Lecture 13:

Global Illumination & Path Tracing

Computer Graphics and Imaging

UC Berkeley CS184

Hard Shadows

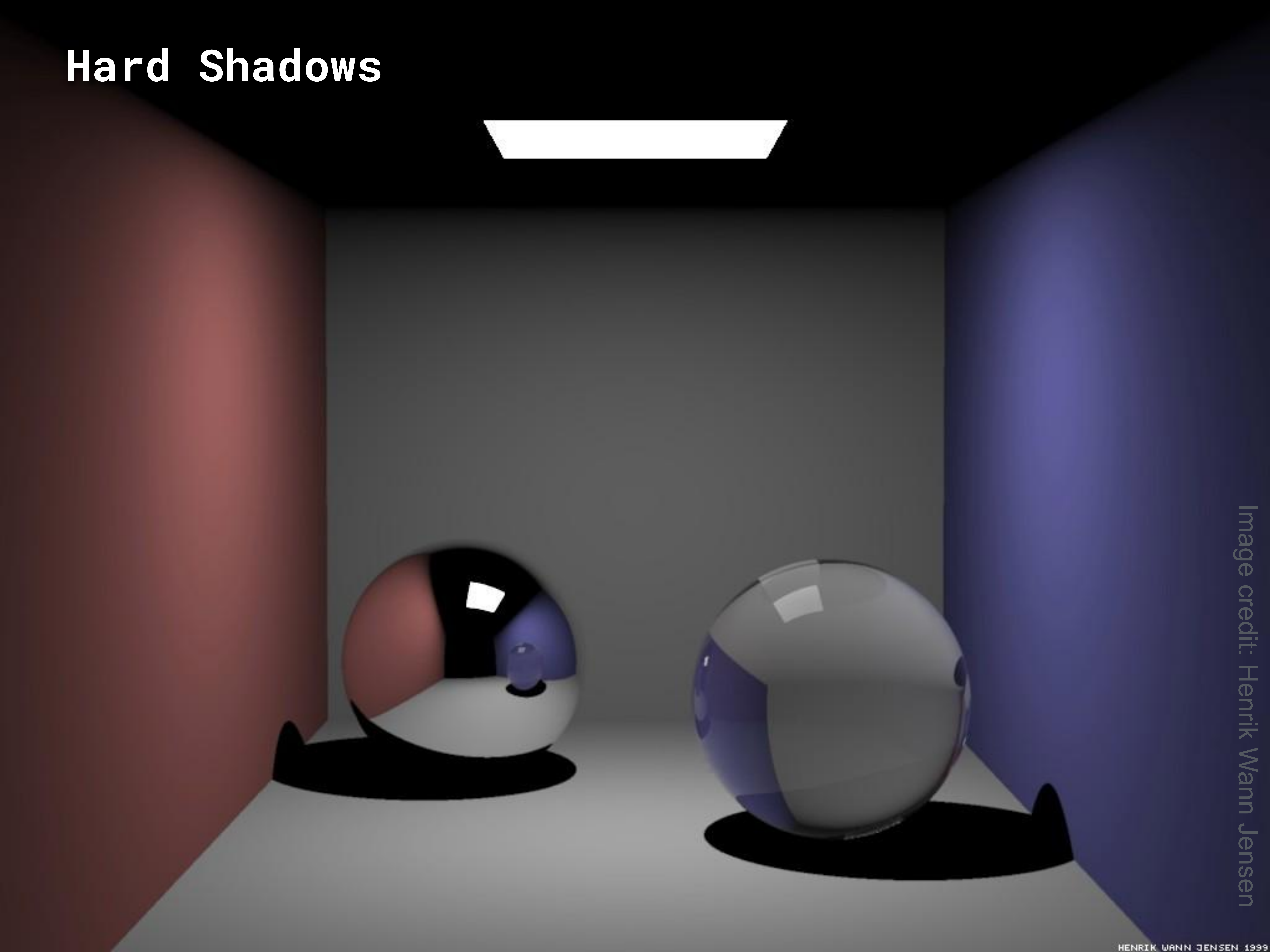


Image credit: Henrik Wann Jensen

Soft Shadows

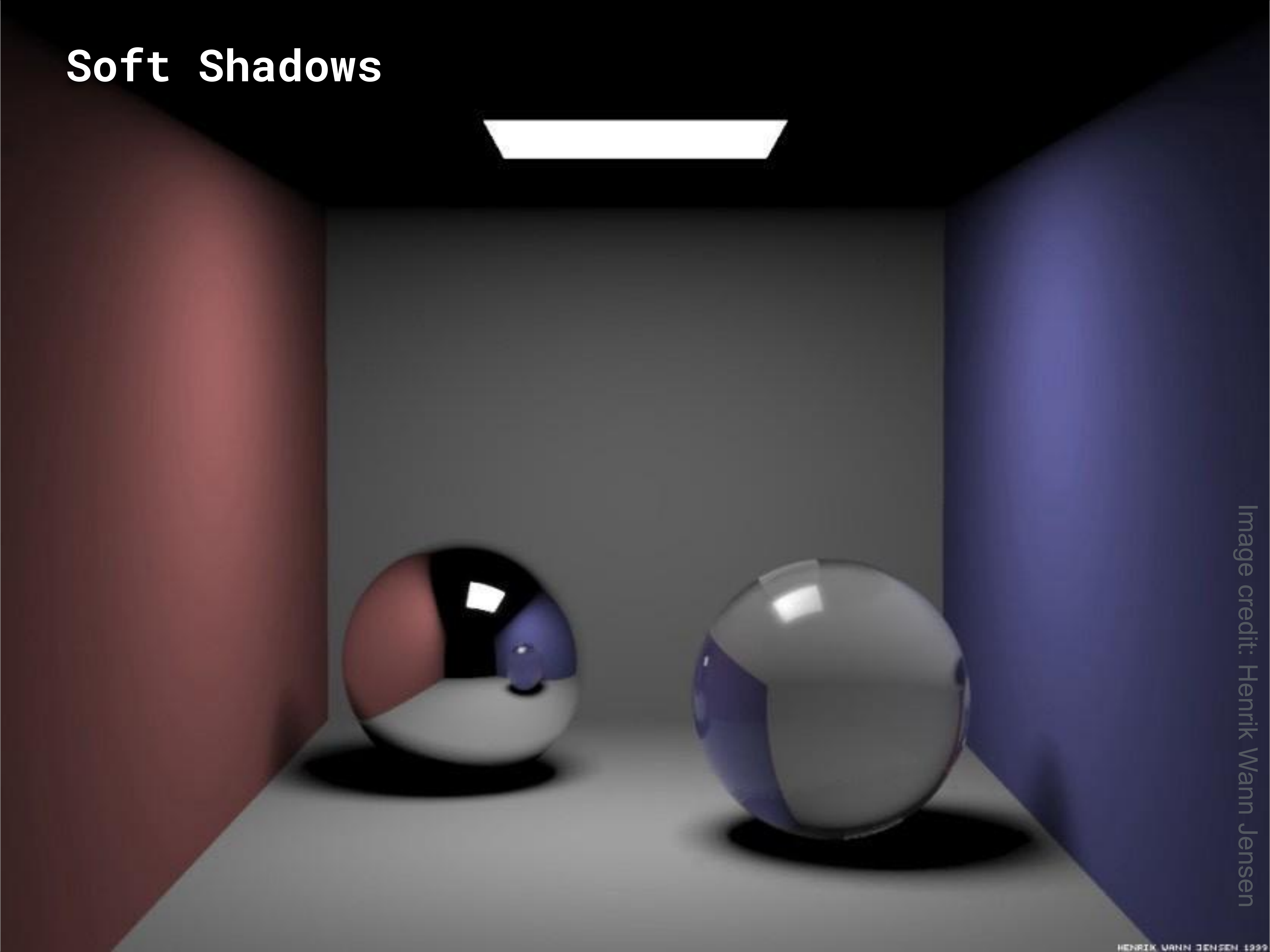


Image credit: Henrik Wann Jensen

+ Caustics

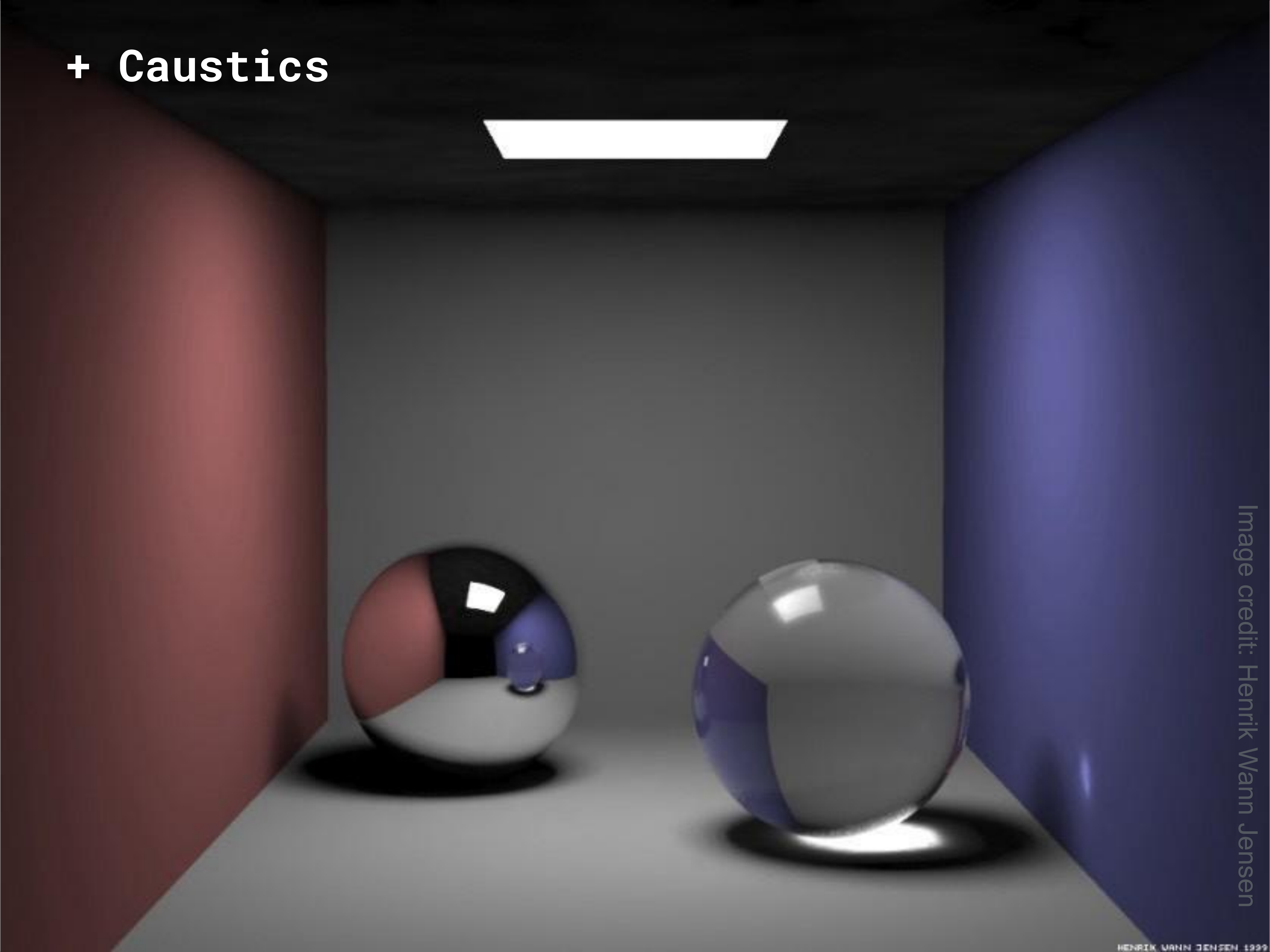


Image credit: Henrik Wann Jensen

+ Inter-Reflections = Global Illumination

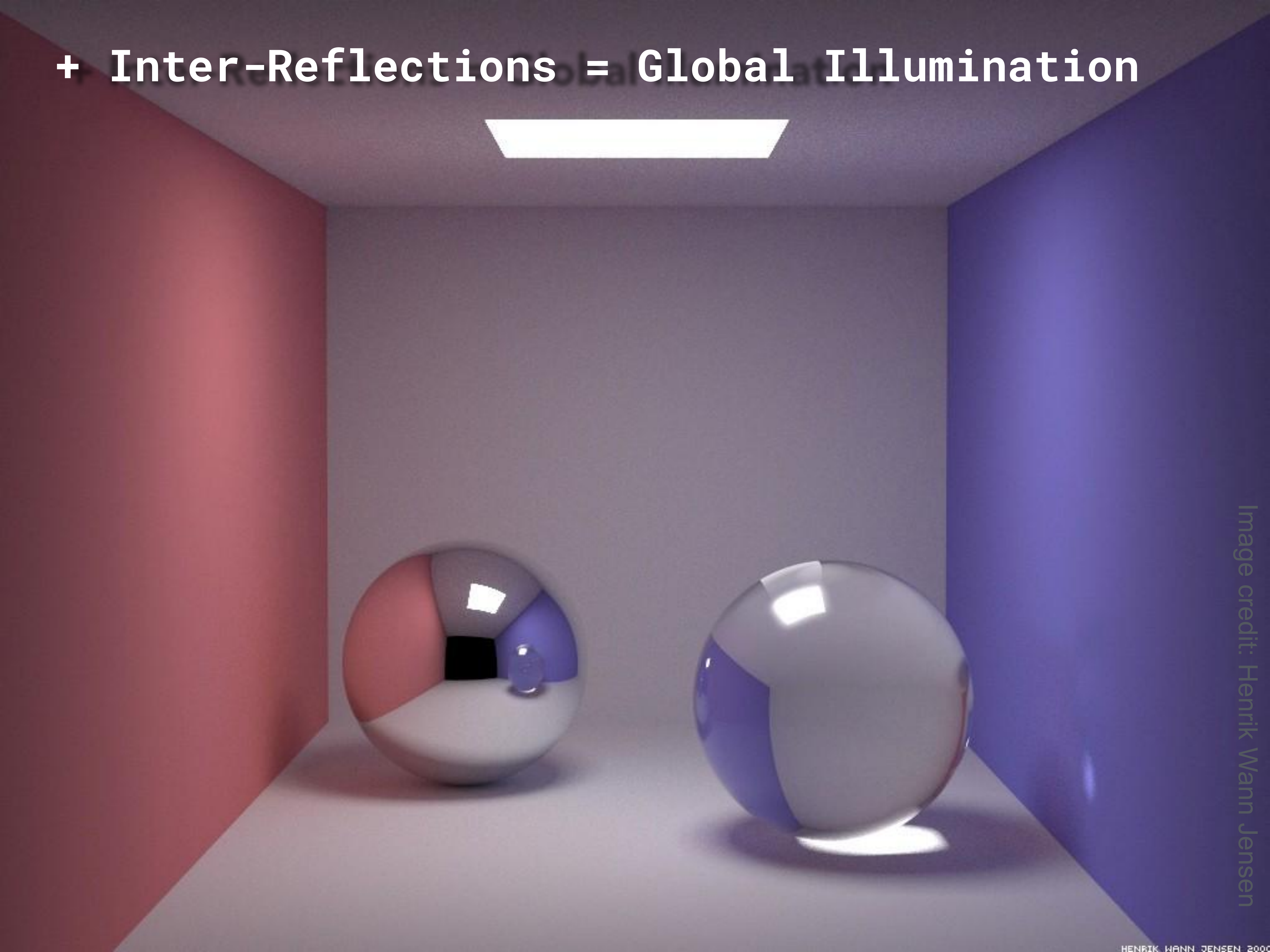
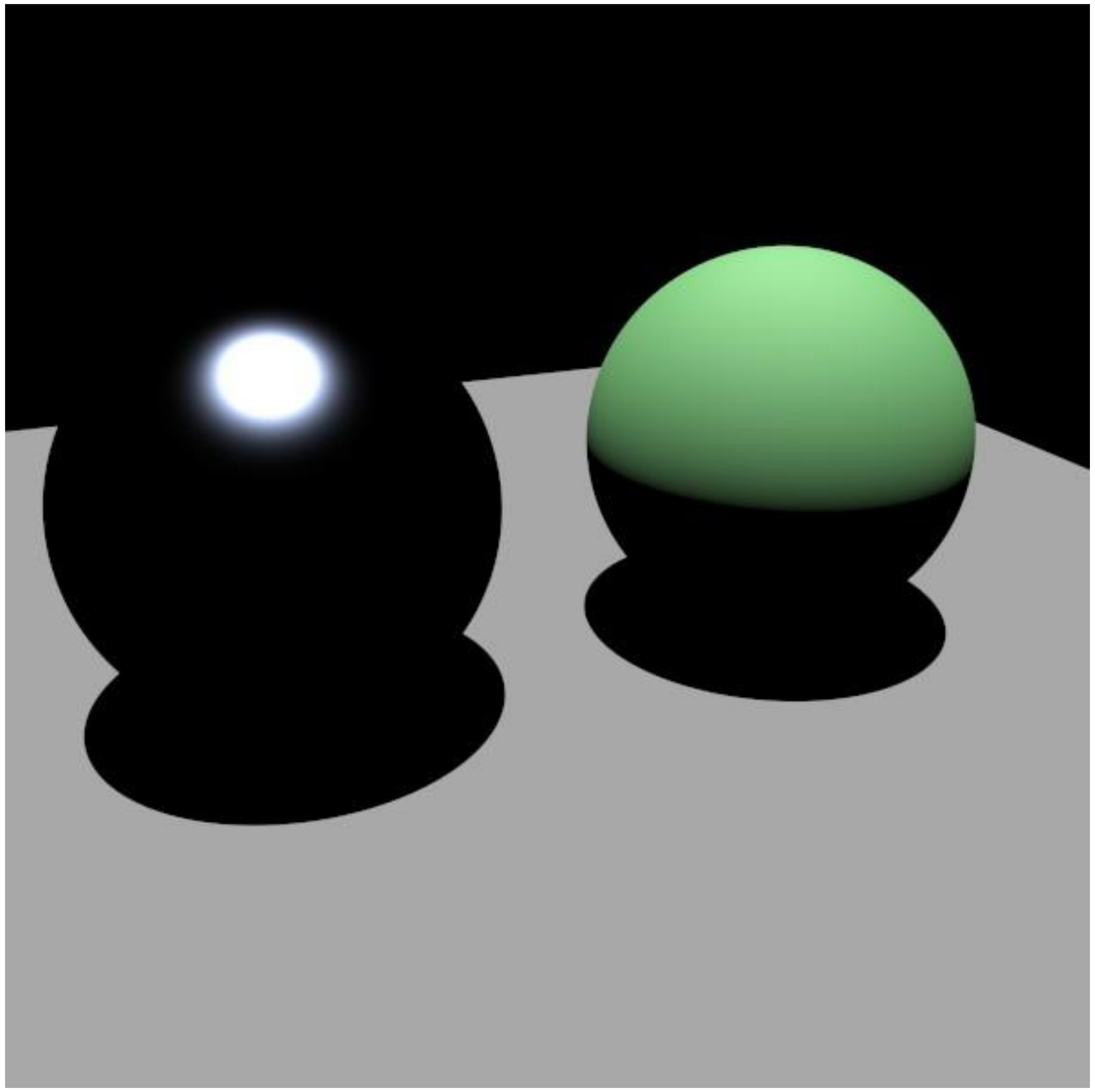


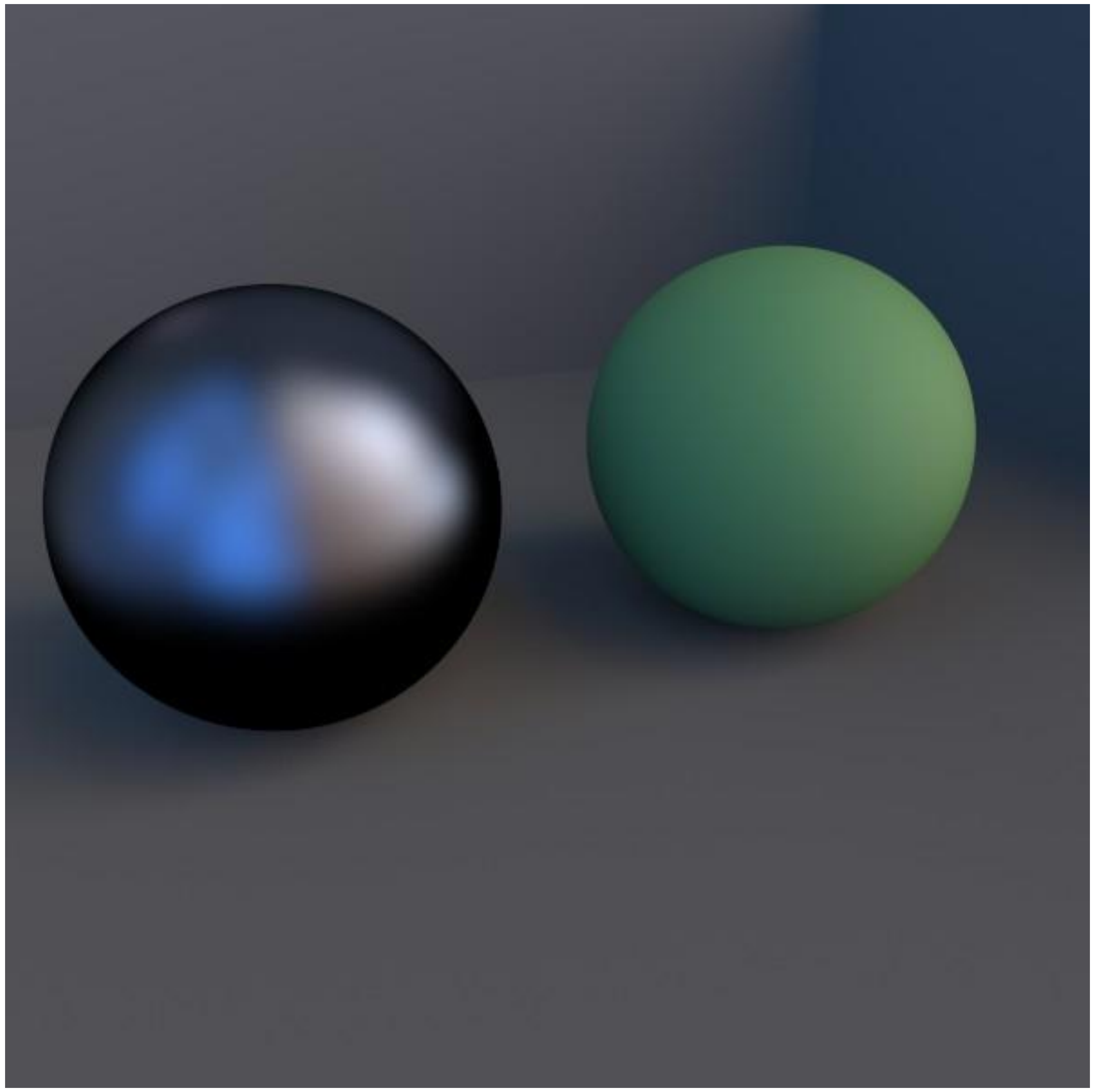
Image credit: Henrik Wann Jensen

Visual Richness from Indirect Lighting

Visual Richness from Complex Lighting



Point Light



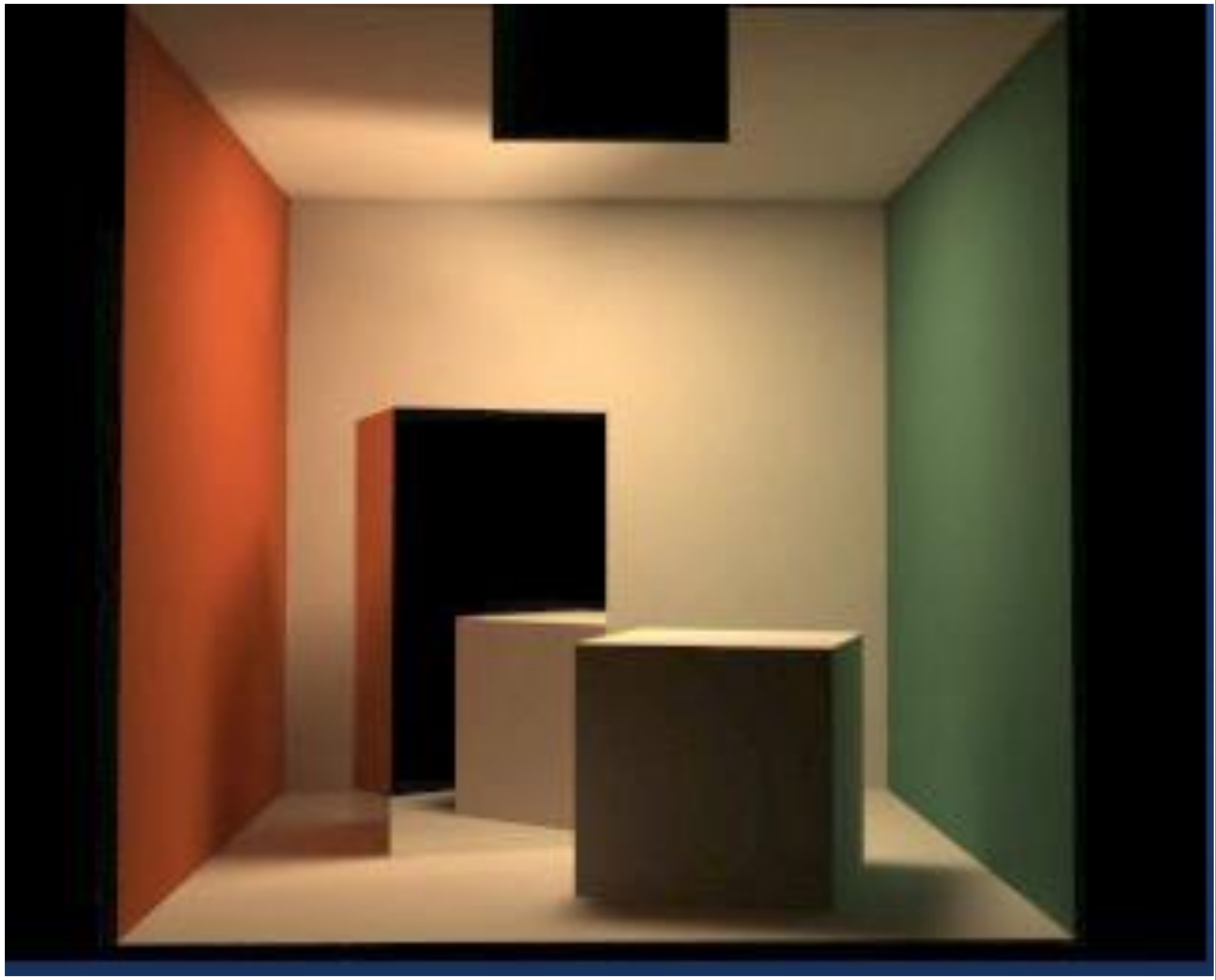
Environment Map Lighting

Visual Richness from Complex Materials

Credit: Bertrand Benoit. “Sweet Feast,” 2009. [Blender /VRay]

