Lecture 13:
Accelerating Ray-Scene Intersection

Computer Graphics and Imaging
UC Berkeley CS184/284A
Pre-Class Discussion: Accelerating Ray-Scene Intersection

~1 million pixels, ~20 million triangles

In pairs, brainstorm accelerations, small or big ideas.
Write down 3-4 ideas.
Pre-Class Discussion: Accelerating Ray-Scene Intersection

Brainstorm 3 or 4 accelerations, small or big ideas.

- Partition space, and check triangles closer first
- Try to reuse ray-object test, if the object is replicated somewhere else (?)
- Condense fine geometry into coarser geometry (level of detail)
- Run in parallel
- Check against the bounding box first.
- Bounding spheres of bounding cubes (faster)
- For faraway triangle, approximate by a sphere
- KD-tree acceleration structure
- Segment objects into clusters, and test against clusters
Bounding Volumes
Bounding Volumes

Quick way to avoid intersections: bound complex object with a simple volume

• Object is fully contained in the volume
• If it doesn’t hit the volume, it doesn’t hit the object
• So test bvol first, then test object if it hits
Ray-Intersection With Box

Could intersect with 6 faces individually

Better way: box is the intersection of 3 slabs
Ray Intersection with Axis-Aligned Box

2D example; 3D is the same! Compute intersections with slabs and take intersection of $t_{\text{min}}/t_{\text{max}}$ intervals

Intersections with $x$ planes
Intersections with $y$ planes
Final intersection result

How do we know when the ray misses the box?
Optimize Ray-Plane Intersection For Axis-Aligned Planes?

General

\[ t = \frac{(p' - o) \cdot N}{d \cdot N} \]

3 subtractions, 6 multiplies, 1 division

Perpendicular to x-axis

\[ t = \frac{p'_x - o_x}{d_x} \]

1 subtraction, 1 division
Uniform Spatial Partitions (Grids)
Preprocess – Build Acceleration Grid

1. Find bounding box
Preprocess – Build Acceleration Grid

1. Find bounding box
2. Create grid
Preprocess – Build Acceleration Grid

1. Find bounding box
2. Create grid
3. Store each object in overlapping cells
Ray-Scene Intersection

Step through grid in ray traversal order (3D line - 3D DDA)

For each grid cell
Test intersection with all objects stored at that cell
Grid Resolution?

One cell
• No speedup
Grid Resolution?

Too many cells

• Inefficiency due to extraneous grid traversal
Grid Resolution?

Heuristic:
- \( \#\text{cells} = C \times \#\text{objs} \)
- \( C \approx 27 \) in 3D
Careful! Objects Overlapping Multiple Cells

What goes wrong here?
- First intersection found (red) is not the nearest!

Solution?
- Check intersection point is inside cell

Optimize
- Cache intersection to avoid re-testing (mailboxing)
Uniform Grids – When They Work Well

Grids work well on large collections of objects that are distributed evenly in size and space
Uniform Grids – When They Fail

“Teapot in a stadium” problem
Non-Uniform Spatial Partitions
Spatial Hierarchies
Spatial Hierarchies
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Spatial Hierarchies
Spatial Partitioning Variants

Oct-Tree  KD-Tree  BSP-Tree

Note: you could have these in both 2D and 3D. In lecture we will illustrate principles in 2D, but for assignment you will implement 3D versions.
KD-Trees

Internal nodes store

- split axis: x-, y-, or z-axis
- split position: coordinate of split plane along axis
- children: reference to child nodes

Leaf nodes store

- list of objects
- mailbox information
KD-Tree Pre-Processing

- Find bounding box
- Recursively split cells, axis-aligned planes
- Until termination criteria met (e.g. max #splits or min #objs)
- Store obj references with each leaf node
KD-Tree Pre-Processing

Only leaf nodes store references to geometry
KD-Tree Pre-Processing

Choosing the split plane

• Simple: midpoint, median split
• Ideal: split to minimize expected cost of ray intersection

Termination criteria?

• Simple: common to prescribe maximum tree depth (empirical $8 + 1.3 \log N$, $N = \#\text{objs}$) [PBRT]
• Ideal: stop when splitting does not reduce expected cost of ray intersection
Top-Down Recursive In-Order Traversal
Top-Down Recursive In-Order Traversal

Internal node: split
Top-Down Recursive In-Order Traversal

Leaf node: intersect all objects
Top-Down Recursive In-Order Traversal

Internal node: split
Top-Down Recursive In-Order Traversal

Leaf node: intersect all objects
Top-Down Recursive In-Order Traversal

Internal node: split

$\min$ $\max$ $\text{split}$
Top-Down Recursive In-Order Traversal

Leaf node: intersect all objects
Top-Down Recursive In-Order Traversal

Intersection found
KD-Trees Traversal – Recursive Step

W.L.O.G. consider x-axis split with ray moving right

\[ t_{\text{split}} = \frac{(x_{\text{split}} - o_x)}{d_x} \]

\[ t_{\text{max}} < t^* \]

Intersect(L, t_{\text{min}}, t_{\text{max}})

Intersect(L, t_{\text{min}}, t^*)

Intersect(R, t^*, t_{\text{max}})

\[ t^* < t_{\text{min}} \]
Object Partitions & Bounding Volume Hierarchy (BVH)
Spatial vs Object Partitions

Spatial partition (e.g. KD-tree)
- Partition space into non-overlapping regions
- Objects can be contained in multiple regions

