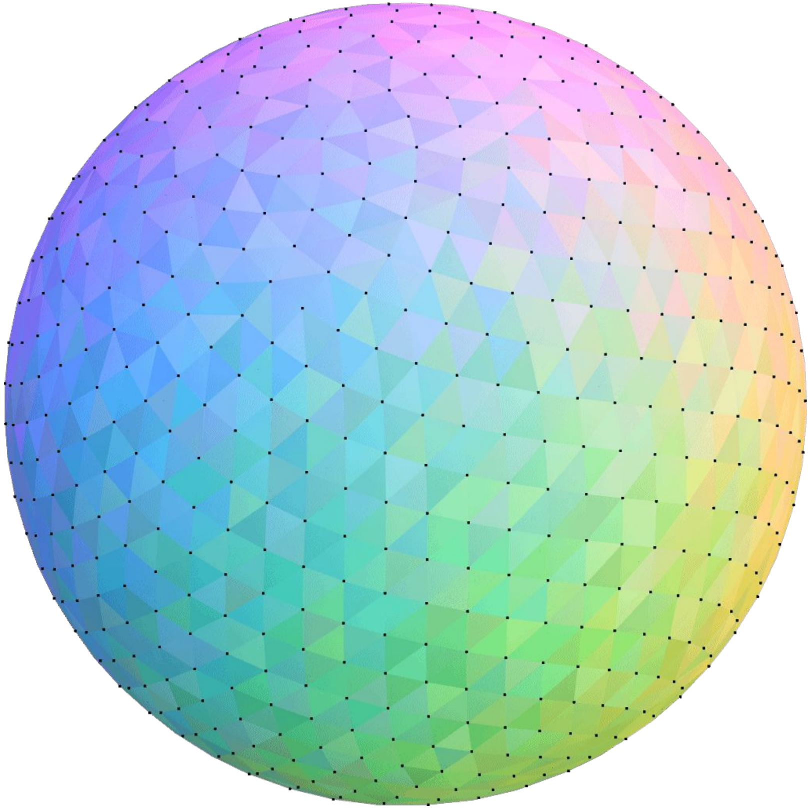
Lecture 2:

Digital Drawing

Computer Graphics and Imaging

UC Berkeley CS184

Drawing Triangles to the Screen by Sampling

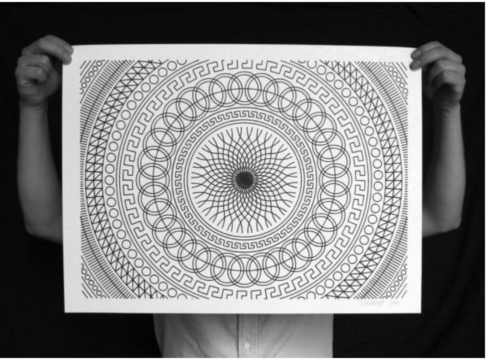


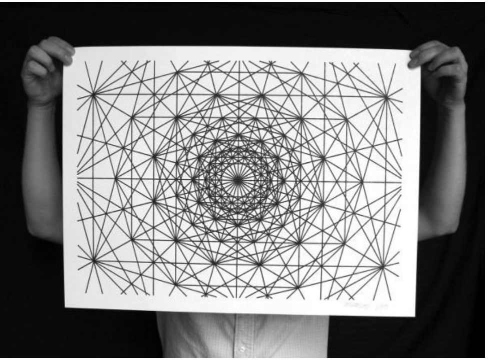
Tarek Elaydi

Drawing Methods

Drawing Machines

CNC Sharpie Drawing Machine





Aaron Panone with Matt W. Moore

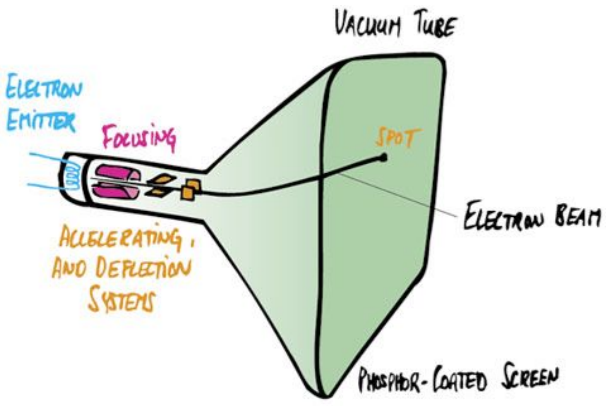
http://44rn.com/projects/numerically-controlled-poster-series-with-matt-w-moore/

Laser Cutters

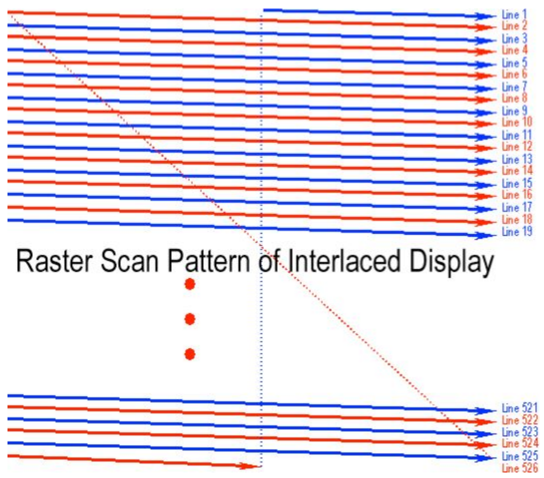
Drawing to a display


SCORE
3963
10 WAVE
SHIELD 9
TOWERS
19
800 POINTS NEXT TOWER
600

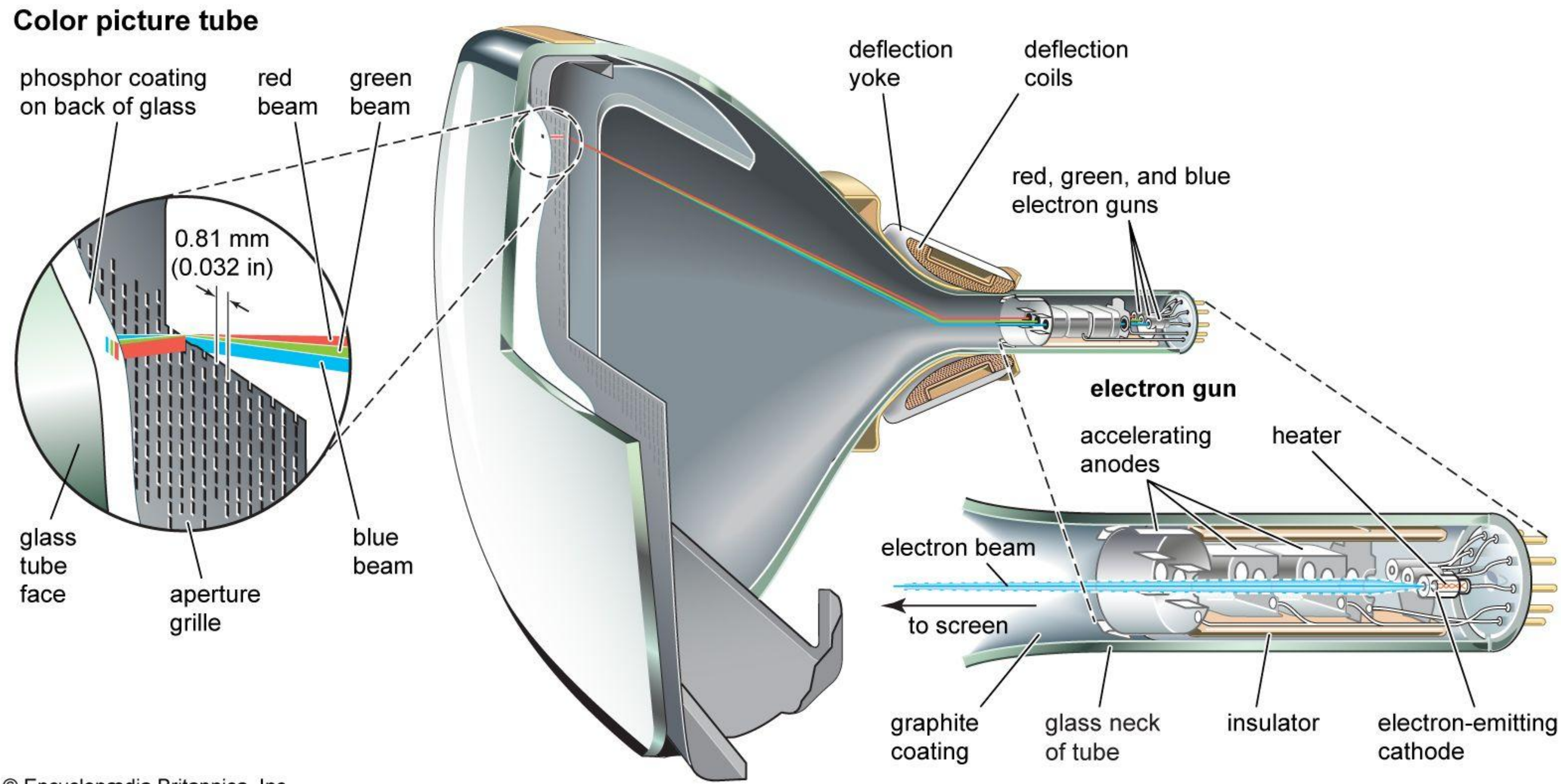
Television - Raster Display CRT



Cathode Ray Tube



Raster Scan
(modulate intensity)


Color picture tube
phosphor coating on back of glass
red beam
green beam
0.81 mm (0.032 in)
glass tube face
aperture grille
blue beam
deflection yoke
deflection coils
red, green, and blue electron guns
electron gun
accelerating anodes
heater
electron beam
to screen
graphite coating
glass neck of tube
insulator
electron-emitting cathode

Different Raster Displays

LED Array Display

Light emitting diode array

LED Array Display

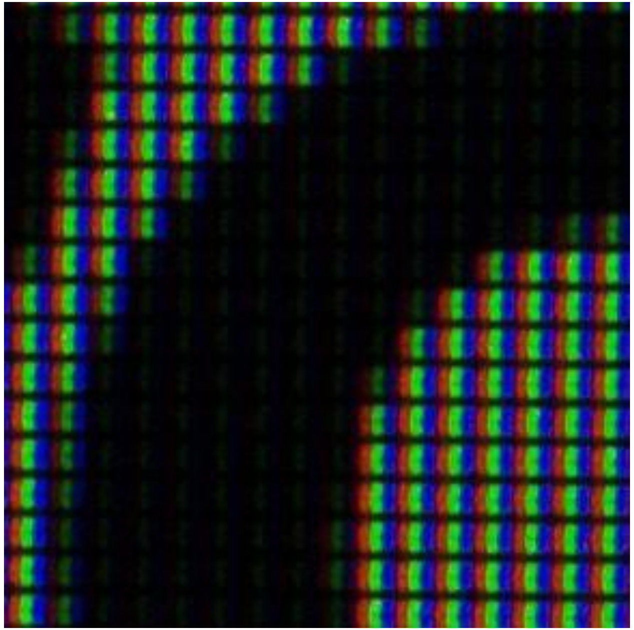
BAMPFA display in Berkeley

Flat Panel Displays

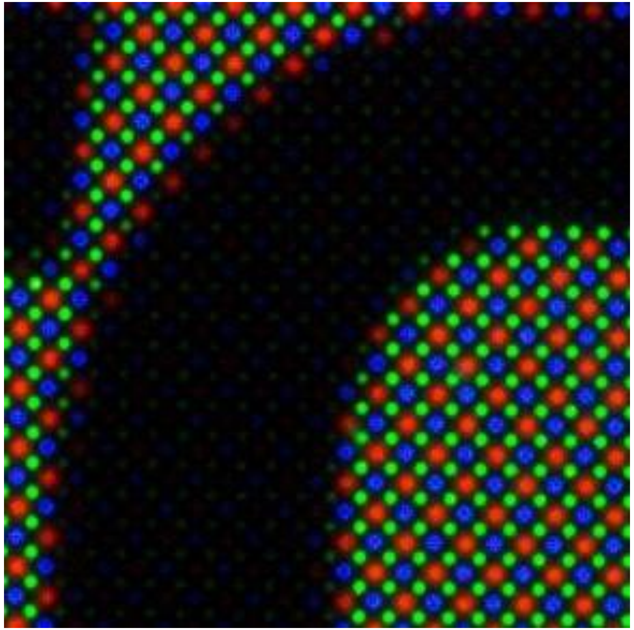
Low-Res LCD Display

Color LCD, OLED, …

Flat Panel Displays



iPhone 6S

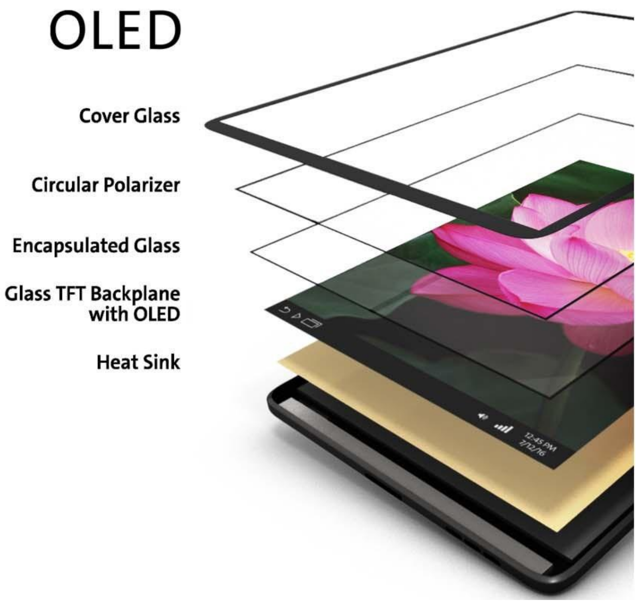


Galaxy S5

Smartphone screen pixels under microscope

LCD vs OLED Displays

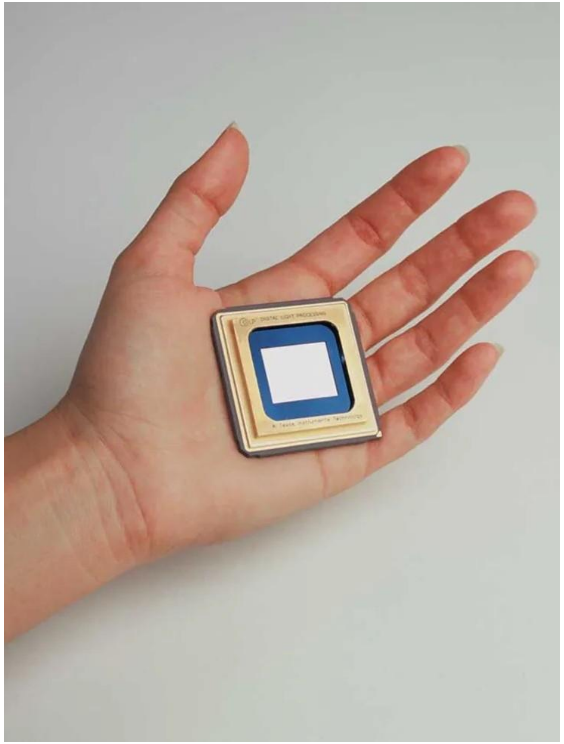
Liquid Crystal Display



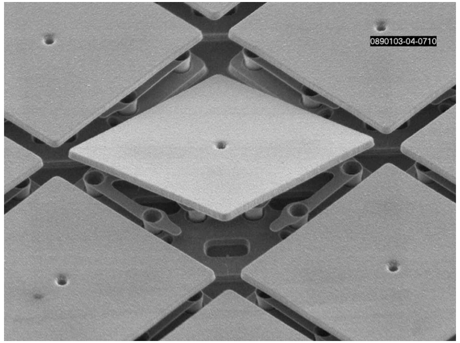
Organic Light Emitting Diode Display

`LCD pixels filter (block) light from uniform backlight; OLED pixels emit light`

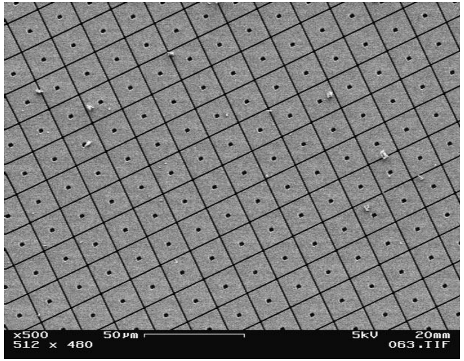
Digital Micromirror Device (DMD/DLP)