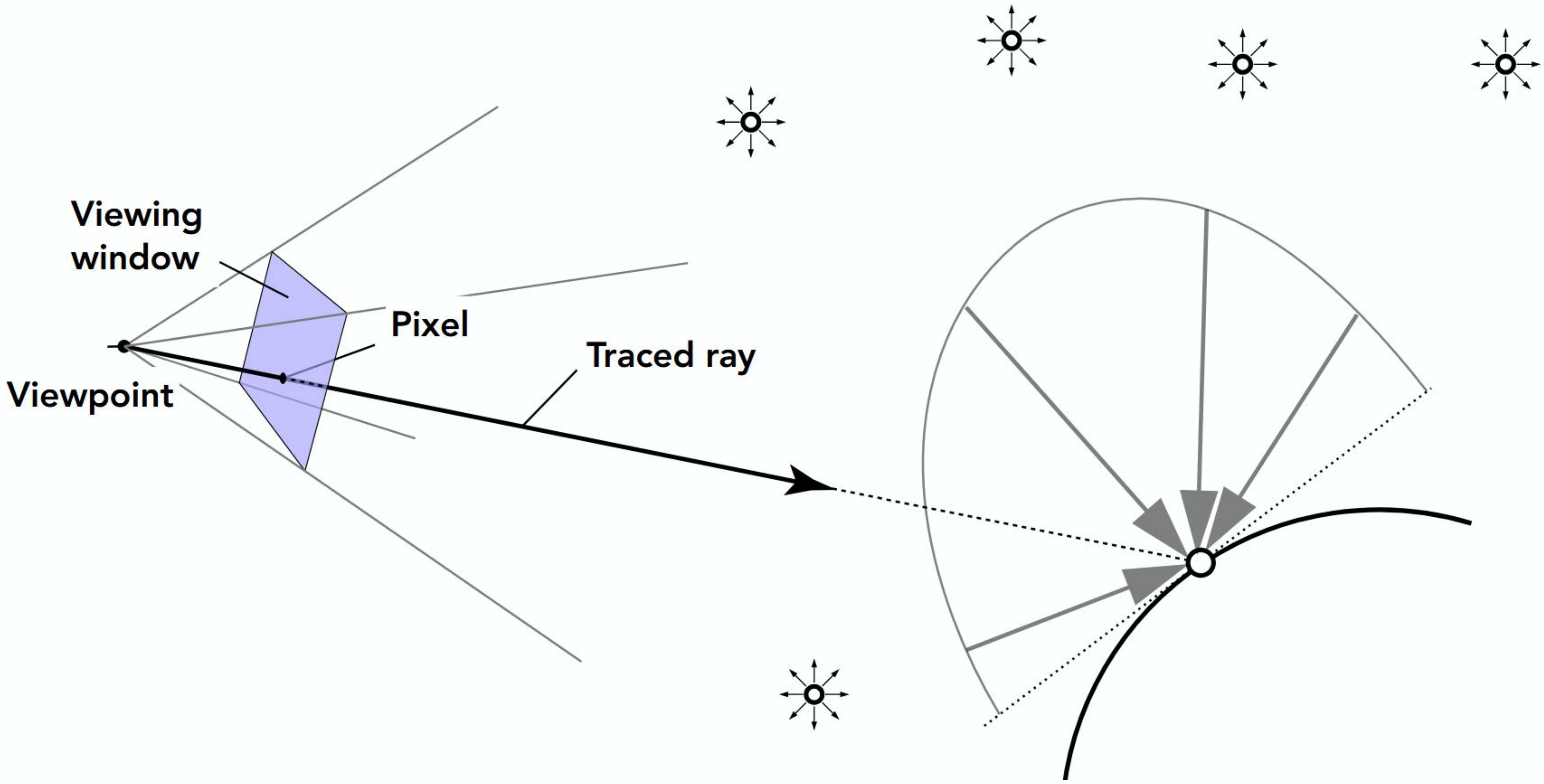
Cornell Box – Photograph vs Rendering

Photograph (CCD)



global illumination (render)

Ray Tracer Samples Radiance Along A Ray



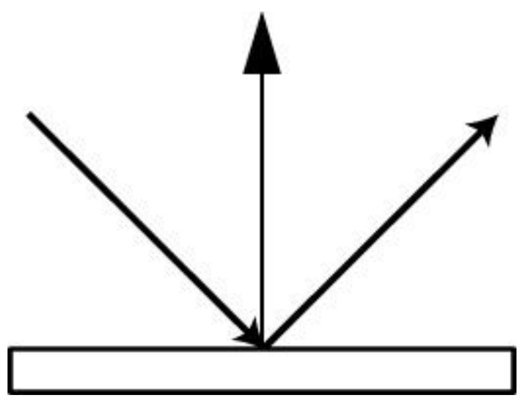
The light entering the pixel is the sum total of the light reflected off the surface into the ray's **reverse** direction

Intro To Material Reflection

Reflection

Definition: reflection is the process by which light incident on a surface interacts with the surface such that it leaves on the incident (same) side without change in frequency.

Categories of Reflection Functions



Ideal specular

- Perfect mirror reflection

Materials: Mirror

Materials: Diffuse

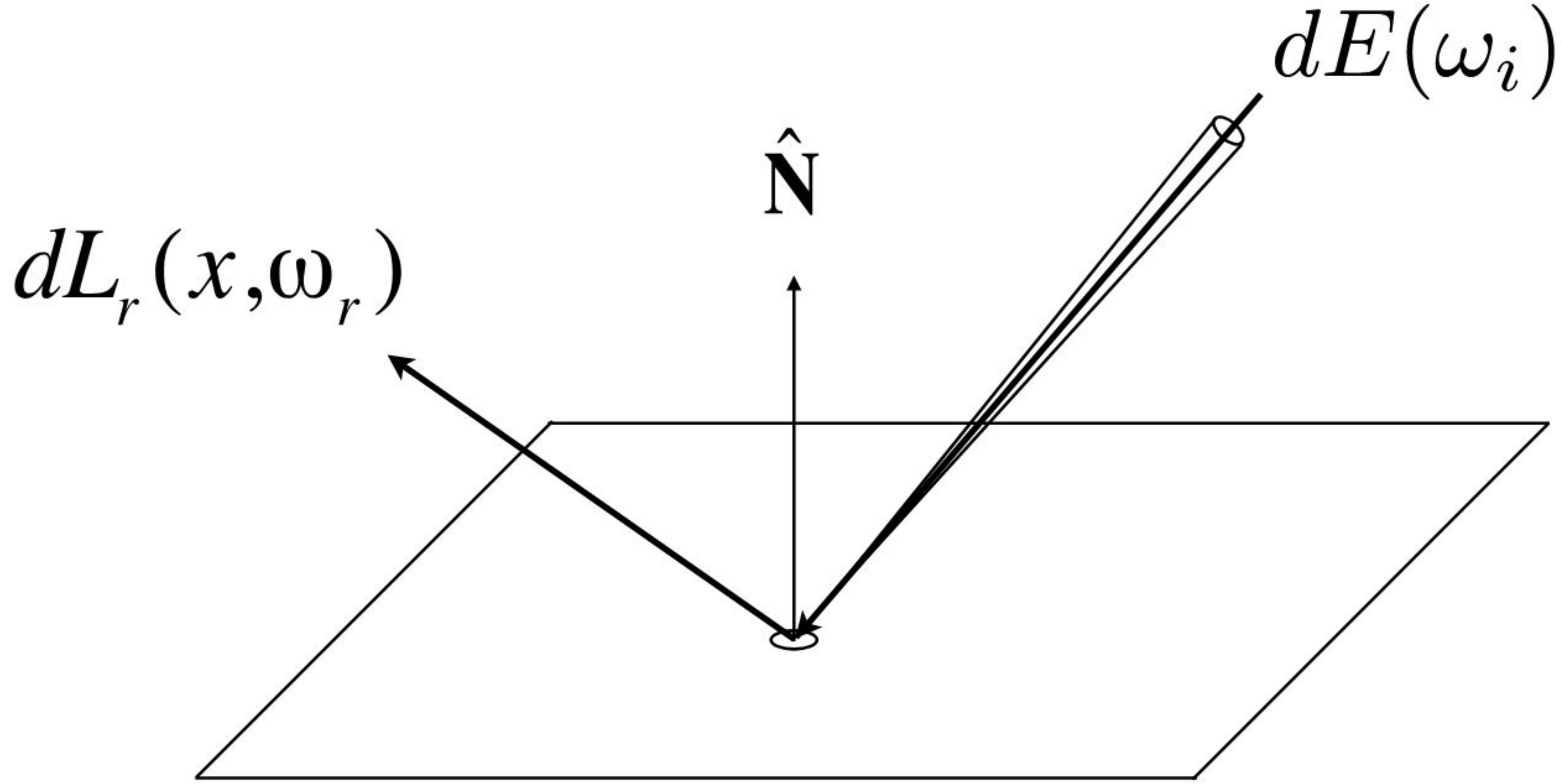
Materials: Gold

Materials: Anisotropic

Materials: Red Semi-Gloss Paint

Materials: Ford Mystic Lacquer Paint

Reflection at a Point



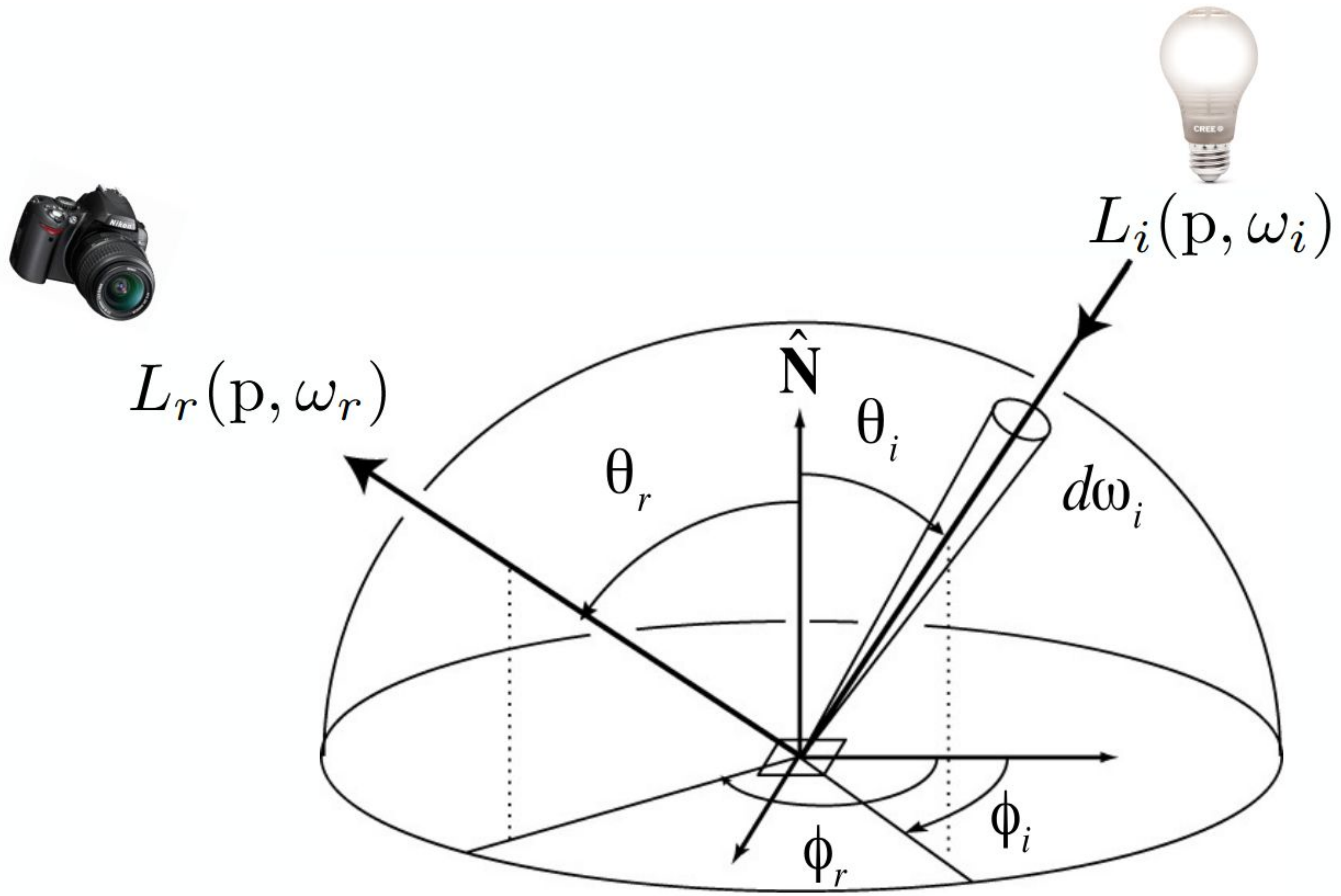
Differential irradiance incoming: $dE(\omega_i) = L(\omega_i)\, cos\, \theta_i\, d\omega_i$

Differential radiance exiting (due to $dE(\omega_i)$**)** $dL_r(\omega_r)$

BRDF

Definition: The bidirectional reflectance distribution function (BRDF) represents how much light is reflected into each outgoing direction ω_r from each incoming direction ω_i

The Reflection Equation

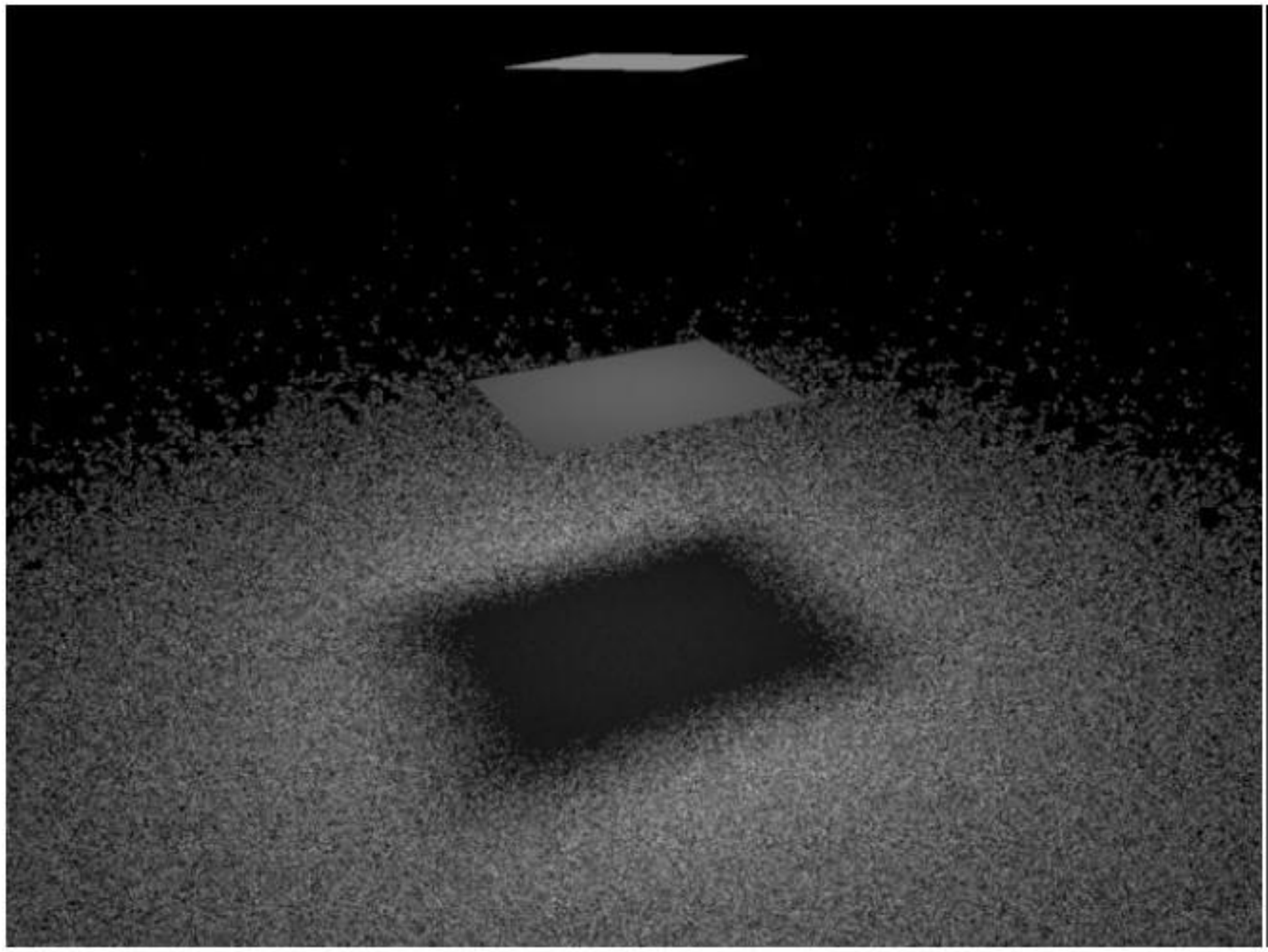


$$L_r(\mathrm{p}, \omega_r) = \int_{H^2} f_r(\mathrm{p}, \omega_i \rightarrow \omega_r)\, L_i(\mathrm{p}, \omega_i)\, \cos\theta_i \,\mathrm{d}\omega_i$$

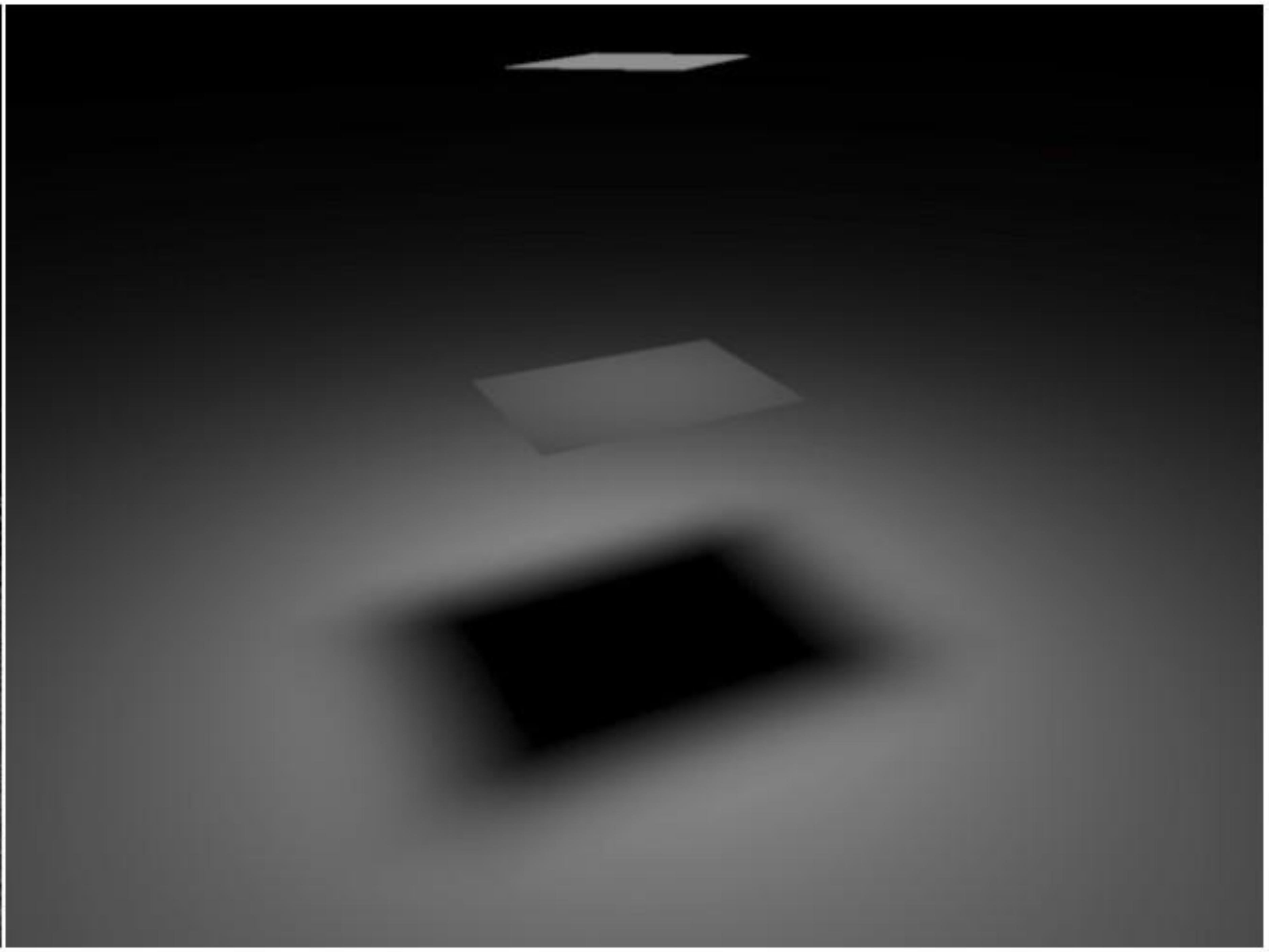
Solving the Reflection Equation

$$L_r(\mathrm{p}, \omega_r) = \int_{H^2} f_r(\mathrm{p}, \omega_i \to \omega_r)\, L_i(\mathrm{p}, \omega_i)\, \cos\theta_i \, \mathrm{d}\omega_i$$

Recall: Hemisphere vs Light Sampling



Sample hemisphere uniformly



Sample points on light

Direct Lighting Pseudocode (Uniform Random Sampling)

```
DirectLightingSampleUniform(p, ωo)
    ωi = hemisphere.sampleUniform();        // uniform random sampling
    pdf = 1.0 / (2 * pi);

    if (scene.shadowIntersection(p, ωi))    // Shadow ray
        return 0;
    else
        L = lights.radiance(intersect(p, ωi), −ωi);
        return L * p.brdf(ωi, ωo) * costheta / pdf;
```

Direct Lighting Pseudocode (Importance Sampling of BRDF)

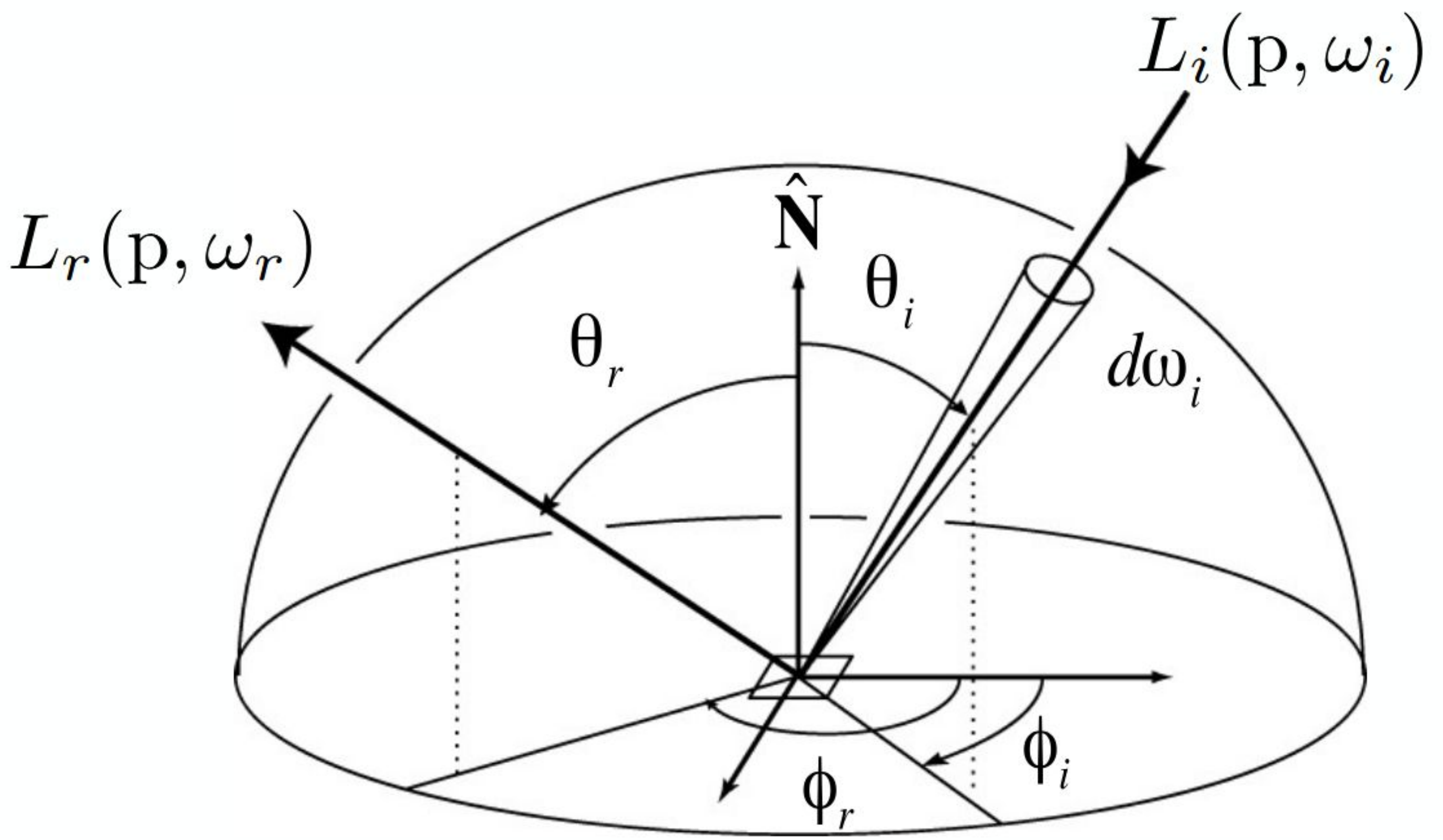
```
DirectLightingSampleBRDF(p, ωo)
    ωi, pdf = p.brdf.sampleDirection(ωo);        // Imp. Sample BRDF


    if (scene.shadowIntersection(p, ωi))         // Shadow ray
        return 0;

    else
        L = lights.radiance(intersect(p, ωi), −ωi);
        return L * p.brdf(ωi, ωo) * costheta / pdf;
```

Direct Lighting Pseudocode (Importance Sampling of Lights)

```
DirectLightingSampleLights(p, ωo)
    L, ωi, pdf = lights.sampleDirection(p);    // Imp. sample lights

    if (scene.shadowIntersection(p, ωi))        // Shadow
        return 0;                               ray
    else
        return L * p.brdf(ωi, ωo) *costheta / pdf;

// Note: only one random sample over all lights.
// Assignment 3-1 asks you to, alternatively, loop over
// multiple lights and take multiple samples
```

