Discussion 10

Color

Computer Graphics and Imaging
UC Berkeley CS 184/284A

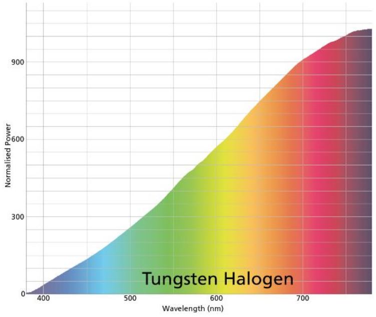
Week 9 Announcements

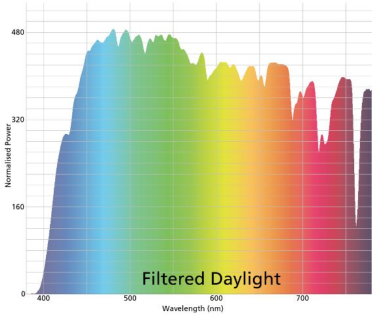
- Homework 3 due 08/01
- Final project milestone due 08/04
- Exam 2 is on 08/07

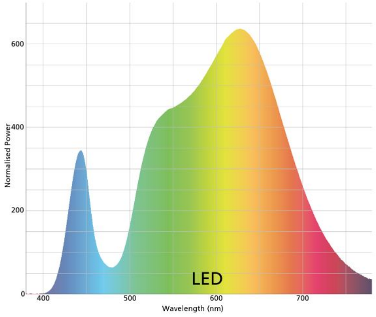
Light

Light

- A light source can be characterized by its <u>spectral power distribution</u>.
- Non-negative function giving the power in a light beam at a given wavelength.
- In *watts/nm*.



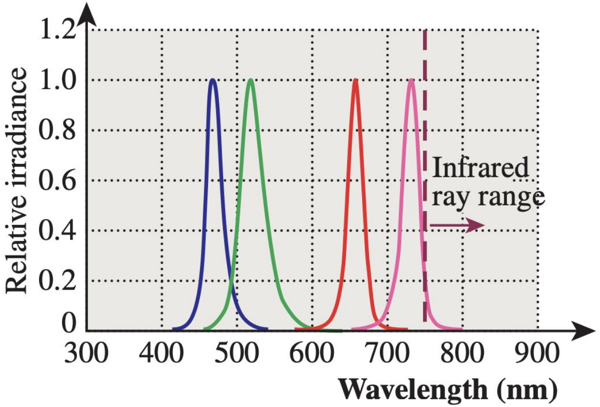




Monospectral Distributions

- Most of the power is near a <u>single wavelength</u>

  ➔ a single color.
- SPDs are additive!
- Example, using red, green, and blue monospectrums:



$$R\, s_R(\lambda) + G\, s_G(\lambda) + B\, s_B(\lambda)$$

red brightness level, [0, 1]

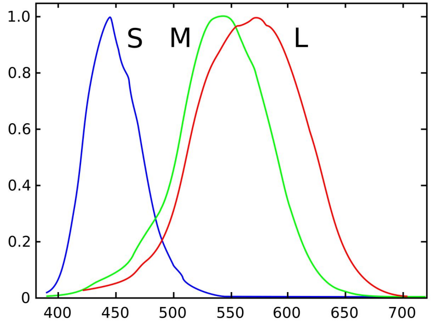
red monospectrum

$$\begin{bmatrix} | & | & | \\ s_R & s_G & s_B \\ | & | & | \end{bmatrix} \begin{bmatrix} R \\ G \\ B \end{bmatrix}$$

Colors

Spectral Response of Human Cone Cells

- Humans have three cone cells, with <u>different sensitivities</u>.
- The L cone's peak response occurs around orange/yellow.
- The M cone around green/yellow.
- The S cone around blue.



The <u>total response</u> of a cone cell as a function of the incoming spectrum is the integral of the <u>response curve</u>!

$$S = \int r_S(\lambda) s(\lambda)\, d\lambda$$

$$M = \int r_M(\lambda) s(\lambda)\, d\lambda$$

$$L = \int r_L(\lambda) s(\lambda)\, d\lambda$$

$$\begin{bmatrix} S \\ M \\ L \end{bmatrix} = \begin{bmatrix} — & r_S & — \\ — & r_M & — \\ — & r_L & — \end{bmatrix} \begin{bmatrix} | \\ s \\ | \end{bmatrix}$$

Question 1

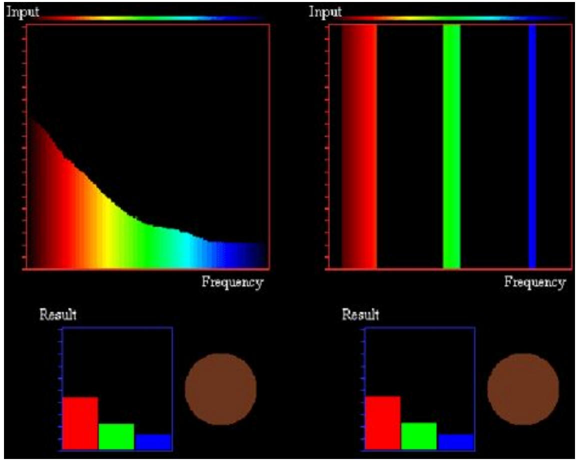
Q1.1 Metamer Magic

1. What is a metamer? Why are metamers useful?

Solution: Metamers are two different spectra that integrate to the same visual (S, M, L) response. They are useful because we can reproduce real-world scenes with hard-to-recreate spectra on screens.