Texas Instruments

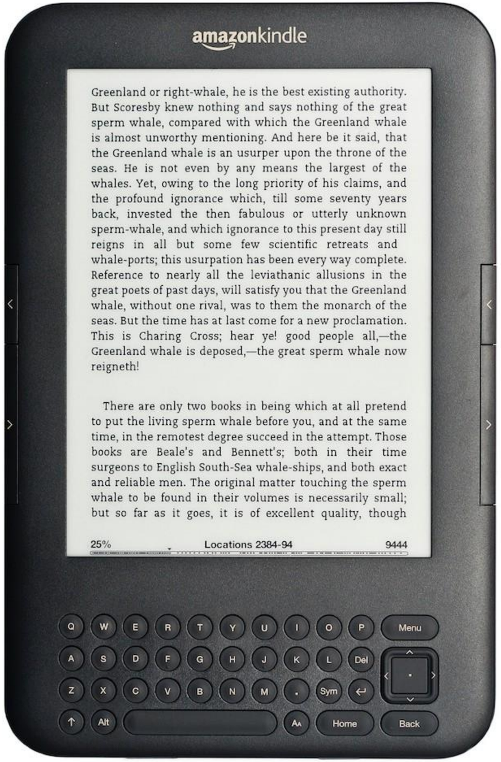


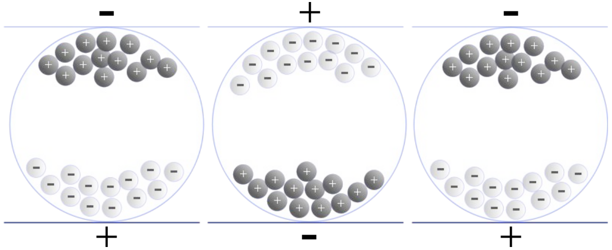
Larry Hornbeck



John Jackson, University of Rochester

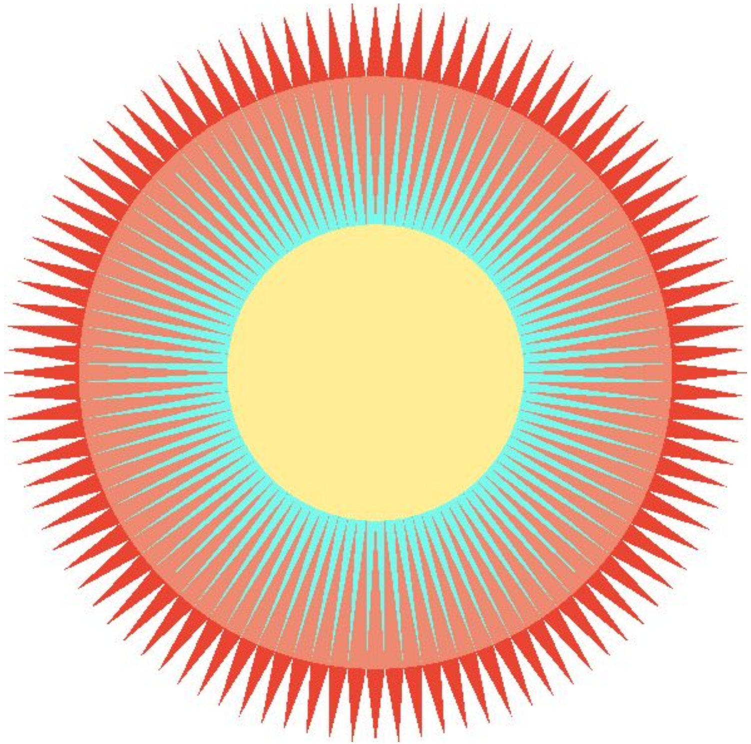
Electrophoretic (Electronic Ink) Display





Drawing to Raster Displays

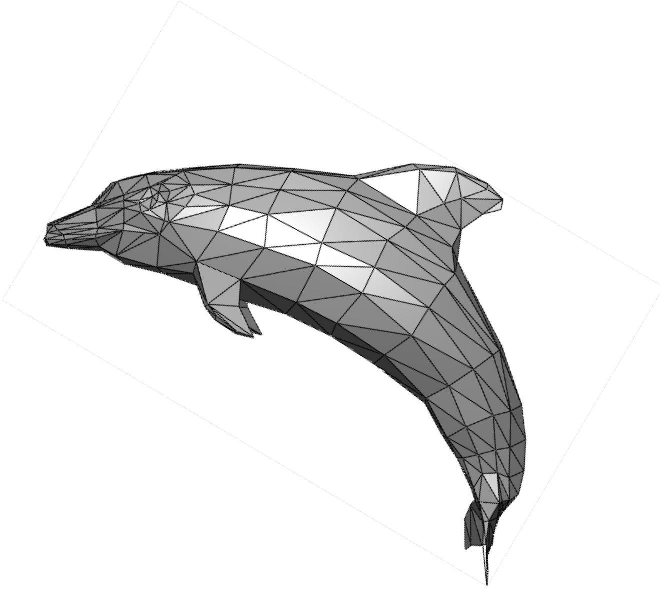
Triangle Meshes



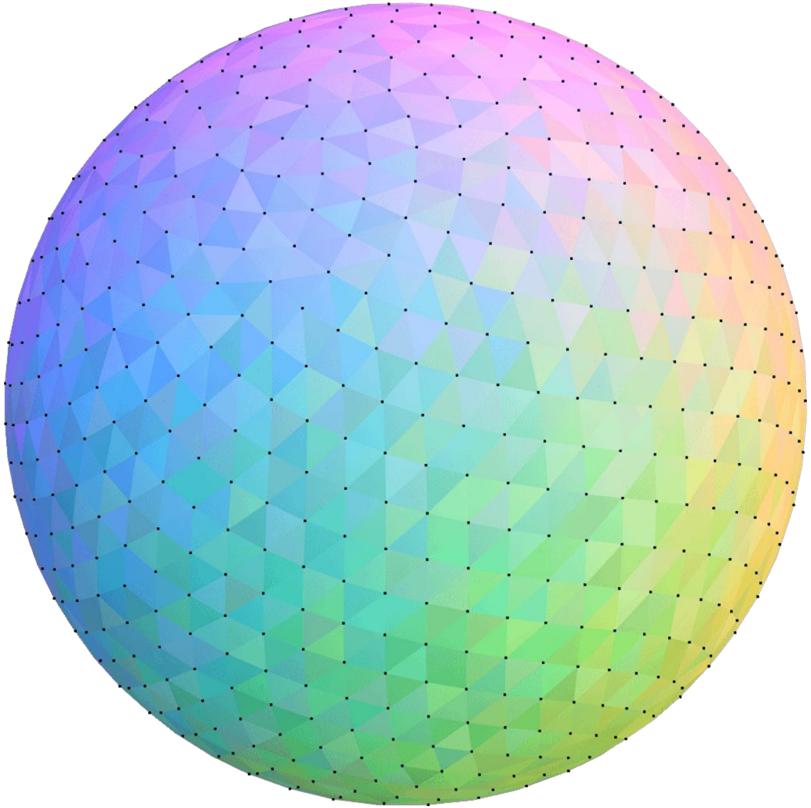
Triangles - Fundamental Area Primitive

Why triangles?

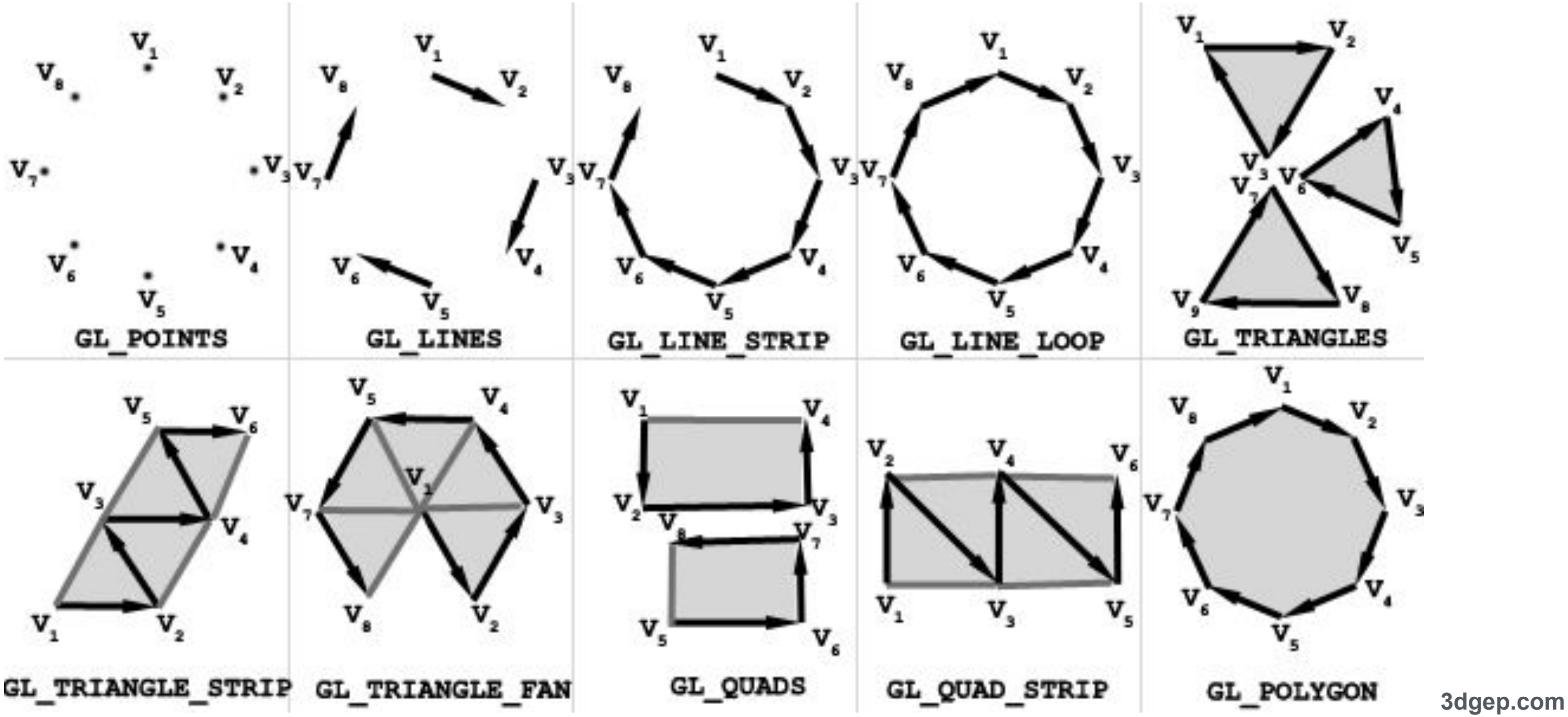
- Most basic polygon
  - Break up other polygons
  - Optimize one implementation
- Triangles have unique properties
  - Guaranteed to be planar
  - Well-defined interior
  - Well-defined method for interpolating values at vertices over triangle (barycentric interpolation)



Triangles Meshes



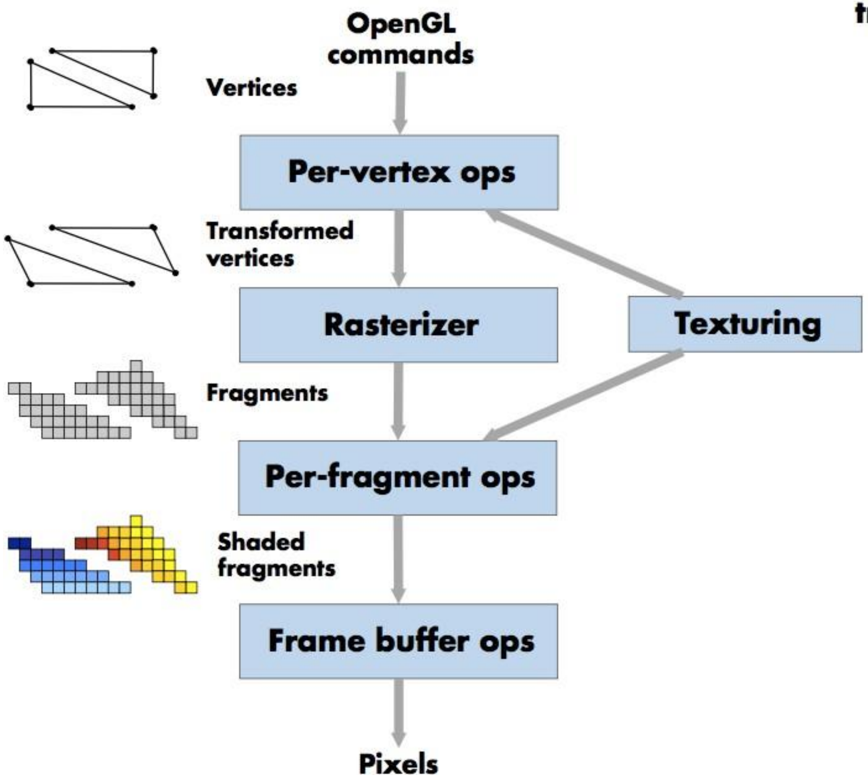
Shape Primitives

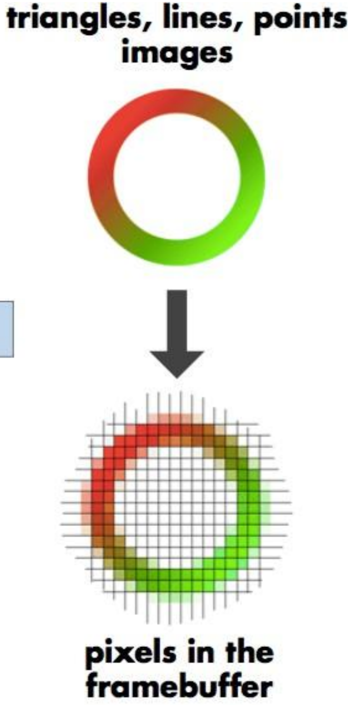


Example shape primitives (OpenGL)