Global Illumination:
Deriving the Rendering Equation

Again: Reflection Equation

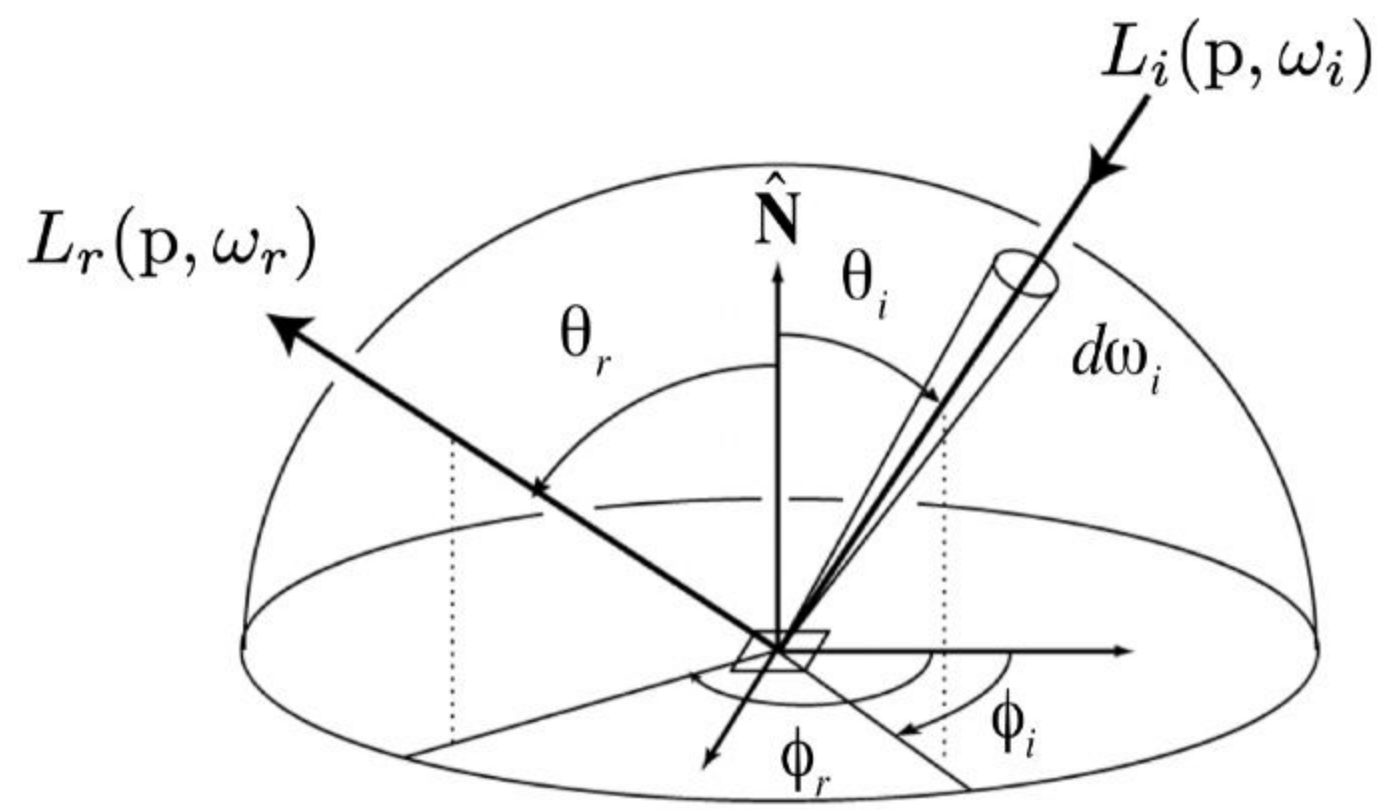


$$L_r(\mathrm{p}, \omega_r) = \int_{H^2} f_r(\mathrm{p}, \omega_i \to \omega_r)\, L_i(\mathrm{p}, \omega_i)\, \cos\theta_i\, \mathrm{d}\omega_i$$

Challenge: This is Actually A Recursive Equation

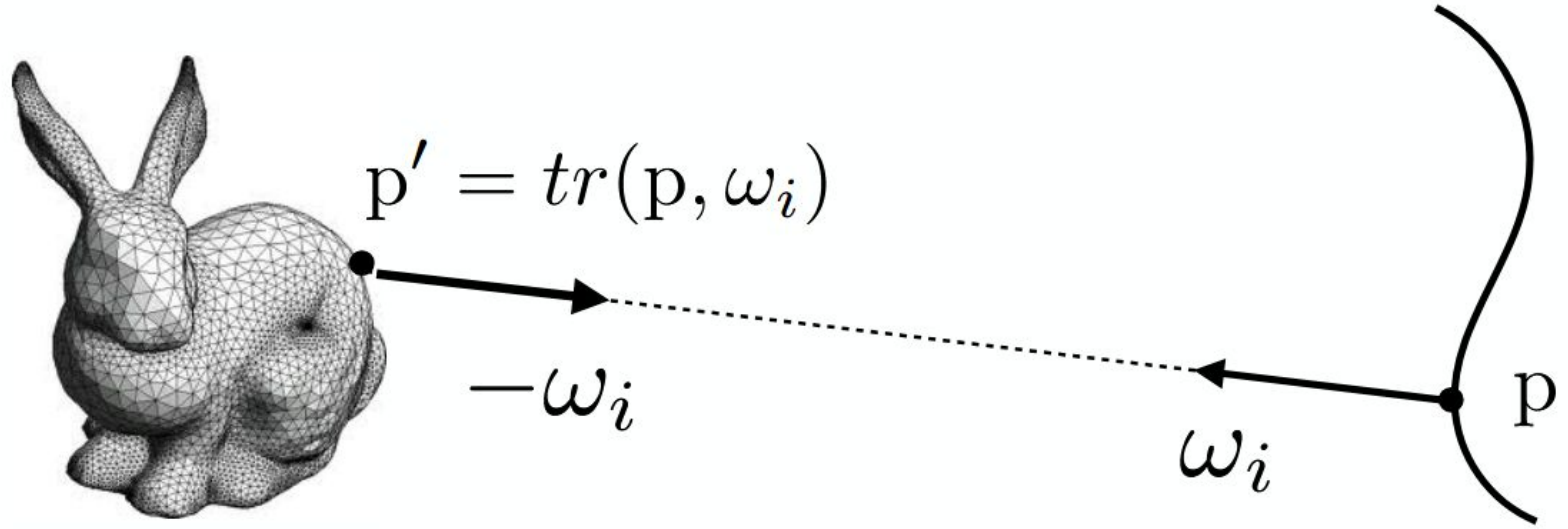
Reflected radiance depends on incoming radiance

$$\boxed{L_r(\mathrm{p}, \omega_r)} = \int_{H^2} f_r(\mathrm{p}, \omega_i \rightarrow \omega_r) \boxed{L_i(\mathrm{p}, \omega_i)} \cos\theta_i \, \mathrm{d}\omega_i$$



Transport Function & Radiance Invariance

Definition: the Transport Function, $tr(\mathrm{p}, \omega)$, returns the first surface intersection point in the scene along ray (p, ω)



The Rendering Equation

Re-write the reflection equation:

$$L_o(\mathrm{p}, \omega_o) = L_e(\mathrm{p}, \omega_o) + \int_{H^2} f_r(\mathrm{p}, \omega_i \to \omega_o)\, L_i(\mathrm{p}, \omega_i)\, \cos\theta_i \, \mathrm{d}\omega_i$$

Using the transport function: $L_i(\mathrm{p}, \omega_i) = L_o(tr(\mathrm{p}, \omega_i), -\omega_i)$

Light Transport Operators

Operators Are Higher-Order Functions

Functions:

$$f, g : (x, \omega) \to \mathbb{R}$$

Operators are higher-order functions:

$$P : ((x, \omega) \to \mathbb{R}) \to ((x, \omega) \to \mathbb{R})$$

$$P(f) = g$$

- Take a function and transform it into another function

Linear Operators

- **Linear operators act on functions like matrices act on vectors**

$$h(x) = (L(f))(x)$$

- **They are linear in that:**

$$L(af + bg) = aL(f) + bL(g)$$

- **Examples of linear operators:**

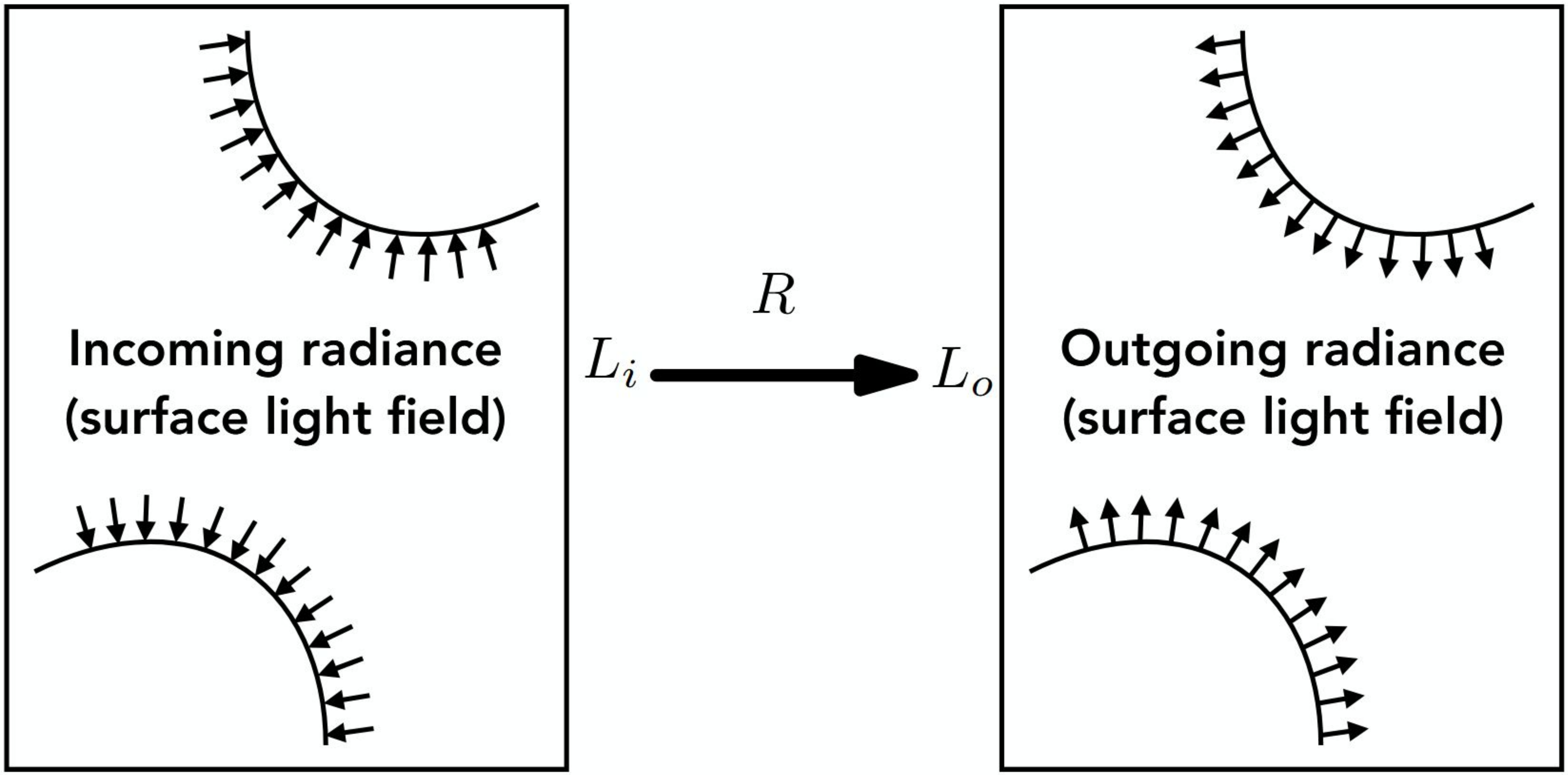
$$H(f)(x) = \int h(x, x')\, f(x')\, \mathrm{d}x'$$

$$D(f)(x) = \frac{\delta f}{\delta x}(x)$$

Light Transport Functions & Operators

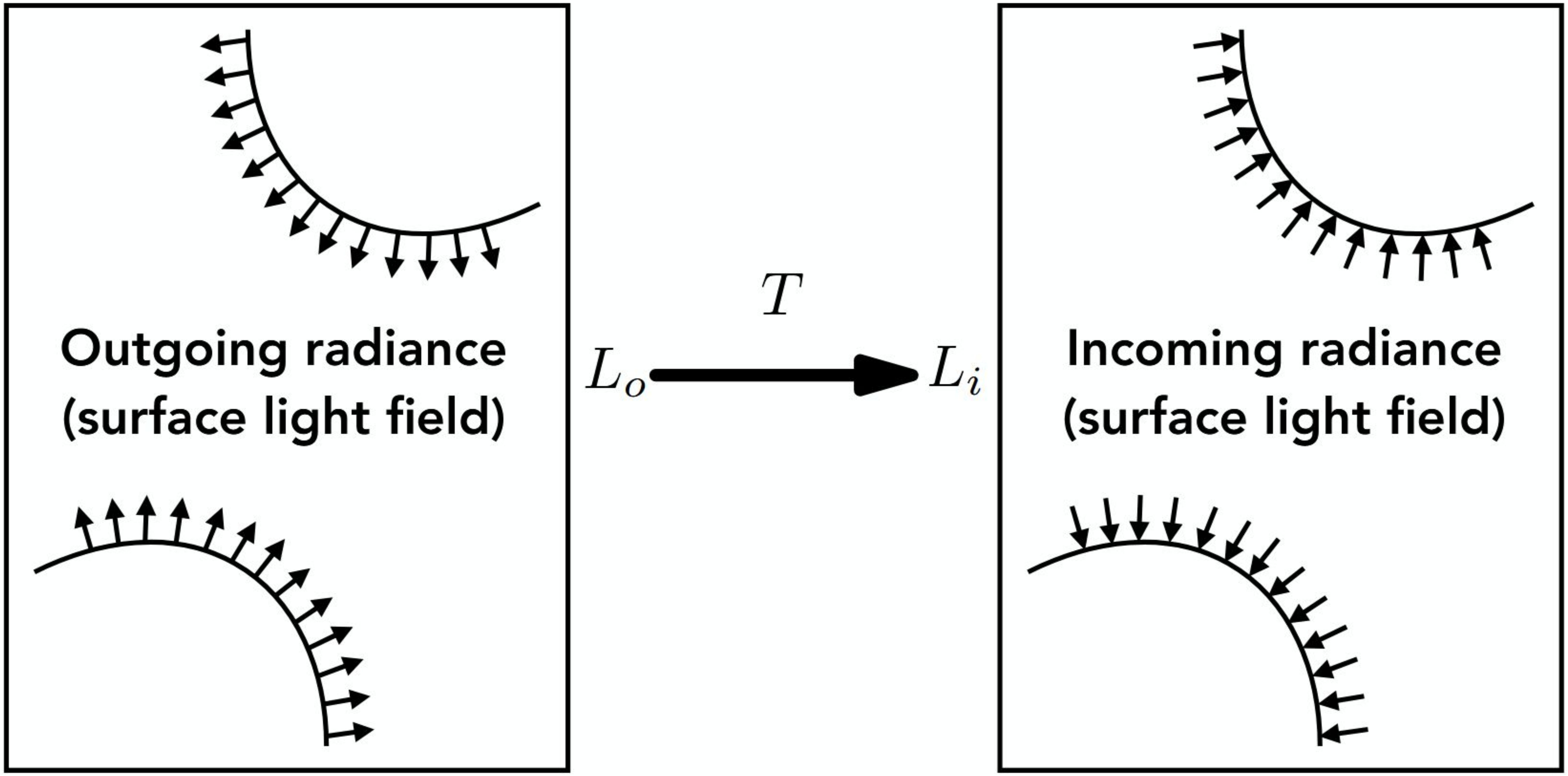
- **Emitted radiance function (all surface points & outgoing directions)** $L_e(\mathrm{p}, \omega)$

Reflection Operator



$$R(g)(\mathrm{p}, \omega_o) \equiv \int_{H^2} f_r(\mathrm{p}, \omega_i \to \omega_o)\, g(\mathrm{p}, \omega_i)\ \cos\theta_i \,\mathrm{d}\omega_i$$

Transport Operator



$$T(f)(\mathrm{p}, \omega_o) \equiv f(tr(\mathrm{p}, \omega_o), -\omega_o)$$
$$T(L_o) = L_i$$

Rendering Equation in Operator Notation

$$L_o(\mathrm{p}, \omega_o) = L_e(\mathrm{p}, \omega_o) + \int_{H^2} f_r(\mathrm{p}, \omega_i \rightarrow \omega_o)\, L_o(tr(\mathrm{p}, \omega_i), -\omega_i)\, \cos\theta_i \, \mathrm{d}\omega_i$$

$$L_o = L_e + (R \circ T)(L_o)$$

Solving the Rendering Equation

Solving the Rendering Equation

- Rendering equation:

$$L = L_e + K(L)$$

$$(I - K)(L) = L_e$$

L **is outgoing reflected**

Solution Intuition

For scalar functions, recall:

$$\frac{1}{1-x} = 1 + x + x^2 + x^3 + \cdots$$

$$\text{converges for } -1 < x < 1$$

Formal Solution

Neumann series:

$$(I - K)^{-1} = \frac{1}{I - K} = I + K + K^2 + K^3 + \cdots$$

Check:

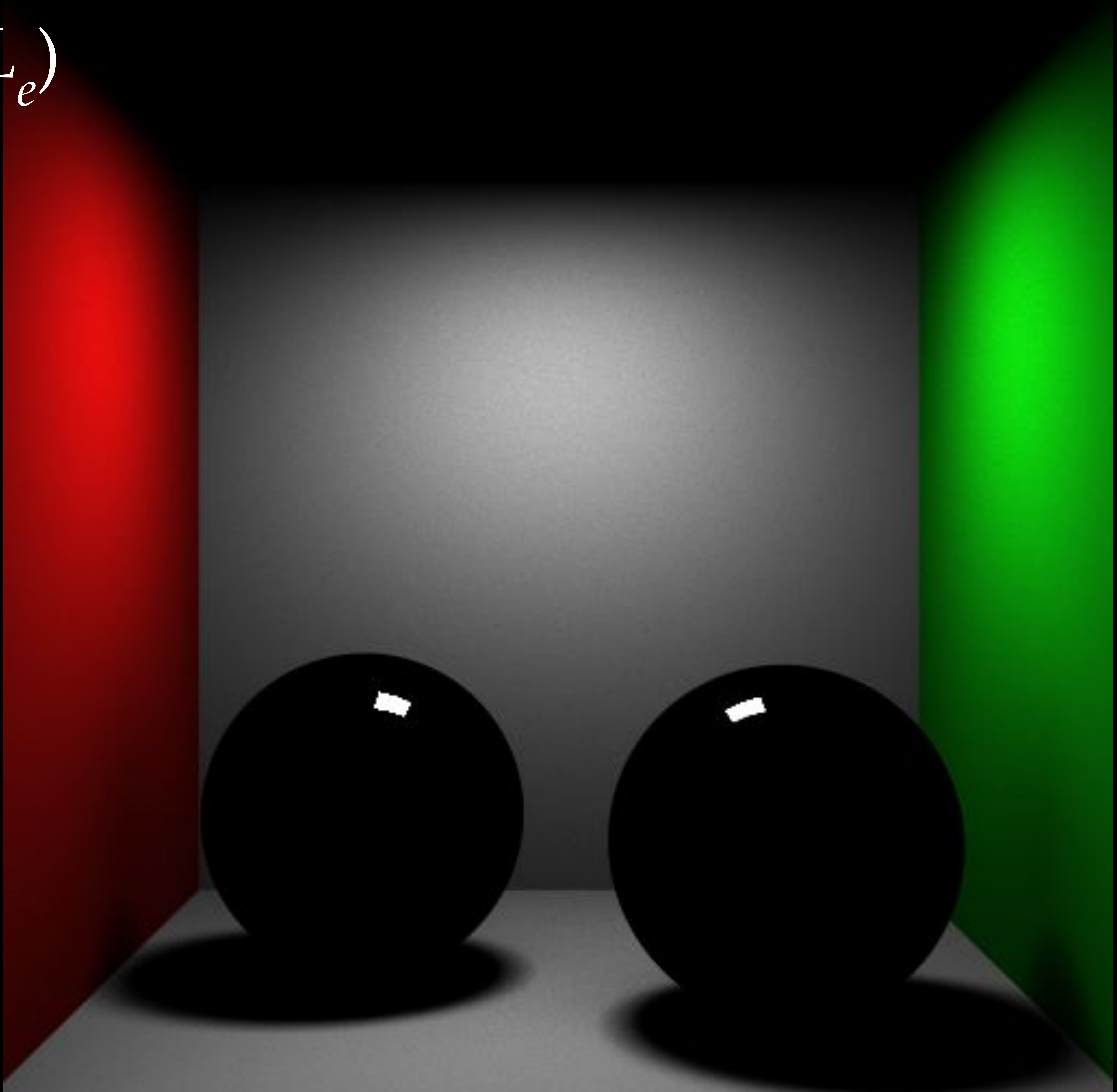
$$\begin{aligned}
&(I - K) \circ (I - K)^{-1} \\
&\quad = (I - K) \circ (I + K + K^2 + K^3 + \cdots) \\
&\quad = (I + K + K^2 + \cdots) - (K + K^2 + \cdots) \\
&\quad = I
\end{aligned}$$

Rendering Equation Solution

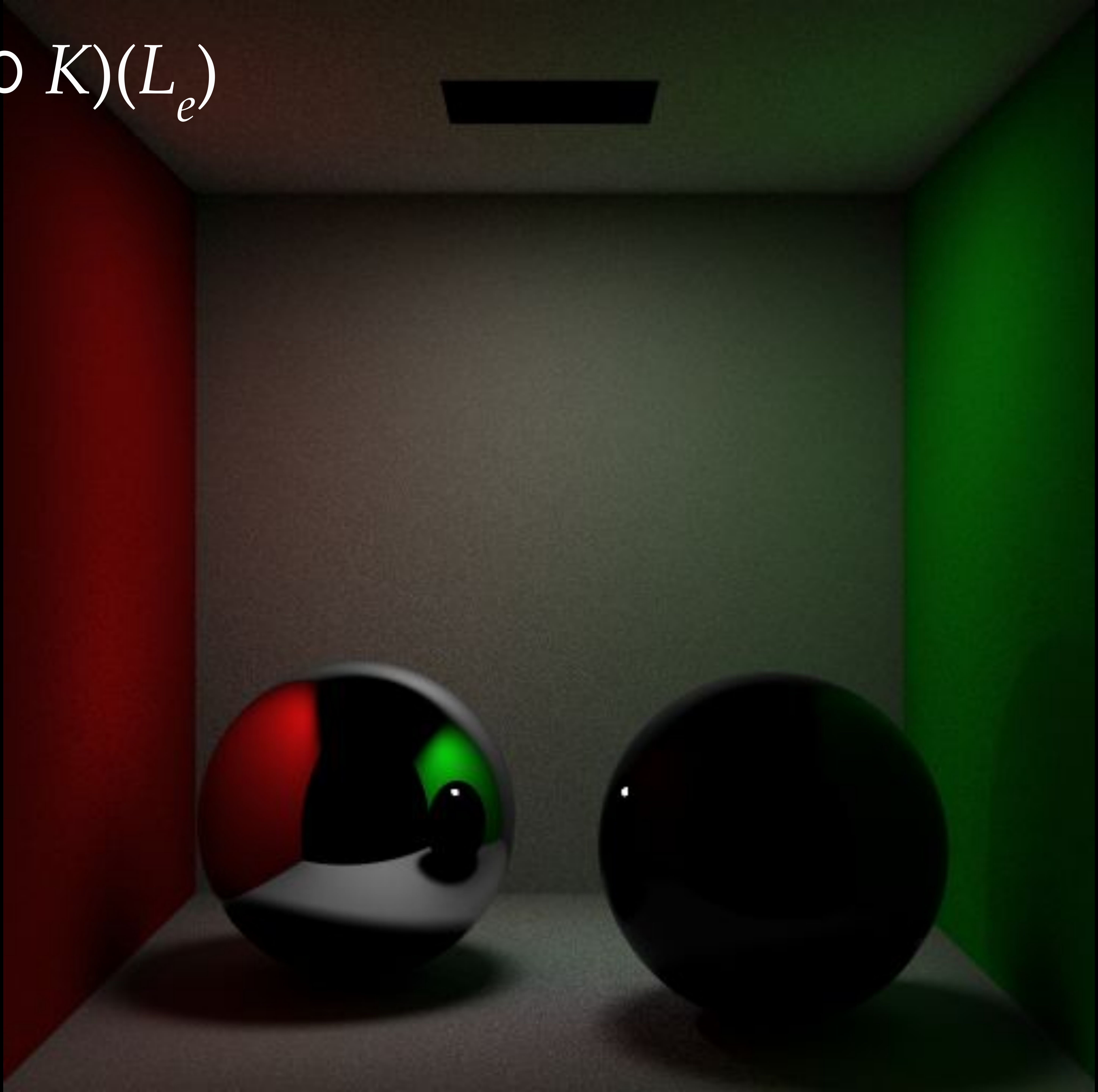
$$
\begin{aligned}
L &= (I - K)^{-1}(L_e) \\
&= (I + K + K^2 + K^3 + \cdots)(L_e) \\
&= L_e + K(L_e) + K^2(L_e) + K^3(L_e) + \cdots
\end{aligned}
$$

(L_e)