Conceptually, they are two equivalent high-dimension signals which project to the same low-dimension signal.

They allow us to reproduce real-world scenes with hard-to-recreate spectra on screens.



Q1.2 Metamer Magic

2. Suppose you have a spectrum, $\boldsymbol{S}_{\text{ref}}$, that is metameric to another spectrum, $\boldsymbol{S}$, to an observer whose spectral sensitivities are represented by

$$\boldsymbol{C} = \begin{bmatrix} \text{——} & r_S & \text{——} \\ \text{——} & r_M & \text{——} \\ \text{——} & r_L & \text{——} \end{bmatrix}$$

Write an equation in terms of $\boldsymbol{S}_{\text{ref}}$, $\boldsymbol{S}$, and $\boldsymbol{C}$ that represents this metameric relationship.

Solution: $\boldsymbol{C}\boldsymbol{S}_{\text{ref}} = \boldsymbol{C}\boldsymbol{S}$.

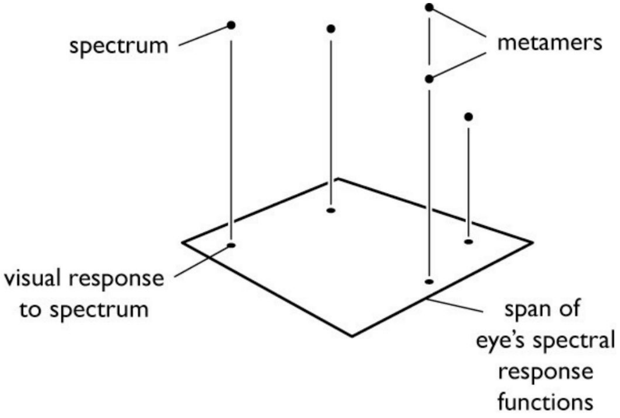
Q1.3 Metamer Magic

3. Show that $\boldsymbol{\Delta} = \boldsymbol{S} - \boldsymbol{S}_{\text{ref}}$ is in the null space of $\boldsymbol{C}$. That is, $\boldsymbol{C}\boldsymbol{\Delta} = 0$. What does this result imply about the differences between metameric spectra?

Solution: $\boldsymbol{C}\boldsymbol{S} - \boldsymbol{C}\boldsymbol{S}_{\text{ref}} = \boldsymbol{C}(\boldsymbol{S} - \boldsymbol{S}_{\text{ref}}) = 0$. The difference between two metameric spectra is perceived by the observer to be zero. In other words, it produces zero perceived color.

The null space is *perpendicular* to the 3D subspace.

The differences between metameric spectra are <u>not detectable</u> when viewing from the 3D subspace.



Q1.4 Metamer Magic

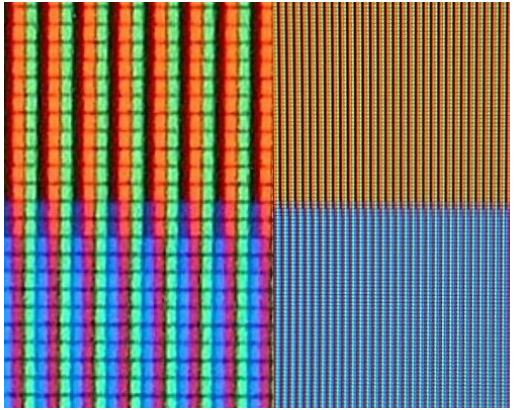
4. You have two matte (diffuse) prints of the same digital photo. At first, you look at the two photos under the sunlight, which has a spectrum $I(\lambda)$. The colors in the prints look identical. However, under the fluorescent lighting of your kitchen, with spectrum $F(\lambda)$, you notice that some of the colors in the two photos now look quite different. What is happening?

Solution: Consider a point that looks different in the two photos under fluorescent light. Call the reflectance at this point $f_1(\lambda)$ and $f_2(\lambda)$, respectively in the first and second photos. Call the spectrum of incoming sunlight, $I(\lambda)$, and the spectrum of the fluorescent light $F(\lambda)$. What is going on is that reflected spectra under sunlight, $f_1(\lambda)I(\lambda)$ and $f_2(\lambda)I(\lambda)$ are metamers (have the same integral projection on the cone response functions). However, the reflected spectra under fluorescent lights, $f_1(\lambda)F(\lambda)$ and $f_2(\lambda)F(\lambda)$ are not metamers. [TBD: add more pedagogical detail here.]