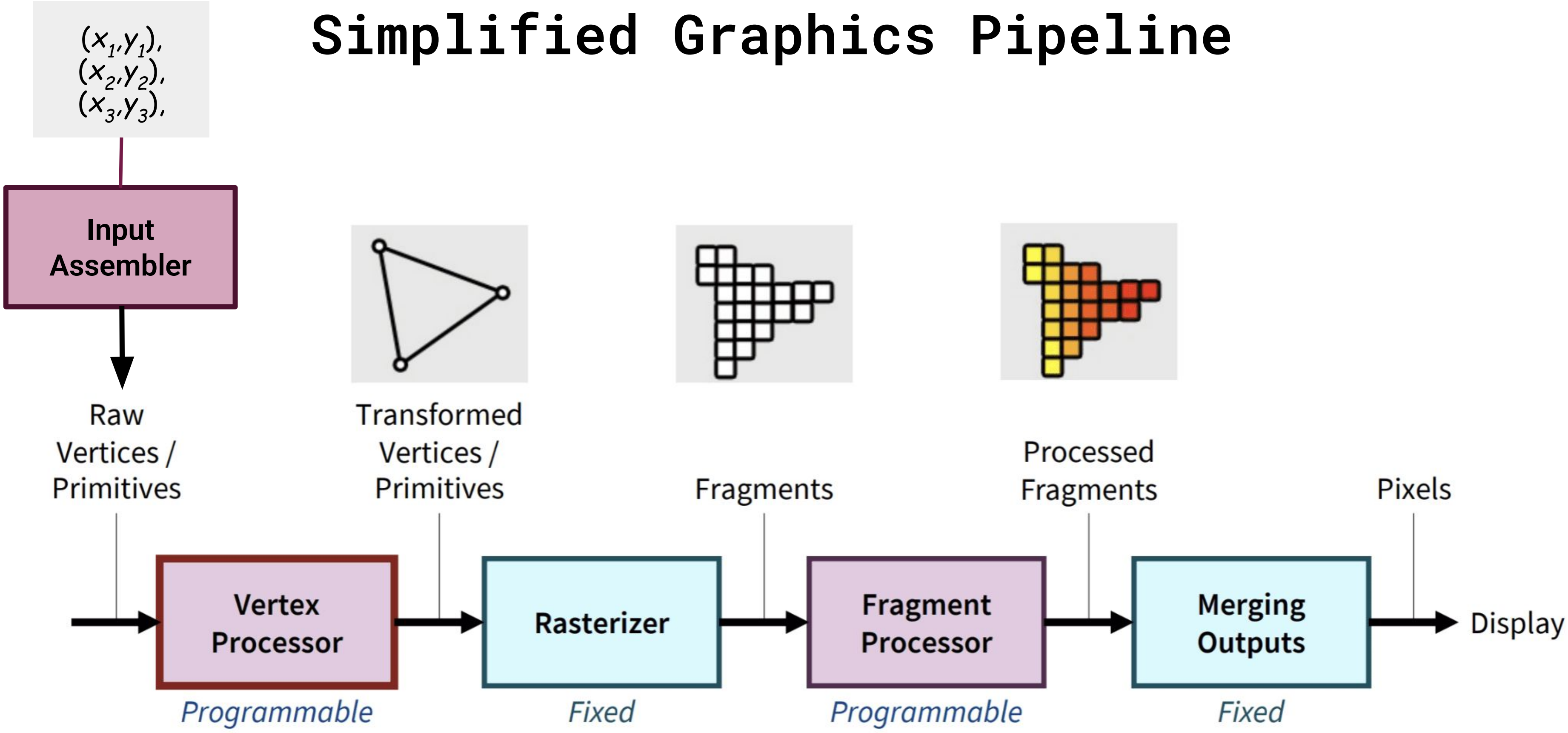
Polygon Meshes

Graphics Pipeline = Abstract Drawing Machine





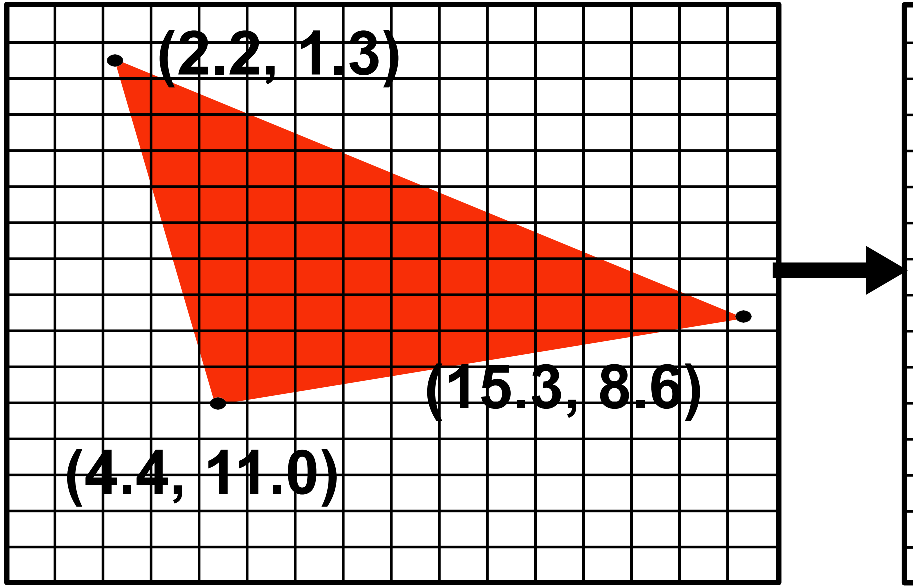
Simplified Graphics Pipeline



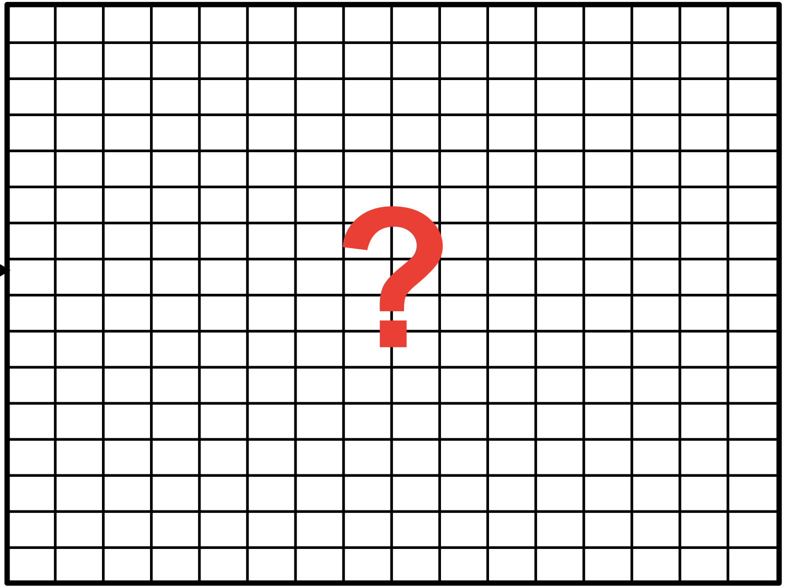
Drawing a Triangle To The Framebuffer

“Rasterization”

What Pixel Values Approximate a Triangle?



Input: position of triangle vertices projected on screen



Output: set of pixel values approximating triangle

Today, Let's Start With A Simple Approach: Sampling

Sampling a Function

Evaluating a function at a point is sampling.

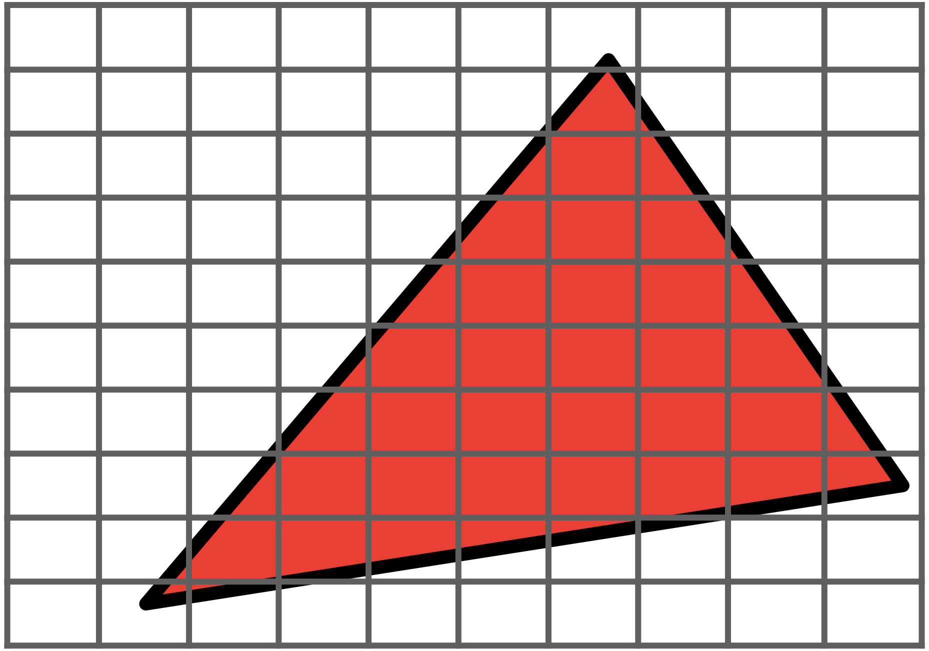
We can discretize a function by periodic sampling.

```
for( int x = 0;    x < xmax;   x++)
    output[x]    = f(x);
```

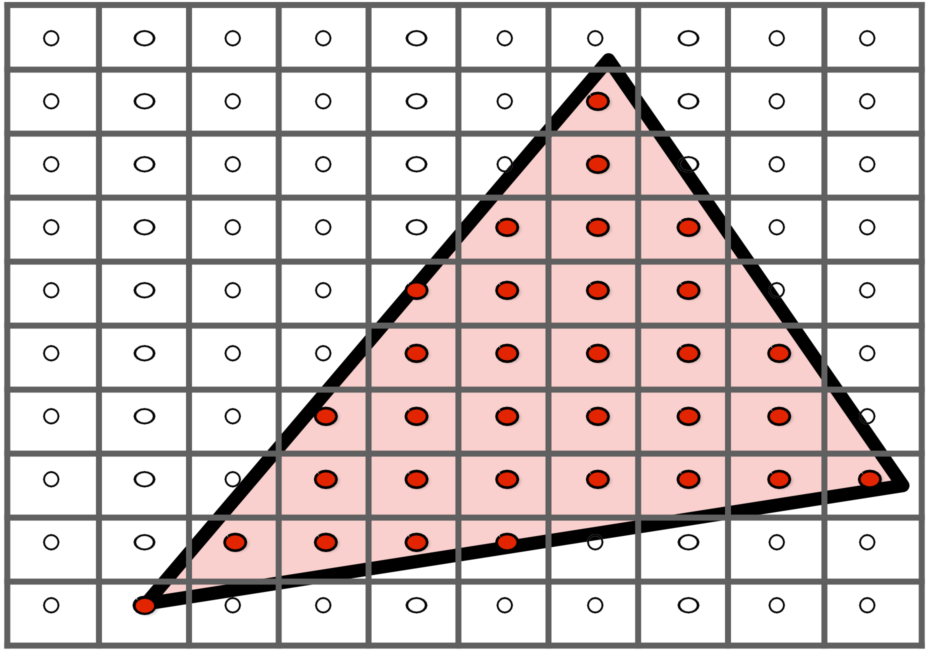
Sampling is a core idea in graphics. We'll sample time (1D), area (2D), angle (2D), volume (3D) …

We'll sample N-dimensional functions, even infinite dimensional functions.

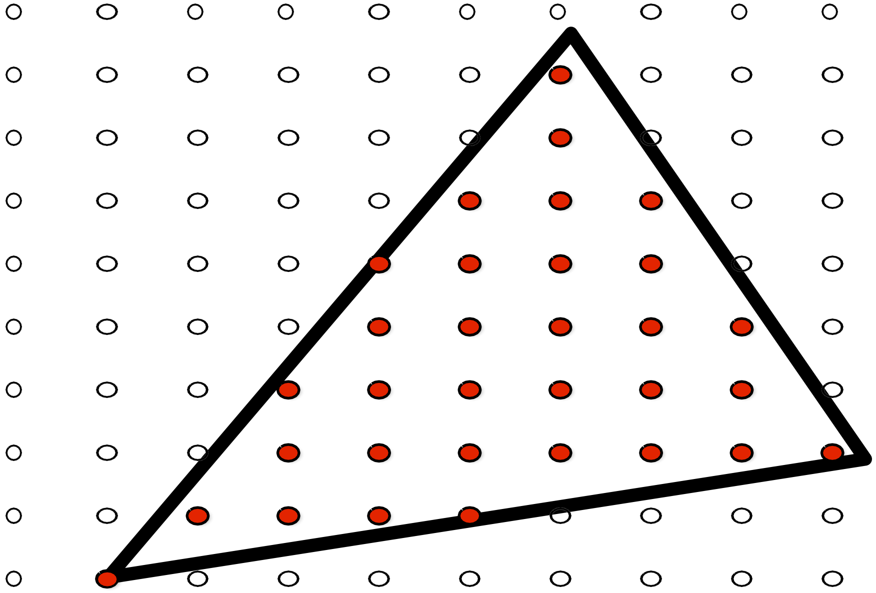
Let’s Try Rasterization As 2D Sampling



Sample If Each Pixel Center Is Inside Triangle

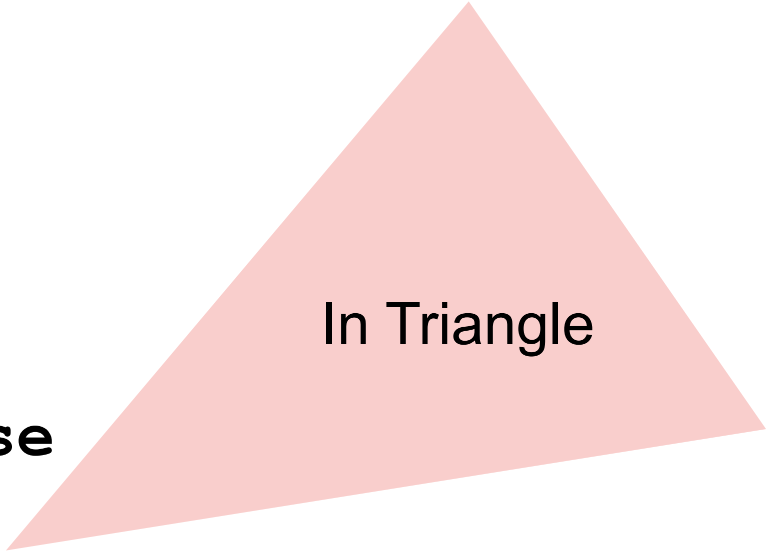


Sample If Each Pixel Center Is Inside Triangle



Define Binary Function: `inside(tri,x,y)`

$$\texttt{inside(t,x,y)} = \begin{cases} \texttt{1} & \texttt{(x,y)} \\ \texttt{0} & \texttt{otherwise} \end{cases}$$



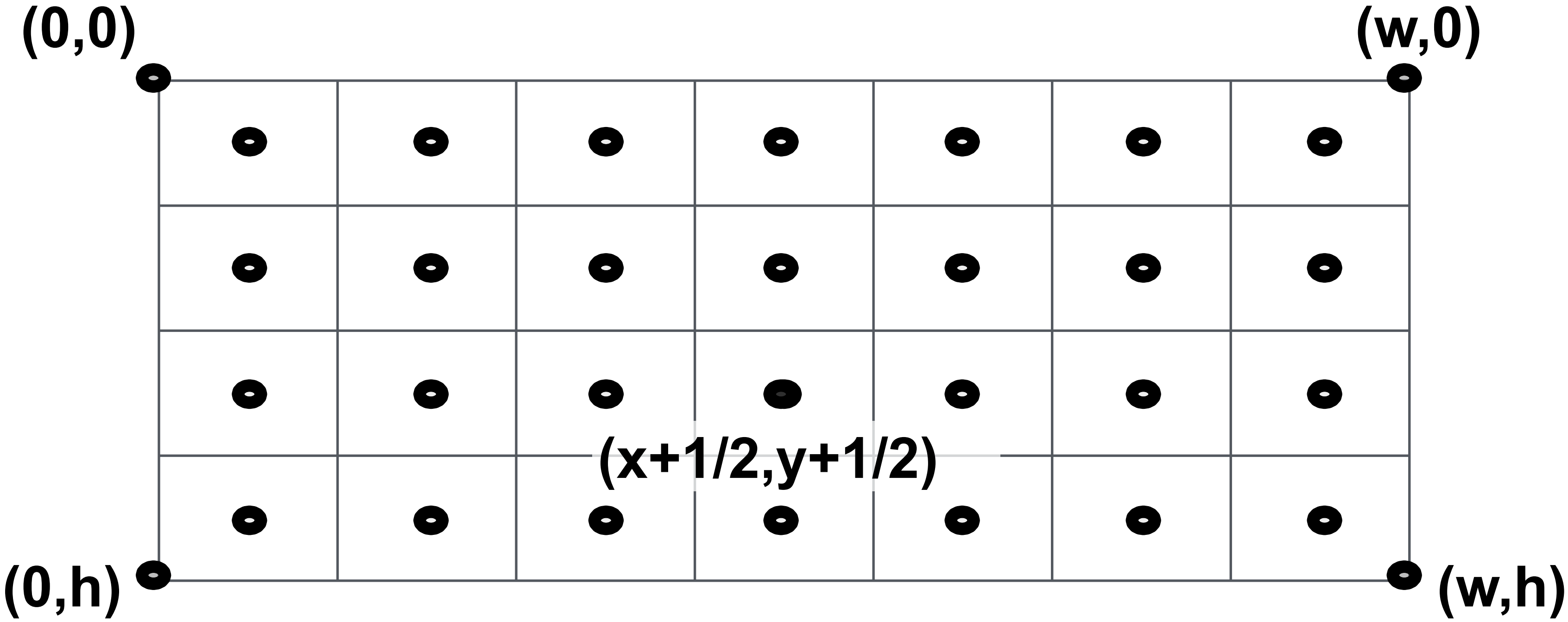
Rasterization = Sampling A 2D Indicator Function

```
for(  int  x  = 0;   x < xmax;   x++  )
    for( int   y =   0; y <  ymax; y++ )
     Image[x][y]   =  f(x + 0.5,  y + 0.5);
```