Object partition (e.g. BVH)
- Partition set of objects into disjoint subsets
- Bounding boxes for each set may overlap in space
Bounding Volume Hierarchy (BVH)
Bounding Volume Hierarchy (BVH)
Bounding Volume Hierarchy (BVH)
Bounding Volume Hierarchy (BVH)
Bounding Volume Hierarchy (BVH)

Internal nodes store

- Bounding box
- Children: reference to child nodes

Leaf nodes store

- Bounding box
- List of objects

Nodes represent subset of primitives in scene

- All objects in subtree
BVH Pre-Processing

- Find bounding box
- Recursively split set of objects in two subsets
- Stop when there are just a few objects in each set
- Store obj reference(s) in each leaf node
BVH Pre-Processing

Choosing the set partition

- Choose a dimension to split or optimize over x,y,z
- Simple #1: Split objects around spatial midpoint
- Simple #2: Split at location of median object
- Ideal: split to minimize expected cost of ray intersection

Termination criteria?

- Simple: stop when node contains few elements (e.g. 5)
- Ideal: stop when splitting does not reduce expected cost of ray intersection
**BVH Recursive Traversal**

Intersect (Ray ray, BVH node)

if (ray misses node.bbox) return;

if (node is a leaf node)
    test intersection with all objs;
    return closest intersection;

hit1 = Intersect (ray, node.child1);
hit2 = Intersect (ray, node.child2);
return closer of hit1, hit2;
Optimizing Hierarchical Partitions (How to Split?)
How to Split into Two Sets? (BVH)
How to Split into Two Sets? (BVH)
How to Split into Two Sets? (BVH)

Split at median element
Child nodes have equal numbers of elements
How to Split into Two Sets? (BVH)

A better split?
Smaller bounding boxes, avoid overlap and empty space
Which Hierarchy Is Fastest?

Key insight: a good partition minimizes the average cost of tracing a ray
Which Hierarchy Is Fastest?

What is the average cost of tracing a ray?

For leaf node:

\[
\text{Cost}(\text{node}) = \text{cost of intersecting all triangles} \\
= C_{\text{isect}} \times \text{TriCount(node)}
\]

- \(C_{\text{isect}}\) = cost of intersecting a triangle
- \(\text{TriCount(node)}\) = number of triangles in node
Which Hierarchy Is Fastest?

What is the average cost of tracing a ray?

For internal node:

\[
\text{Cost(node)} = C_{\text{trav}} + \text{Prob(hit L)} \times \text{Cost(L)} + \text{Prob(hit R)} \times \text{Cost(R)}
\]

- \(C_{\text{trav}}\): cost of traversing a cell
- \(\text{Cost(L)}\): cost of traversing left child
- \(\text{Cost(R)}\): cost of traversing right child
Estimating Cost with Surface Area Heuristic (SAH)

Probabilities of ray intersecting a node

• If assume uniform ray distribution, no occlusions, then probability is proportional to node’s surface area

Cost of processing a node

• Common approximation is #triangles in node’s subtree

\[
\text{Cost(cell)} = C_{\text{trav}} + SA(L) \times \text{TriCount}(L) + SA(R) \times \text{TriCount}(R)
\]

\[
SA(\text{node}) = \text{surface area of bbox of node}
\]
\[
C_{\text{trav}} = \text{ratio of cost to traverse vs. cost to intersect tri}
\]
  \[
  C_{\text{trav}} = 1:8 \text{ in PBRT [Pharr & Humphreys]}
  \]
  \[
  C_{\text{trav}} = 1:1.5 \text{ in a highly optimized version}
  \]
Ray Intersection Probability

The probability of a random ray hitting a convex shape A enclosed by another convex shape B is the ratio of their surface areas, \( \frac{S_A}{S_B} \).

\[
P(\text{hit}A|\text{hit}B) = \frac{S_A}{S_B}
\]
Partition Implementation

Constrain search to axis-aligned spatial partitions

- Choose an axis
- Choose a split plane on that axis
- Partition objects into two halves by centroid
- $2N - 2$ candidate split planes for node with $N$ primitives. (Why?)
Partition Implementation (Efficient)

Efficient modern approximation: split spatial extent of primitives into B buckets (B is typically small: B < 32)

For each axis: x, y, z:
- Initialize buckets
- For each object p in node:
  - b = compute_bucket(p.centroid)
  - b.bbox.union(p.bbox);
  - b.prim_count++;
- For each of the B-1 possible partitioning planes evaluate SAH
- Execute lowest cost partitioning found (or make node a leaf)
Cost-Optimization Applies to Spatial Partitions Too

• Discussed optimization of BVH construction
• But principles are general and apply to spatial partitions as well
• E.g. to optimize KD-Tree construction
  • Goal is to minimize average cost of intersecting ray with tree
  • Can still apply Surface Area Heuristic
• Note that surface areas and number of nodes in children differ from BVH
Things to Remember

Linear vs logarithmic ray-intersection techniques

Many techniques for accelerating ray-intersection

• Spatial partitions: Grids and KD-Trees
• Object partitions: Bounding Volume Hierarchies

Optimize hierarchy construction based on minimizing cost of intersecting ray against hierarchy

• Leads to Surface Area Heuristic for best partition
Acknowledgments

Thanks to Pat Hanrahan, Kayvon Fatahalian, Mark Pauly and Steve Marschner for lecture resources.