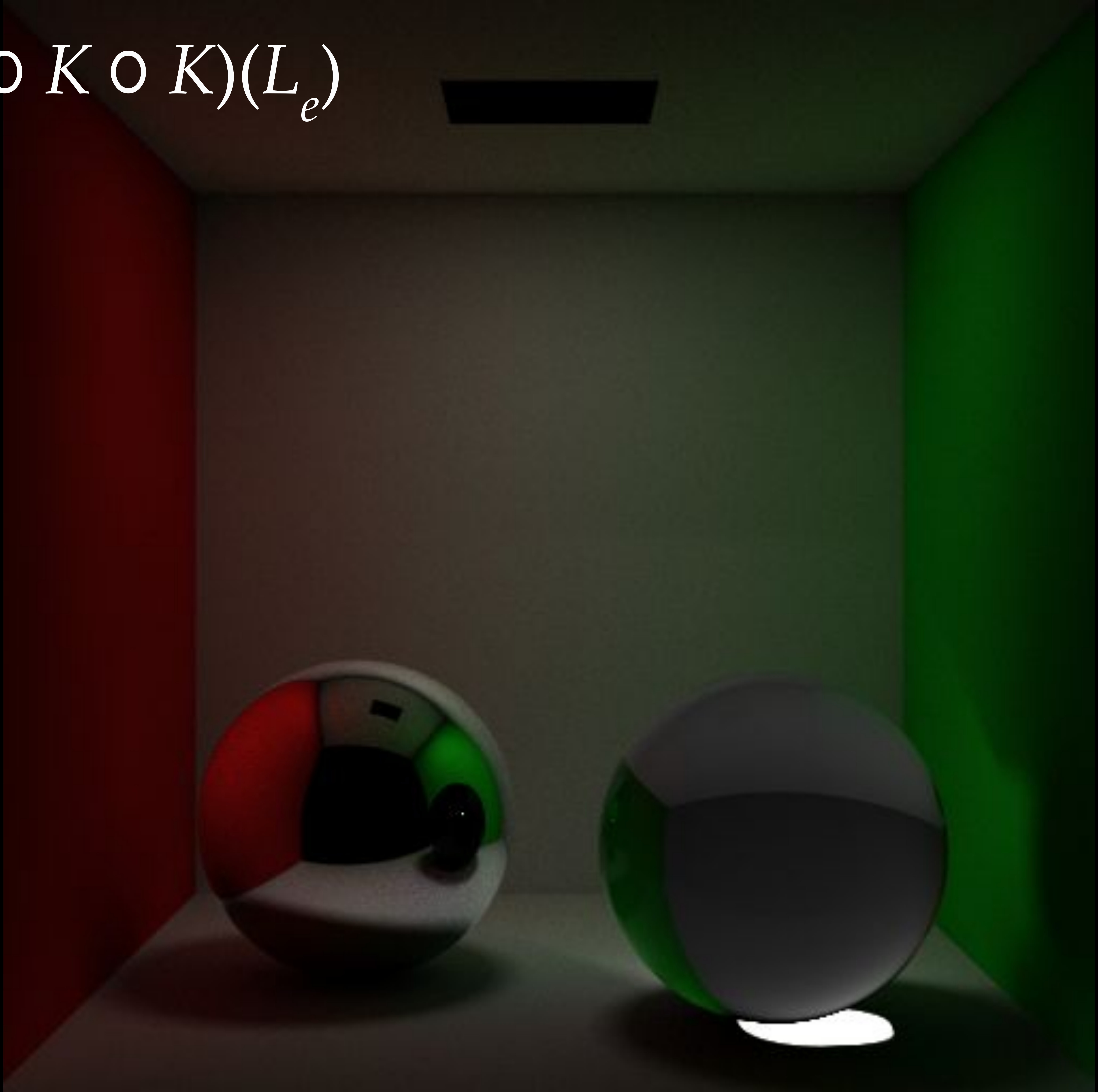
$K(L_e)$



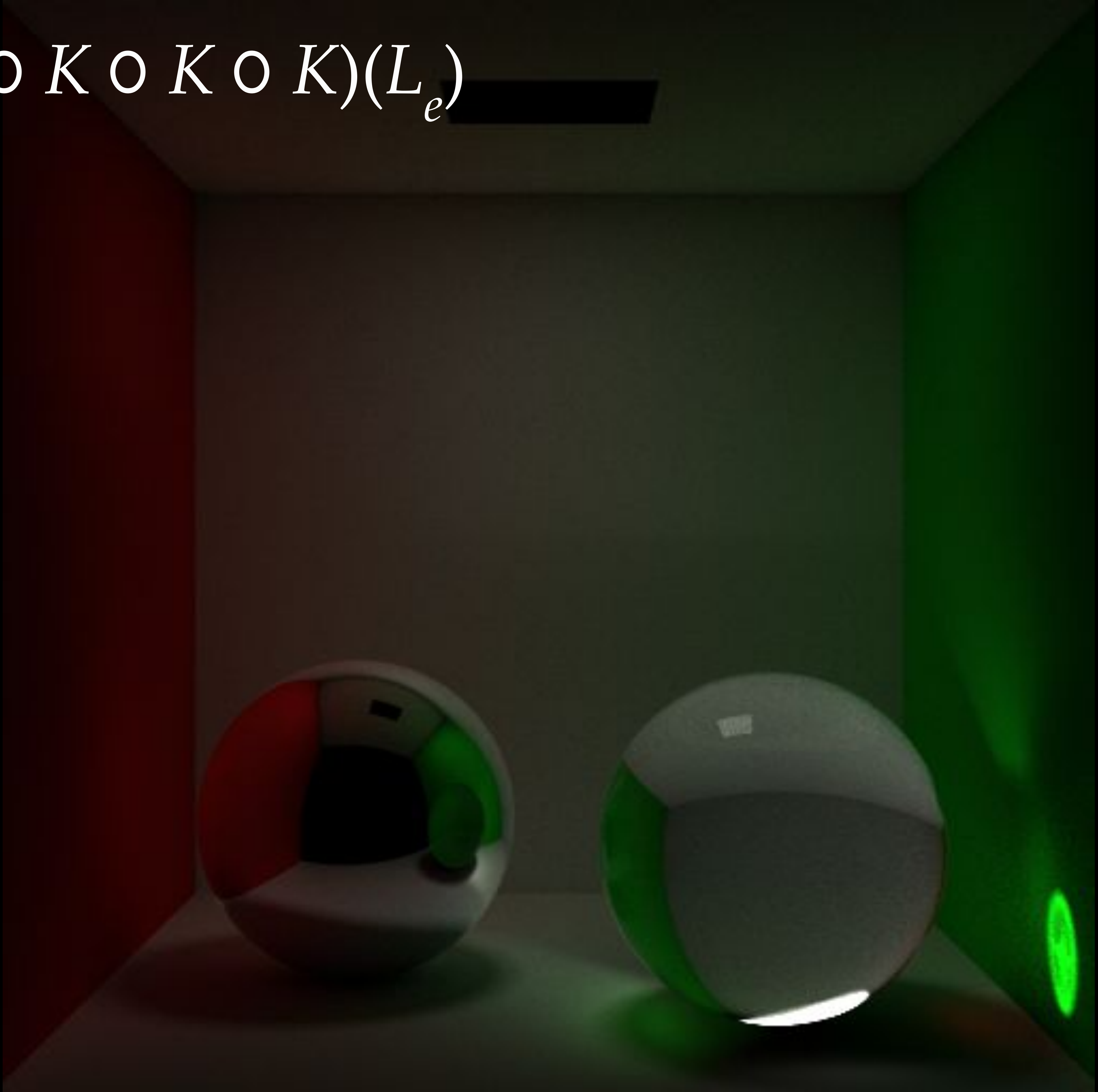
$(K \circ K)(L_e)$



$(K \circ K \circ K)(L_e)$



$$(K \circ K \circ K \circ K)(L_e)$$

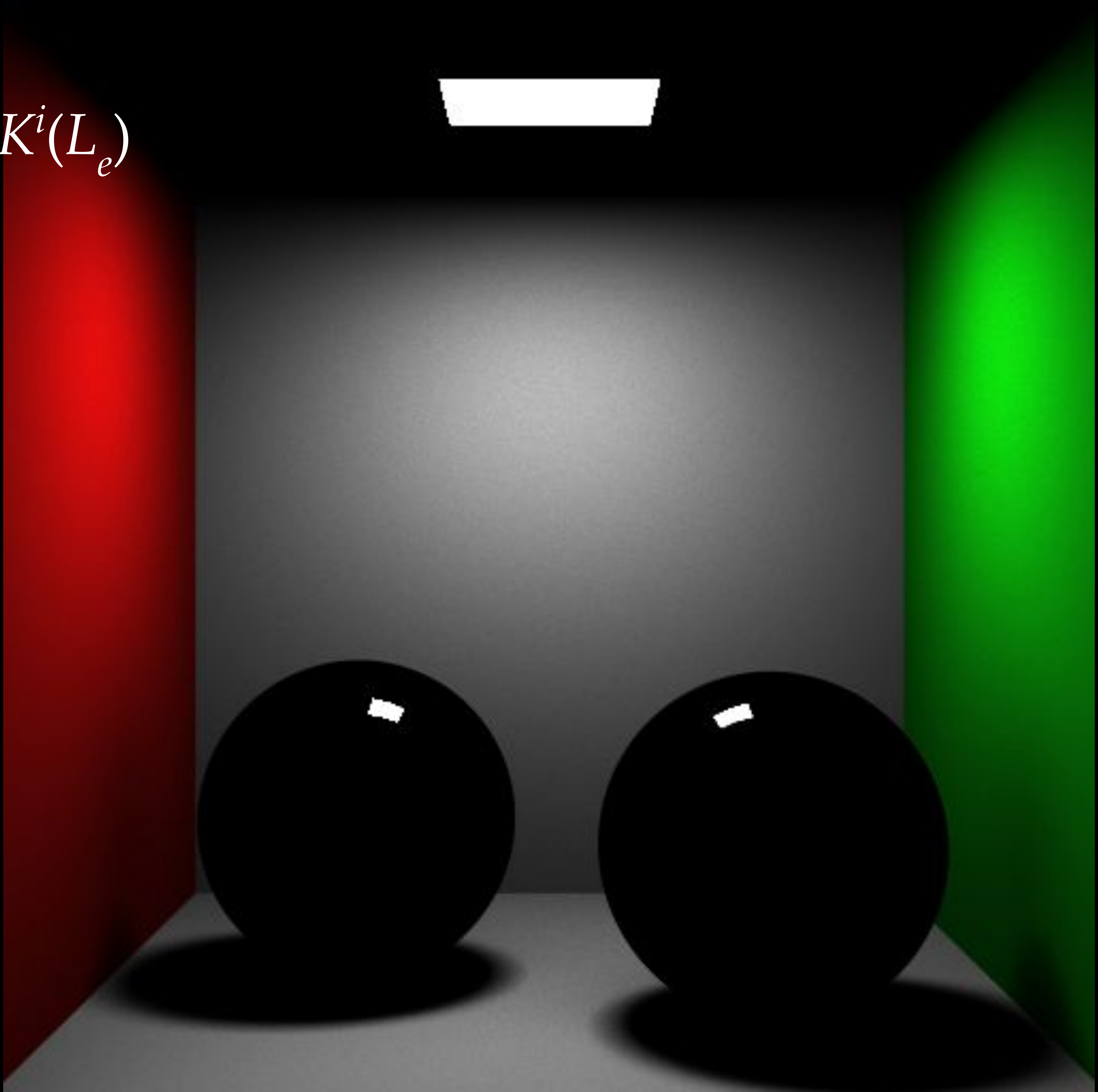


$$(K \circ K \circ K \circ K \circ K)(L_e)$$

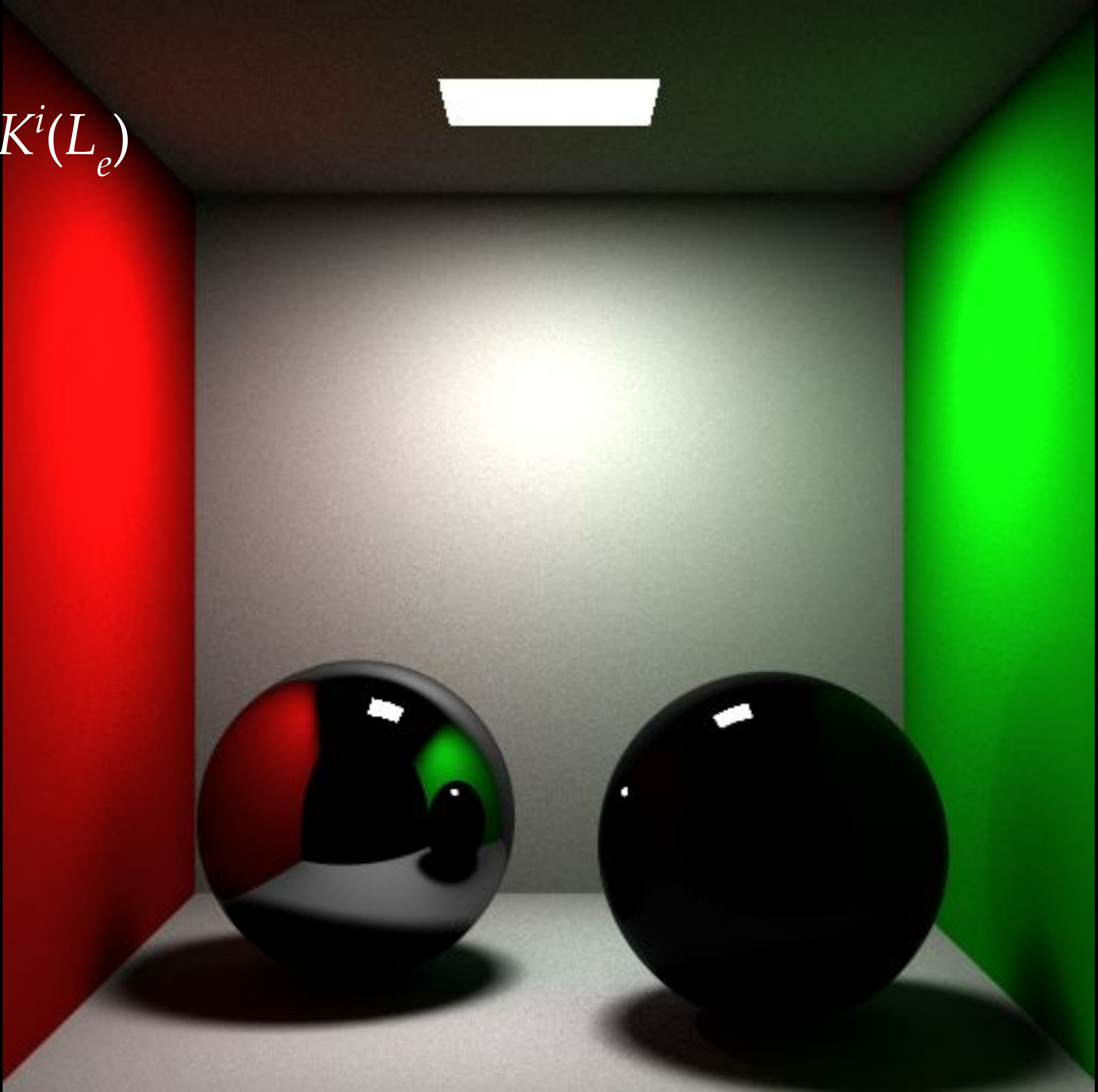
$$(K \circ K \circ K \circ K \circ K \circ K)(L_e)$$

(L_e)