Rasterize triangle `tri` by sampling the function

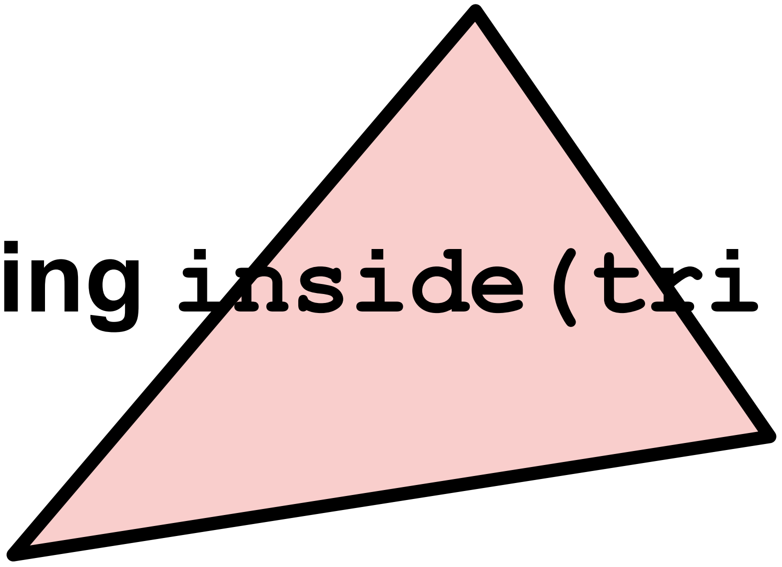
```
f(x,y)  =  inside(tri,x,y)
```

Implementation Detail: Sample Locations

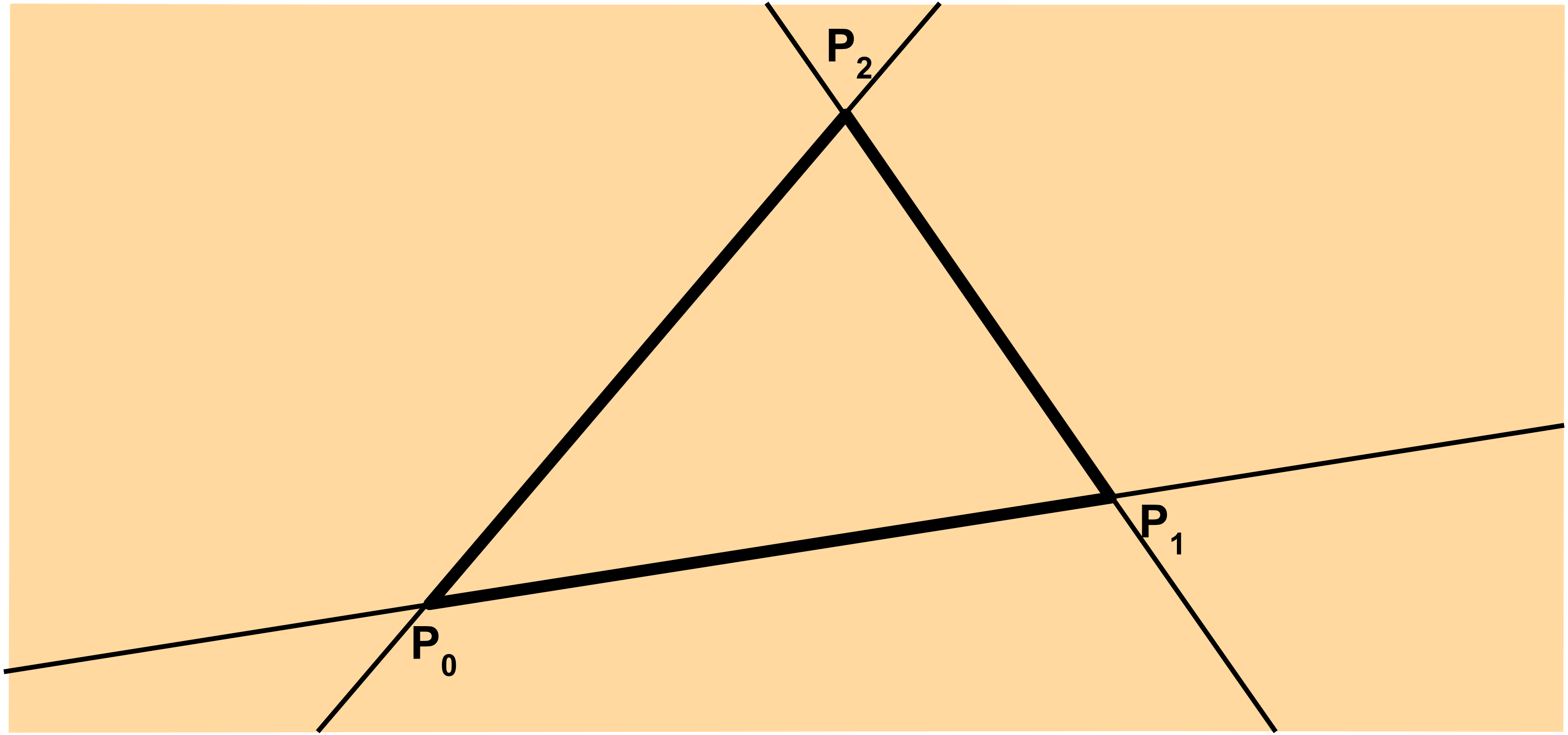


Sample location for pixel (x,y)

Evaluating `inside(tri,x,y)`



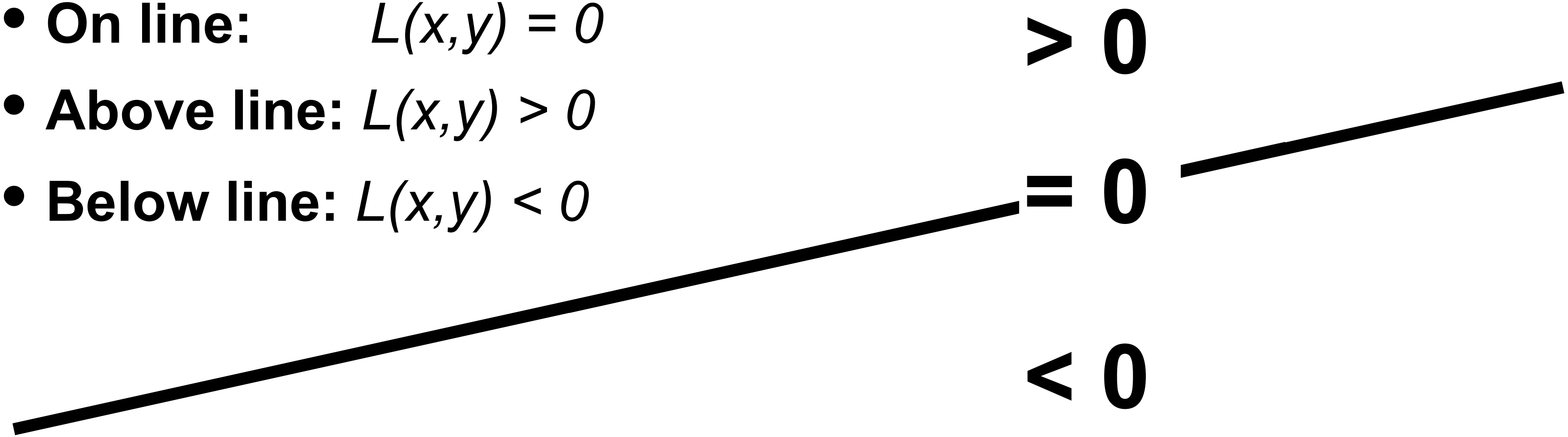
Triangle = Intersection of Three Half Planes



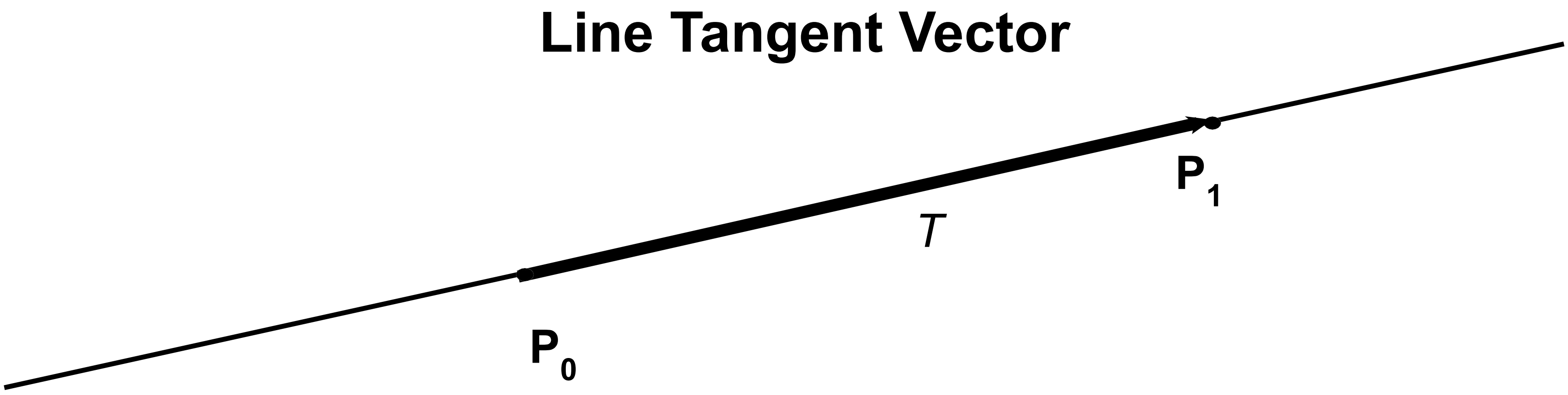
Each Line Defines Two Half-Planes

Implicit line equation

- $L(x,y) = Ax + By + C$
- **On line:** $L(x,y) = 0$
- **Above line:** $L(x,y) > 0$
- **Below line:** $L(x,y) < 0$

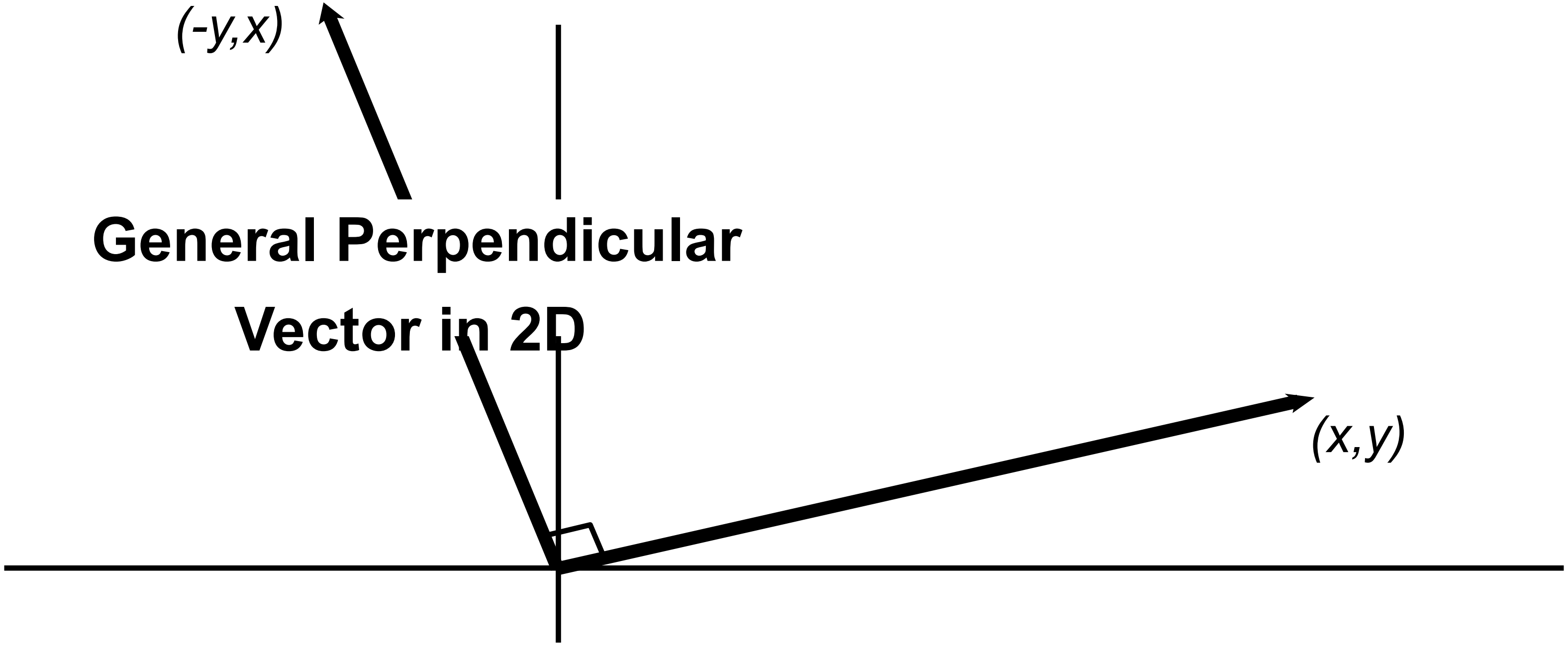


Line Equation Derivation



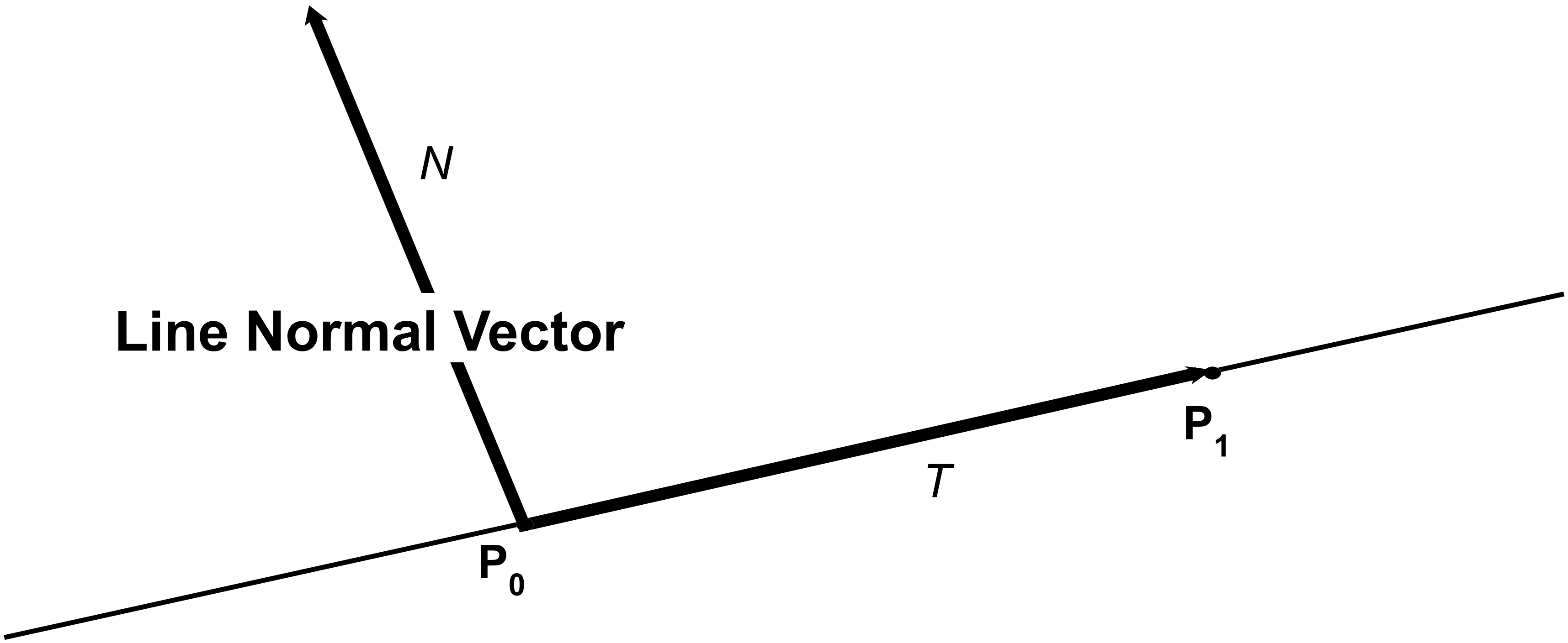
$$T = P_1 - P_0 = (x_1 - x_0, y_1 - y_0)$$