Color Reproduction

Color Reproduction

- Most digital displays use RGB the model, with red, green, and blue as the <u>three primaries</u> (monospectral distributions).
- Colors in the gamut of the display can be expressed as ***(R, G, B)*** color triples.



$$s_{\text{disp}}(\lambda) = R\, s_R(\lambda) + G\, s_G(\lambda) + B\, s_B(\lambda)$$

$$\begin{bmatrix} | \\ s_{\text{disp}} \\ | \end{bmatrix} = \begin{bmatrix} | & | & | \\ s_R & s_G & s_B \\ | & | & | \end{bmatrix} \begin{bmatrix} R \\ G \\ B \end{bmatrix}$$

Color Reproduction

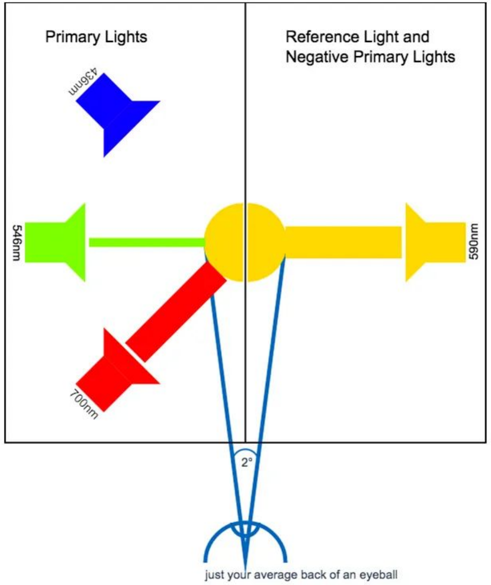
- Goal: reproduce the color that we see in the world into a color on the display.
  - Given: Human cone sensitivities r_S, r_M, r_L, and SPD $s(\lambda)$.
  - Solve for: Display (R, G, B) values.
- Solution: Use metamers!

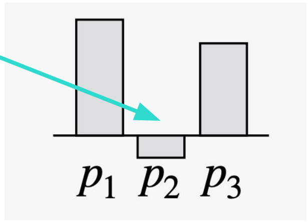
$$\overbrace{\begin{bmatrix} — & r_S & — \\ — & r_M & — \\ — & r_L & — \end{bmatrix} \begin{bmatrix} | & | & | \\ s_R & s_G & s_B \\ | & | & | \end{bmatrix} \begin{bmatrix} R \\ G \\ B \end{bmatrix}}^{\text{color we perceive on display}} = \overbrace{\begin{bmatrix} — & r_S & — \\ — & r_M & — \\ — & r_L & — \end{bmatrix} \begin{bmatrix} | \\ s \\ | \end{bmatrix}}^{\text{color we perceive in real life}}$$

Color Matching Experiment

Maxwell’s Color Matching Experiment

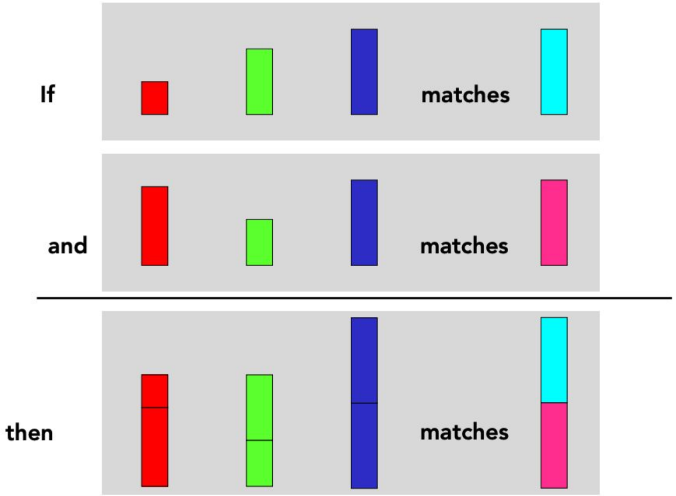
- Primaries do not need to be red, green, and blue
- Goal: Match primary lights to the provided test light
  - If it is not possible to match the test color, then it is **out of the gamut**
    - **Gamut**: Range of colors that can be displayed on a device
    - You’d have to add a **“negative”** amount of the color (add some of the primary into the test side)





Color Matching Experiment Findings

- The color matching experiment is linear

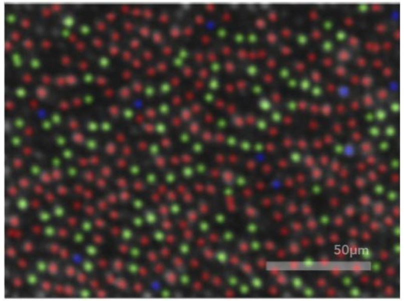


- Three primary colors are necessary and sufficient for “normal” color vision
- Two primary colors are necessary and sufficient for red-green colorblindness