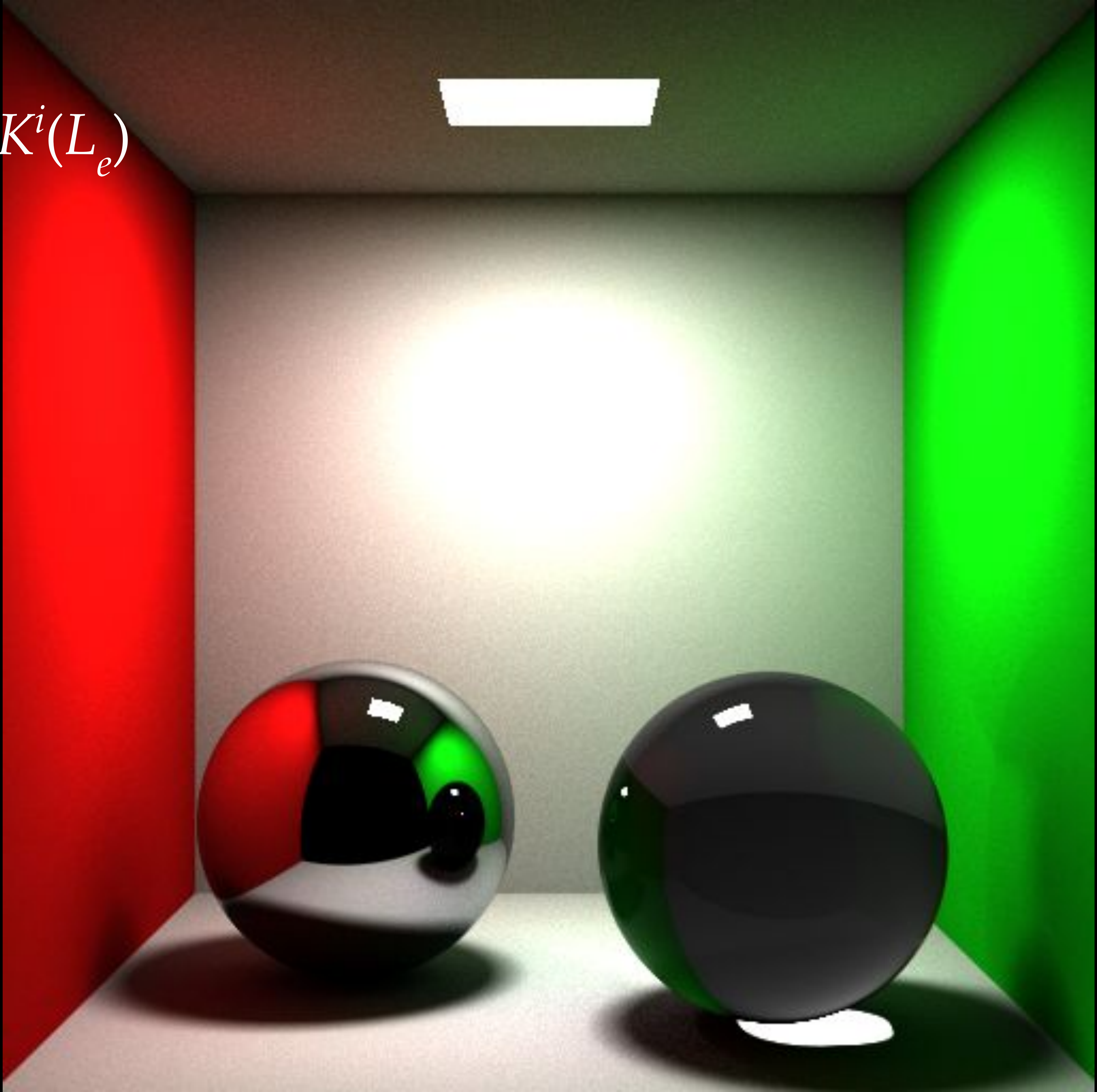
$$\sum_{i=0}^{1} K^i(L_e)$$



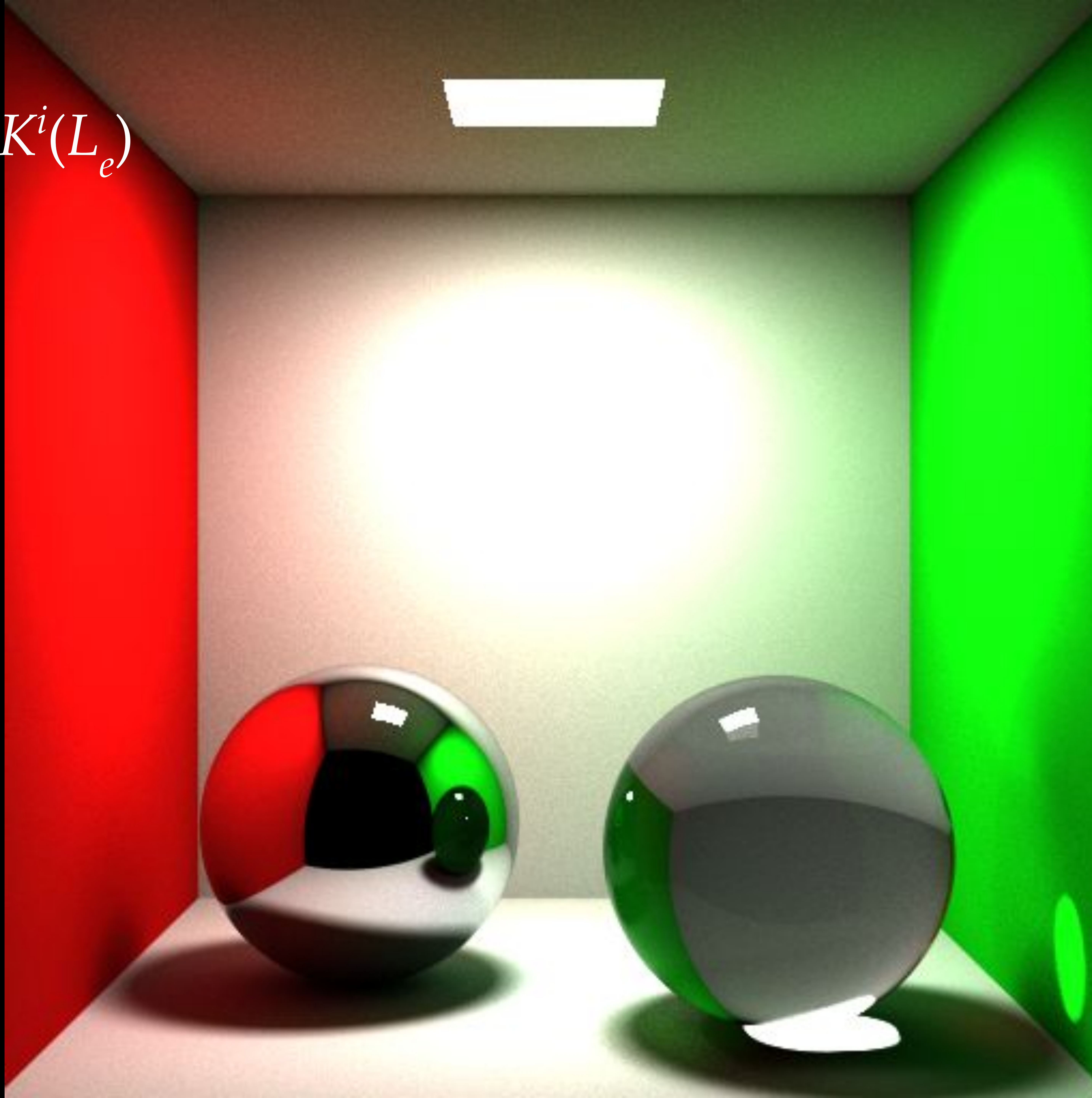
$$\sum_{i=0}^{2} K^i(L_e)$$



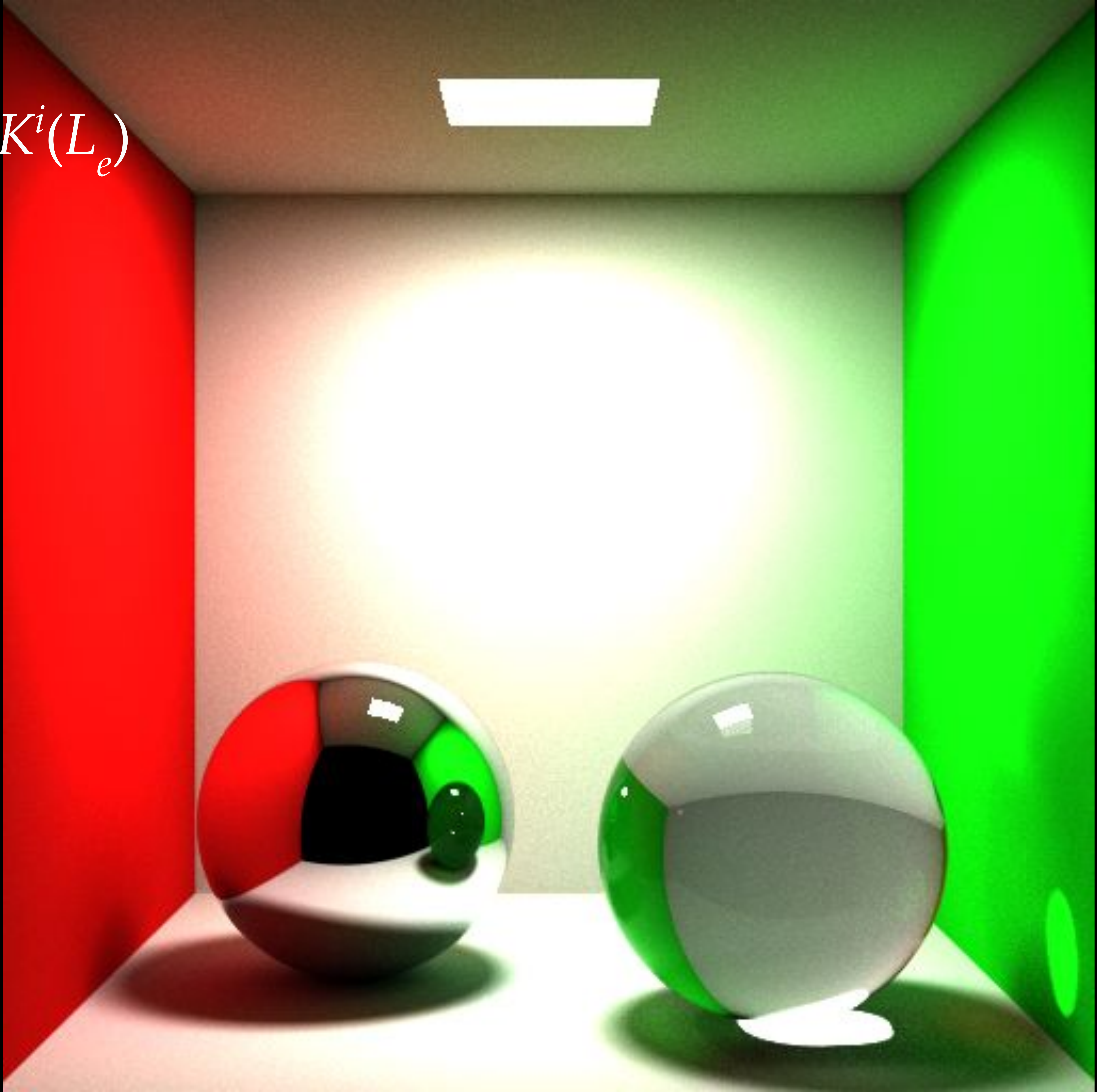
$$\sum_{i=0}^{3} K^i(L_e)$$



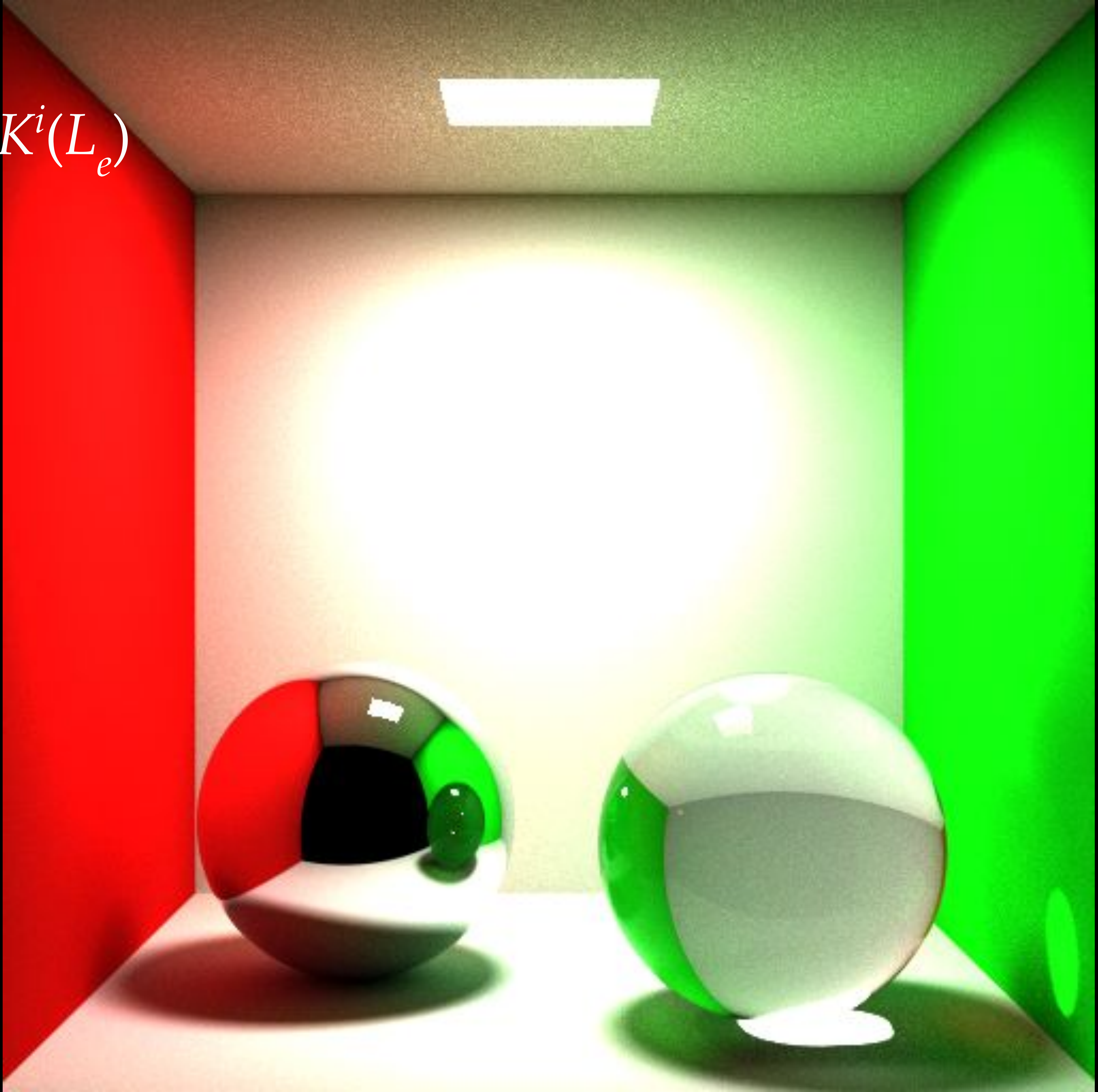
$$\sum_{i=0}^{4} K^i(L_e)$$



$$\sum_{i=0}^{5} K^i(L_e)$$



$$\sum_{i=0}^{6} K^i(L_e)$$




p
Direct illumination

p
One-bounce global illumination


p
Two-bounce global illumination

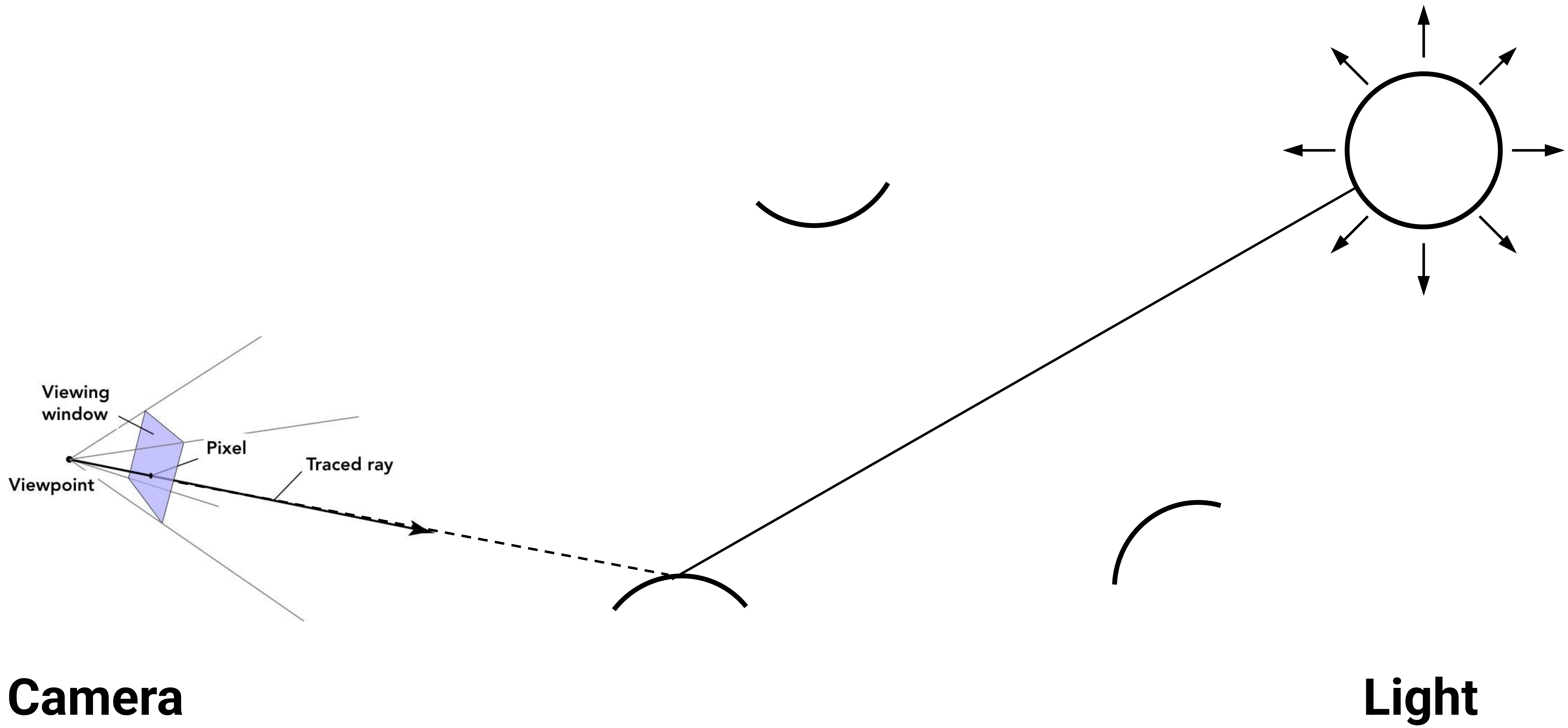
•p
Four-bounce global illumination

p
Eight-bounce global illumination

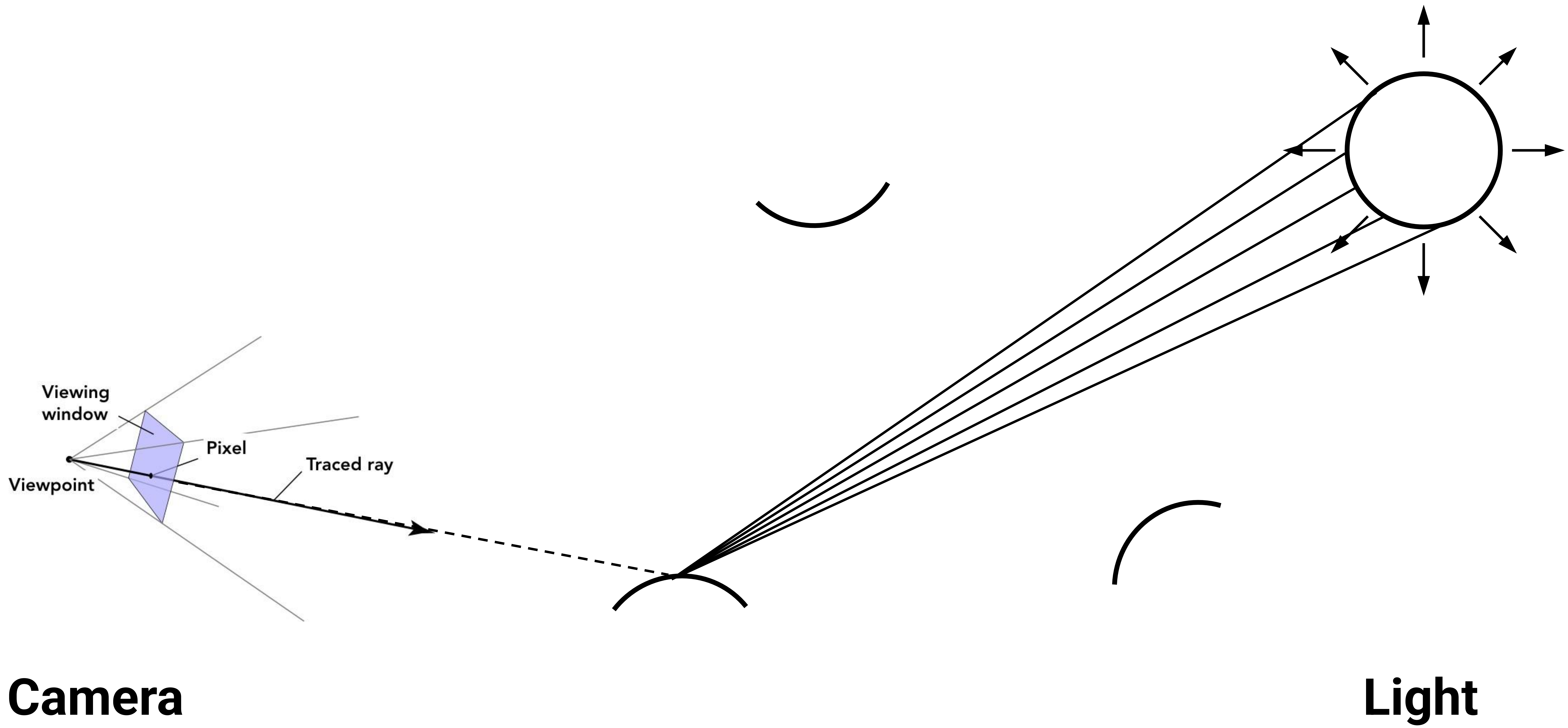

•p
Sixteen-bounce global illumination

Light Paths

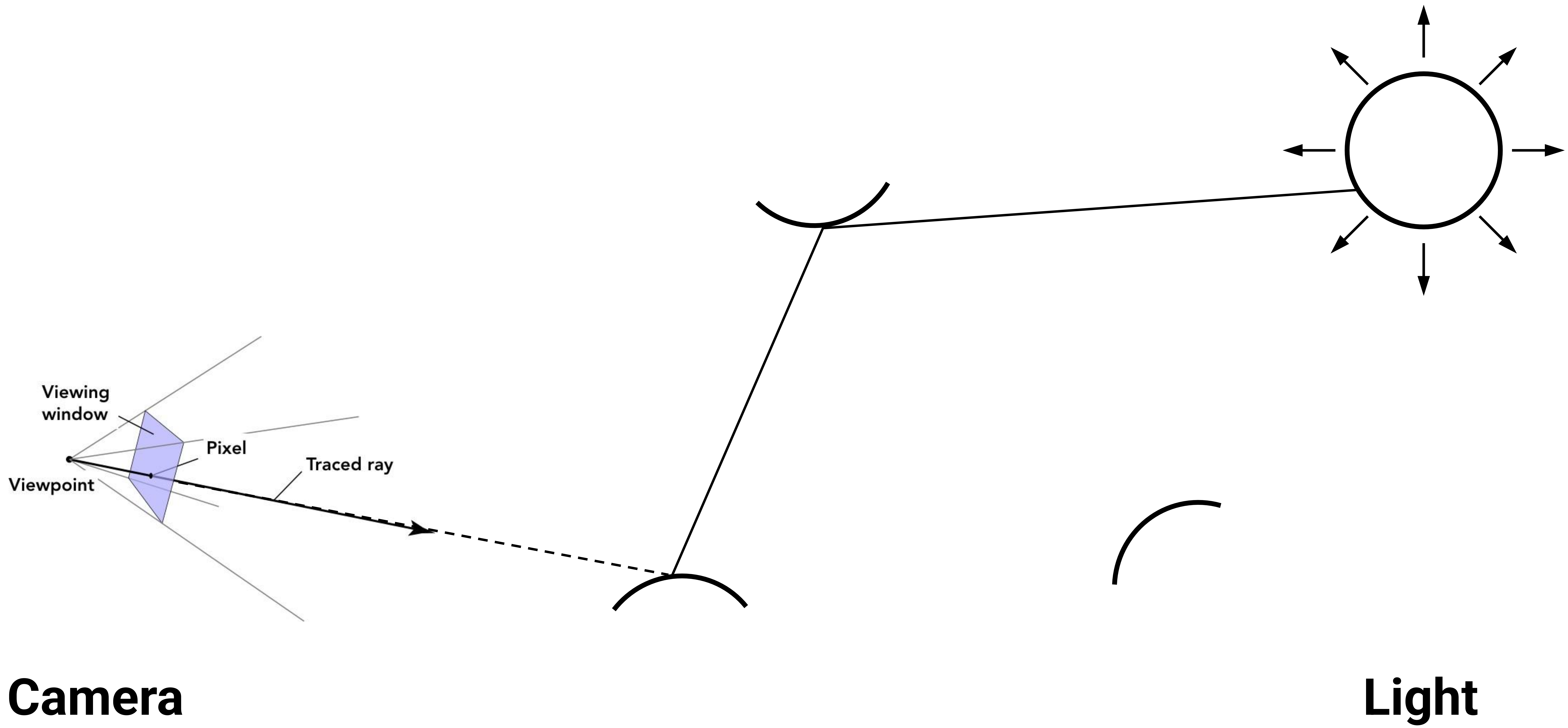
1-Bounce Path Connecting Ray to Light



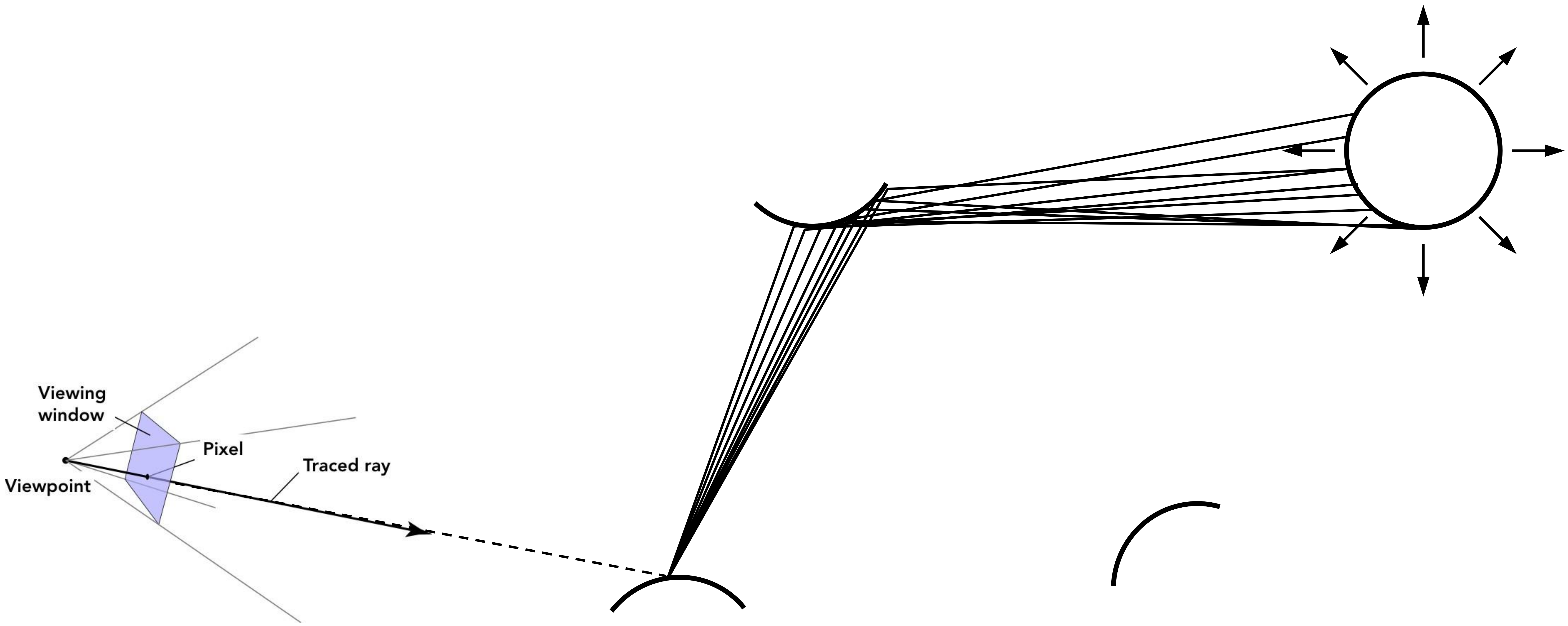
1-Bounce Paths Connecting Ray to Light



2-Bounce Path Connecting Ray to Light



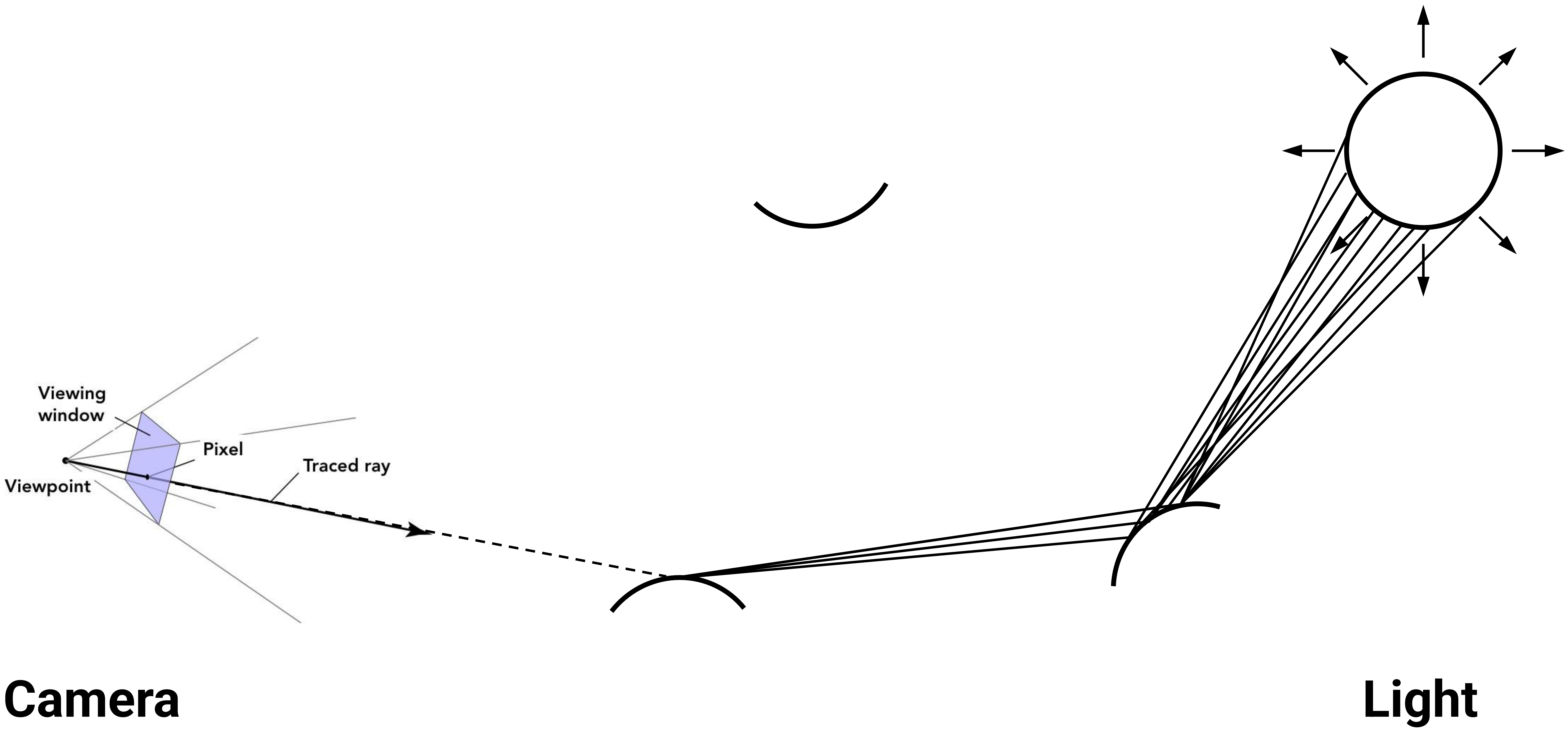
2-Bounce Paths Connecting Ray to Light



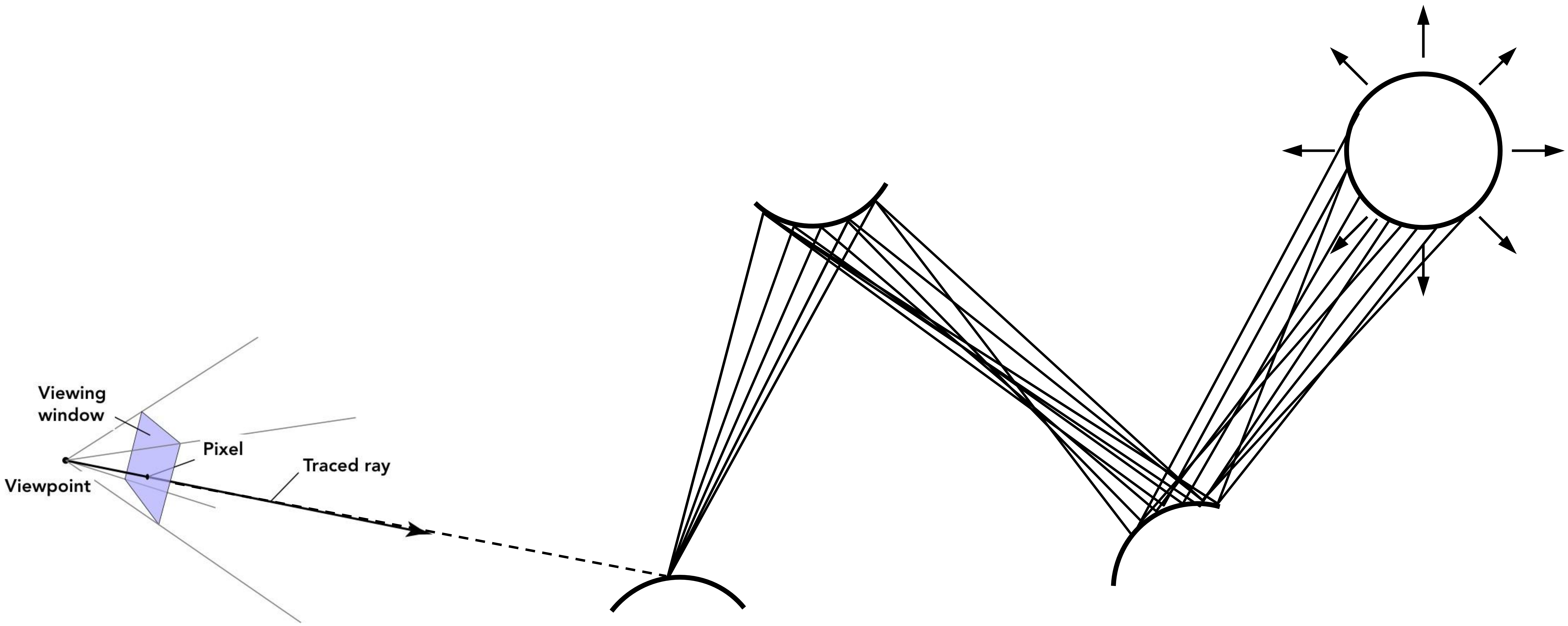
Camera

Light

2-Bounce Paths Connecting Ray to Light



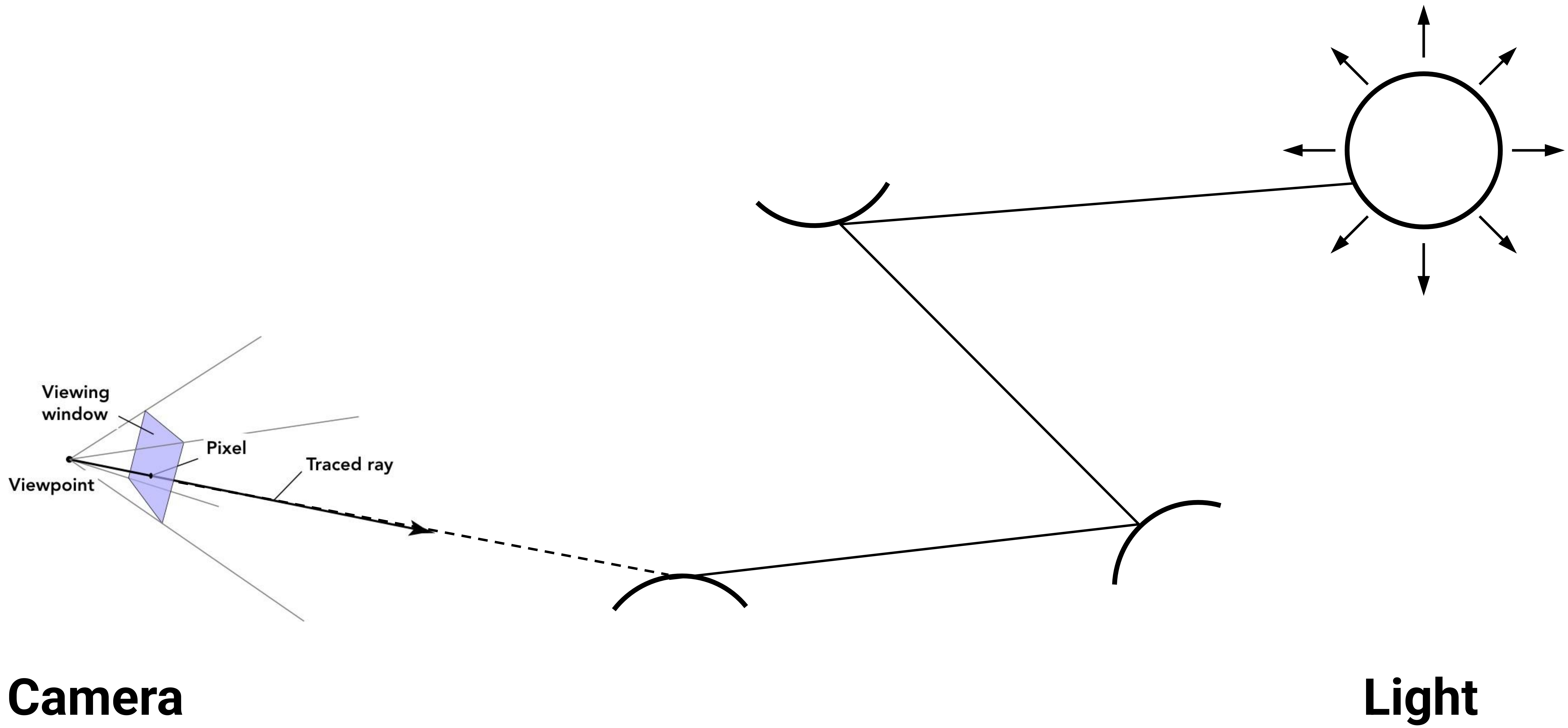
3-Bounce Paths Connecting Ray to Light



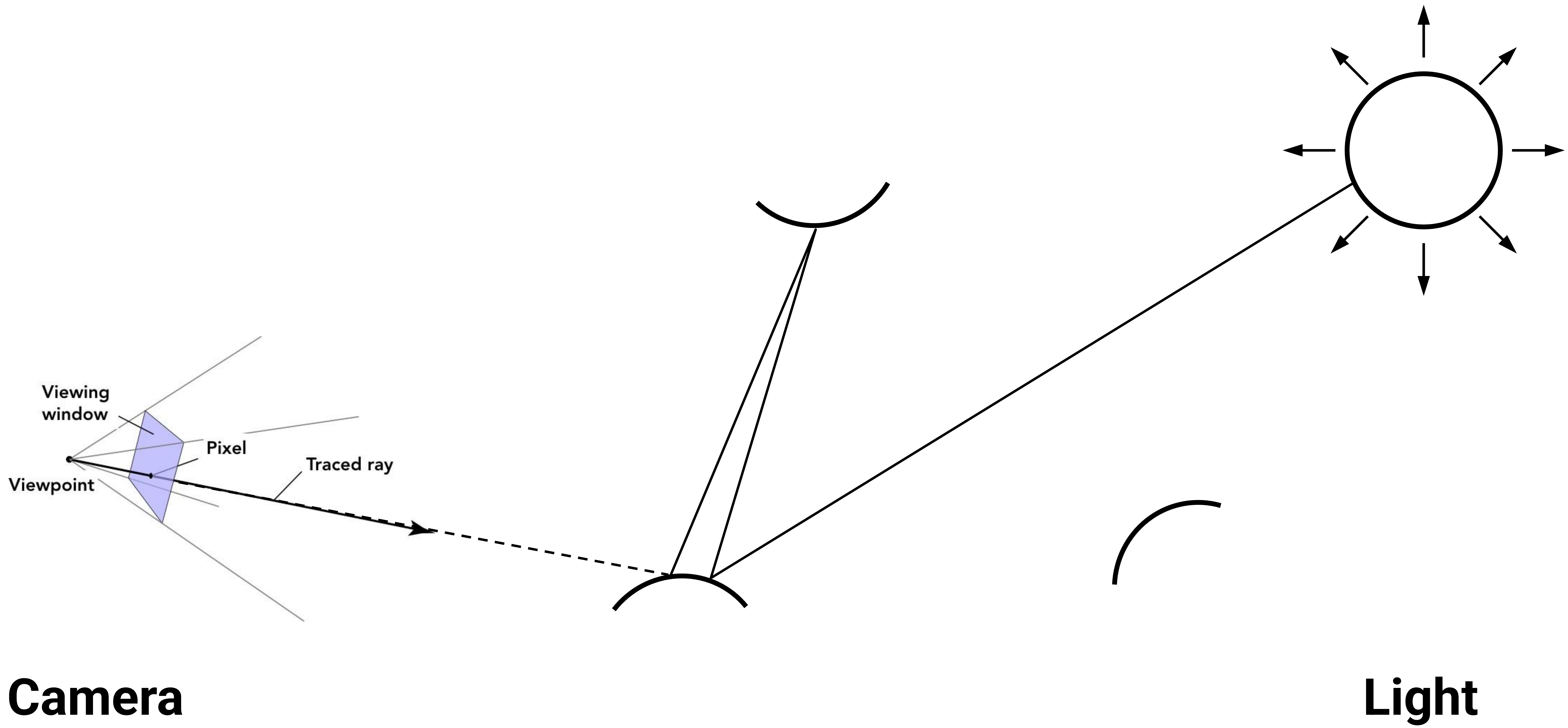
Camera

Light

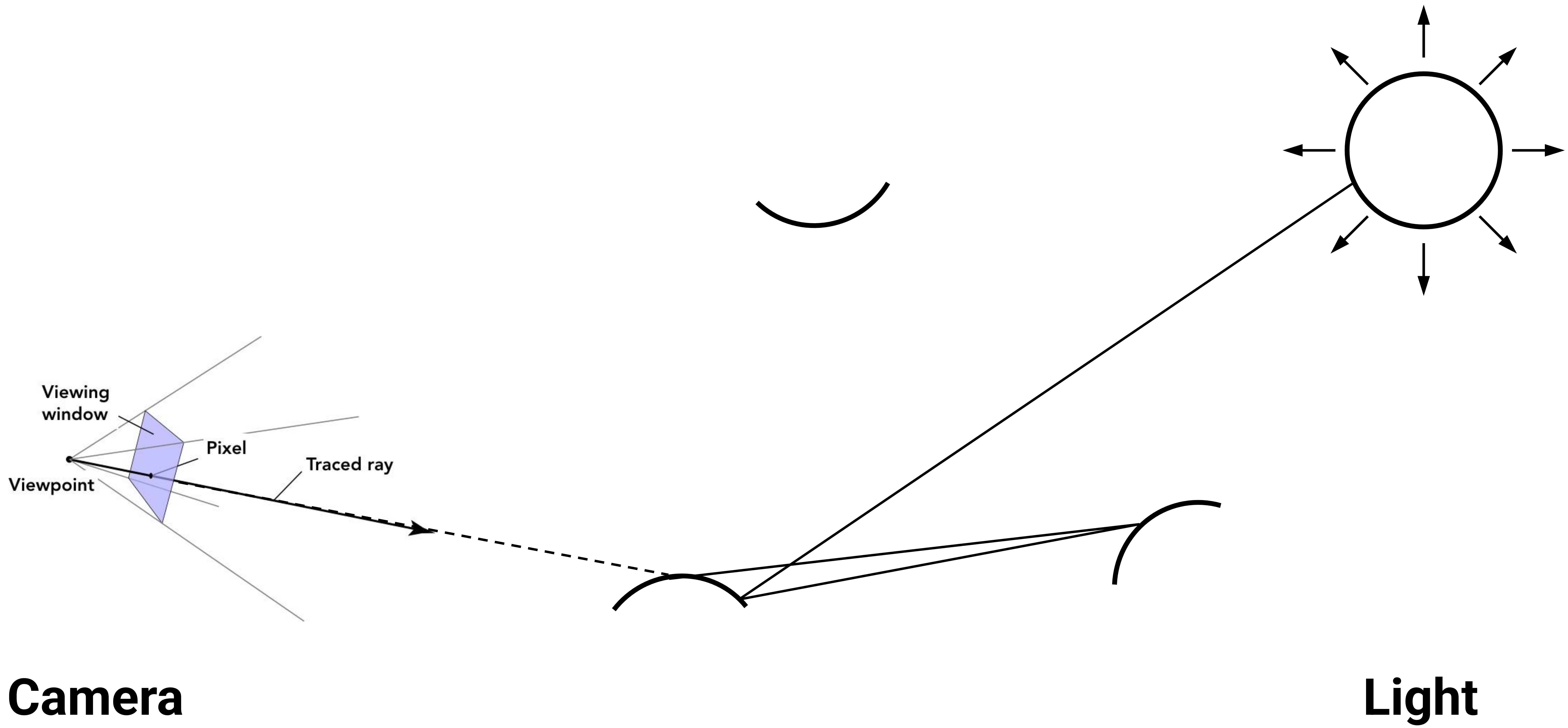
3-Bounce Paths Connecting Ray to Light



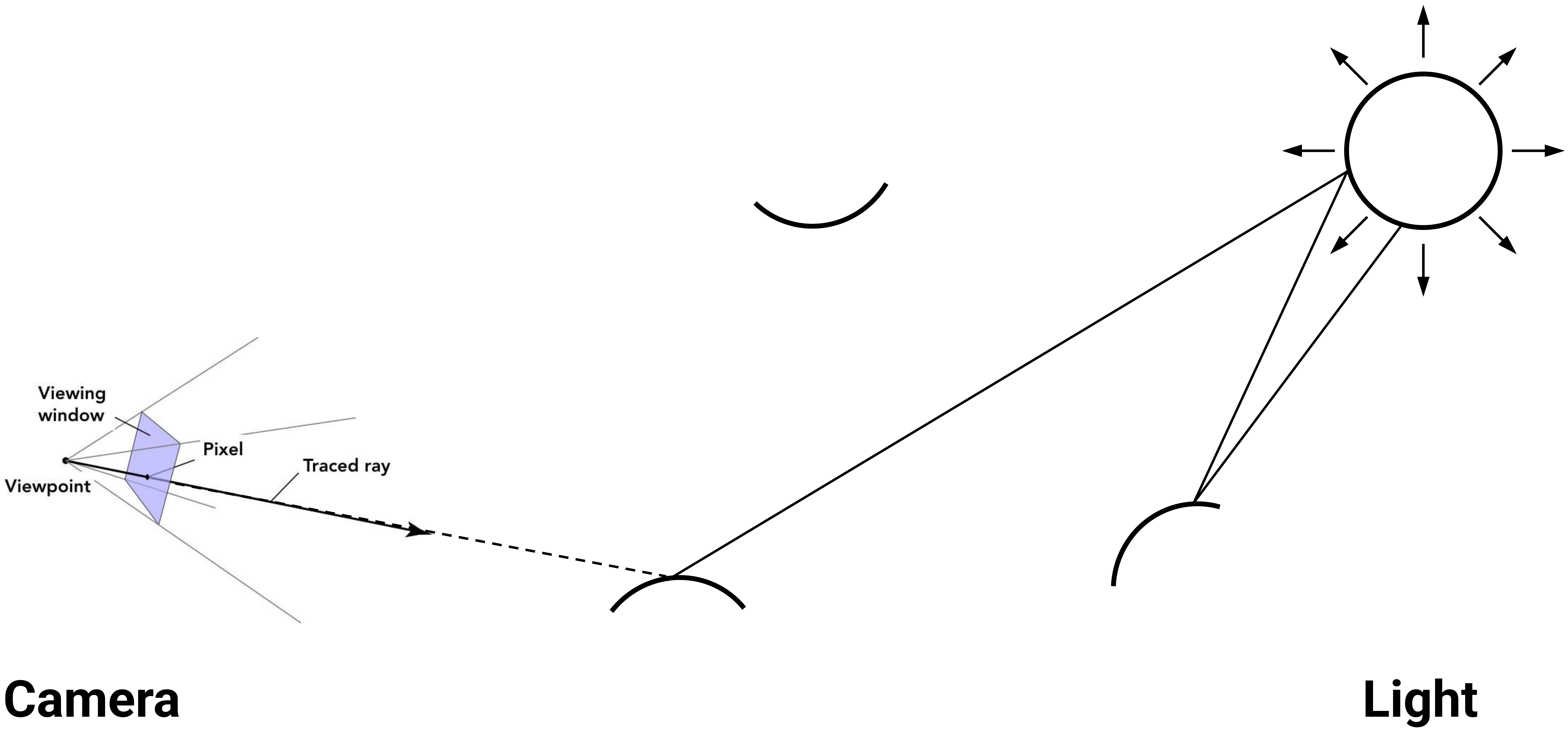
3-Bounce Paths Connecting Ray to Light



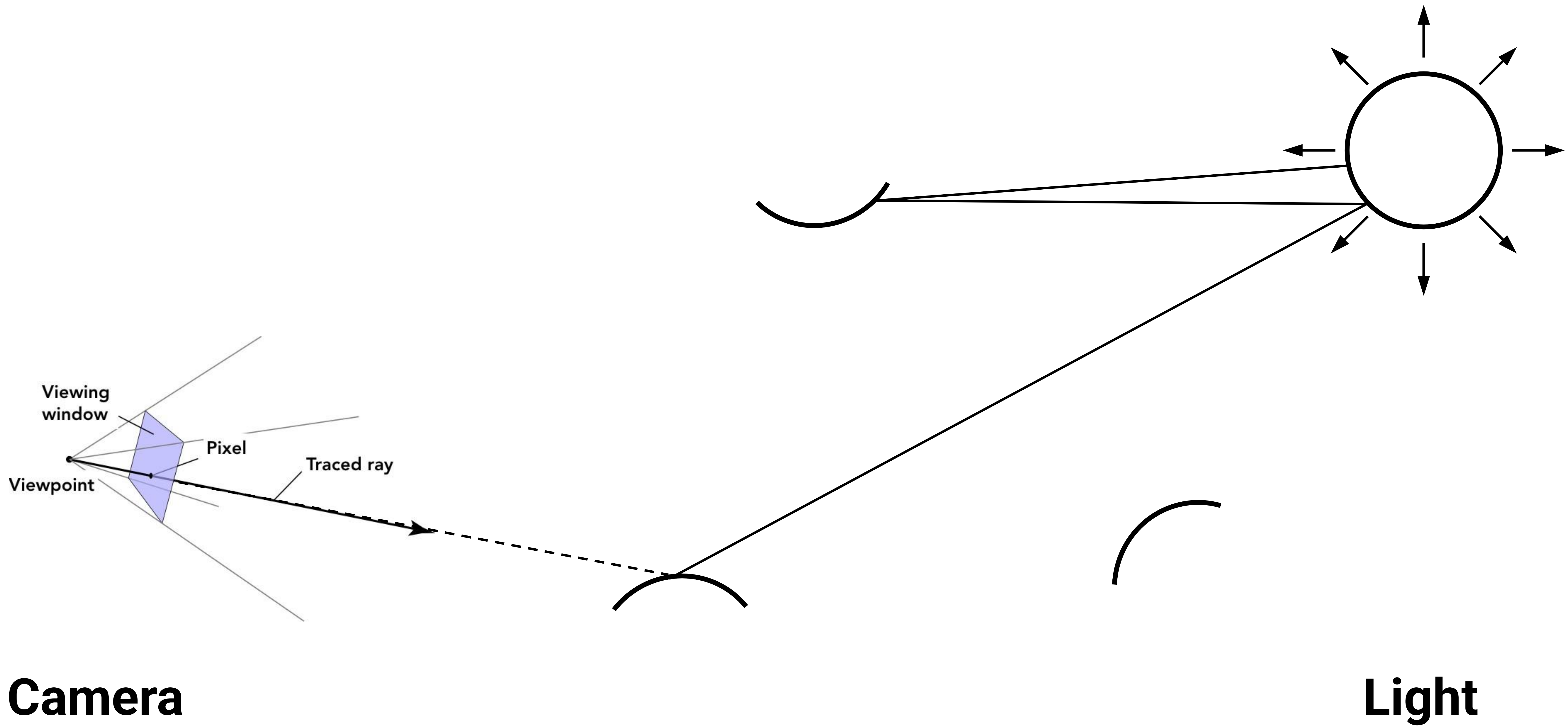
3-Bounce Paths Connecting Ray to Light



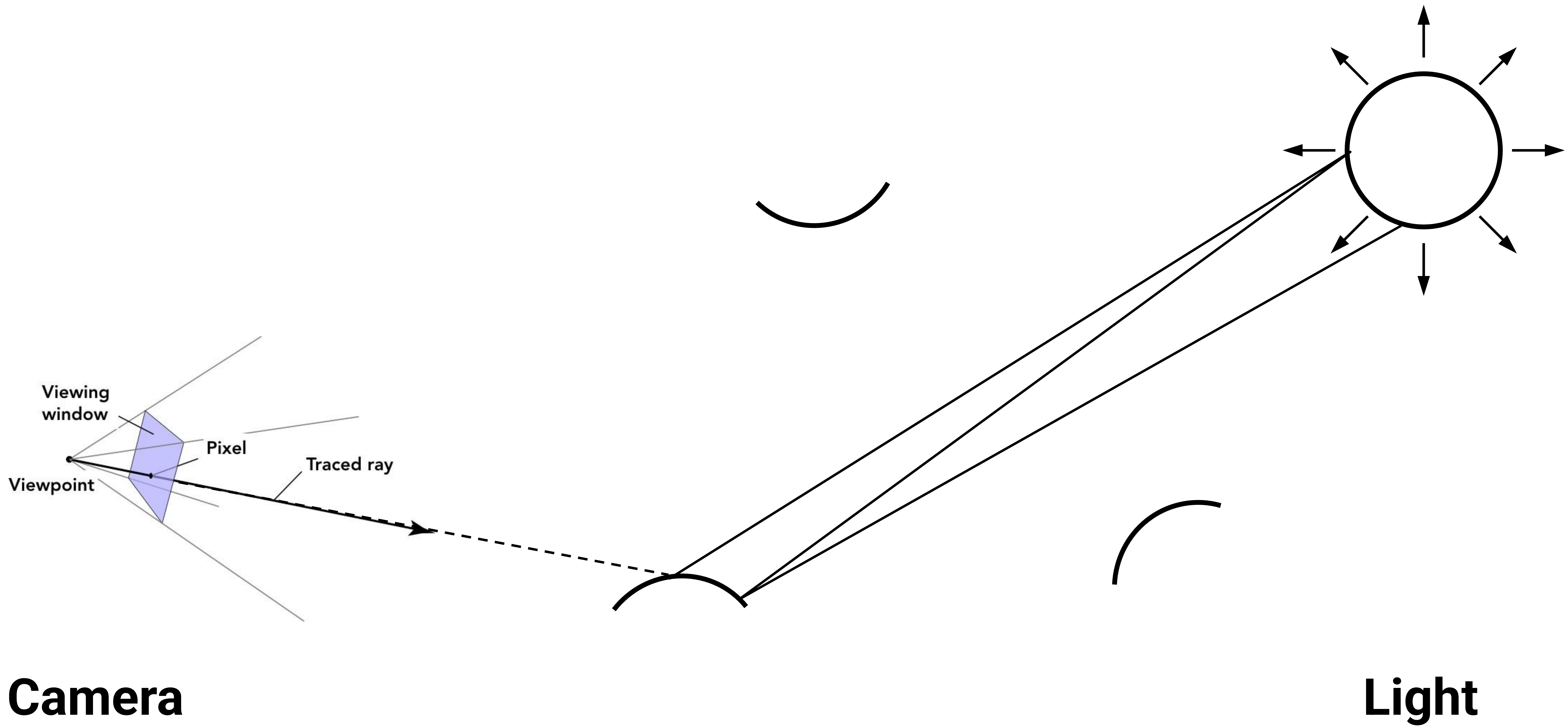
3-Bounce Paths Connecting Ray to Light



3-Bounce Paths Connecting Ray to Light



3-Bounce Paths Connecting Ray to Light



Global Illumination Rendering

GI is the sum of light over all paths, of ∞ lengths