Line Equation Derivation



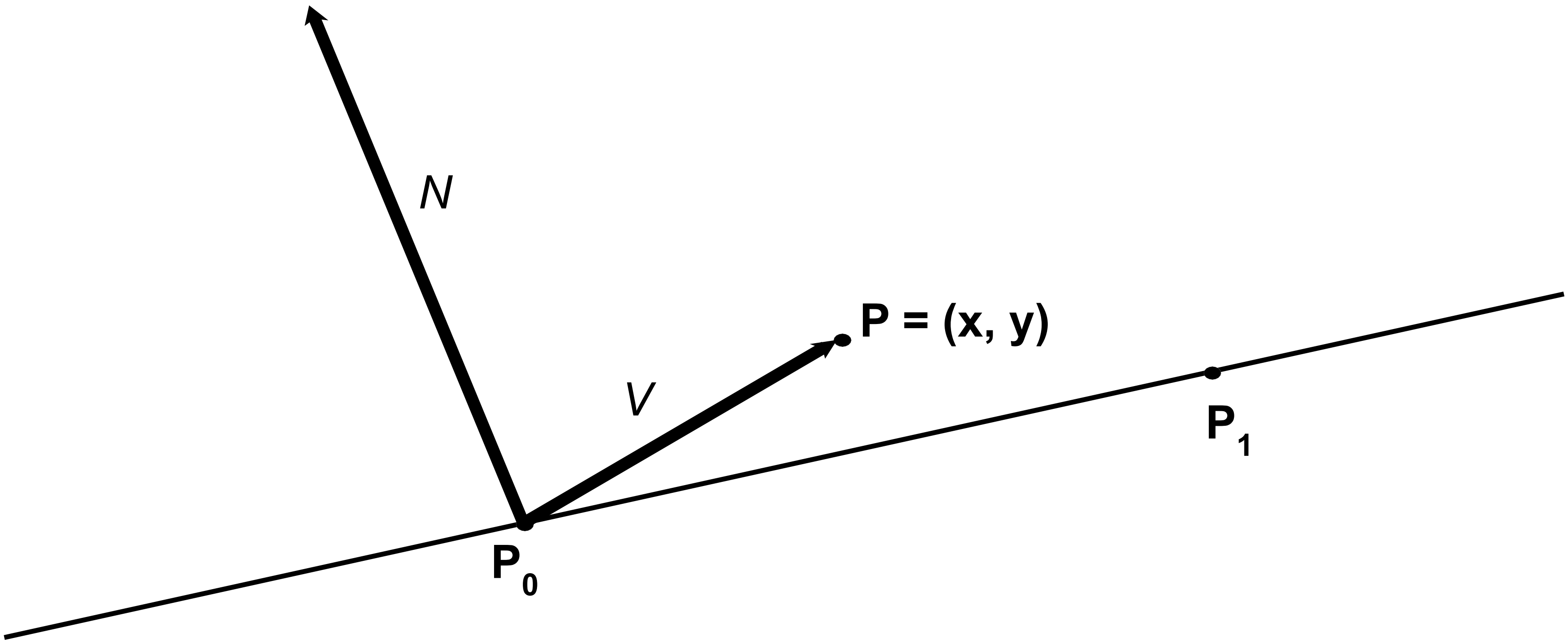
$$\text{Perp}(x, y) = (-y, x)$$

Line Equation Derivation



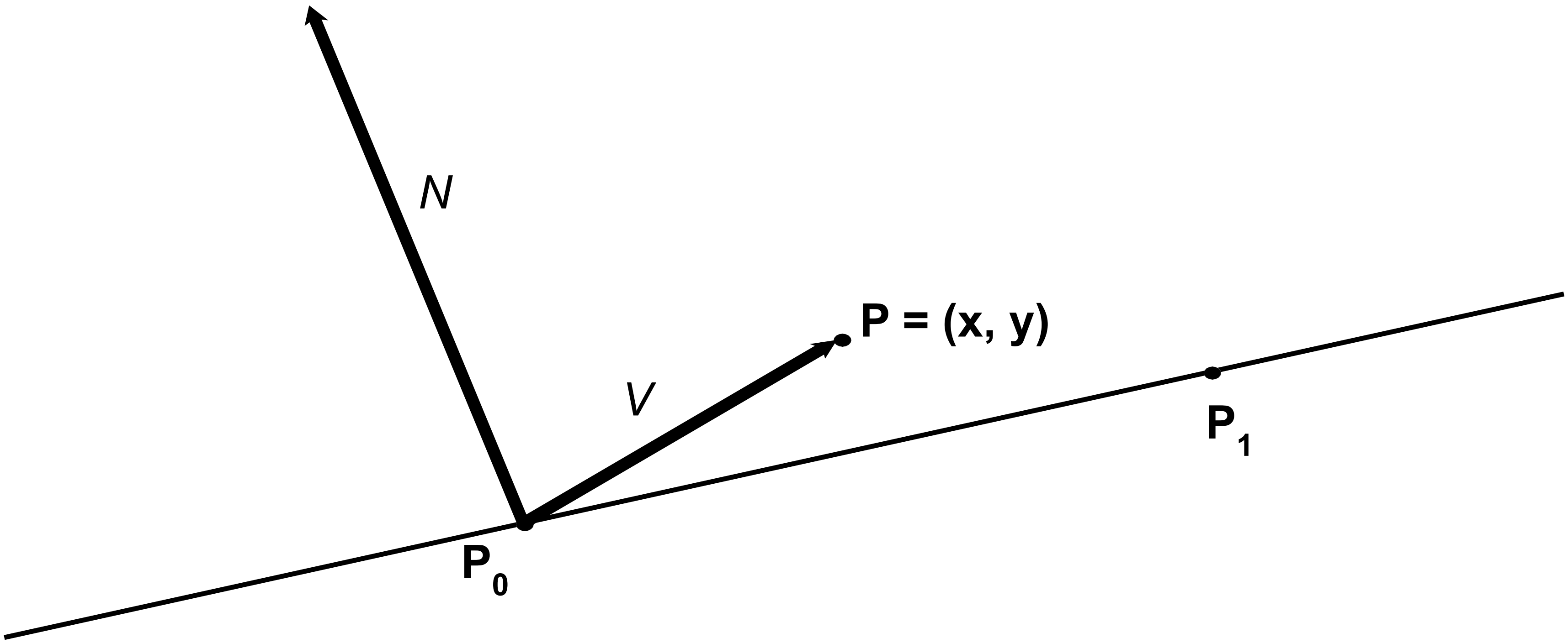
$$N = \text{Perp}(T) = (-(y_1 - y_0), x_1 - x_0)$$

Line Equation Derivation



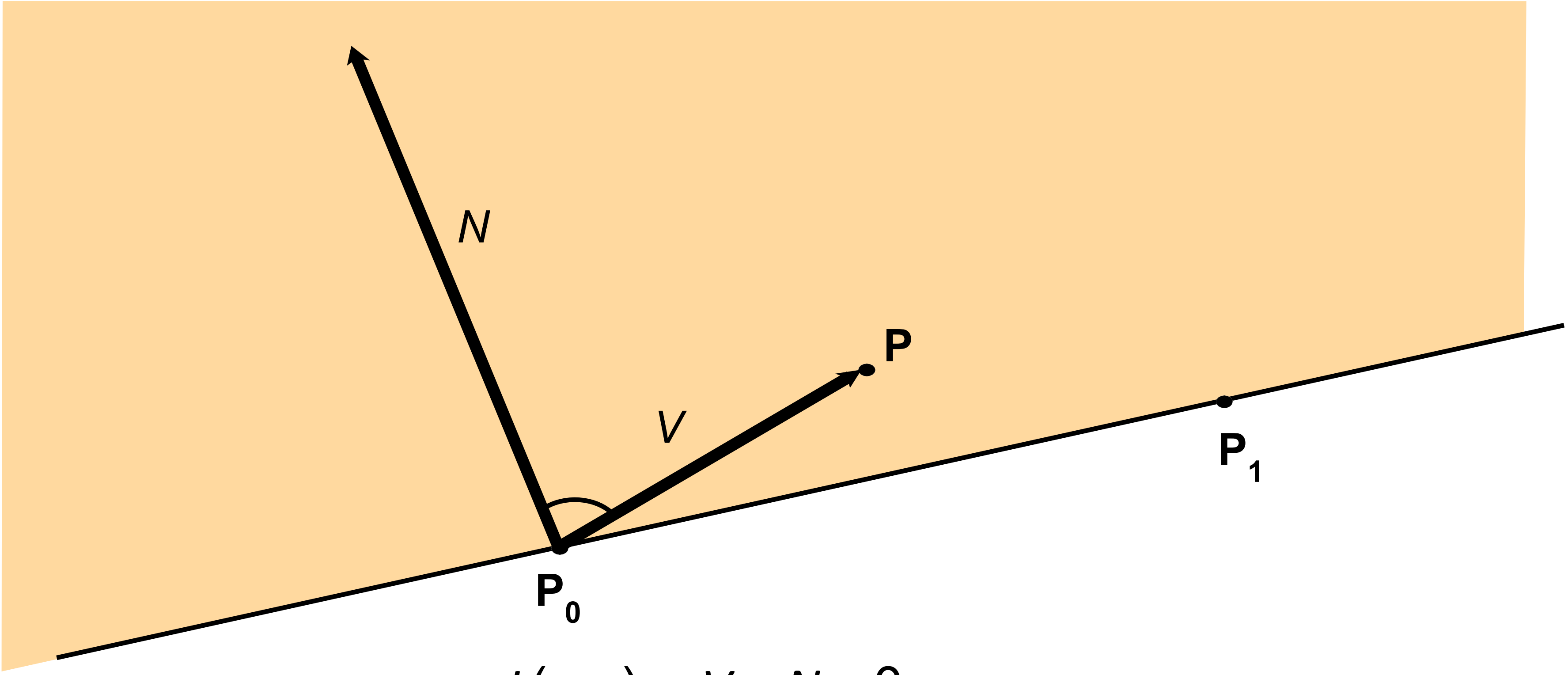
$$V = P - P_0 = (x - x_0, y - y_0)$$

Line Equation



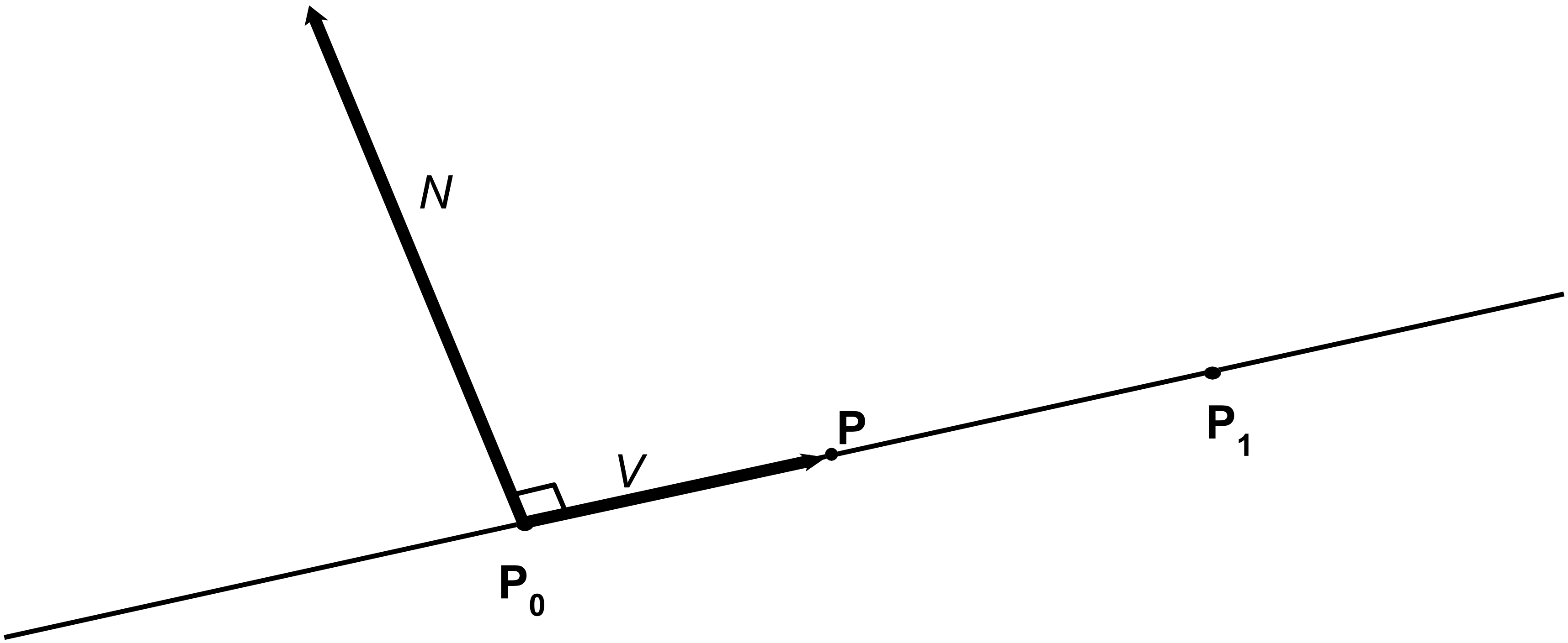
$$L(x, y) = V \cdot N = -(x - x_0)(y_1 - y_0) + (y - y_0)(x_1 - x_0)$$

