \\ B_A \end{bmatrix}$$

$$\begin{bmatrix} R_A \\ G_A \\ B_A \end{bmatrix} = \left(\begin{bmatrix} --r_S-- \\ --r_P-- \\ --r_L-- \end{bmatrix} \begin{bmatrix} | & | & | \\ S_R & S_G & S_B \\ | & | & | \end{bmatrix} \right)^{-1} \begin{bmatrix} --r_S-- \\ --r_P-- \\ --r_L-- \end{bmatrix} \begin{bmatrix} | \\ s \\ | \end{bmatrix}$$

7a.iv. (2 points) The reprogrammed smartphones are very popular amongst the aliens, but nobody succeeds in reprogramming the smartphones to achieve exact color reproduction for both humans and aliens at the same time. Give a brief technical explanation of why this cannot be achieved.

Solution: This can't be achieved because of the difference between the P and M cone cells. The activation for the aliens will always be slightly different except for where/if the two curves intersect and will give the brain cues that the colors are different.

7a.v. (3 points) It turns out that there are aliens with color vision deficiency (CVD), similar to humans, where they are effectively missing their L cone cells (they have only S and P cones). In an engineering brainstorming session at your company, someone proposes that you may be able to program an RGB TV's color processing so that it provides exact color appearance for an audience of CVD aliens as well as CVD humans who were missing M cone cells. Do you agree, and if so, briefly how would you achieve this? (It is an engineering meeting, so there are no marketing people to complain that there may not be a large market for CVD aliens and CVD humans who want to watch a movie together.)

Solution: Yes it can be done - if we setup a system of equations for the S, P cones for aliens and for the S, L cones for humans we can then solve for a set of RGB such that both are activated equally.