COLORS 10

CS 184: FOUNDATIONS OF COMPUTER GRAPHICS

1 Metamer Magic

1. What is a metamer? Why are metamers useful?

> **Solution:** Metamers are two different spectra that integrate to the same visual (S, M, L) response. They are useful because we can reproduce real-world scenes with hard-to-recreate spectra on screens.

2. Suppose you have a spectrum, $\boldsymbol{S}_{\text{ref}}$, that is metameric to another spectrum, $\boldsymbol{S}$, to an observer whose spectral sensitivities are represented by

$$\boldsymbol{C} = \begin{bmatrix} \text{———} & r_S & \text{———} \\ \text{———} & r_M & \text{———} \\ \text{———} & r_L & \text{———} \end{bmatrix}$$

Write an equation in terms of $\boldsymbol{S}_{\text{ref}}$, $\boldsymbol{S}$, and $\boldsymbol{C}$ that represents this metameric relationship.

> **Solution:** $\boldsymbol{C}\boldsymbol{S}_{\text{ref}} = \boldsymbol{C}\boldsymbol{S}$.

3. Show that $\boldsymbol{\Delta} = \boldsymbol{S} - \boldsymbol{S}_{\text{ref}}$ is in the null space of $\boldsymbol{C}$. That is, $\boldsymbol{C}\boldsymbol{\Delta} = 0$. What does this result imply about the differences between metameric spectra?

> **Solution:** $\boldsymbol{C}\boldsymbol{S} - \boldsymbol{C}\boldsymbol{S}_{\text{ref}} = \boldsymbol{C}(\boldsymbol{S} - \boldsymbol{S}_{\text{ref}}) = 0$. The difference between two metameric spectra is perceived by the observer to be zero. The changes are perpendicular to the 3D subspace.

4. You have two matte (diffuse) prints of the same digital photo. At first, you look at the two photos under the sunlight, which has a spectrum $I(\lambda)$. The colors in the prints look identical. However, under the fluorescent lighting of your kitchen, with spectrum $F(\lambda)$, you notice that some of the colors in the two photos now look quite different. What is happening?

> **Solution:** Consider a point that looks different in the two photos under fluorescent light. Call the reflectance at this point $f_1(\lambda)$ and $f_2(\lambda)$, respectively in the first and second photos. Call the spectrum of incoming sunlight, $I(\lambda)$, and the spectrum of the fluorescent light $F(\lambda)$. What is going on is that reflected spectra under sunlight, $f_1(\lambda)I(\lambda)$ and $f_2(\lambda)I(\lambda)$ are metamers (have the same integral projection on the cone response functions). However, the reflected spectra under fluorescent lights, $f_1(\lambda)F(\lambda)$ and $f_2(\lambda)F(\lambda)$ are not metamers.

2 Hue Knew?

Answer true or false to the following statements. Provide an explanation for your answer.

1. Gamuts are device-dependent.

   Solution: True. By definition, a gamut is the range of colors that a display can produce.

2. In the CIE chromaticity diagram, the colors are least saturated at the edge of the plot, and become more saturated towards the centroid at the plot.

   Solution: False. Pure (spectral) colors are at the edge of the plot and become more desaturated as you go towards the centroid of the plot.

3. The gamut of a display that uses combinations of three distinct colors (three primaries) is visualizable as a triangle in the CIE chromaticity diagram.

   Solution: True.

4. The CIE chromaticity diagram includes the color black.

   Solution: False. CIE chromaticity only visualizes hue and saturation. Black is not on this diagram because it is purely a chromaticity diagram - luminance values are dealt with separately.

5. In the CIELAB color space, equal distances in the color space correspond to roughly equal perceived differences in color.

   Solution: True. CIELAB was designed to be perceptually uniform. Perceptual uniformity across colors. L* is lightness, a* is red-green, and b* is blue-yellow.

6. In Maxwell's color matching experiment, the color appearance of any input light could be matched by adjusting the brightness of three primary lights.

   Solution: False, it may be necessary to add some primary light to the input light if it is out of the gamut of the three primaries.