Line Equation Tests



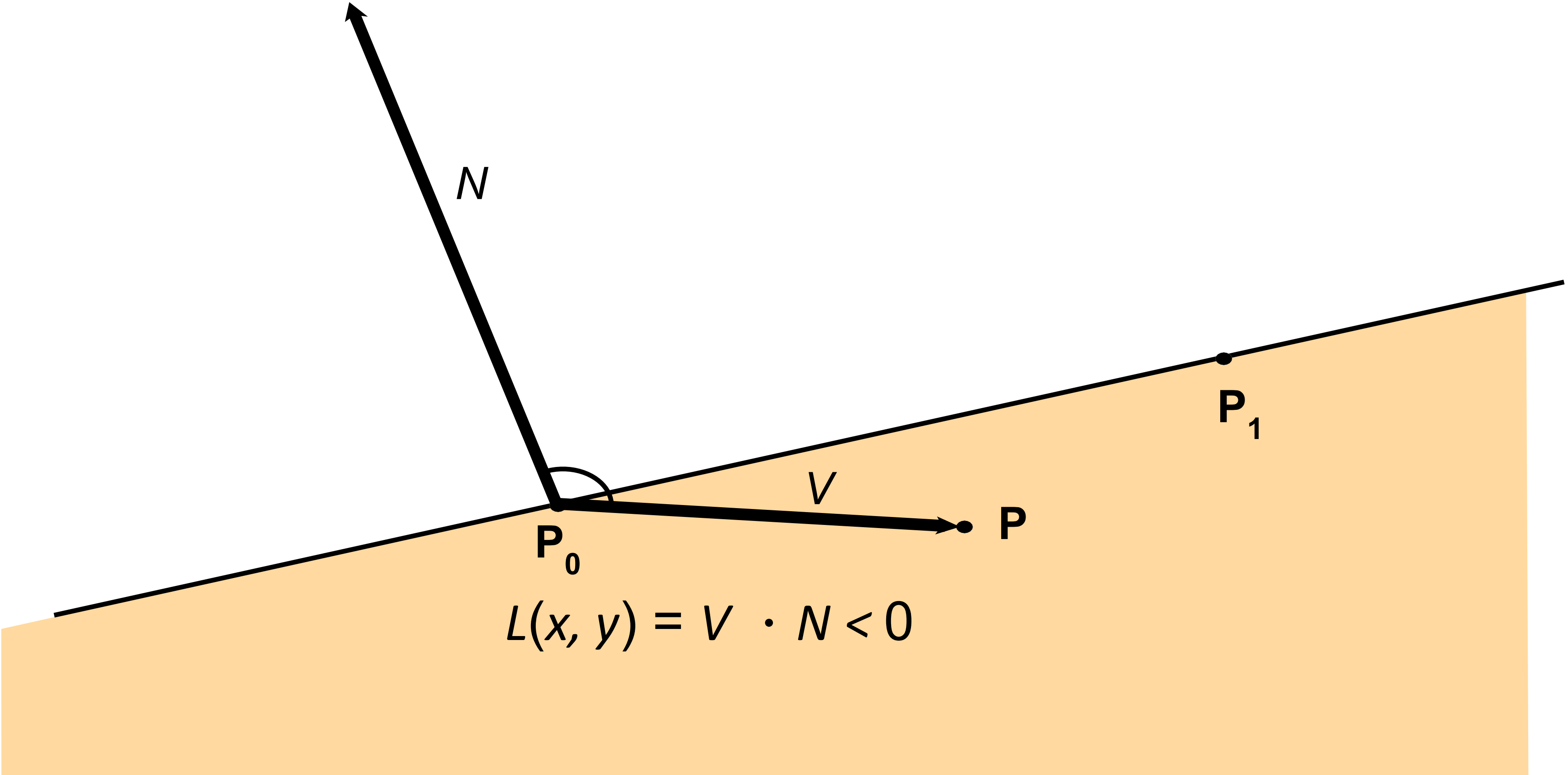
$$L(x, y) = V \cdot N > 0$$

Line Equation Tests



$$L(x, y) = V \cdot N = 0$$

Line Equation Tests



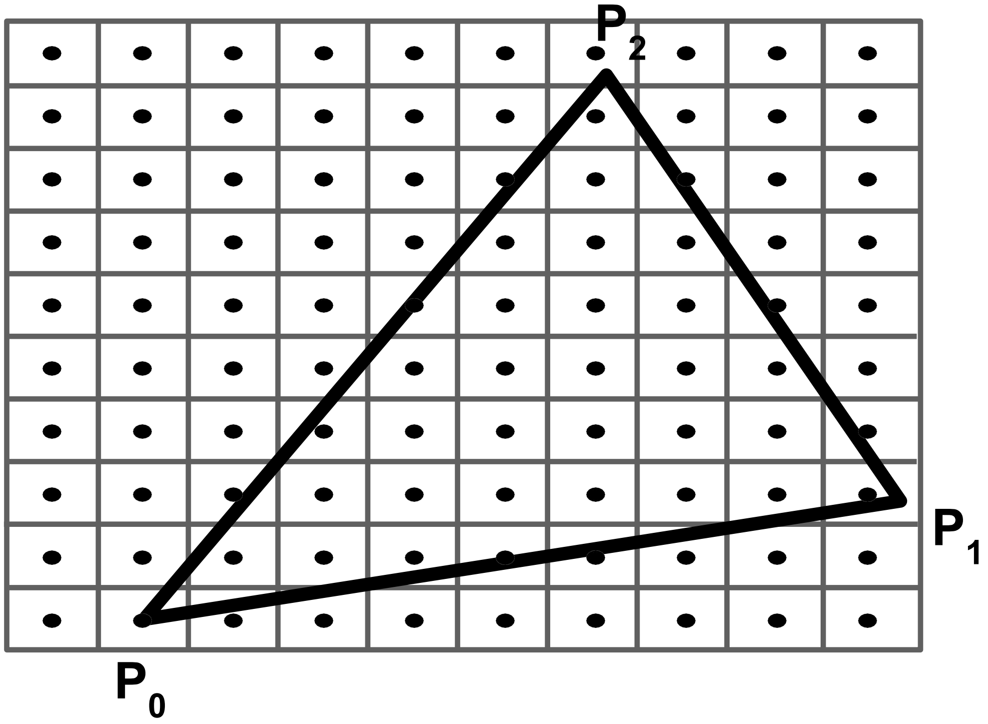
$$L(x, y) = V \cdot N < 0$$

Point-in-Triangle Test: Three Line Tests

$P_i = (X_{i,} Y_i)$

$dX_i = X_{i+1} - X_i \, dY_i$
$= Y_{i+1} - Y_i$
$L_i(x, y) \quad = -(x - X_i)\, dY_i \quad + (y - Y_i)\, dX_i$
$= A_i x + B_i y + C_i$

$L_i(x, y) = 0$: point on edge
< 0 : outside edge
> 0 : inside edge



Compute line equations from pairs of vertices

Point-in-Triangle Test: Three Line Tests

$P_i = (X_i, Y_i)$

$dX_i = X_{i+1} - X_i \; dY_i$
$= Y_{i+1} - Y_i$

$L_i(x, y) \quad = -(x - X_i)\, dY_i \;\; + (y - Y_i)\, dX_i$
$= A_i x + B_i y + C_i$

$L_i(x, y) = 0$: point on edge
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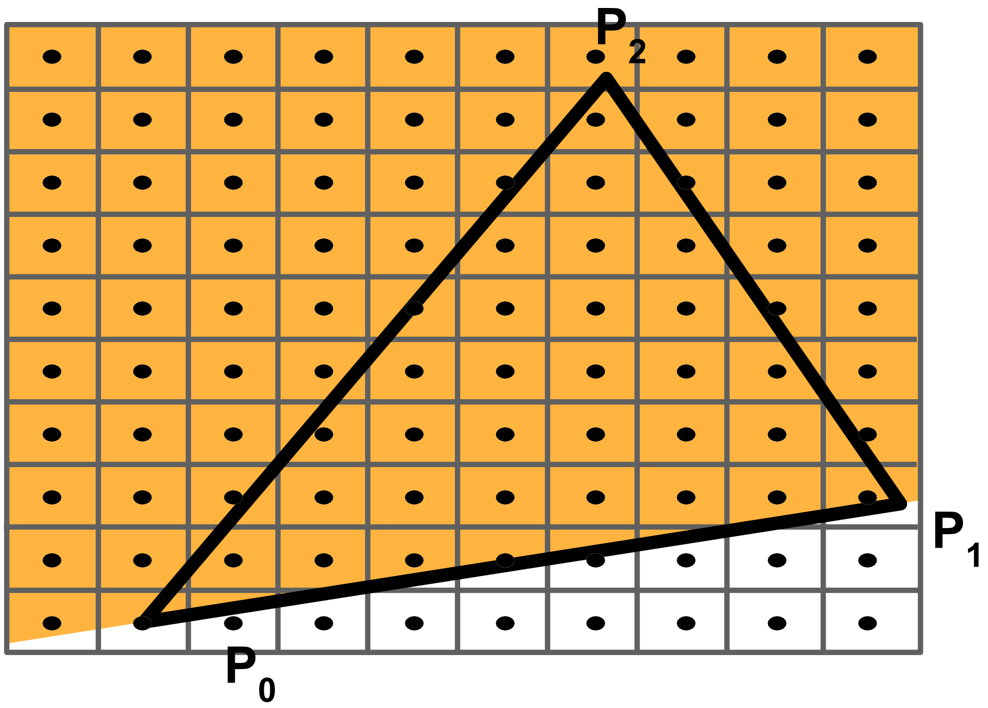


$L_0(x, y) > 0$

Point-in-Triangle Test: Three Line Tests

$P_i = (X_i, Y_i)$

$dX_i = X_{i+1} - X_i \; dY_i$

$= Y_{i+1} - Y_i$

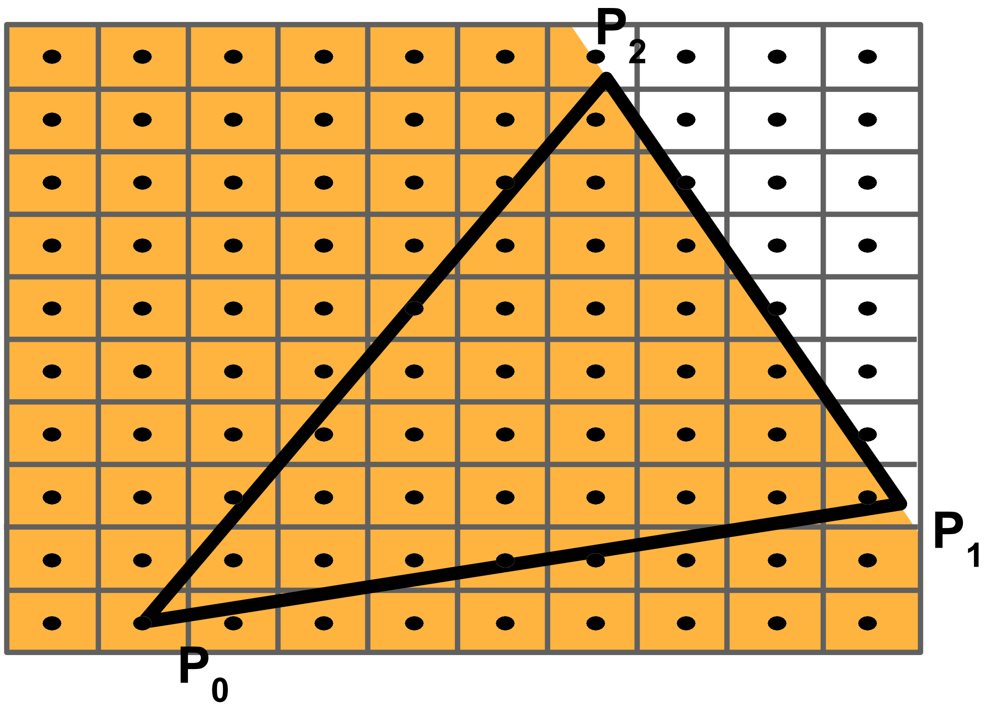
$L_i(x, y) = -(x - X_i)\, dY_i + (y - Y_i)\, dX_i$

$= A_i x + B_i y + C_i$

$L_i(x, y) = 0$: point on edge

< 0 : outside edge

> 0 : inside edge



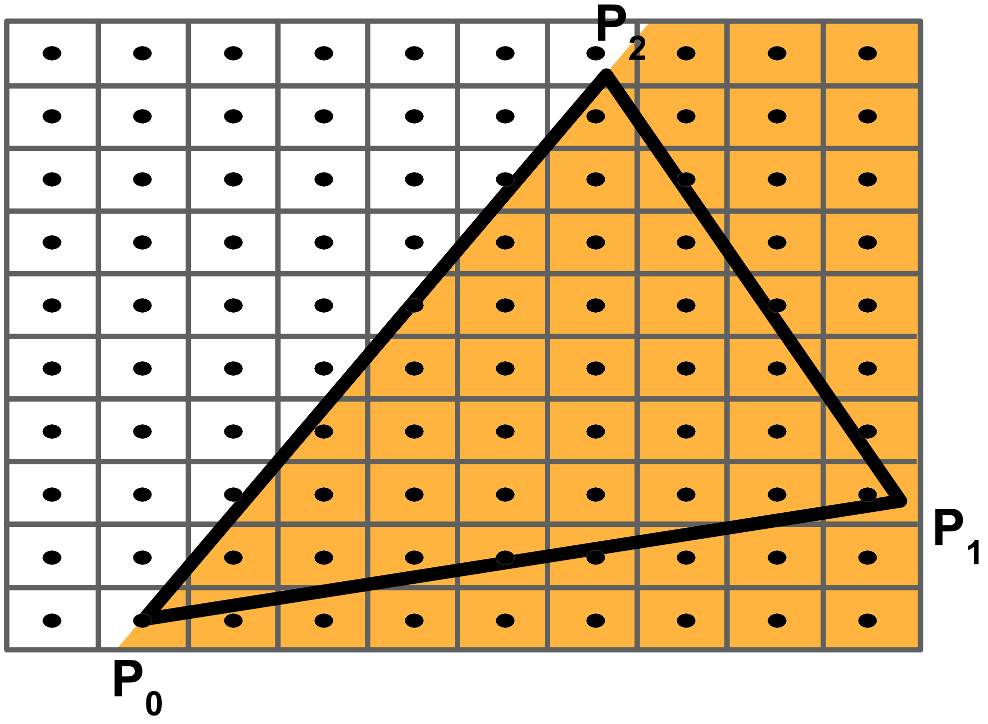
$L_1(x, y) > 0$

Point-in-Triangle Test: Three Line Tests

$P_i = (X_{i,} Y_i)$

$dX_i = X_{i+1} - X_i dY_i$
$= Y_{i+1} - Y_i$
$L_i(x, y) \quad = -(x - X_i)\, dY_i \quad + (y - Y_i)\, dX_i$
$= A_i x + B_i y + C_i$