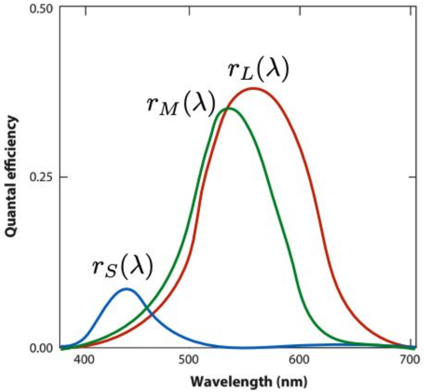
Some ~~brief~~ biology

Human Eyes

- Two types of retinal photoreceptors
  - Rods: primarily used in low light conditions
    - Allows you to perceive only shades of gray, no color
  - Cones: “photopic” receptors, primarily used in typical light levels
    - Provides the sensation of color
    - Three types of cones = three detectors!
      - Short
      - Medium
      - Long



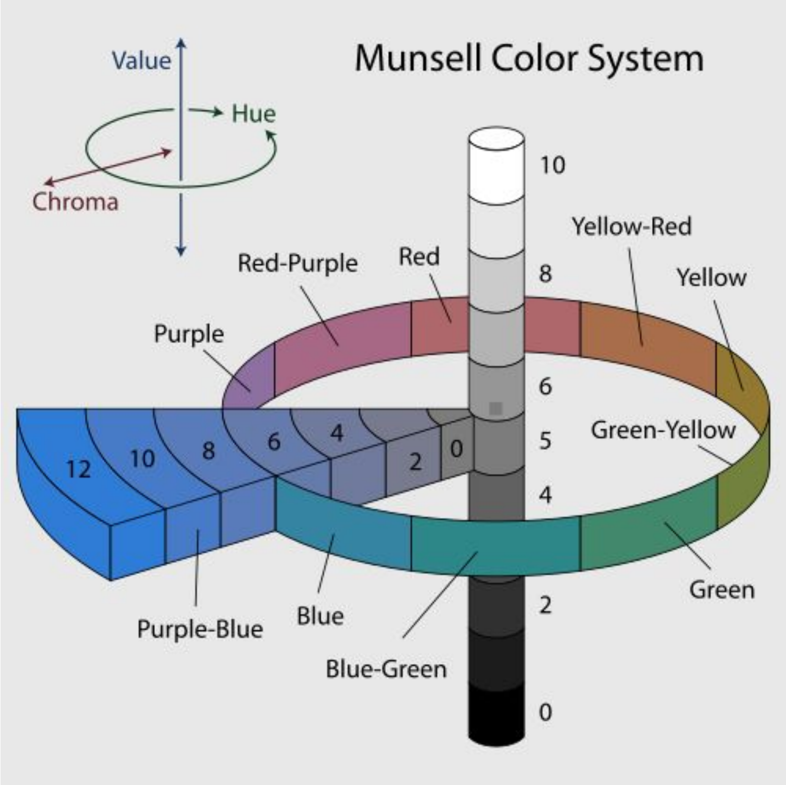
Sabesan Lab, UW. Pandiyan et al. 2020.



Brainard, Color and the Cone Mosaic, 2015.

Color Space

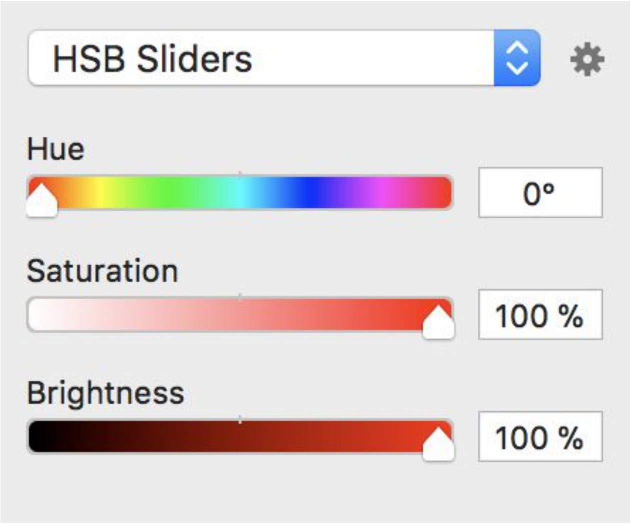
Hue, Saturation, Value (HSV)