Challenges:

- How to generate all possible paths?
- How to sample (integrate) the space of paths efficiently?

Sum Over Paths

Try 1: Monte Carlo Sum over Paths

```
EstRadianceIn(x, ω)
    p = intersectScene(x, ω);
    L = p.emittedLight(−ω);
    ωi, pdf = p.brdf.sampleDirection(−ω);
    L += EstRadianceIn(p, ωi) * p.brdf(ωi, −ω) * costheta / pdf;
    return L;
```

Problem: Infinite Bounces of Light

How to integrate over infinite dimensions?

- Note: if energy dissipates, contributions from higher bounces decrease exponentially

Idea: just use N bounces

Russian Roulette

Russian Roulette - Unbiased Random Termination

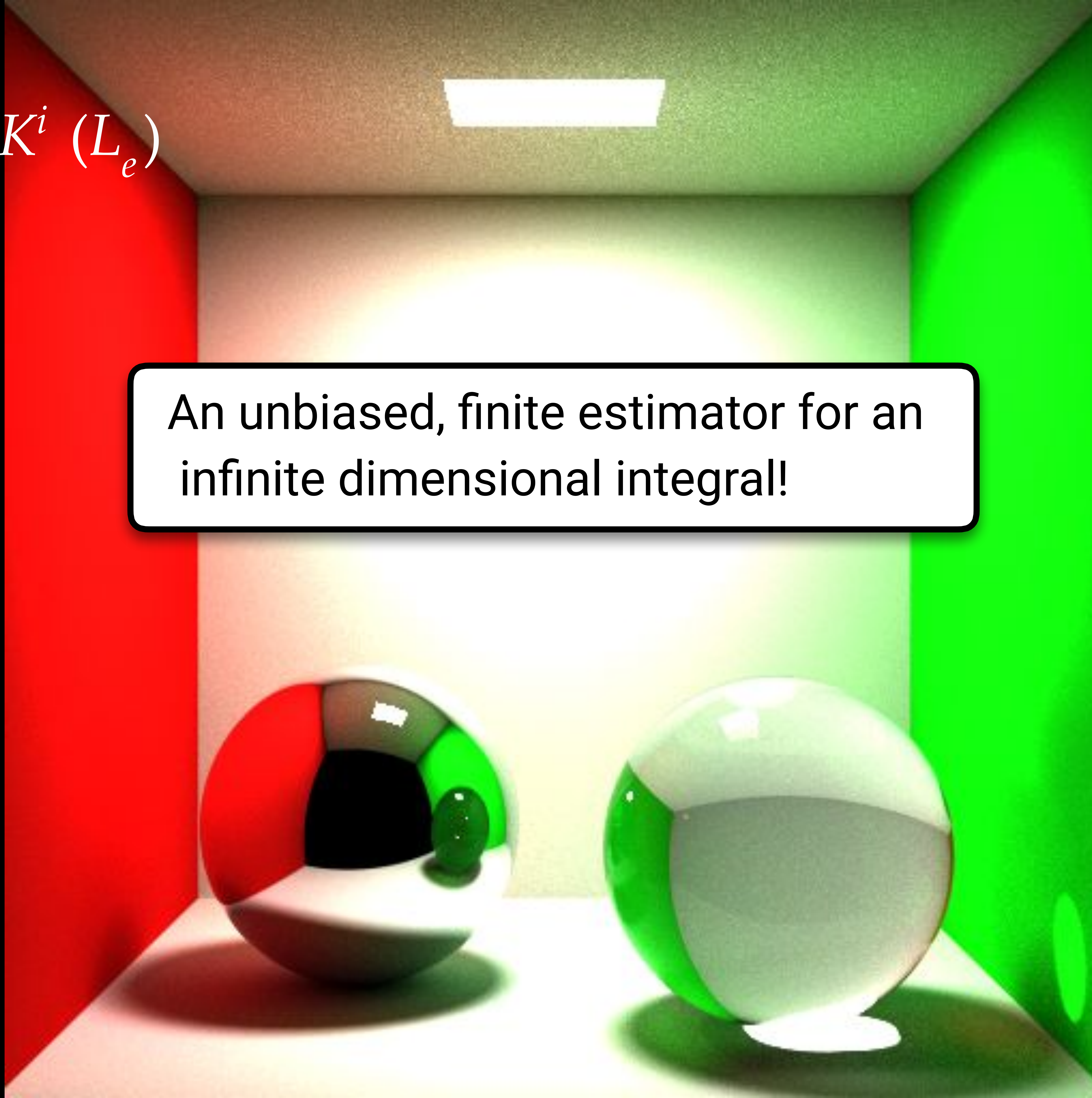
Idea: probabilistic termination of recursion

Russian Roulette: Unbiased Random Termination

New estimator: evaluate original estimator with probability p_{rr}, reweighted. Otherwise ignore.

$$\text{Let } X_{\mathrm{rr}} = \begin{cases} \frac{X}{p_{\mathrm{rr}}}, & \text{with probability } p_{\mathrm{rr}} \\ 0, & \text{otherwise} \end{cases}$$

$$\sum_{i=0}^{6} K^i (L_e)$$



An unbiased, finite estimator for an infinite dimensional integral!

Try 2: Russian Roulette Monte Carlo over Paths

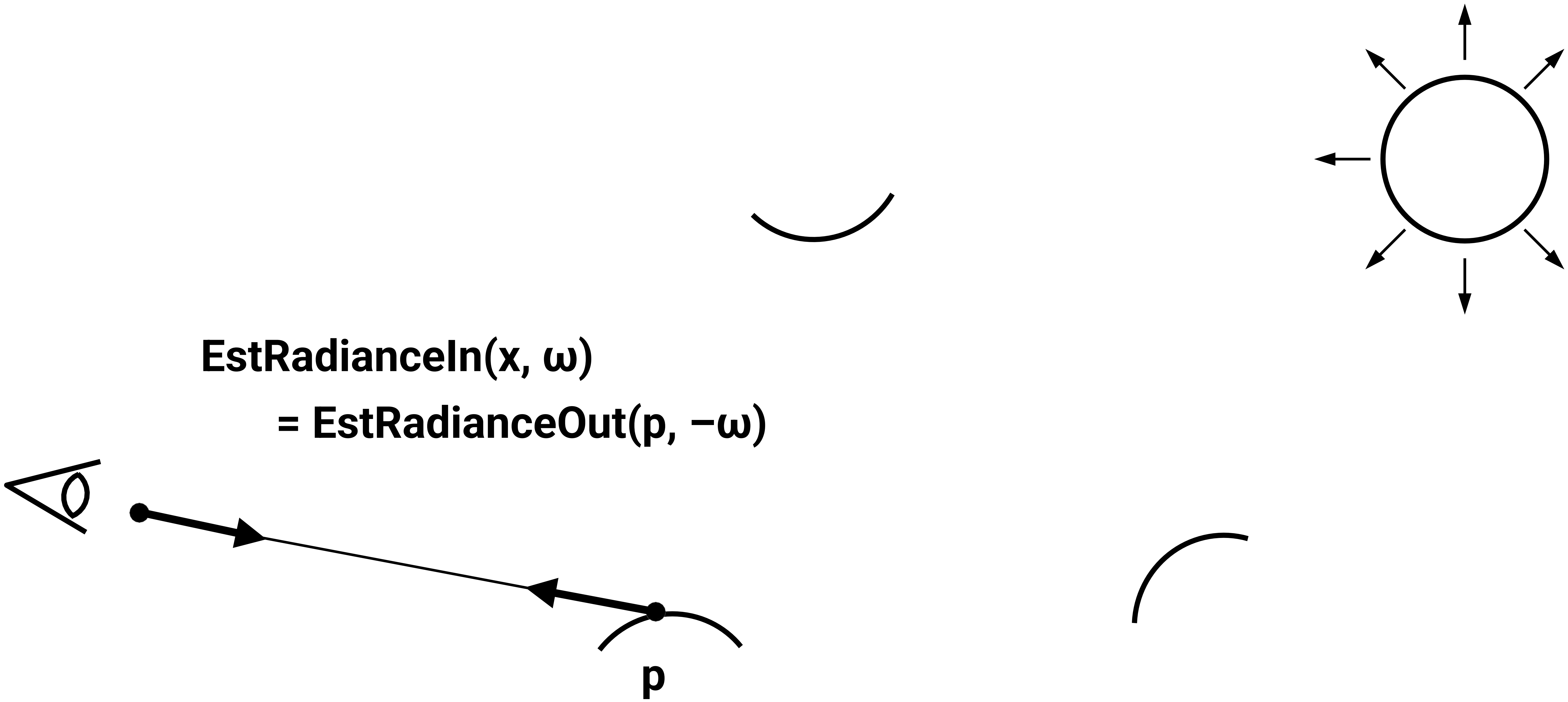
```
EstRadianceIn(x, ω)
    p = intersectScene(x, ω);
    L = p.emittedLight(−ω);
    ωi, pdf = p.brdf.sampleDirection(−ω);
    cpdf = continuationProbability(p.brdf, ωi);
    if (random01() < cpdf)                         // Russian Roulette
        L += EstRadianceIn(p, ωi)                  // Recursion
             * p.brdf(ωi, −ω) * costheta / pdf / cpdf;
    return L;

// Unbiased, computation terminates, but still extremely noisy!
```