$L_i(x, y) = 0$: point on edge
< 0 : outside edge
> 0 : inside edge



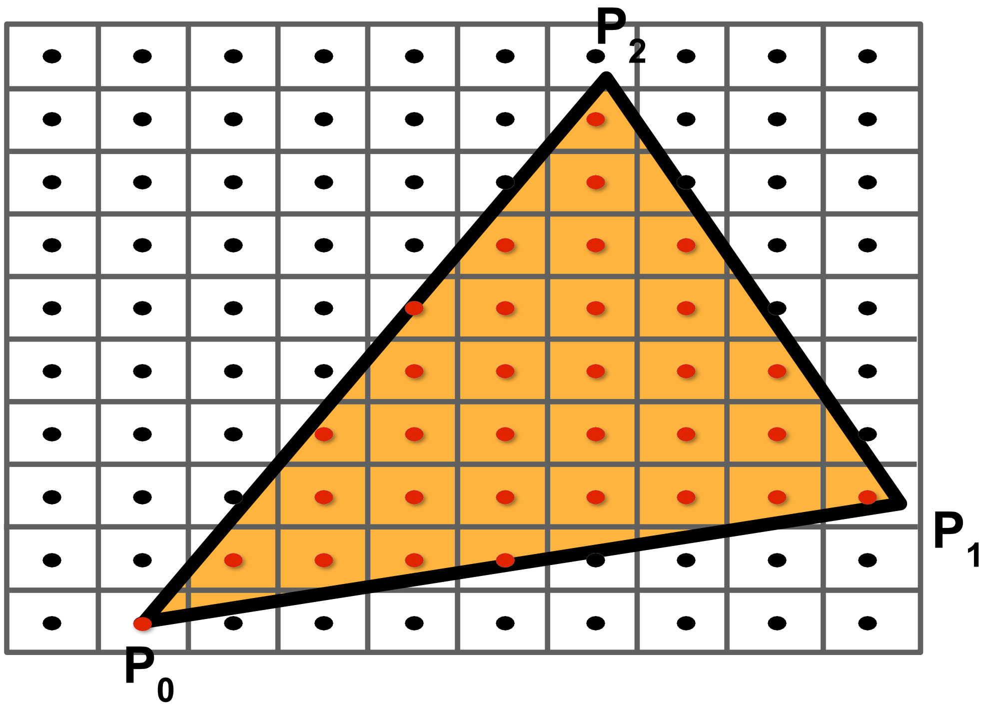
$L_2(x, y) > 0$

Point-in-Triangle Test: Three Line Tests

Sample point $s = (sx, sy)$ **is inside the triangle if it is inside all three lines.**

$$
\begin{aligned}
&inside(sx, sy) = \\
&\quad L_0(sx, sy) > 0 \;\&\& \\
&\quad L_1(sx, sy) > 0 \;\&\& \\
&\quad L_2(sx, sy) > 0;
\end{aligned}
$$

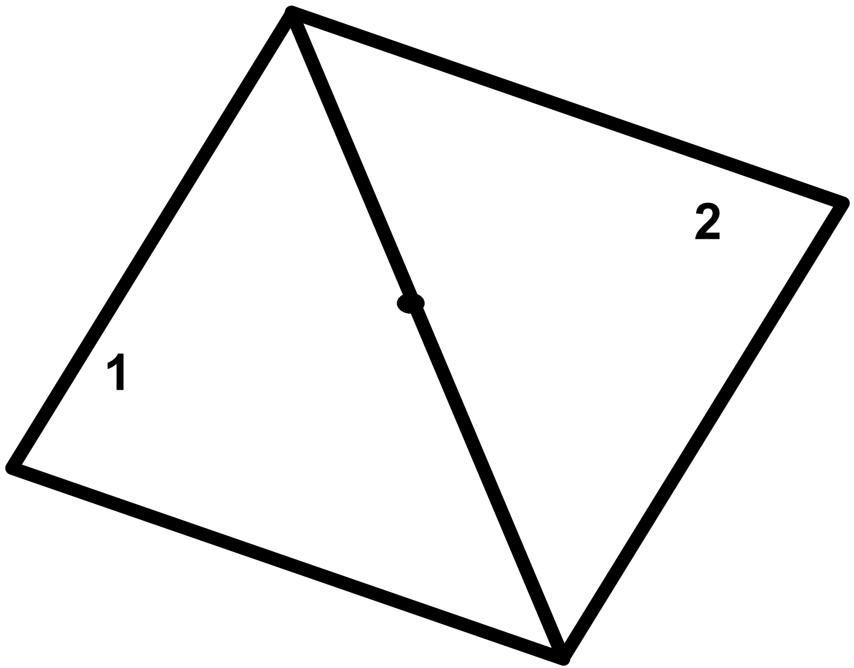
Note: actual implementation of $inside(sx,sy)$ **involves ≤ checks based on edge rules**



Some Details

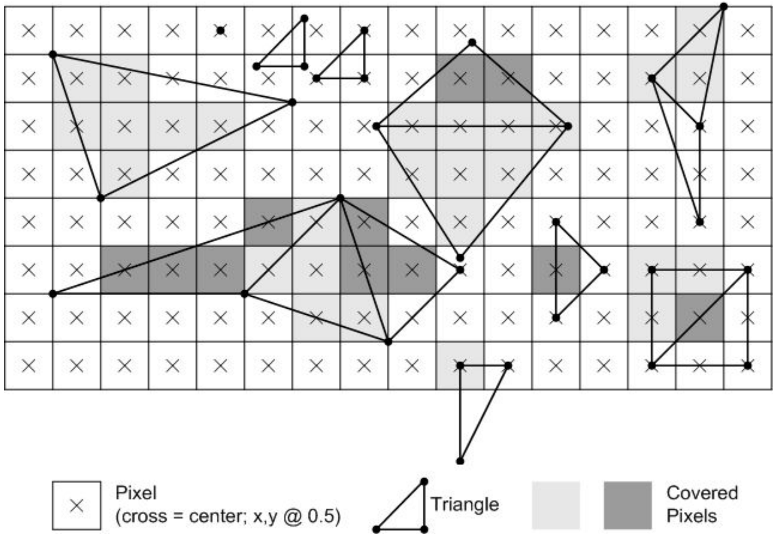
Edge Cases (Literally)

Is this sample point covered by triangle 1, triangle 2, or both?



OpenGL/Direct3D Edge Rules

When sample point falls on an edge, the sample is classified as within triangle if the edge is a "top edge" or "left edge"

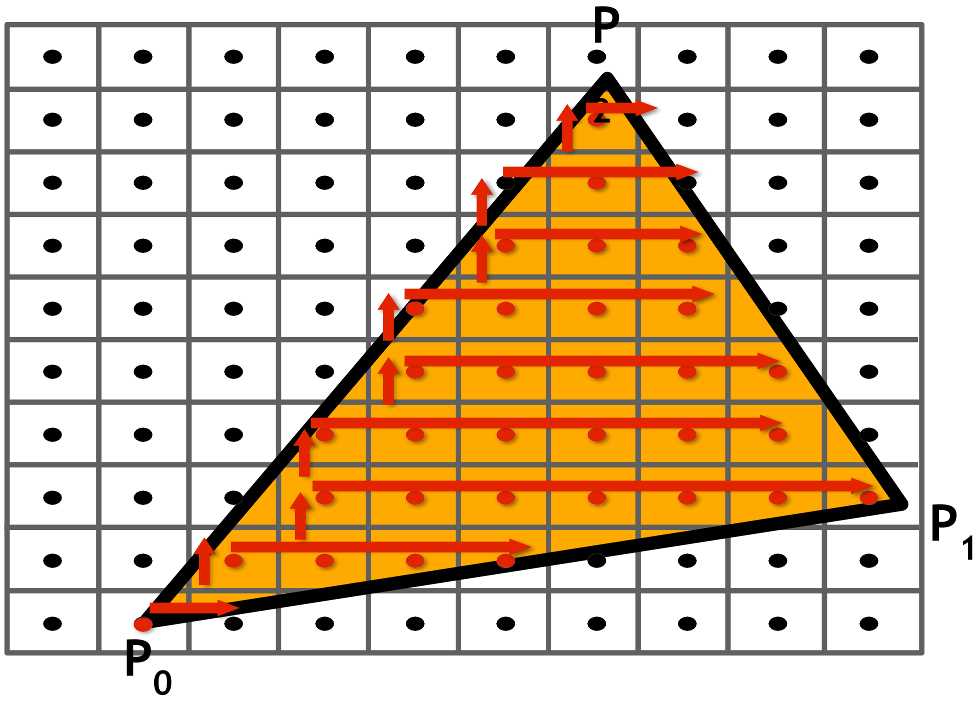


Source: Direct3D Programming Guide, Microsoft

```
Top edge: horizontal edge that is above
all other edges
```

```
Left edge: an edge that is not exactly
horizontal and is on the left side of the
triangle. (triangle can have one or two
left edges)
```

Incremental Triangle Traversal (Faster?)



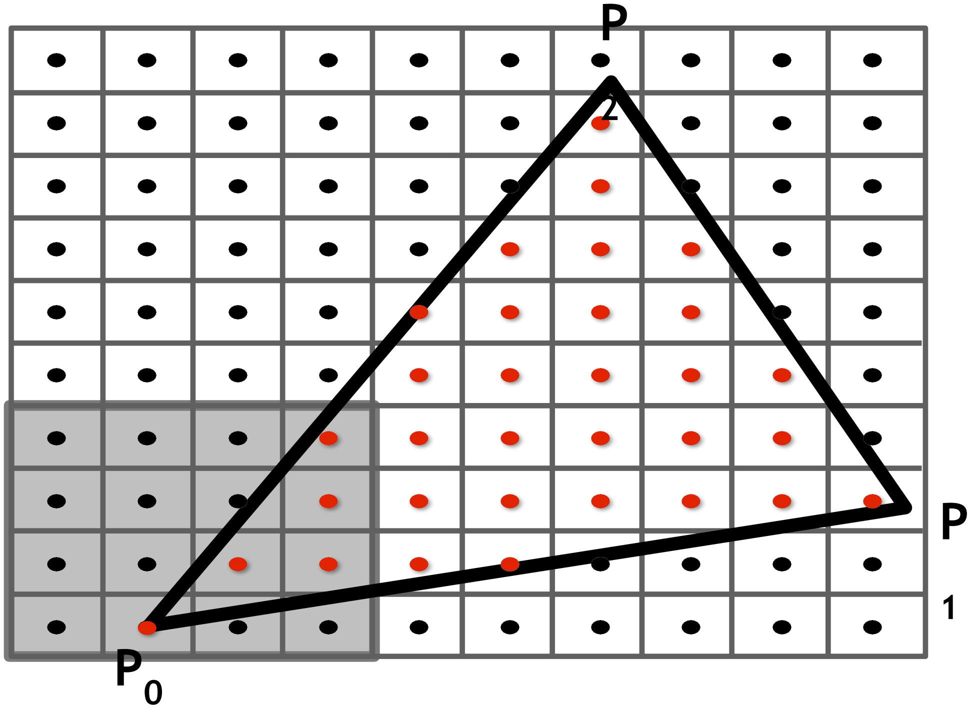
Modern Approach: Tiled Triangle Traversal

Traverse triangle in blocks

Test all samples in block in parallel

Advantages:

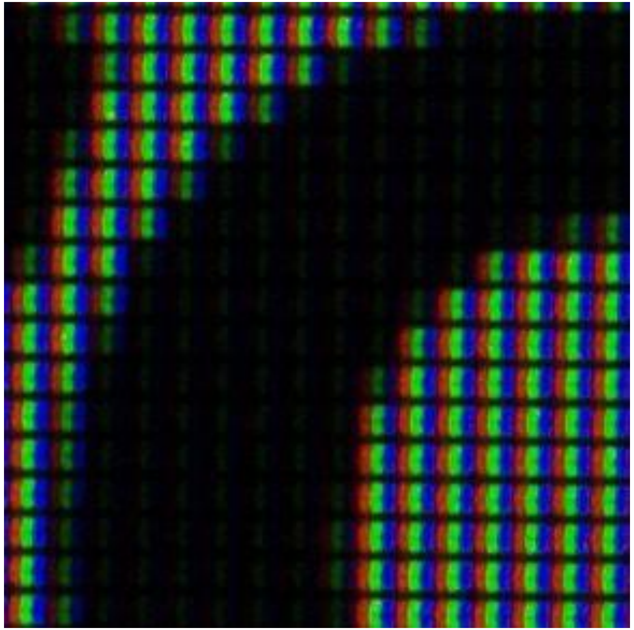
- Simplicity of wide parallel execution overcomes cost of extra point-in-triangle tests (most triangles cover many samples, especially when super-sampling)
- Can skip sample testing work: entire block not in triangle ("early out"), entire block entirely within triangle ("early in")



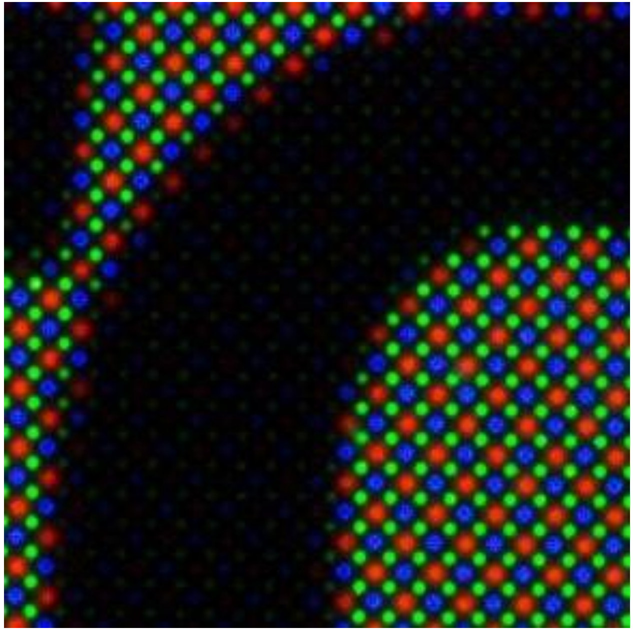
All modern GPUs have special-purpose hardware for efficient point-in-triangle tests

Signal Reconstruction
on Real Displays

Real LCD Screen Pixels (Closeup)



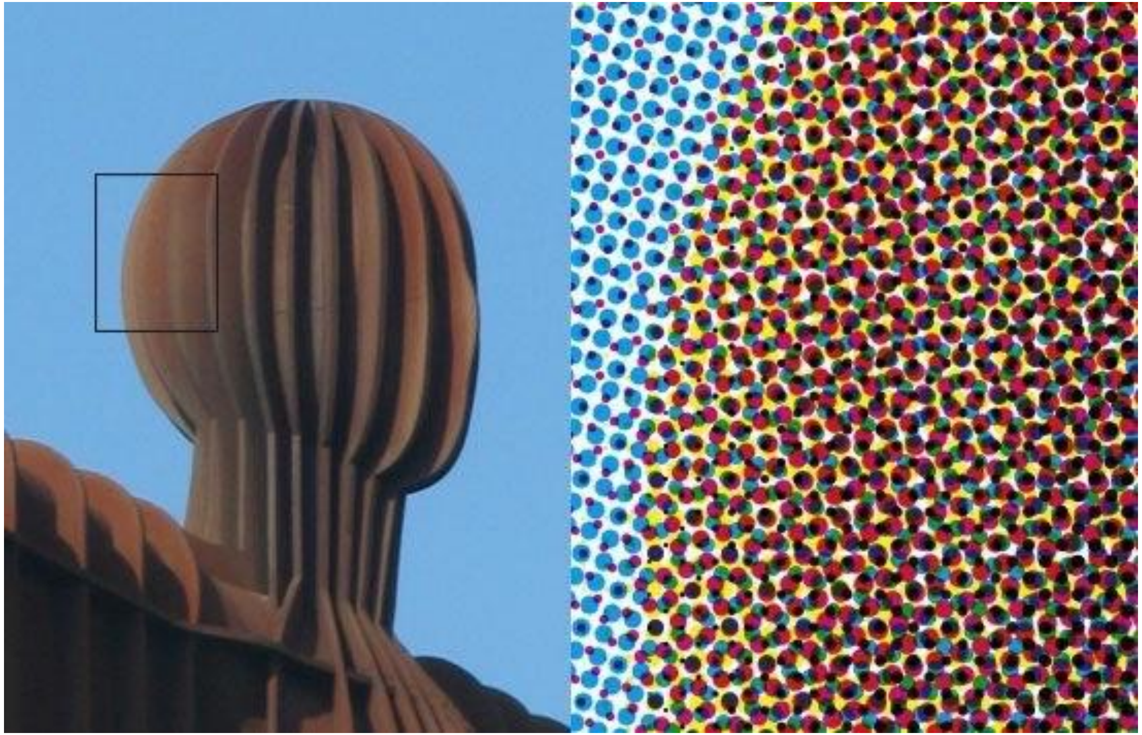
iPhone 6S



Galaxy S5

Notice R,G,B pixel geometry! But in this class, we will assume a colored square full-color pixel.

Aside: What About Other Display Methods?