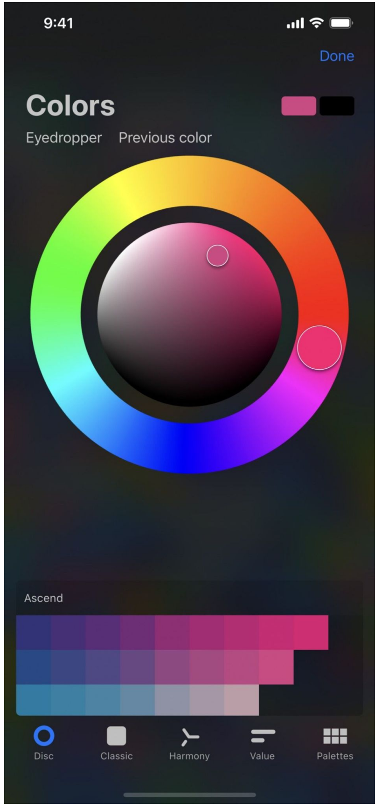


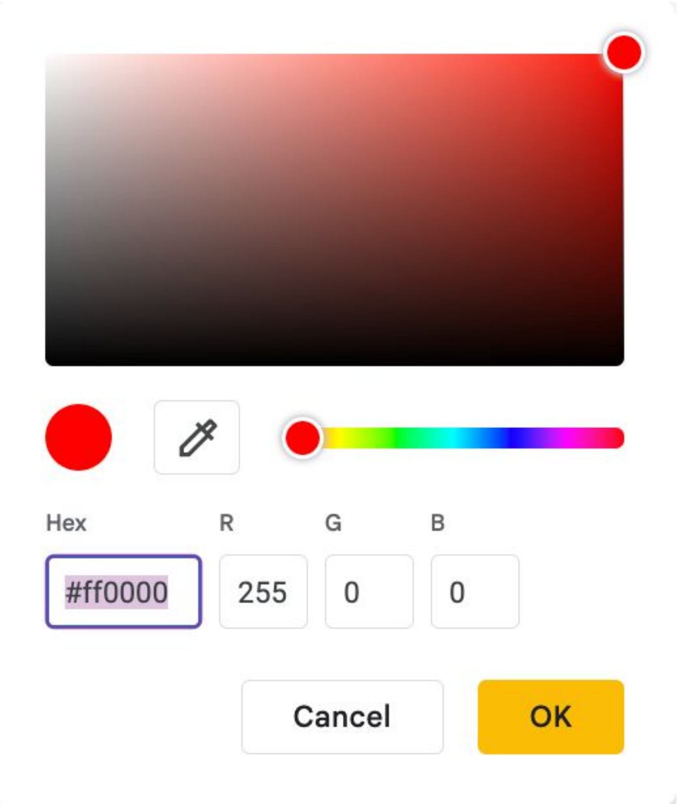
From 1905!

- Hue: the dominant wavelength, what color do you call it (red/blue/green, etc.)
- Saturation: the “colorfulness”, how vivid the color is
- Value: the amount of light

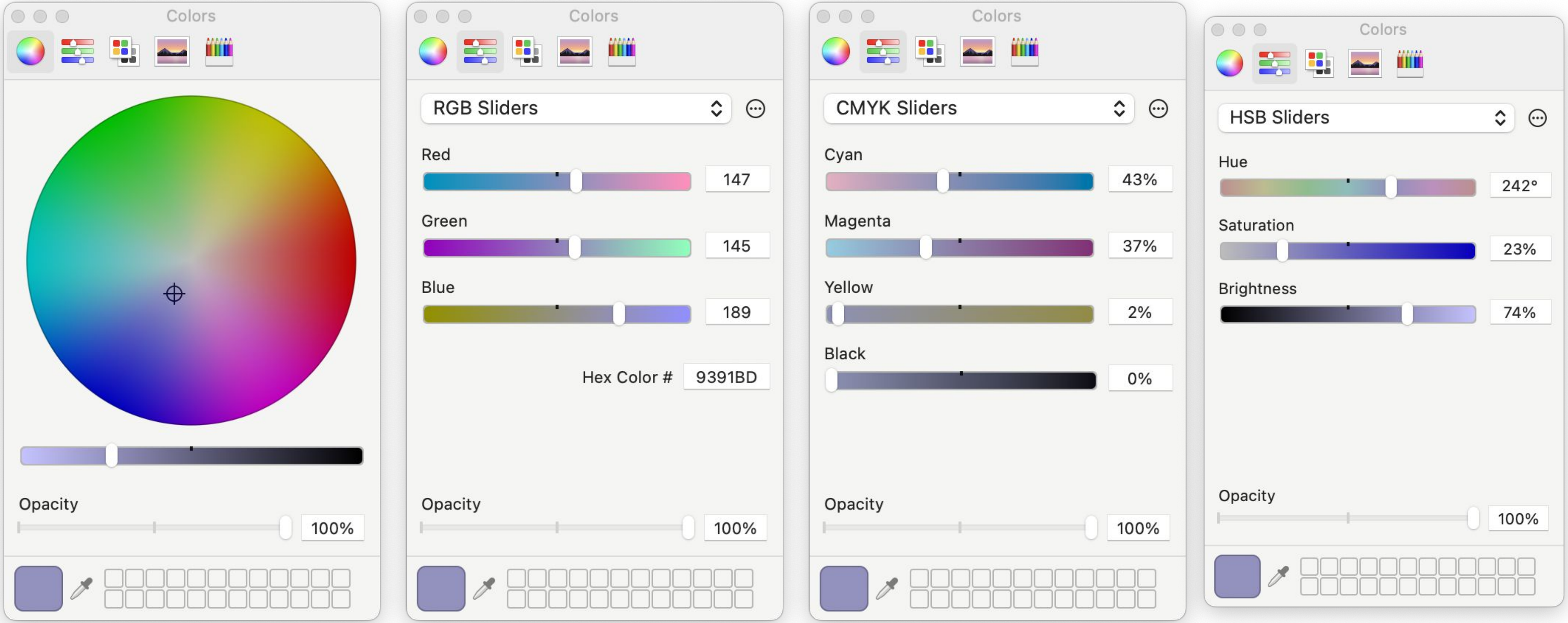
Hue, Saturation, Value (HSV)