Path Tracing

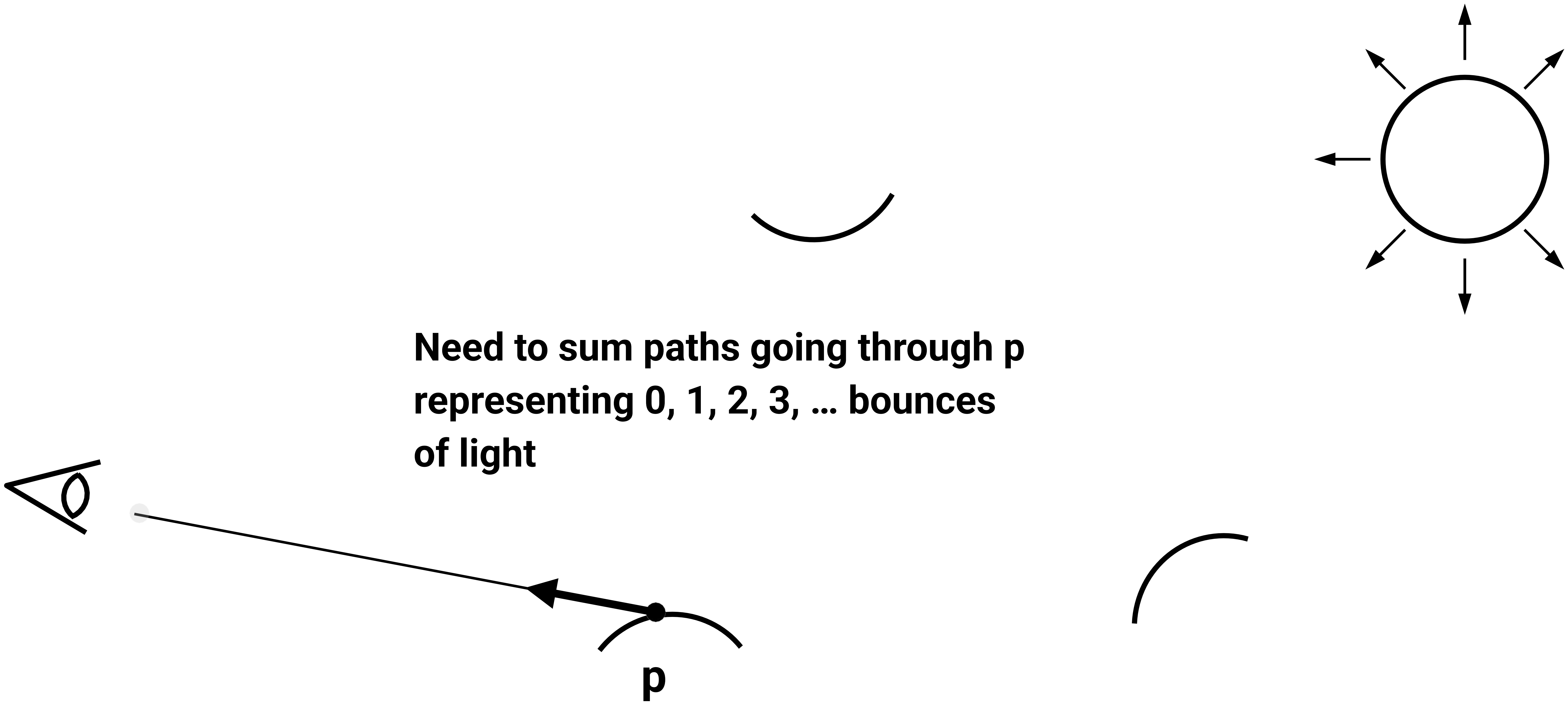
Path Tracing Overview

Terminate paths randomly with Russian Roulette then Partition the recursive radiance evaluation into two parts:

Partitioning the Rendering Equation

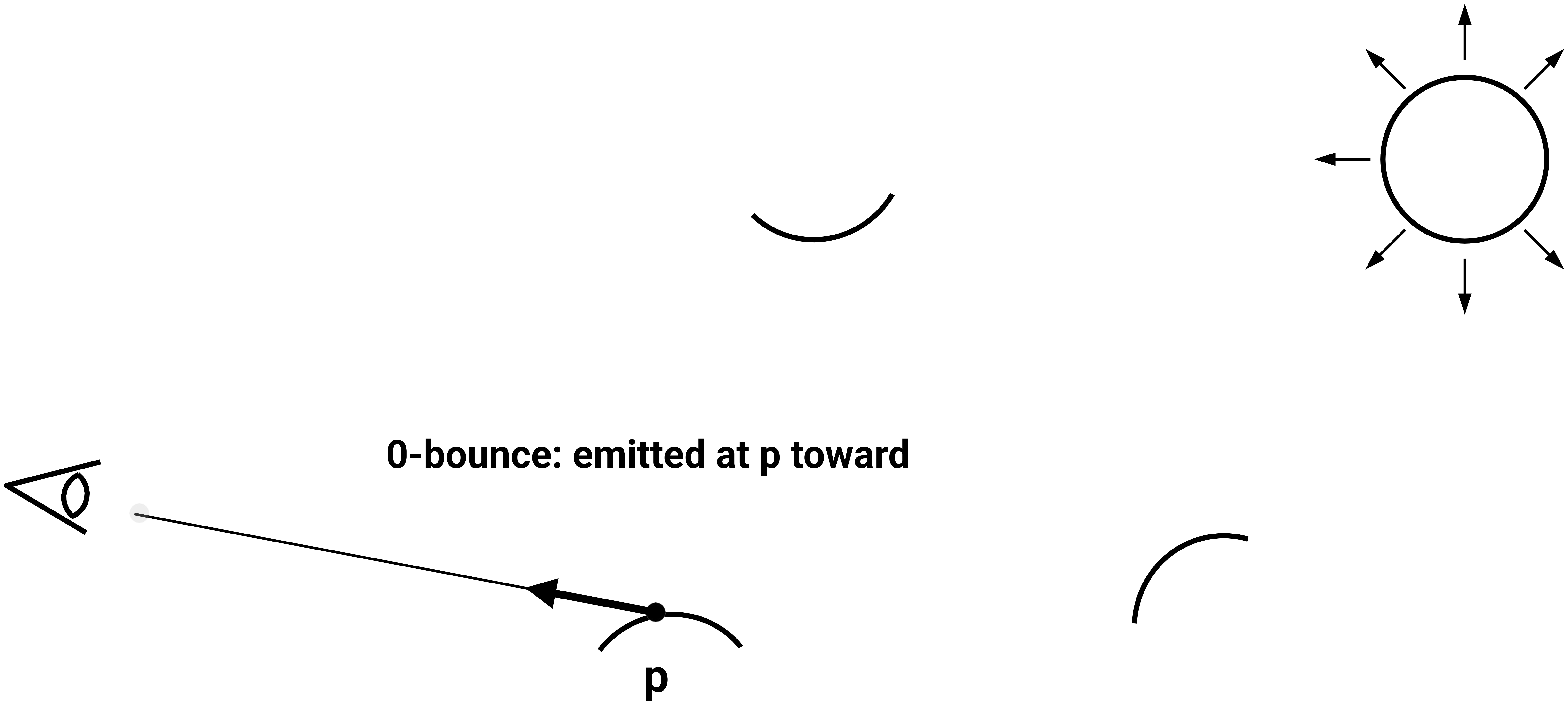


Partitioning the Rendering Equation



Need to sum paths going through p representing 0, 1, 2, 3, … bounces of light

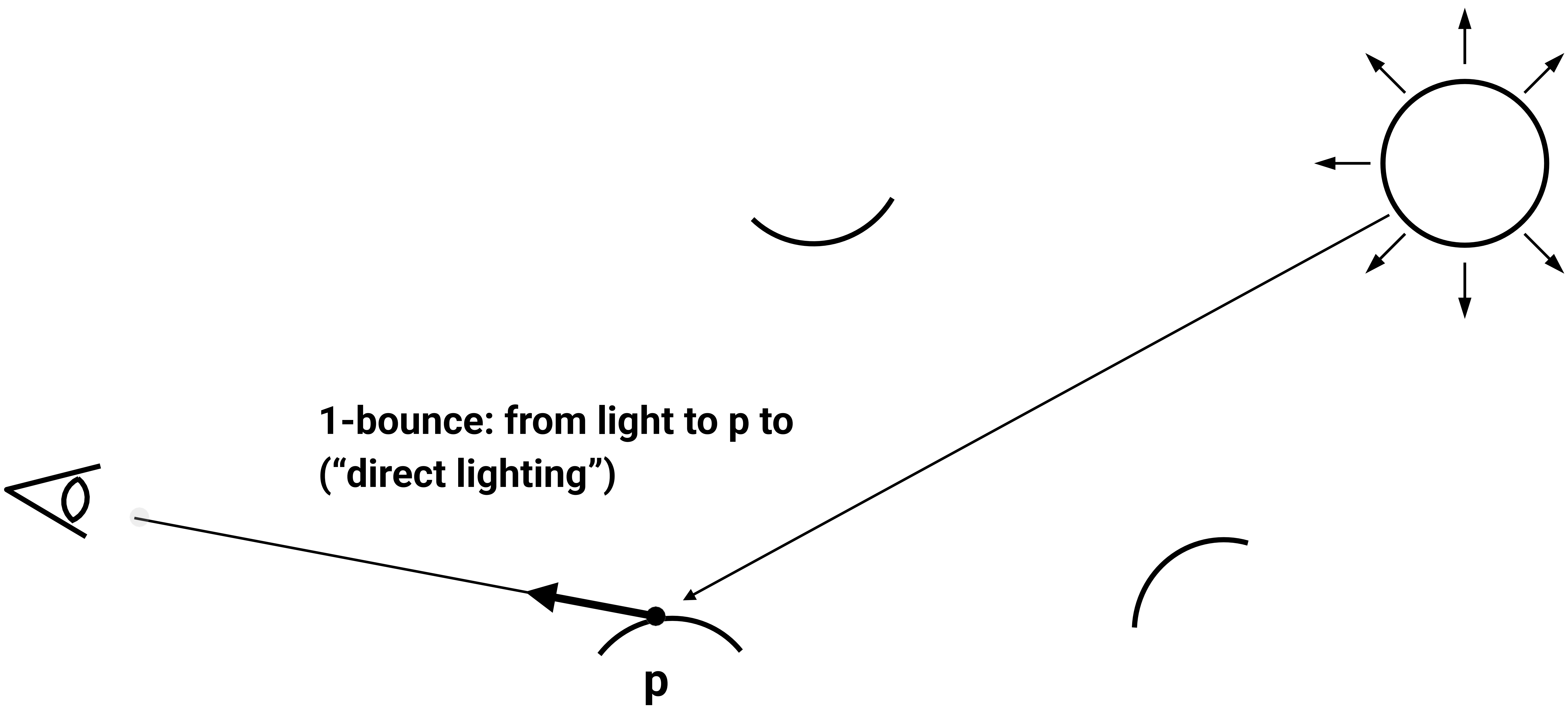
Partitioning the Rendering Equation



At p, consider light contributions from paths of varying bounce-length

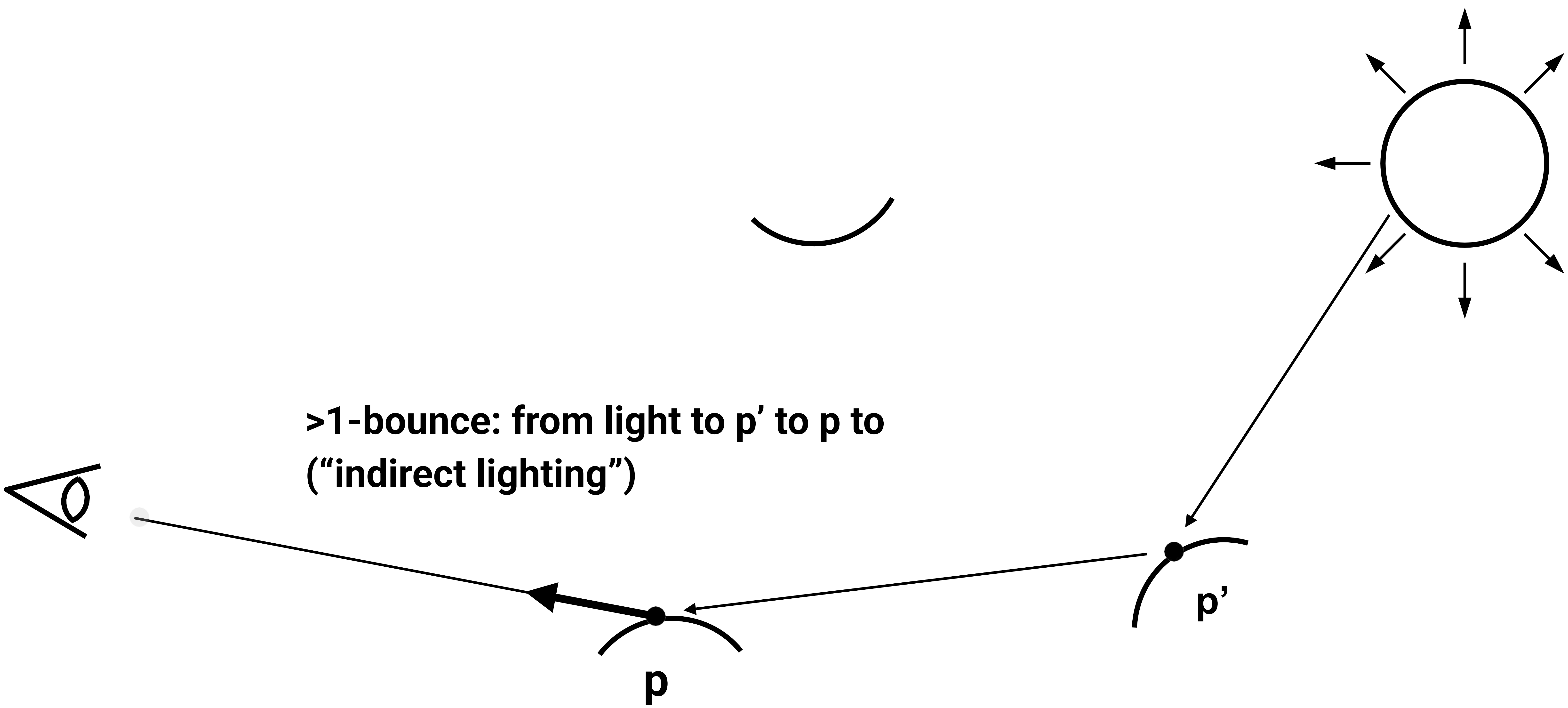
- 0-bounce: light emitted from p (p is on a light source)

Partitioning the Rendering Equation



At p, consider light contributions from paths of varying bounce-length
- **0-bounce: light emitted from p (p is on a light source)**
- **1-bounce: from light to p to x ("direct illumination")**

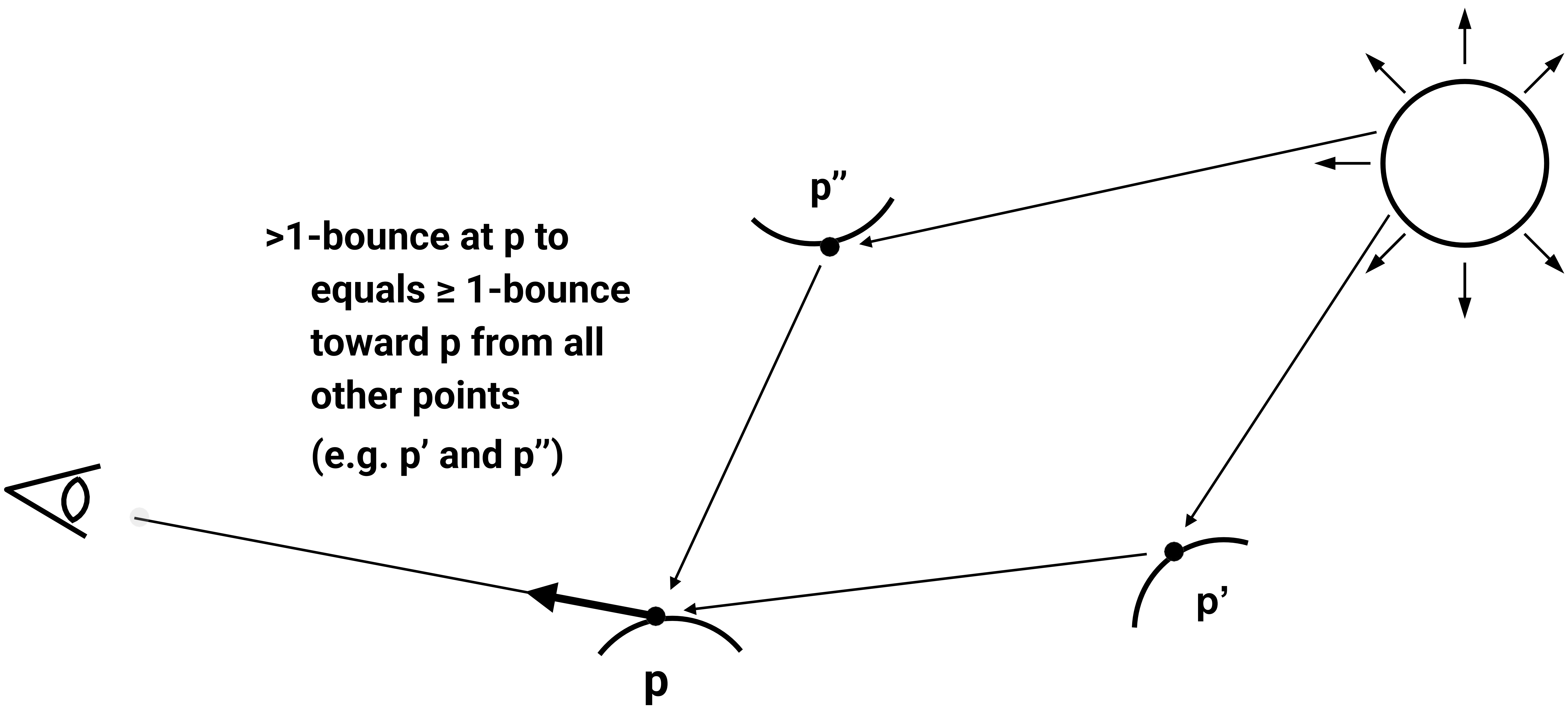
Partitioning the Rendering Equation



At p, consider light contributions from paths of varying bounce-length

- **0-bounce: light emitted from p (p is on a light source)**
- **1-bounce: from light to p to x (“direct illumination”)**
- **>1-bounce: from light to at least one other point to p to x (“indirect illumination”)**

Consider Evaluation of >1 Bounce of Light



>1-bounce at p to equals ≥ 1-bounce toward p from all other points (e.g. p' and p'')

At p, consider light contributions from paths of varying bounce-length

- 0-bounce: light emitted from p (p is on a light source)
- 1-bounce: from light to p to x ("direct illumination")
- >1-bounce: from light to at least one other point to p to x ("indirect illumination")

Path Tracing Pseudocode

```
EstRadianceIn(x, ω)    // incoming at x from dir ω
    p = intersectScene(x, ω);
    return ZeroBounceRadiance(p, −ω)
            + AtLeastOneBounceRadiance(p, −ω);

ZeroBounceRadiance(p, ωo)    // outgoing at p in dir ωo
return p.emittedLight(ωo);
```

Path Tracing Pseudocode

```
AtLeastOneBounceRadiance(p, ωo)                       // out at p, dir ωo
    L = OneBounceRadiance(p, ωo);                     // direct illum

    ωi, pdf = p.brdf.sampleDirection(ωo);             // Imp. sampling
    p’ = intersectScene(p, ωi);

    cpdf = continuationProbability(p.brdf, ωi, ωo);
    if (random01() < cpdf)                            // Russ. Roulette
        L += AtLeastOneBounceRadiance(p’, −ωi)        // Recursive est. of
        * p.brdf(ωi, ωo) * costheta / pdf / cpdf;     // indirect illum
    return L;


OneBounceRadiance(p, ωo)                   // out at p, dir ωo return
    DirectLightingSampleLights(p, ωo);     // direct illum
```

Direct Lighting Pseudocode (Lights)

```
DirectLightingSamplingLights(p, ωo)
    L, ωi, pdf = lights.sampleDirection(p);     // Imp. sampling

    if (scene.shadowIntersection(p, ωi))        // Shadow ray
        return 0;
    else
        return L * p.brdf(ωi, ωo) * costheta / pdf;

// Note: only one random sample over all lights.
// Assignment 3-1 asks you to, alternatively, loop over
// multiple lights and take multiple samples (later slide)
```

Direct Lighting (0 and 1 Bounce Only)

Path Tracing (All Bounces of Light)

One sample (path) per pixel

32 samples (paths) per pixel

1024 samples (paths) per pixel

Summary of Intuition on Global Illumination Path Tracing

Summary of Intuition on G.I. & P.T.

- Operator notation leads to insight that solution is adding successive bounces of light
- Trace N paths through a pixel, sample radiance
- Build paths by recursively tracing to next surface point and choosing a random reflection direction. At each surface, sum emitted light and reflected light
- How to terminate paths? We use Russian Roulette to probabilistically stop the recursion
- How to reduce noise? Use importance sampling in choosing random direction. Two ways: importance sample the lights, and importance sample the BRDF.

Implementation Notes

Multiple Light Sources

Consider multiple lights in direct lighting estimate

One strategy:

- Loop over all N lights, sum Monte-Carlo estimates for each light
- For each light: compute Monte Carlo estimate with M samples taken with importance sampling

Needs N * M samples

This is what the assignment asks you to implement.

Multiple Light Sources (Single Sample)

Consider random sampling of multiple lights with a single sample

- **Randomly choose light i, with probability pi**
- **Randomly sample over that light's directions, with probability pL**
- **Probability of choosing sample is (pi * pL)**
- **Weight the lighting calculation by 1/(pi * pL)**
- **Is this estimator unbiased? Yes!**
- **How would you importance sample intelligently? Can of course average N such samples**

Point Lights / Ideal Specular Materials – Issues

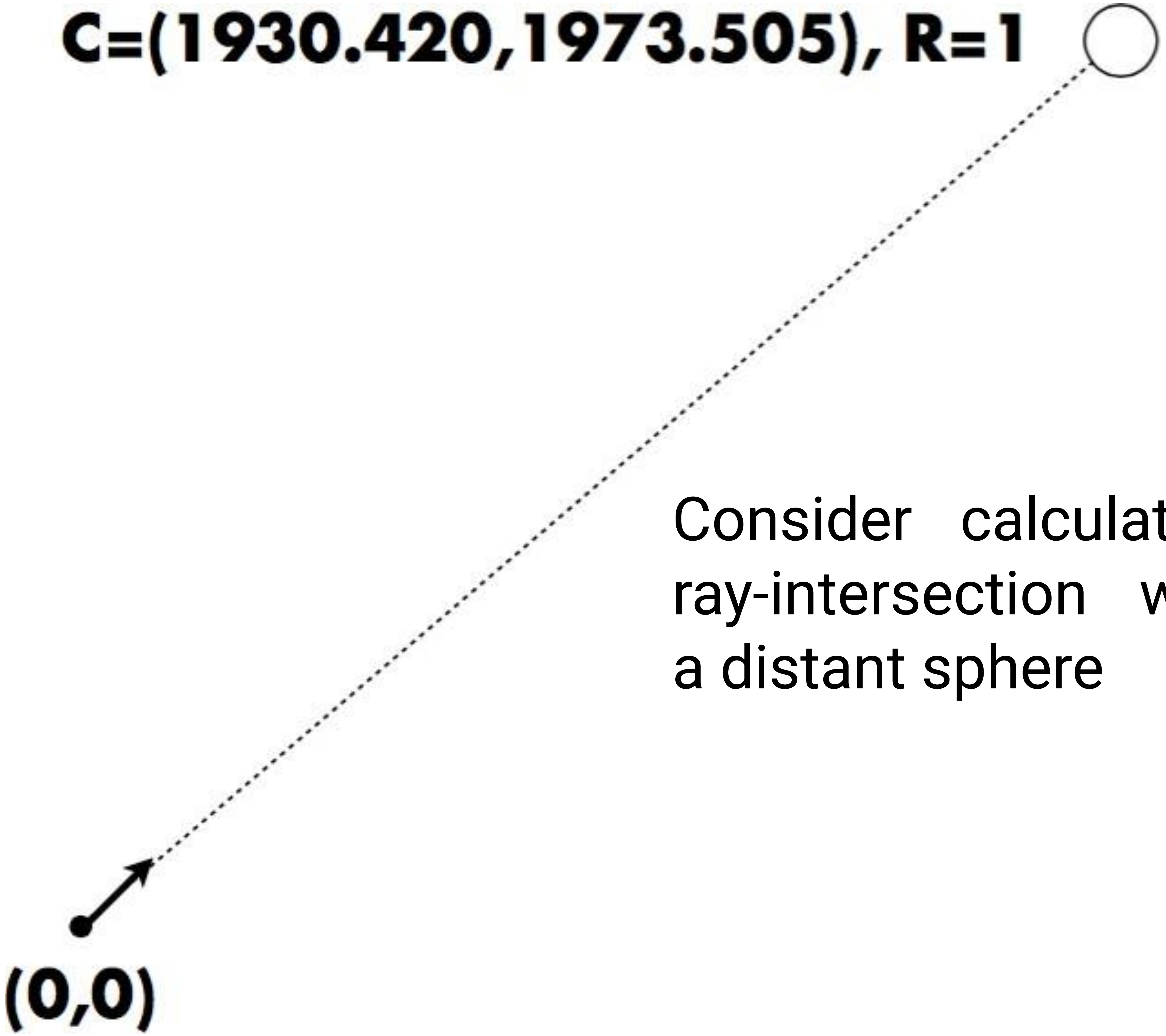
Sampling problems

- When sampling directions randomly, we have zero probability of matching exact direction of a point light or mirror reflection / specular refraction

Remedy

- In direct lighting, importance sample point lights by generating a single sample pointing directly at the light (only one sample needed)
- In indirect lighting, importance sample specular BRDFs by generating a single sample point directly along the specular refraction / transmission direction