3 Color Blindness Virus!

A terrible virus spreads rapidly through the world. It affects the M cone cells in our retinas, turning them into L cone cells. Now all humans only have S and L cones! You realize there's a silver lining: it might be possible to reproduce the appearance of "full" (which is now 2D) color using just the red and blue pixels on your phone.

Throughout this problem, assume that the spectral response curves of the remaining human cone cells, as functions of wavelength, are given by $r_S(\lambda)$ and $r_L(\lambda)$.

1. First, consider a target light with spectral power distribution (SPD) $I(\lambda)$ that we wish to reproduce. Write down expressions for the scalar response of each cone cell when exposed to $I(\lambda)$.

   $S_{\text{target}} =$

   $L_{\text{target}} =$

   Solution:

   $$S_{\text{target}} = \int r_S(\lambda) I(\lambda) d\lambda$$
   $$L_{\text{target}} = \int r_L(\lambda) I(\lambda) d\lambda$$

2. Now consider the red and blue pixels on your phone, with SPDs given by functions $s_R(\lambda)$ and $s_B(\lambda)$. If we set the brightness of these pixels by scalar values R and B, respectively, write down the scalar response from each cone cell type when exposed to the resulting light.

   $S_{\text{display}} =$

   $L_{\text{display}} =$

   Solution:

   $$S_{\text{display}} = \int r_S(\lambda) \left(R s_R(\lambda) + B s_B(\lambda)\right) d\lambda$$
   $$L_{\text{display}} = \int r_L(\lambda) \left(R s_R(\lambda) + B s_B(\lambda)\right) d\lambda$$

3. Note that we can re-write the result from part (ii) in matrix form:

   $$\begin{bmatrix} S_{\text{display}} \\ L_{\text{display}} \end{bmatrix} = M \begin{bmatrix} R \\ B \end{bmatrix} = \begin{bmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{bmatrix} \begin{bmatrix} R \\ B \end{bmatrix} \tag{1}$$

   Write down expressions for the elements of this matrix:

   $m_{11} =$ $\qquad\qquad$ $m_{12} =$

$m_{21} =$ $m_{22} =$

Solution: Note that each element of the matrix is given by the integral of the product of the cone sensitivity and the pixel's SPD. That is,

$$m_{11} = \int r_S(\lambda) s_R(\lambda)\, d\lambda, \quad m_{12} = \int r_S(\lambda) s_B(\lambda)\, d\lambda,$$

$$m_{21} = \int r_L(\lambda) s_R(\lambda)\, d\lambda, \quad m_{22} = \int r_L(\lambda) s_B(\lambda)\, d\lambda.$$

Each matrix element represents how much a given pixel color (red or blue) stimulates a given cone type. For example, m_{11} captures how much the red pixel stimulates the S cone.

4. Finally, to complete the color matching procedure for the color-blind humans using only red and blue pixels, determine how to choose values for R and B so that the display light matches the perceived color of the input SPD $I(\lambda)$. Assume that $I(\lambda)$ is within the gamut of the red and blue pixels.

   Write your answer as a one-line matrix expression for R and B. You may use any variables defined in previous parts of this question and standard matrix operations (e.g., transpose, inverse).

   Hint: Match the target and display responses!

   $R =$

   $B =$

Solution: To match the target cone responses, we want:

$$\begin{bmatrix} S_{\text{display}} \\ L_{\text{display}} \end{bmatrix} = \begin{bmatrix} S_{\text{target}} \\ L_{\text{target}} \end{bmatrix}.$$

From earlier, we know that the display responses are given by:

$$\begin{bmatrix} S_{\text{display}} \\ L_{\text{display}} \end{bmatrix} = M \begin{bmatrix} R \\ B \end{bmatrix}.$$

So to match the responses, we solve:

$$M \begin{bmatrix} R \\ B \end{bmatrix} = \begin{bmatrix} S_{\text{target}} \\ L_{\text{target}} \end{bmatrix}, \quad \text{which gives} \quad \begin{bmatrix} R \\ B \end{bmatrix} = M^{-1} \begin{bmatrix} S_{\text{target}} \\ L_{\text{target}} \end{bmatrix}.$$

The variables R and B represent the brightness values (or intensity scalars) that you set for the red and blue pixels on the display.