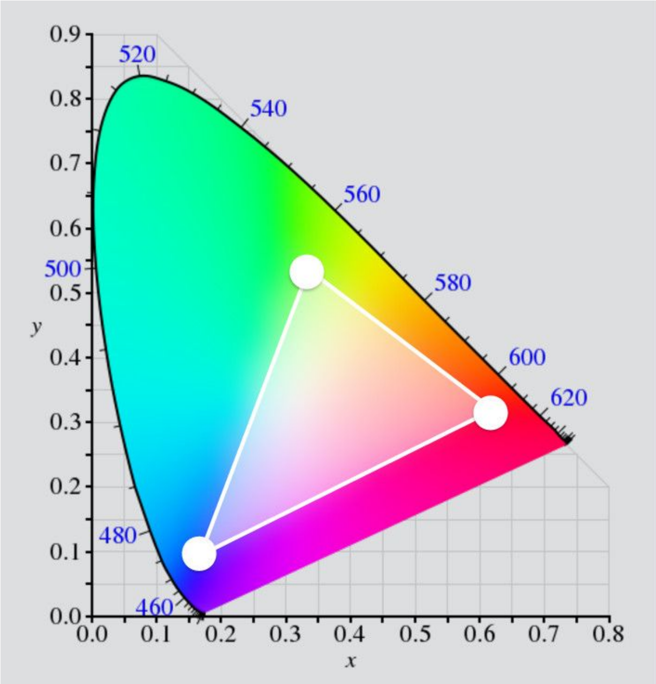
Hue, Saturation, Value (HSV)



CIE Chromaticity

- Pure (spectral) colors are at the edge of the plot and become more desaturated as you go towards the centroid of the plot.

- Black is not on this diagram because it is purely a chromaticity diagram - luminance values are dealt with separately.



Problems defining a Color Space

- Device-dependent: different devices will show different colors
- Limited gamut: limited range of colors you can show

$$\begin{bmatrix} R \\ G \\ B \end{bmatrix} = \begin{bmatrix} r_S \cdot s_R & r_S \cdot s_G & r_S \cdot s_B \\ r_M \cdot s_R & r_M \cdot s_G & r_M \cdot s_B \\ r_L \cdot s_R & r_L \cdot s_G & r_L \cdot s_B \end{bmatrix}^{-1} \begin{bmatrix} — & r_S & — \\ — & r_M & — \\ — & r_L & — \end{bmatrix} \begin{bmatrix} | \\ s \\ | \end{bmatrix}$$

Color Repr … color corresponding to a scene color and you get negative color values, what does it mean and what should you do?

Problems defining a Color Space

$$\begin{bmatrix} R \\ G \\ B \end{bmatrix} = \begin{bmatrix} r_S \cdot s_R & r_S \cdot s_G & r_S \cdot s_B \\ r_M \cdot s_R & r_M \cdot s_G & r_M \cdot s_B \\ r_L \cdot s_R & r_L \cdot s_G & r_L \cdot s_B \end{bmatrix}^{-1} \begin{bmatrix} — & r_S & — \\ — & r_M & — \\ — & r_L & — \end{bmatrix} \begin{bmatrix} | \\ s \\ | \end{bmatrix}$$

Color Reproduction: If you solve the equations above to get the RGB color corresponding to a scene color and you get negative color values, what does it mean and what should you do?

- Answer: Get a better display, or find a different color vector that maps to the same SML values (the gamut of this devices is not enough to display this solution for the scene color)!

Question 2

Q2.1 & 2.2 Hue Knew?

1. Gamuts are device-dependent.

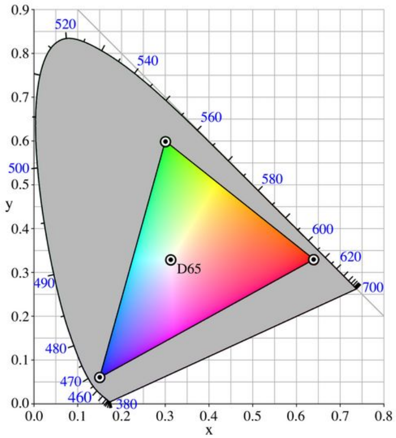
Solution: True. By definition, a gamut is the range of colors that a display can produce.

2. In the CIE chromaticity diagram, the colors are least saturated at the edge of the plot, and become more saturated towards the centroid at the plot.

Solution: False. Pure (spectral) colors are at the edge of the plot and become more desaturated as you go towards the centroid of the plot.

Q2.3 & 2.4 Hue Knew?

3. The gamut of a display that uses combinations of three distinct colors (three primaries) is visualizable as a triangle in the CIE chromaticity diagram.



Solution: True.

4. The CIE chromaticity diagram includes the color black.

Solution: False. CIE chromaticity only visualizes hue and saturation. Black is not on this diagram because it is purely a chromaticity diagram - luminance values are dealt with separately.