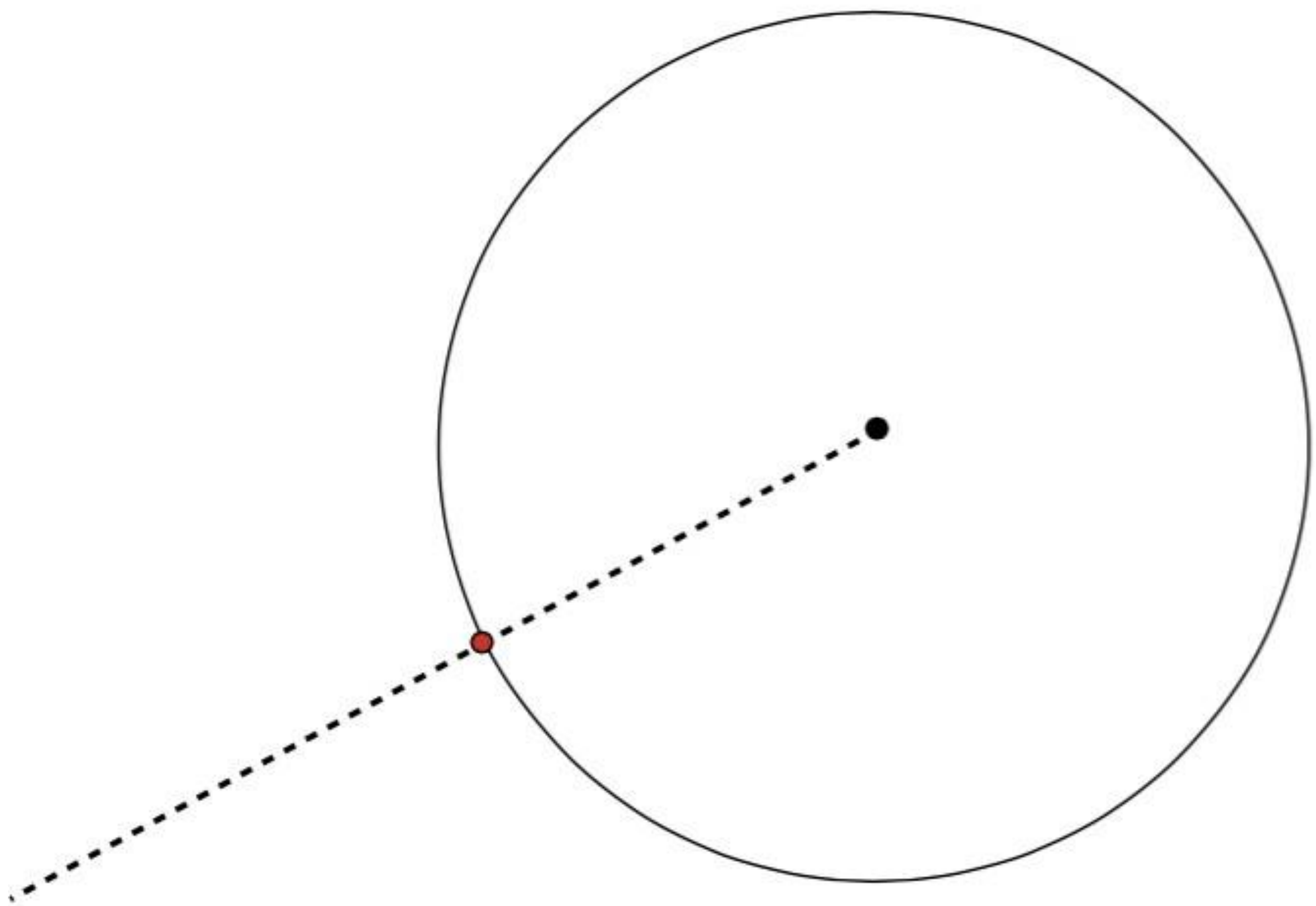
Numerical Precision Issues



Consider calculating ray-intersection with a distant sphere

Numerical Precision Issues

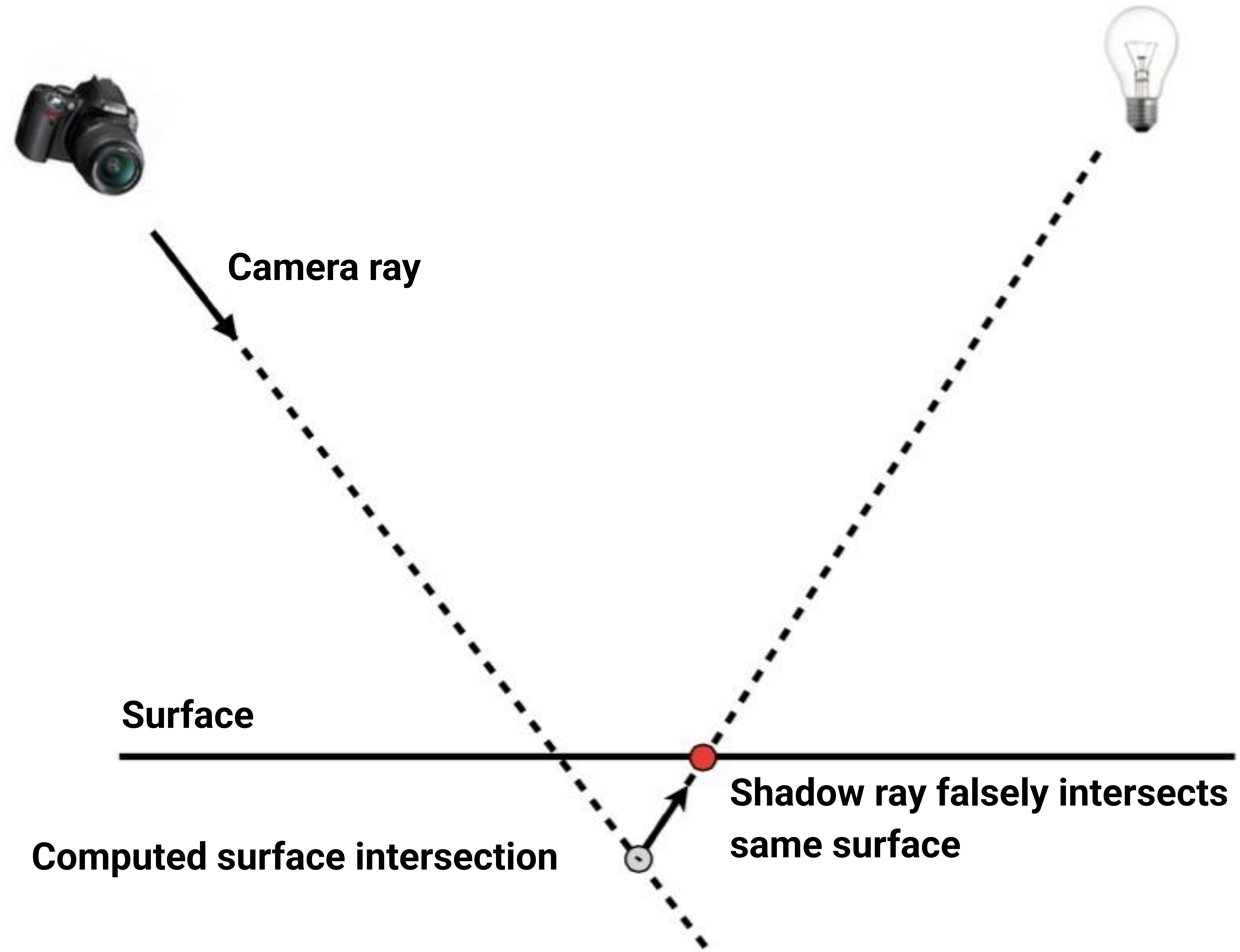
C=(1930.420,1973.505) R=1

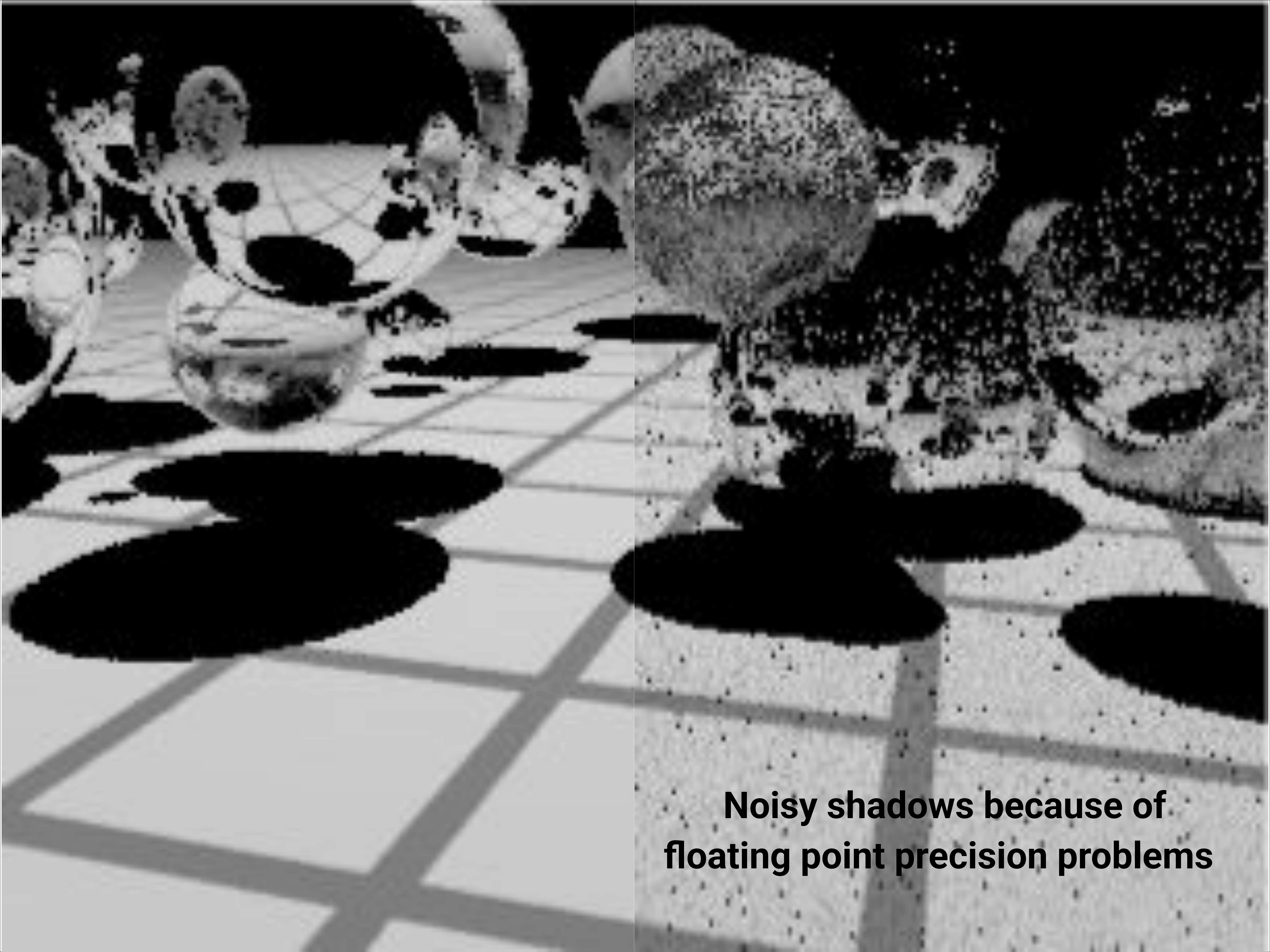


True Intersection: (1929.7203..., 1972.7897...)

Computed Intersection: (1930.4196..., 1973.5054...)

Noisy Shadows

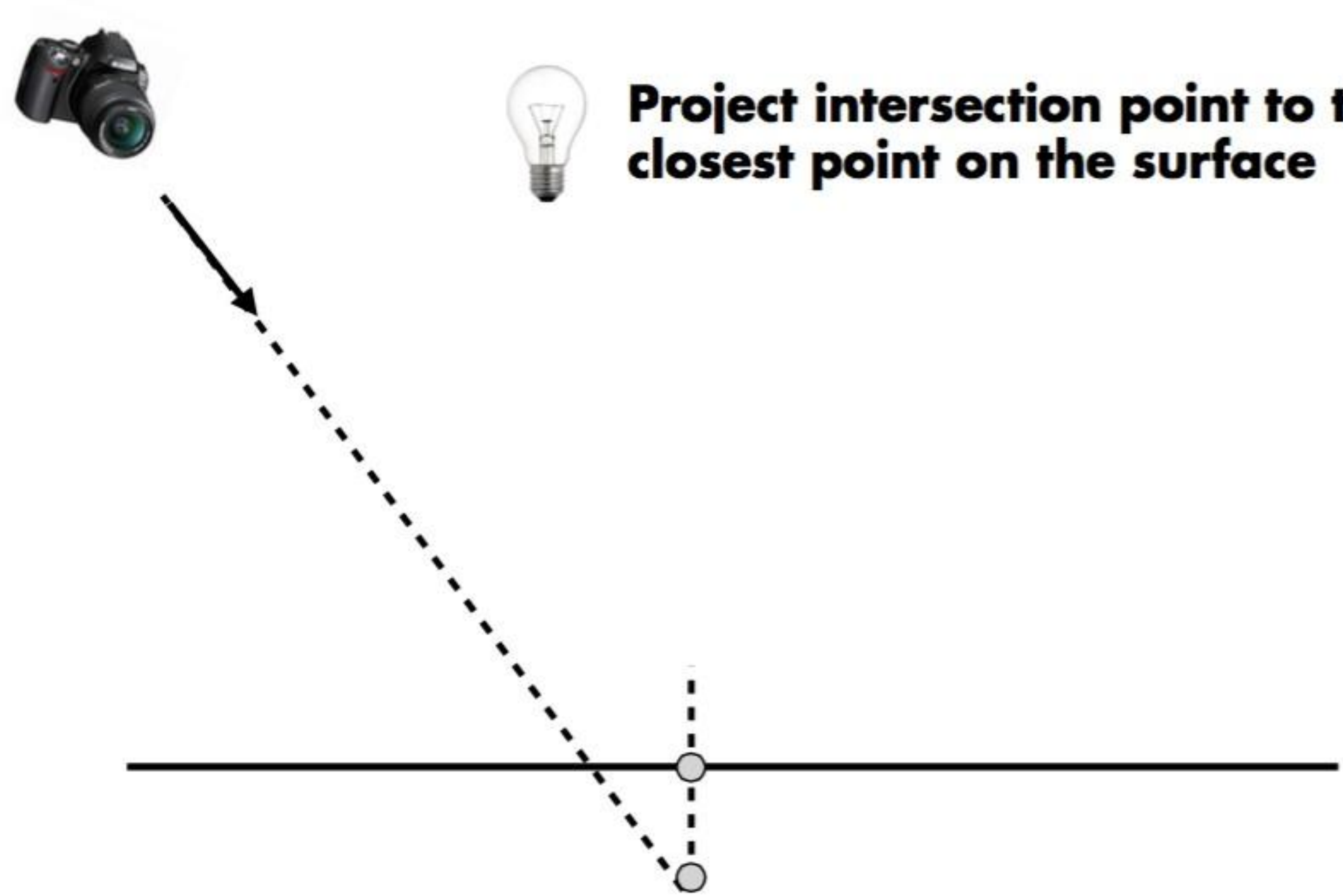



Noisy shadows because of
floating point precision problems

Floating-Point Precision Remedies

1. double (fp64) rather than float (fp32)
   - 53-bits of precision instead of 24-bits
   - Increase memory footprint
2. Ignore re-intersection with the last object hit
   - Only works for flat objects (e.g. triangles)
   - No help if model has coincident triangles
3. Offset origin along ray to ignore close intersections
   - Hard to choose offset that isn't too small or too big

Remedy: Project Intersection Point to Surface



Project intersection point to the closest point on the surface

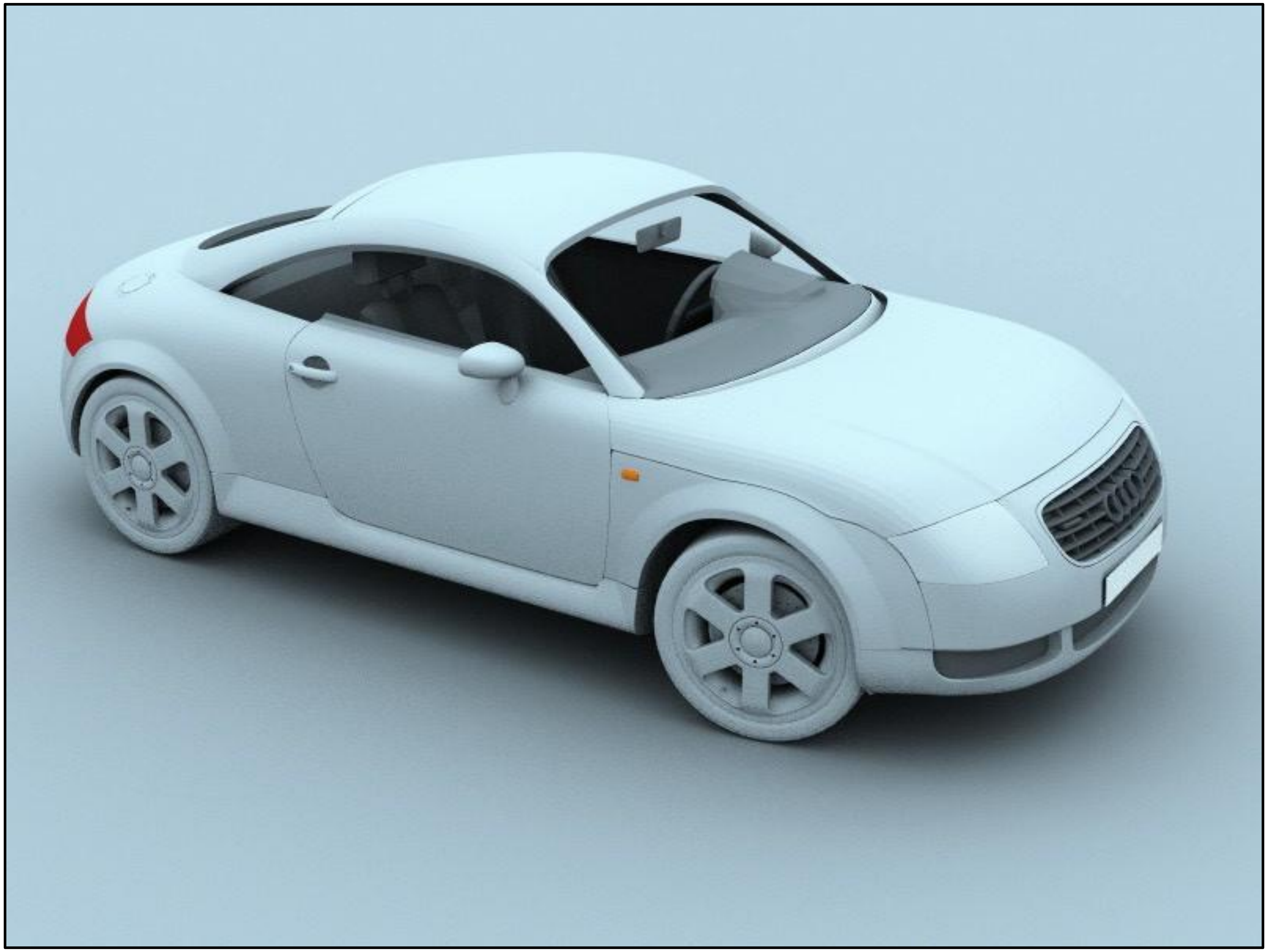
Good Scenes for Path Tracing (Diffuse, Sky Lighting)

M. Fajardo, Arnold Path Tracer

Good Scenes for Path Tracing (Diffuse, Sky Lighting)

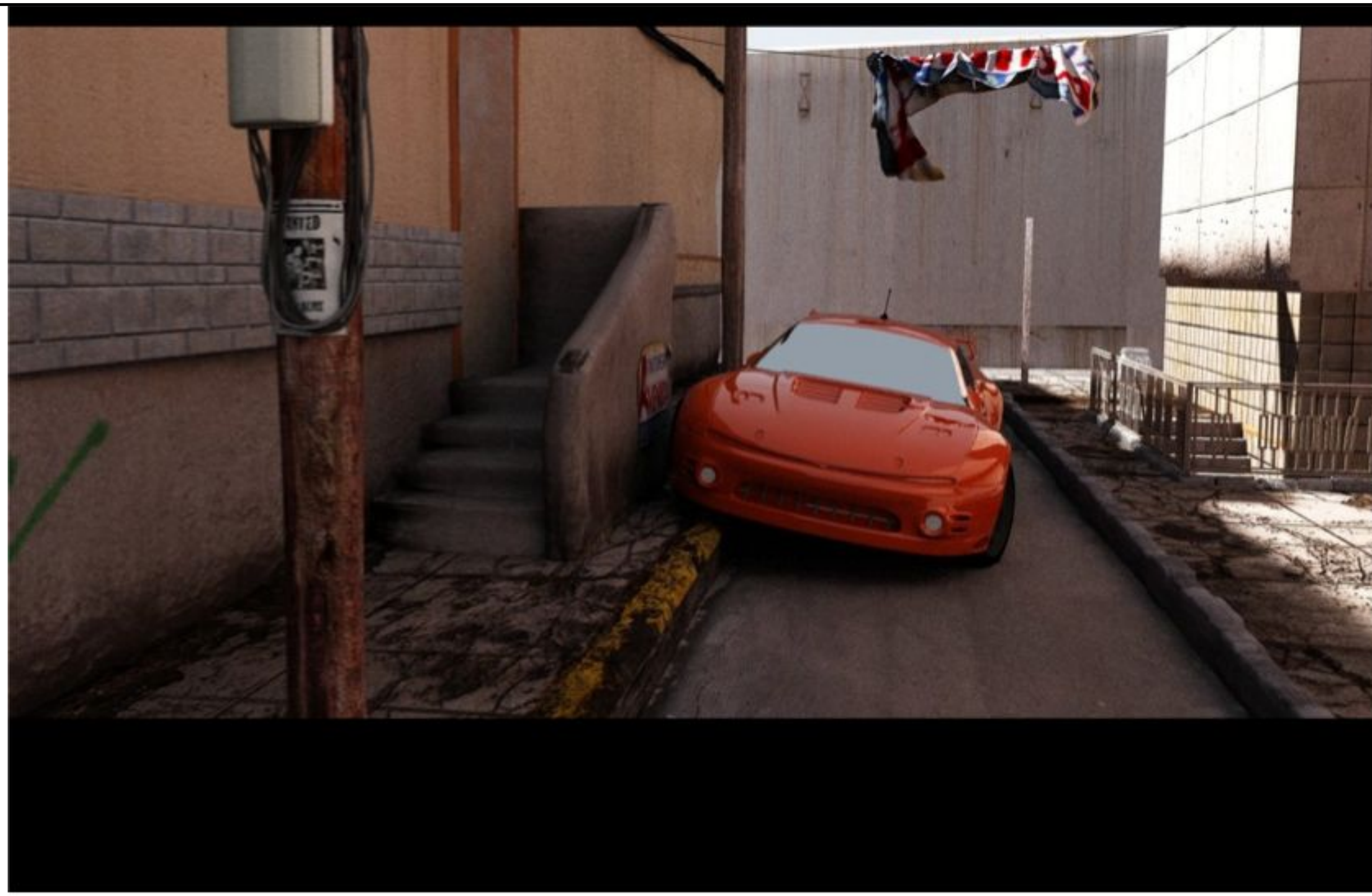
M. Fajardo, Arnold Path Tracer

Good Scenes for Path Tracing (Diffuse, Sky Lighting)



M. Fajardo, Arnold Path Tracer

Good Scenes for Path Tracing (Diffuse, Sky Lighting)

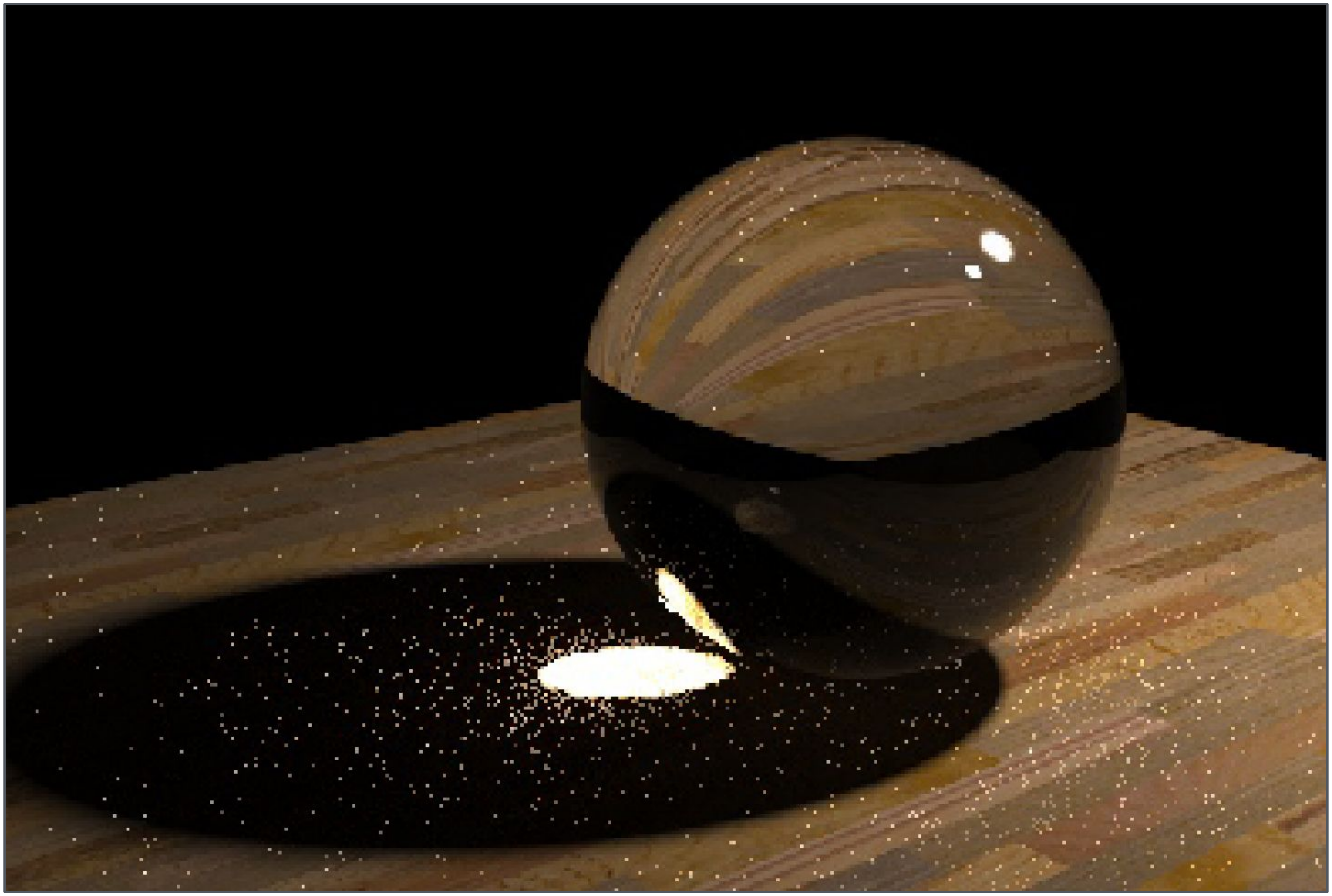


Street scene 1

1536x654, 16 paths/pixel, 2 bounces, 250,000 faces, 18 min., dual PIII-800

M. Fajardo, Arnold Path Tracer

A Challenging Scene for Path Tracing – Why?



Henrik Wann Jensen

1000 paths / pixel

Things to Remember

Global illumination challenge: recursive light transport

Reflection & rendering equations, operator notation Neumann solution of rendering equation:

- Sum successive bounces of light, infinite series Pathtracing
- Russian Roulette for unbiased finite estimate of infinite series (infinite dimensional integral)
- Partition into direct and indirect illumination
- Importance sampling of **lighting and BRDF**

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