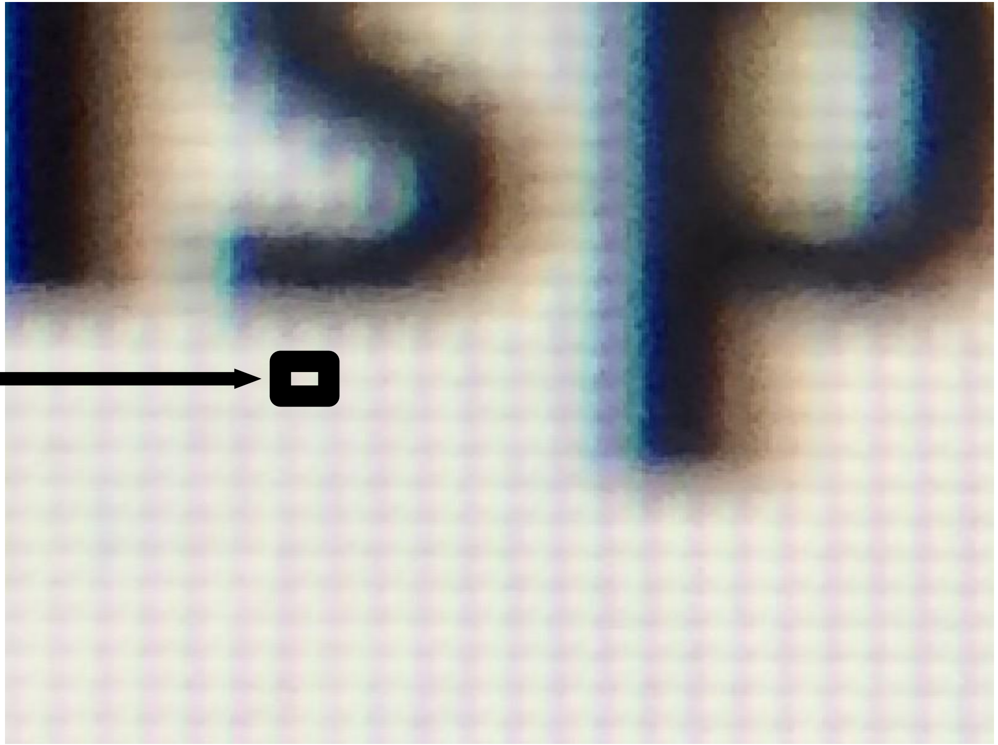


Color print: observe half-tone pattern

Assume Display Pixels Emit Square of Light

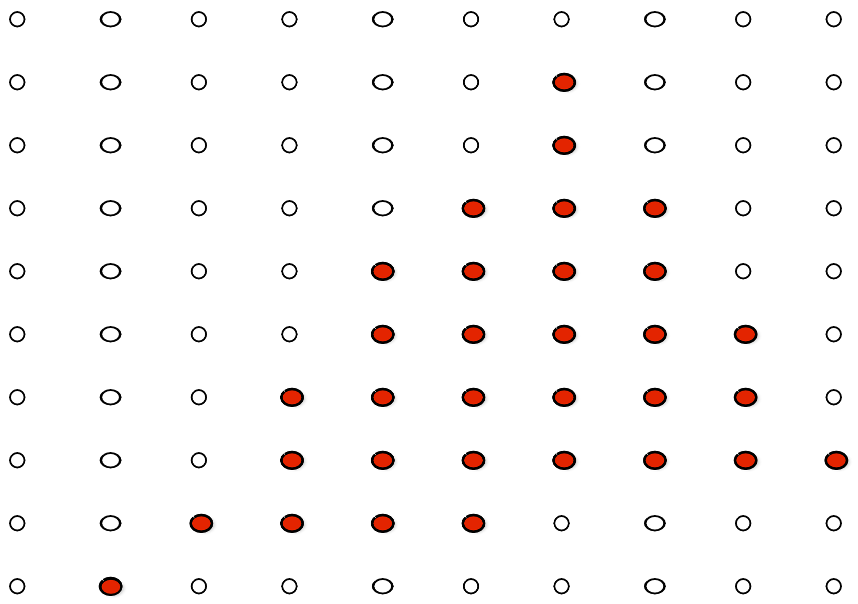
Each image sample sent to the display is converted into a little square of light of the appropriate color: (a pixel = picture element)

`LCD pixel`
`on laptop`

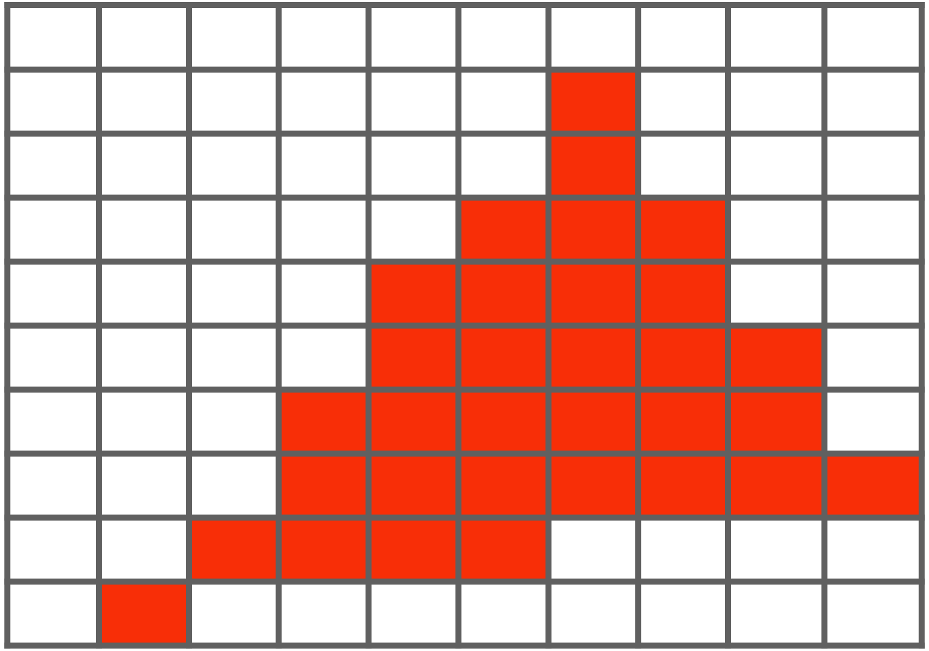


*** LCD pixels do not actually emit light in a square uniform color, but this approximation suffices for our current discussion**

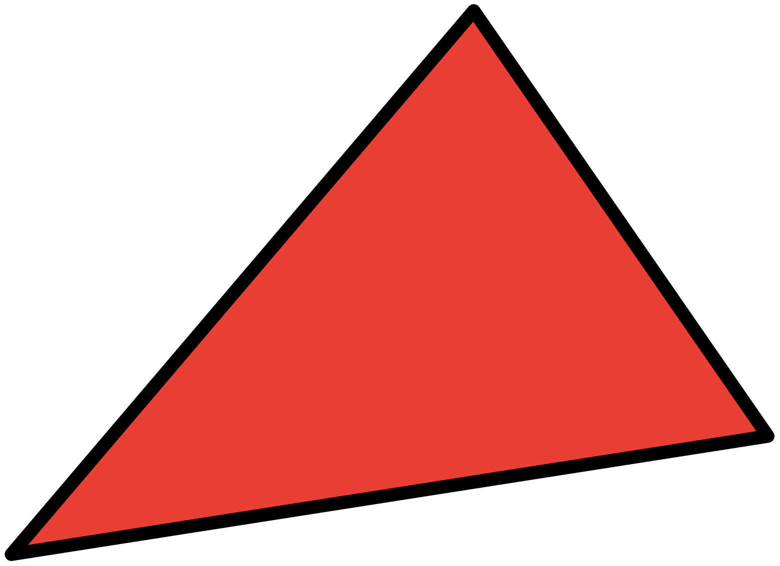
So, If We Send The Display This Sampled Signal



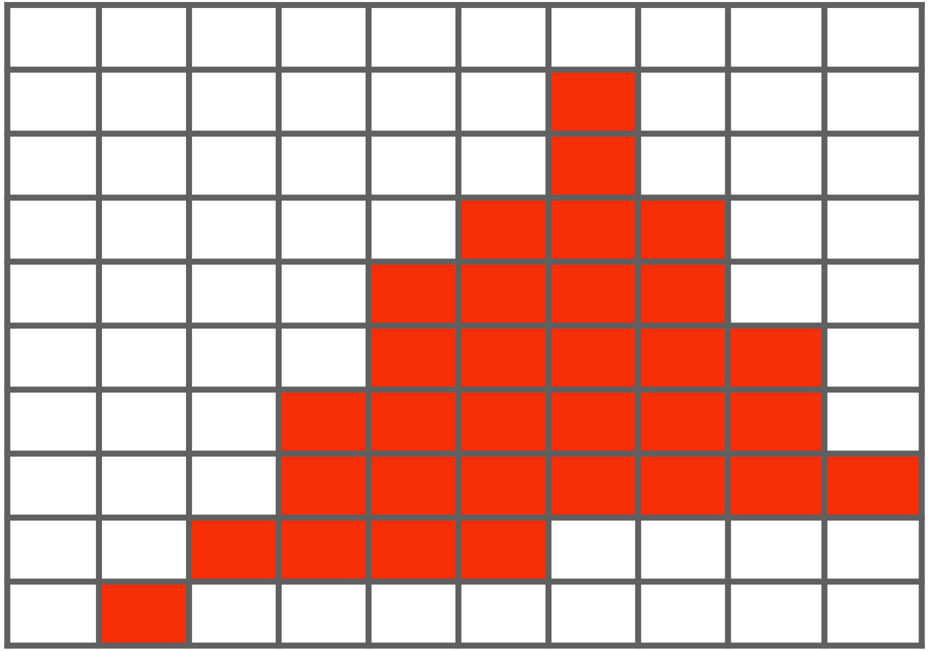
The Display Physically Emits This Signal



Compare: The Continuous Triangle Function

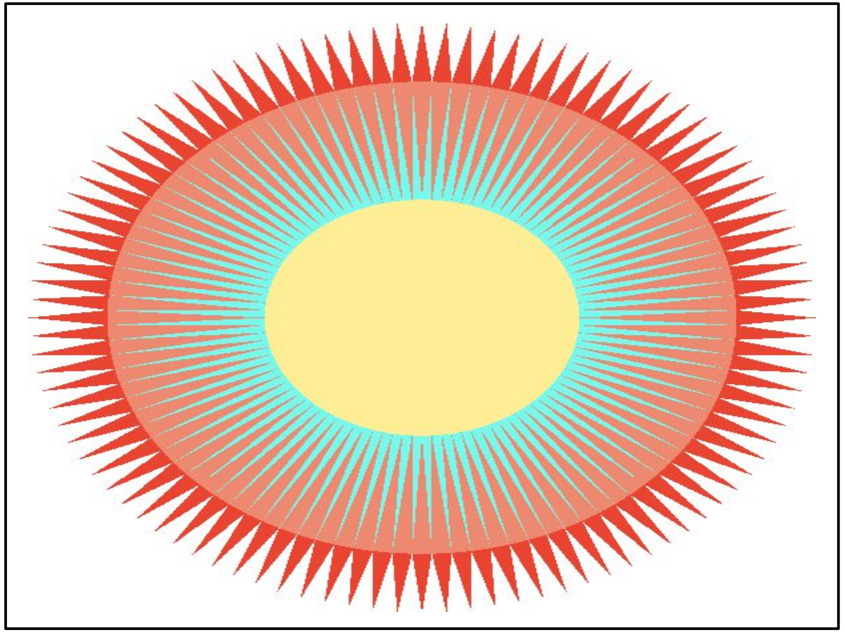


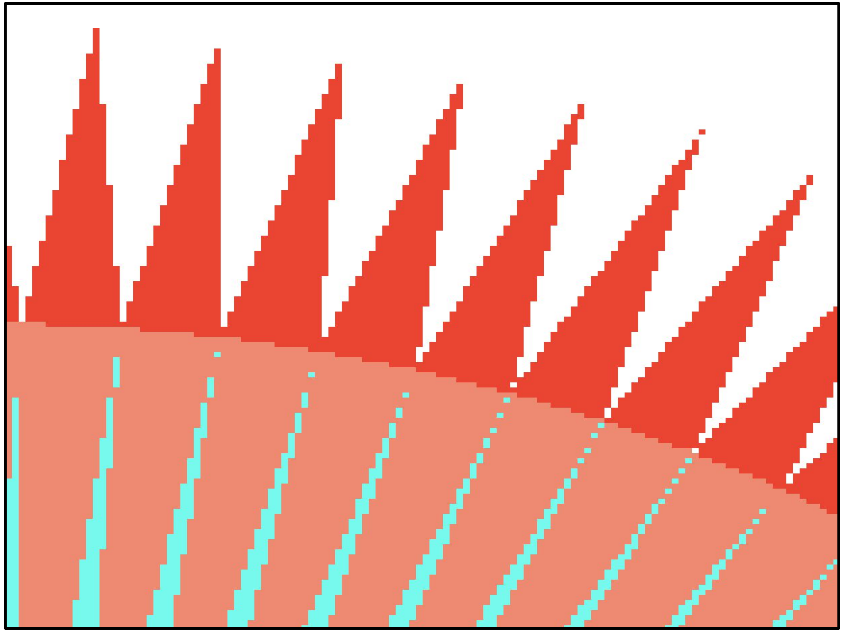
What's Wrong With This Picture?



Jaggies!

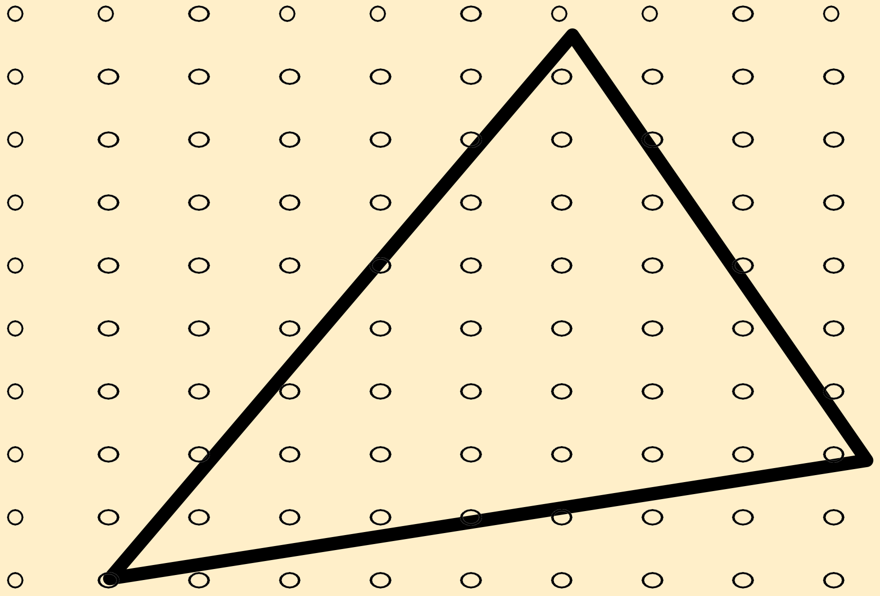
Jaggies (Staircase Pattern)





Is this the best we can do?

Discussion: What Value Should a Pixel Have?



Potential topics for your pair discussion:

- **Ideas for "higher quality" pixel formula?**
- **What are all the relevant factors?**
- **What's right/wrong about point sampling?**
- **Why do jaggies look "wrong"?**

Things to Remember

Drawing machines

- Many possibilities
- Why framebuffers and raster displays?
- Why triangles?

We posed rasterization as a 2D sampling process

- Test a binary function inside(triangle,x,y)
- Evaluate triangle coverage by 3 point-in-edge tests
- Finite sampling rate causes “jaggies” artifact (next time we will analyze in more detail)

Acknowledgments

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Frame Buffer: Memory for a Raster Display