Q2.5 & 2.6 Hue Knew?

5. In the CIELAB color space, equal distances in the color space correspond to roughly equal perceived differences in color.

Solution: True. CIELAB was designed to be perceptually uniform. Perceptual uniformity across colors. L* is lightness, a* is red-green, and b* is blue-yellow.

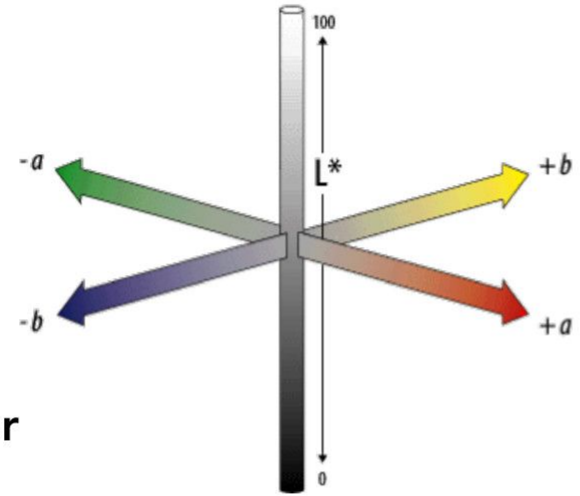
6. In Maxwell's color matching experiment, the color appearance of any input light could be matched by adjusting the brightness of three primary lights.

Solution: False, it may be necessary to add some primary light to the input light if it is out of the gamut of the three primaries.

Q1.4 What is the goal of the CIELAB color space?

A commonly used color space that strives for perceptual uniformity

- L* is lightness
- a* and b* are color-opponent pairs
  - a* is red-green, and b* is blue-yellow
- A gamma transform is used for warping because perceived brightness is proportional to scene intensity$^{\gamma}$, where $\gamma \approx 1/3$

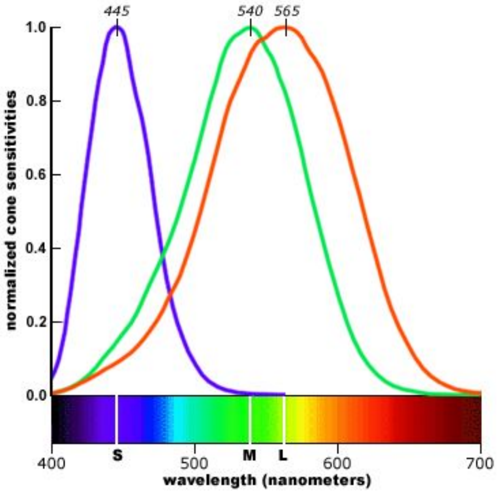


Color Blindness

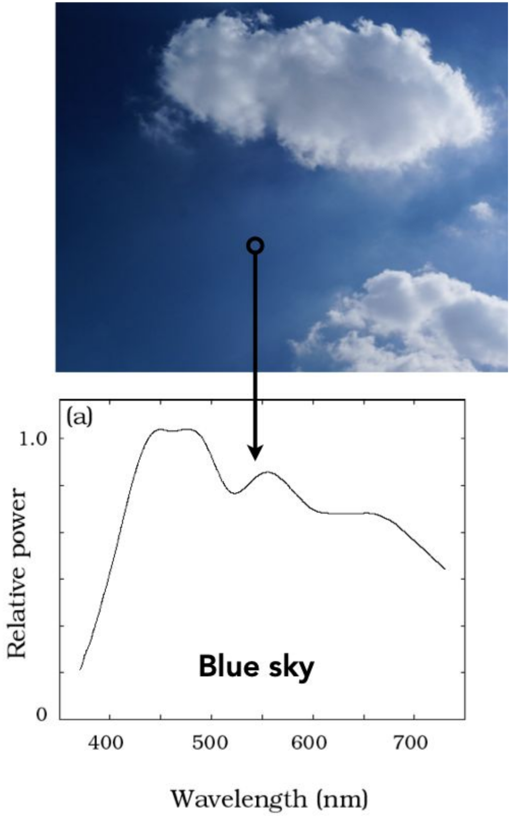
Normal color perception

Red/Green color blindness

Trichromat cone responses



●



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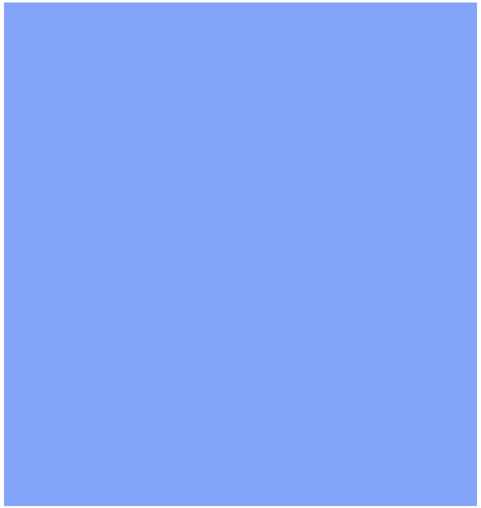

Demo

Question 3

3 Color Blindness Virus!

A terrible virus spreads rapidly through the world. It affects the M cone cells in our retinas, turning them into L cone cells. As a result, everyone becomes red-green color-blind, since the M and L cones normally help distinguish nearby wavelengths in the red-green range. Now all humans only have S and L cones!

As the last remaining humans adapt to this new form of vision, you realize there's a silver lining: with only two cone types remaining, it might be possible to reproduce the appearance of "full" (which is now 2D) color using just the red and blue pixels on your phone.

Throughout this problem, assume that the spectral response curves of the remaining human cone cells, as functions of wavelength, are given by $S(\lambda)$ and $L(\lambda)$.

1. First, consider a target light with spectral power distribution (SPD) $I(\lambda)$ that we wish to reproduce. Write down expressions for the scalar response of each cone cell when exposed to $I(\lambda)$.

$s_{\text{target}} =$

$l_{\text{target}} =$

$$s_{\text{target}} = \int S(\lambda)I(\lambda)d\lambda$$

$$l_{\text{target}} = \int L(\lambda)I(\lambda)d\lambda$$

2. Now consider the red and blue pixels on your phone, with SPDs given by functions $R(\lambda)$ and $B(\lambda)$. If we set the brightness of these pixels by scalar values r and b, respectively, write down the scalar response from each cone cell type when exposed to the resulting light.

$s_{\text{display}} =$

$l_{\text{display}} =$

$$s_{\text{display}} = \int S(\lambda)\,(rR(\lambda) + bB(\lambda))\,d\lambda$$
$$l_{\text{display}} = \int L(\lambda)\,(rR(\lambda) + bB(\lambda))\,d\lambda$$

3. Note that we can re-write the result from part (ii) in matrix form:

$$\begin{bmatrix} s_{\text{display}} \\ l_{\text{display}} \end{bmatrix} = M \begin{bmatrix} r \\ b \end{bmatrix} = \begin{bmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{bmatrix} \begin{bmatrix} r \\ b \end{bmatrix} \tag{1}$$

Solution: Note that each element of the matrix is given by the integral of the product of the cone sensitivity and the pixel's SPD. That is,

$$m_{11} = \int S(\lambda)R(\lambda)\,d\lambda, \quad m_{12} = \int S(\lambda)B(\lambda)\,d\lambda,$$

$$m_{21} = \int L(\lambda)R(\lambda)\,d\lambda, \quad m_{22} = \int L(\lambda)B(\lambda)\,d\lambda.$$

Each matrix element represents how much a given pixel color (red or blue) stimulates a given cone type. For example, m_{11} captures how much the red pixel stimulates the S cone.

4. Finally, to complete the color matching procedure for the color-blind humans using only red and blue pixels, determine how to choose values for r and b so that the display light matches the perceived color of the input SPD $I(\lambda)$. Assume that $I(\lambda)$ is within the gamut of the red and blue pixels.

   Write your answer as a one-line matrix expression for r and b. You may use any variables defined in previous parts of this question and standard matrix operations (e.g., transpose, inverse).

   Hint: Match the target and display responses!

   Solution: To match the target cone responses, we want:

$$\begin{bmatrix} s_{\text{display}} \\ l_{\text{display}} \end{bmatrix} = \begin{bmatrix} s_{\text{target}} \\ l_{\text{target}} \end{bmatrix}.$$

   From earlier, we know that the display responses are given by:

$$\begin{bmatrix} s_{\text{display}} \\ l_{\text{display}} \end{bmatrix} = M \begin{bmatrix} r \\ b \end{bmatrix}.$$

   So to match the responses, we solve:

$$M \begin{bmatrix} r \\ b \end{bmatrix} = \begin{bmatrix} s_{\text{target}} \\ l_{\text{target}} \end{bmatrix}, \quad \text{which gives} \quad \begin{bmatrix} r \\ b \end{bmatrix} = M^{-1} \begin{bmatrix} s_{\text{target}} \\ l_{\text{target}} \end{bmatrix}.$$

   The variables r and b represent the brightness values (or intensity scalars) that you set for the red and blue pixels on the display.

5. Someone affected by the virus does the computation above and says, “Awesome! Why don’t we just make displays with only red and blue pixels now?” Given that humans now only have S and L cones, this idea might seem reasonable, as we’ve shown above. However, why might there still be a benefit to keeping the green pixels? Explain your reasoning.

Solution: Even with only S and L cone cells, green pixels can still stimulate those cones in a way that red and blue pixels cannot. This additional variation allows for more flexibility in reproducing different combinations of cone responses. In other words, the display’s effective gamut—the range of perceptually distinguishable colors—would likely be larger with three primaries than with only two.

This is analogous to how, in trichromatic vision, adding a fourth primary (like yellow) can increase the gamut compared to a standard RGB display.

For Fun: Color Theory

Primary Colors

Georges Seurat

- 19th century.
- Pointillism. Divide colors into their components.

Let's Take Attendance.

- Be sure to select Week 13 and input your TA's secret word 😉
- Any feedback? Let us know!